Parameter Estimation with Physics Informed Neural Networks

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Debating the Potential of Machine Learning in Astronomical Surveys
IAP (Oct. 2021)

Structure

Part I: Motivation and Problem Statement

Part II: Neural Network Solution and Validation

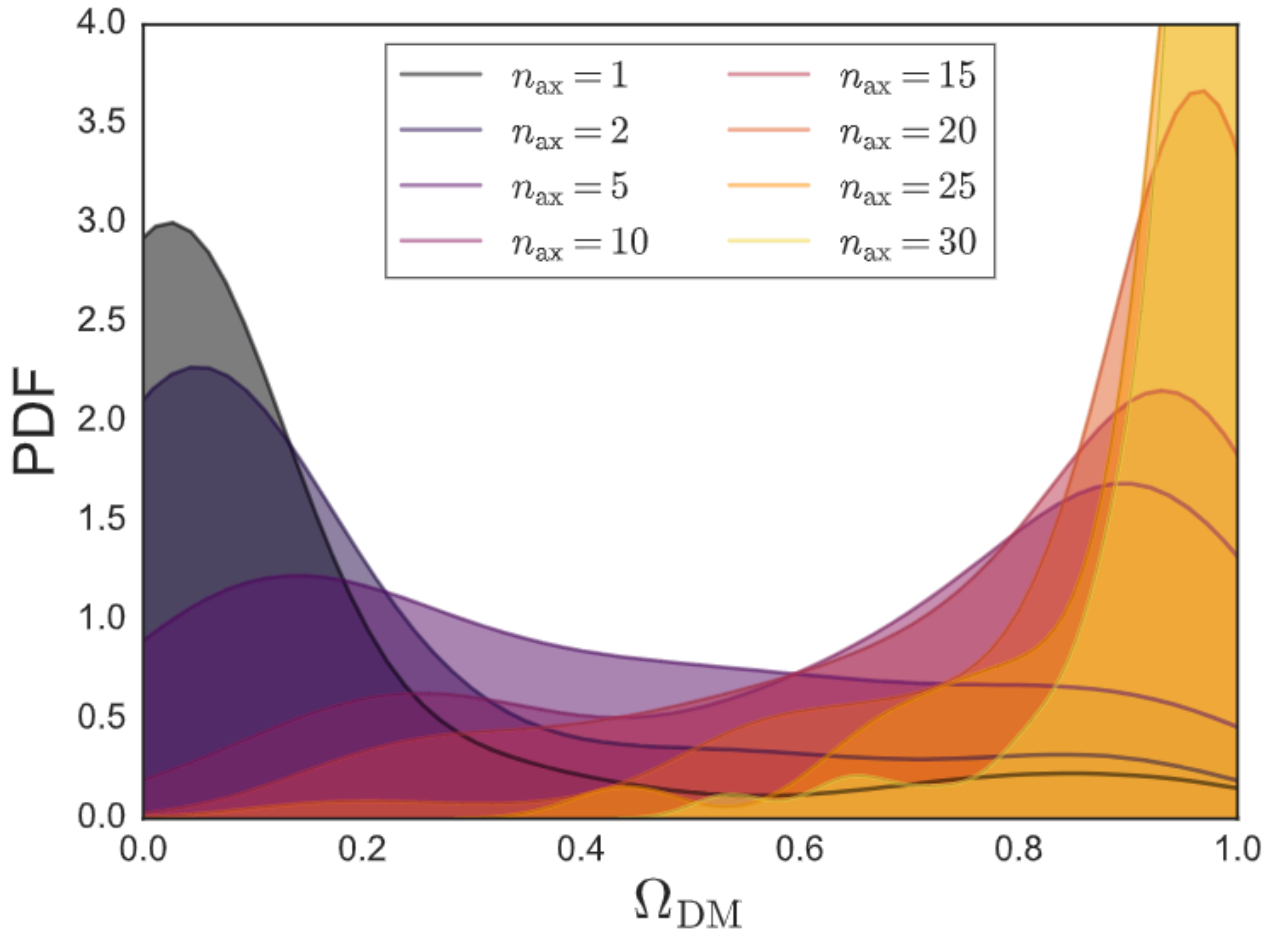
Ultralight Axions

Particle Physics Origin

- QCD axion too heavy
- Turn to axions from string theory
- Axiverse of many axions ($n_{\rm ax} > 1$)
- May compose part of DM

