Classicalization of primordial perturbations by Continuous Spontaneous Localization, a quantum collapse model, after PLANCK and BICEP2

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Inflationary Paradigm: Success

- Solves Big Bang pathologies
- Generates primordial perturbations → seeds for large scale structures, CMB anisotropy
- Predicts (Single field models):
  1. Almost scale invariant scalar power spectrum: $n_s = 0.9635 \pm 0.0094$
  2. Almost Gaussian distribution of primordial perturbations: $f_{NL} < 2.7 \pm 5.8$
  3. Consistency relation: $r = -8n_T$
Lingering conceptual issues

A. Trans-Planckian issue: Largest observable modes were below Planck length during inflation
   - Solutions: Alternatives to inflation

B. Quantum to Classical transition of primordial perturbations: origin of perturbations are quantum but observed structures are classical
   - Solutions:
     1. Does not modify the basic mechanism of QM: Decoherence
     2. Modifies basic mechanism of QM: Collapse models [Continuous Spontaneous Localization (CSL) Model]
Continuous Spontaneous Localization

- Modifies Schrödinger equation by adding non-linear stochastic terms:

\[
d\psi_t = \left[ -\frac{i}{\hbar} H dt + \frac{\sqrt{\gamma}}{m_0} \int dx (M(x) - \langle M(x) \rangle_t) dW_t(x) - \frac{\gamma}{2m_0^2} \int dx (M(x) - \langle M(x) \rangle_t)^2 dt \right] \psi_t
\]

- Non-linear terms break the superposition of wave functions

- Amplification Mechanism

\[
\gamma(m) = \gamma_0 \left( \frac{m}{m_N} \right) ^\beta, \quad \gamma(m) = n^2 \gamma_0 \left( \frac{m}{m_N} \right) ^\beta
\]

- Hamiltonian not conserved due to non-Hermitian evolution

Non-conservation of energy

\[
\langle E \rangle = \frac{3\gamma_0 \hbar^2}{4m} t
\]
Schrödinger picture of inflation

- Scalar perturbations in terms of Mukhanov-Sasaki variable
  \[ \zeta(\tau, \mathbf{x}) = a \left[ \delta \varphi^g + \varphi' \frac{\Phi_B}{H} \right] \]

- Quantum state wavefunctional satisfy functional Schrödinger equation
  \[ i \frac{\partial \psi^{R,I}_{k}}{\partial \tau} = \hat{H}^{R,I}_{k} \psi^{R,I}_{k} \]

- Hamiltonian that of harmonic oscillator
  \[ \hat{H}^{R,I}_{k} = -\frac{1}{2} \frac{\partial^2}{\partial (\zeta^{R,I}_{k})^2} + \frac{1}{2} \omega^2 \left( \zeta^{R,I}_{k} \right)^2, \quad \omega^2 \equiv k^2 - \frac{a''}{a} \]

- Solution of functional Schrödinger equation is a functional Gaussian State
  \[ \psi^{R,I}_{k} \left[ \tau, \zeta^{R,I}_{k} \right] = \sqrt{N_k(\tau)} \exp \left( -\frac{\Omega_k(\tau)}{2} \left( \zeta^{R,I}_{k} \right)^2 \right) \]
Wigner Function & Squeezing

- Wigner function recognises the correlation between position (field) and its momentum (conjugate to field)

\[
W(\zeta_k^R, \zeta_k^I, p_k^R, p_k^I) = \frac{1}{(2\pi)^2} \int dxdy \Psi^* \left( \zeta_k^R - \frac{x}{2}, \zeta_k^I - \frac{y}{2} \right) e^{-ip_k^R x - ip_k^I y} \Psi \left( \zeta_k^R + \frac{x}{2}, \zeta_k^I + \frac{y}{2} \right)
\]

\[
= \frac{1}{\pi^2} e^{-Re \Omega_k (\zeta_k^R^2 + \zeta_k^I^2)} e^{-\frac{(p_k^R + Im \Omega_k \zeta_k^R)^2}{Re \Omega_k} - \frac{(p_k^I + Im \Omega_k \zeta_k^I)^2}{Re \Omega_k}}
\]

- During inflation \( \rightarrow \) on superhorizon scales \( Re \Omega_k \rightarrow 0 \)

\[
W(\zeta_k^R, \zeta_k^I, p_k^R, p_k^I) \rightarrow \frac{Re \Omega_k}{\pi} e^{-Re \Omega_k (\zeta_k^R^2 + \zeta_k^I^2)} \delta (p_k^R) \delta (p_k^I)
\]
• Highly squeezed in momentum direction and spread in field direction

• Observation shows classicality in field direction

  Expect 'collapse models' to squeeze the modes in field direction

  \[ \text{Re } \Omega_k \rightarrow \infty \]
CSL-like modification with constant $\gamma$

- Modify functional Schrödinger equation with 'CSL-like' terms

\[
d\Psi_{k}^{R,I} = \left[ -i\hat{H}_{k}^{R,I} d\tau + \sqrt{\gamma} \left( \hat{c}_{k}^{R,I} - \langle \hat{c}^{R,I}_{k} \rangle \right) dW_{\tau} - \frac{\gamma}{2} \left( \hat{c}_{k}^{R,I} - \langle \hat{c}^{R,I}_{k} \rangle \right)^{2} d\tau \right]
\]

