Planck inflation constraints: search for features in the power spectrum

Jan Hamann
CERN

on behalf of the Planck Collaboration
Natural scatter or signs of new physics?

large angle deficit  ell = 20-30 feature  other features
• Standard slow-roll inflation
  almost scale-invariant spectrum of scalar perturbations
  (i.e., power-law, small running)
• Deviations from almost-scale invariance ("features")
  can be caused by:
  • Non-standard initial conditions (curvature, matter, kinetic
    energy of inflaton)
  • Non-Bunch-Davies vacuum
  • Features in the inflaton potential
  • Multi-field dynamics
  • ...
Inflation

$P_R(k)$

Convolution with transfer functions
**Bottom-up**

- Reconstruct shape of primordial power spectrum from measurement of the CMB angular power spectrum

- Planck 2014: three different reconstruction approaches

**Top-down**

- Fit a specific physical features model or parameterised features spectrum to the data

- Planck 2014: four different parameterised features models [plus axion monodromy case study]
Consider deviations from power-law spectrum

\[ P_R(k) = P_R^{(0)}(k) [1 + f(k)] \]

• Take discrete \( f(k) \), interpolate with B-splines
• Add a likelihood penalty

\[
\begin{align*}
    f^T R(\lambda, \alpha) f &= \lambda \int dk \left( \frac{\partial^2 f(k)}{\partial k^2} \right)^2 \\
    &\quad + \alpha \int_{-\infty}^{k_{\min}} dk \ f^2(k) + \alpha \int_{k_{\max}}^{+\infty} dk \ f^2(k)
\end{align*}
\]

suppresses small structures

drive \( f(k) \) to zero at the largest and smallest scales

• Maximise penalised likelihood with respect to \( f_i(k), h, \Omega_b h^2, \Omega_c h^2 \)
• Extra degrees of freedom* = \( N_{\text{bins}} - 2 \)

* with respect to a power-law spectrum
Temperature data

Deviation from power-law for different smoothness penalties

- The deviation from power-law is constrained to be within a few per cent
- The feature at \( \ell \approx 1800 \) reported in 2013 papers no longer present (improved understanding of 4K cooler systematics)
- Inclusion of polarisation data increases resolution and reduces scatter

Cosmological parameter values are remarkably stable under changes to primordial spectrum
Deviation from power-law for different smoothness penalties

- The deviation from power-law is constrained to be within a few percent
- The feature at $\ell \approx 1800$ reported in 2013 papers no longer present (improved understanding of 4K cooler systematics)
- Inclusion of polarisation data increases resolution and reduces scatter

Cosmological parameter values are remarkably stable under changes to primordial spectrum
Method 2: Bayesian reconstruction with cubic splines on fixed knots

- Primordial power spectrum taken as cubic spline interpolation between fixed logarithmically spaced knots
- Extra degrees of freedom = $N_{\text{knots}} - 2$
- Bayesian method
- MCMC analysis, varying $P_i(k)$, tensor amplitude (assumed to be power-law), cosmological and foreground parameters
- Using slow-roll relations, can also reconstruct inflaton potential $V(\phi)$
Method 2:
Bayesian reconstruction with cubic splines on fixed knots

![Graph showing power spectrum with confidence intervals for various models.](https://example.com/graph.png)
Method 2: Bayesian reconstruction with cubic splines on fixed knots

Corresponding reconstructed inflaton potentials

Compare with Bayesian reconstruction of potential using n-th order polynomial expansion of $V(\phi)$
Method 3: Bayesian reconstruction with linear splines and variable knot positions

- Primordial power spectrum taken as linear interpolation between knots with variable positions
- Bayesian method
- Varying all primordial, cosmological and foreground parameters, using PolyChord sampler (nested sampling)
- Use Bayesian evidence to decide how many knots to add
- Extra degrees of freedom = 2 $N_{\text{knots}}$
Bayesian evidence does not favour the introduction of extra knots
Temperature data best-fit power spectra

Cutoff model
(inflation starts from kinetic stage)

Step model
(step in inflaton potential)

Log oscillation model
(e.g., non-BD vacuum, axion monodromy)