FDM: Impose $\Omega_a = \Omega_d \approx 0.3$

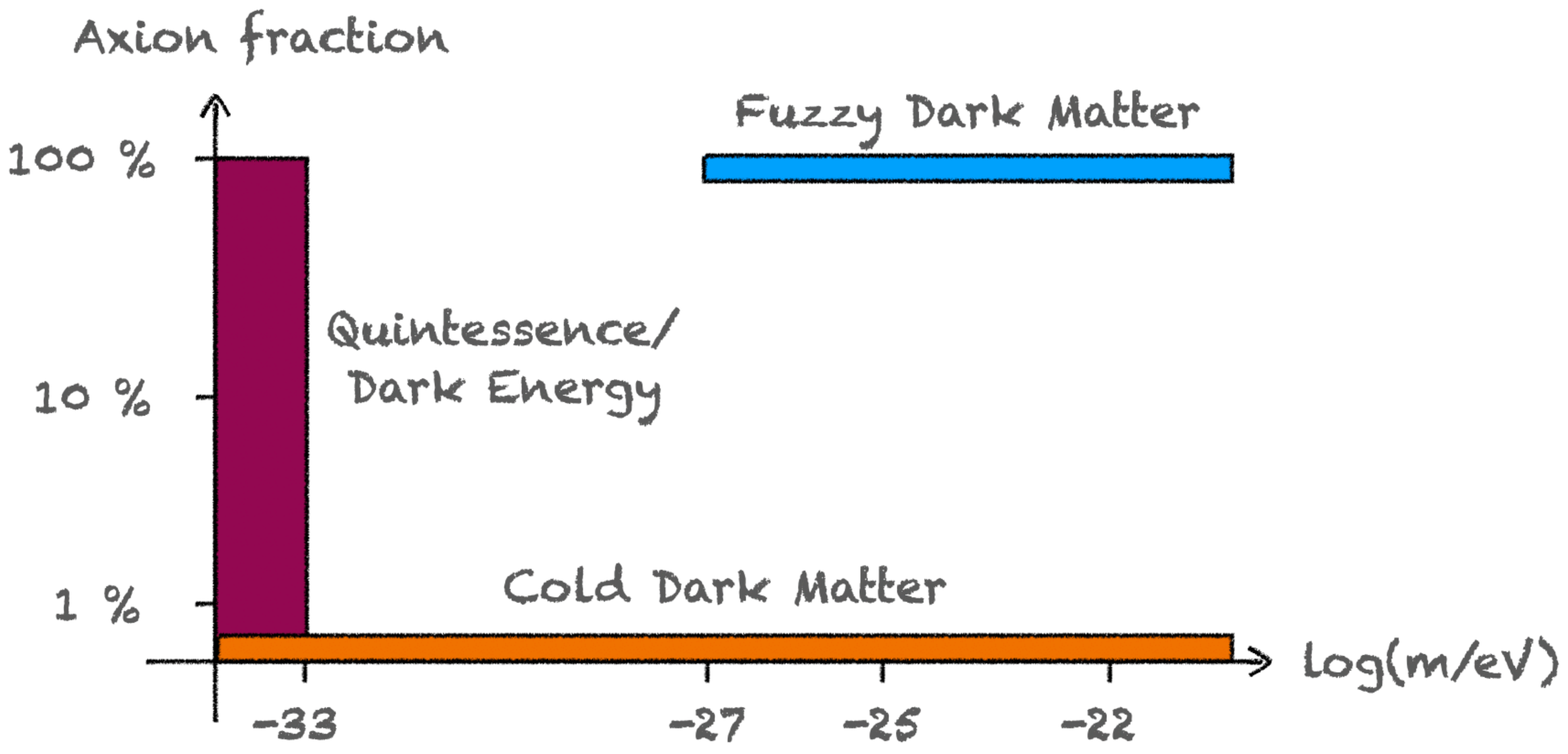
ULA: Ω_a **from HEP**



Svrček & Witten (2006) Arvanitaki *et al.* (2010) Hui *et al.* (2016) Stott *et al.* (2017)

ULA Model

Two free parameters: $m_a, \Omega_a/\Omega_d$



- Can't add 100s of parameters to LCDM!
- Minimal extension:
  - 1 ULA, rest of DM behaves as CDM
- Focus on **deviations** from CDM

Multiple Axions

Many models including Early DE

Gravitational lensing H_0 tension from ultralight axion galactic cores

Kfir Blum[1,*] and Luca Teodori[1,†]