- Frequency of the Harmonic Oscillator Hamiltonian becomes time dependent and complex

\[
\omega^{2} = k^{2} - 2i\gamma - \frac{a''}{a}
\]
Smaller modes \((2\gamma \ll k^2)\)

\[
\Re \Omega_k \approx 2k(-k\tau)^2 \to 0, \quad P_R(k) = \frac{H^2}{16\pi^2\epsilon M_{P1}^2}
\]

- Wigner function not affected by \(\gamma\)
- Squeezing in momentum direction (can’t explain classicality)
- Power spectrum scale-independent (good for observation)

Larger modes \((2\gamma \gg k^2)\)

\[
\Re \Omega_k \approx \frac{2\gamma}{k}(-k\tau) \to 0, \quad P_R(k) = \frac{H^2 k^3}{16\pi^2\epsilon M_{P1}^2} e^{-\Delta N}
\]

- Wigner function affected by \(\gamma\) (which we wanted !!!)
- Squeezing in momentum direction (can’t explain classicality)
- Power spectrum scale-dependent (bad for observation)
Modification by scale-dependent $\gamma$

- Modes behave more classically as they start crossing the horizon.

- $\gamma$ should discriminate between different modes according to their physical length scales as they grow stronger as a mode starts crossing the horizon during inflation.

- $\gamma$ should be a function of time

$$\gamma = \frac{\gamma_0(k)}{(-k\tau)\alpha}, \quad 0 < \alpha < 2$$
on superhorizon scales

\[ \text{Re } \Omega_k \approx \frac{k}{2} \left( -k \tau \right)^{1-\alpha} \left( \frac{2\gamma_0(k)}{k^2} \right) \]

0 \( < \alpha < 1 \) \( \rightarrow \) \( \text{Re } \Omega_k \rightarrow 0 \)

No macro-objectification

1 \( < \alpha < 2 \) \( \rightarrow \) \( \text{Re } \Omega_k \rightarrow \infty \)

Macro-objectification occurs
Scale-invariance of power spectrum

- To obtain a scale-invariant power spectrum make \( \gamma \) mode dependent

\[
\gamma_0(k) = \tilde{\gamma}_0 \left( \frac{k}{k_0} \right)^\beta
\]

- The power spectrum becomes

\[
P_R(k) \propto k^{3+\alpha-\beta}
\]

- \( \beta = 3 + \alpha \) yields scale-invariant power spectrum
Macro-objectification of tensor modes

- Tensor modes $\rightarrow$ Traceless and transverse part of metric fluctuations $\rightarrow$ associated with two helicity states $+$ and $\times$

- Each helicity states identical to a massless scalar in de Sitter space

- Previous analysis of scalar perturbations applicable to each helicity states to obtain macro-objectification of tensor modes

- Assumption: CSL-modified dynamics is essentially same for gravitons and inflatons
Observables

- **Scalar power spectrum**
  \[
  \mathcal{P}_R = \frac{1}{8\pi^2 M_{Pl}^2} \frac{k_0^2 H^2}{\tilde{\gamma}_0} e^{-(1+\alpha)\Delta N} \left( \frac{k_*}{k_0} \right)^{3+\alpha-\beta} \left( \frac{k}{k_*} \right)^{3+\alpha-\beta+2\eta-3\epsilon} \\
  \equiv A_s(k_*) \left( \frac{k}{k_*} \right)^{n_s-1}
  \]

- **Tensor power spectrum**
  \[
  \mathcal{P}_T = \frac{2}{\pi^2 M_{Pl}^2} \frac{k_0^2 H^2}{\tilde{\gamma}_0} e^{-(1+\alpha)\Delta N} \left( \frac{k_*}{k_0} \right)^{3+\alpha-\beta} \left( \frac{k}{k_*} \right)^{3+\alpha-\beta-2\epsilon} \\
  \equiv A_T(k_*) \left( \frac{k}{k_*} \right)^{n_T}
  \]

- **Tensor amplitude depends upon collapse parameter** → **Stronger collapse parameter can bring down the scale of inflation**
- Scalar spectral index

\[ n_s - 1 = \delta + 2\eta - 4\epsilon \]

- \( \delta \) can be of the order of slow-roll parameters

- Tensor spectral index

\[ n_T = \delta - 2\epsilon \]

- Tensor-to-Scalar ratio

\[ r = -8n_T + 8\delta \]

- Accurate measurements of \( r \) and \( n_T \) would be able to distinguish this scenario with the generic one
Summary

- Scale dependent collapse parameter can yield micro-objectification of modes, both scalar and tensor
- Wave-number dependence of collapse parameter yield nearly scale-invariance of power spectrum
- Collapse dynamics changes the consistency relation of single-field model
- Accurate measurement of tensor-to-scalar ratio and tensor spectral index can distinguish this dynamics from the generic scenario
Thank you