Linear oscillation model
(e.g., boundary EFT)

\[ V(\phi) = \frac{m^2}{2} \phi^2 \left[ 1 + c \tanh \left( \frac{\phi - \phi_c}{d} \right) \right] \]

\[ P^\log_R(k) = P^0_R(k) \left[ 1 + A_{\log} \cos \left( \omega_{\log} \ln \left( \frac{k}{k_*} \right) + \varphi_{\log} \right) \right] \]

\[ P^{\text{lin}}_R(k) = P^0_R(k) \left[ 1 + A_{\text{lin}} \left( \frac{k}{k_*} \right)^{n_{\text{lin}}} \cos \left( \omega_{\text{lin}} \frac{k}{k_*} + \varphi_{\text{lin}} \right) \right] \]
Search for parameterised features

- Linear oscillation model
- Log oscillation model
- Cutoff model
- Step model
- Linear oscillation model
“What are the relative probabilities of the features models compared to power-law $\Lambda$CDM?”

- Compute Bayesian evidences with MultiNest, varying primordial and other cosmological parameters (foregrounds fixed)
Search for parameterised features: Bayesian analysis

“What are the relative probabilities of the features models compared to power-law $\Lambda$CDM?”

<table>
<thead>
<tr>
<th>Feature</th>
<th>Parameters</th>
<th>$T$</th>
<th>$T+E$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>cutoff</strong></td>
<td>1 extra parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \chi^2$</td>
<td>-2.1</td>
<td>-1.8</td>
<td></td>
</tr>
<tr>
<td>$\ln B_{01}$</td>
<td>0.8</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td><strong>step</strong></td>
<td>3 extra parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \chi^2$</td>
<td>-8.2</td>
<td>-6.6</td>
<td></td>
</tr>
<tr>
<td>$\ln B_{01}$</td>
<td>0.1</td>
<td>-0.3</td>
<td></td>
</tr>
<tr>
<td><strong>log oscillations</strong></td>
<td>3 extra parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \chi^2$</td>
<td>-9.2</td>
<td>-9.3</td>
<td></td>
</tr>
<tr>
<td>$\ln B_{01}$</td>
<td>1.3</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td><strong>linear oscillations</strong></td>
<td>4 extra parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \chi^2$</td>
<td>-7.3</td>
<td>-11.1</td>
<td></td>
</tr>
<tr>
<td>$\ln B_{01}$</td>
<td>0.9</td>
<td>0.4</td>
<td></td>
</tr>
</tbody>
</table>

Caveat: Bayes factors are prior dependent!

![Planck logo](Planck.png)
“What would be the typical improvement in the fit if the underlying model was power-law $\Lambda$CDM?”

- Simulate Planck-like power spectra, using the power-law $\Lambda$CDM best-fit as fiducial model
- For each simulated data set:
  - Find power-law $\Lambda$CDM best-fit effective $\chi^2$ and parameters
  - Find features models best-fit effective $\chi^2$ (varying only primordial parameters, other cosmological parameters fixed to their respective best-fit values)
  - Evaluate effective $\Delta \chi^2$ (conservative, i.e., underestimates the maximum obtainable value)
- Compare distribution of simulated effective $\Delta \chi^2$ with observed effective $\Delta \chi^2$
Search for parameterised features: frequentist analysis

**Cutoff**
1 extra parameter

![Cutoff Distribution](image)

**Step**
3 extra parameters

![Step Distribution](image)

**Log Oscillations**
3 extra parameters

![Log Oscillations Distribution](image)

**Linear Oscillations**
4 extra parameters

![Linear Oscillations Distribution](image)
Conclusions

- Planck data are consistent with a smooth, power-law primordial spectrum as generically predicted by the simplest models of inflation.
- Particularly strong constraints on features for wavenumbers $0.008 \text{ Mpc}^{-1} < k < 0.2 \text{ Mpc}^{-1}$
- Different ways of reconstructing the primordial power spectrum from Planck data yield results in agreement with each other.
- Observed features at large scales could in principle be explained by (inflationary) models predicting features in the primordial spectrum, but no strong statistical evidence.
The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada.

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.