Gravitational lensing time delays offer an avenue to measure the Hubble parameter H_0, with some analyses suggesting a tension with early-type probes of H_0. The lensing measurements must mitigate systematic uncertainties due to the mass modelling of lens galaxies. In particular, a core component in the lens density profile would form an approximate local mass sheet degeneracy and could bias H_0 in the right direction to solve the lensing tension. We consider ultralight dark matter as a possible mechanism to generate such galactic cores. We show that cores of roughly the required properties could arise naturally if an ultralight axion of mass $m \sim 10^{-25}$ eV makes up a fraction of order ten percent of the total cosmological dark matter density. A relic abundance of this order of magnitude could come from vacuum misalignment. Stellar kinematics measurements of well-resolved massive galaxies (including the Milky Way) may offer a way to test the scenario. Kinematics analyses aiming to test the core hypothesis in massive elliptical lens galaxies should not, in general, adopt the perfect mass sheet limit, as ignoring the finite extent of an actual physical core could lead to significant systematic errors.

Simulations

First Cosmological Simulations of ULAs + CDM

Solve Schrödinger Equations for Axions

Axions

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_a a^2}\nabla^2\psi + m_a V\psi, \quad |\psi|^2 = \rho_{\rm ax}$$

Simulations

First Cosmological Simulations of ULAs + CDM

Solve Schrödinger-Poisson system and
N-body CDM dynamics

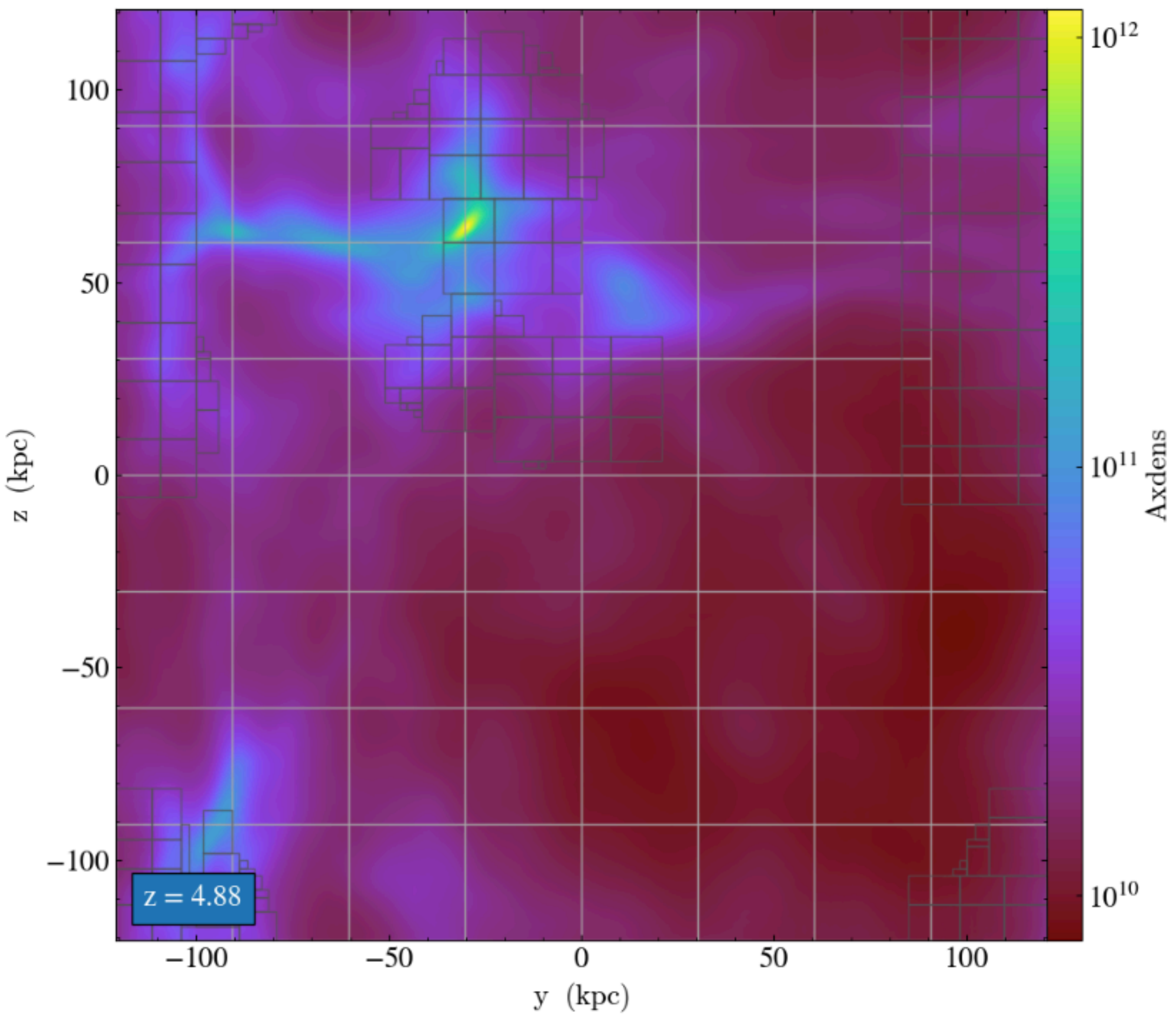
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_a a^2}\nabla^2\psi + m_a V\psi$$

$$\nabla^2 V = \frac{4\pi G}{a}\left(|\psi|^2 + \rho_{\mathrm{CDM}} - \bar{\rho}_{\mathrm{tot}}\right)$$

Schwabe & **Laguë** (in prep)

Simulations

Find Density Profile of CDM + ULAs



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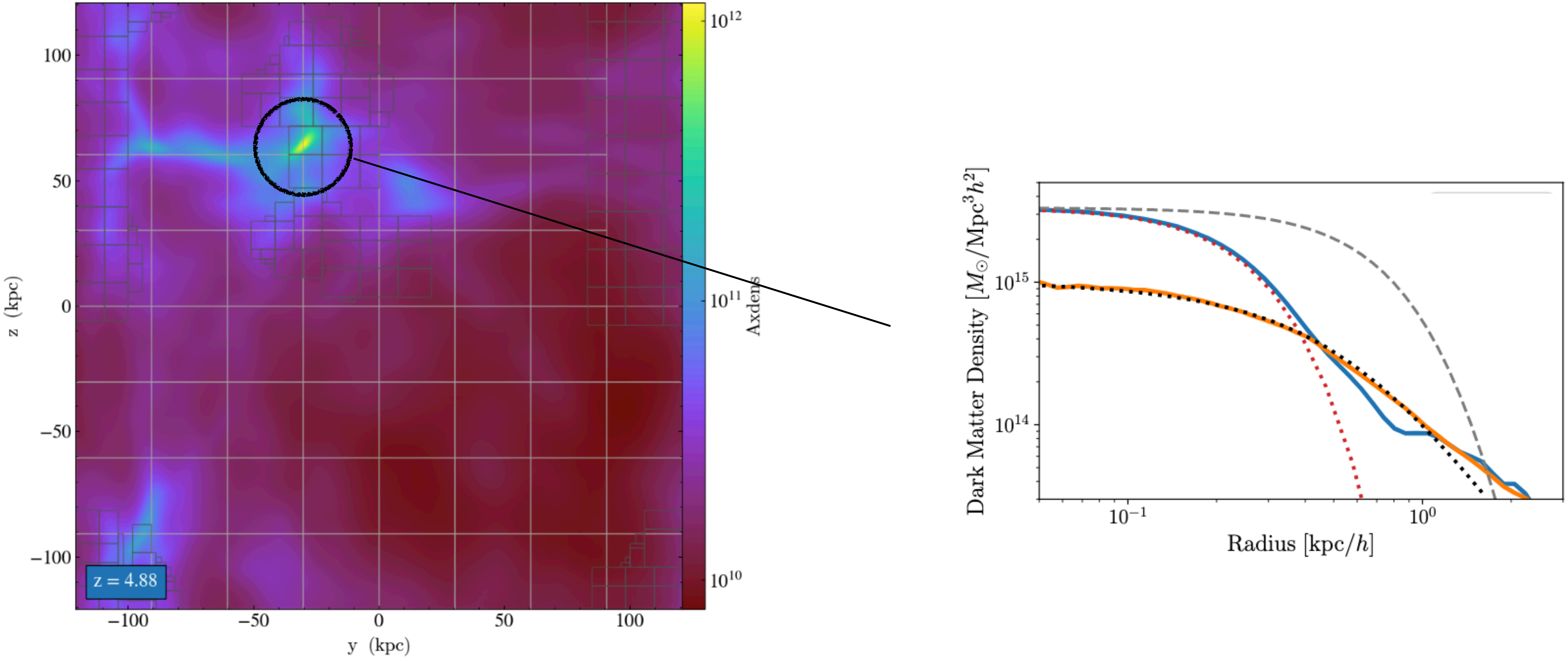
Simulations

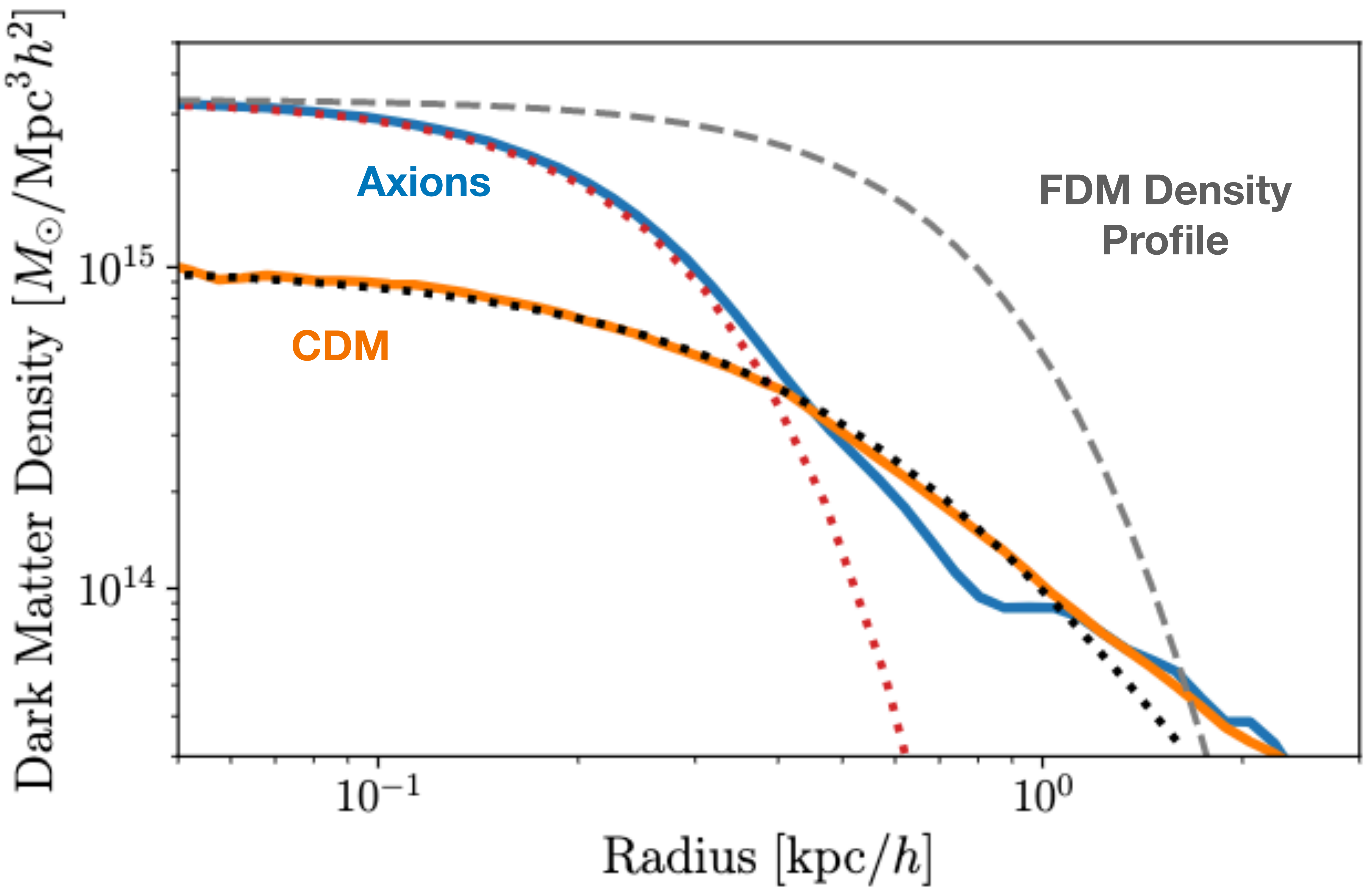
Halo Density



Schwabe & **Laguë** (in prep)

Constraining Mass from Sims

Halo Density Profiles



Schwabe & **Laguë** (in prep)

Can we infer m_a from a measurement of $\rho(r)$?

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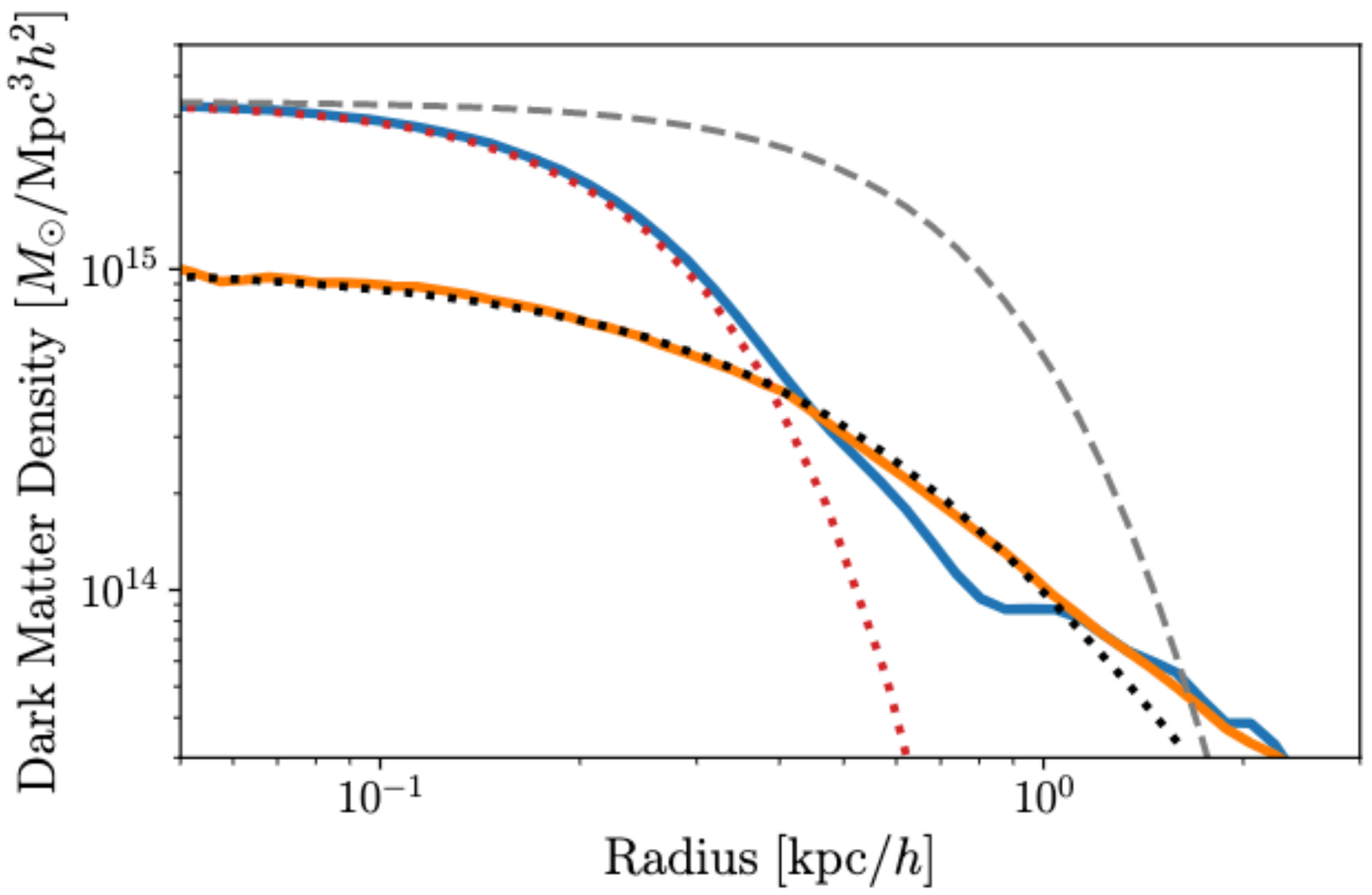
Forward problem: Specify parameters + PDE to get solution

Inverse problem: Specify PDE + solution to infer parameters

Non-Linear Modelling

A Useful Simplification



Equilibrium Solution

$$\psi = e^{-i\gamma t/\hbar}\phi$$

$$\frac{\partial^2(r\phi)}{\partial r^2} = 2r\left(\frac{m_a^2}{\hbar^2}V - \frac{m_a}{\hbar^2}\gamma\right)\phi$$

$$\frac{\partial^2(rV)}{\partial r^2} = 4\pi G r\left(\phi^2 + \rho_{\mathrm{CDM}}\right)$$

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Non-Linear Modelling

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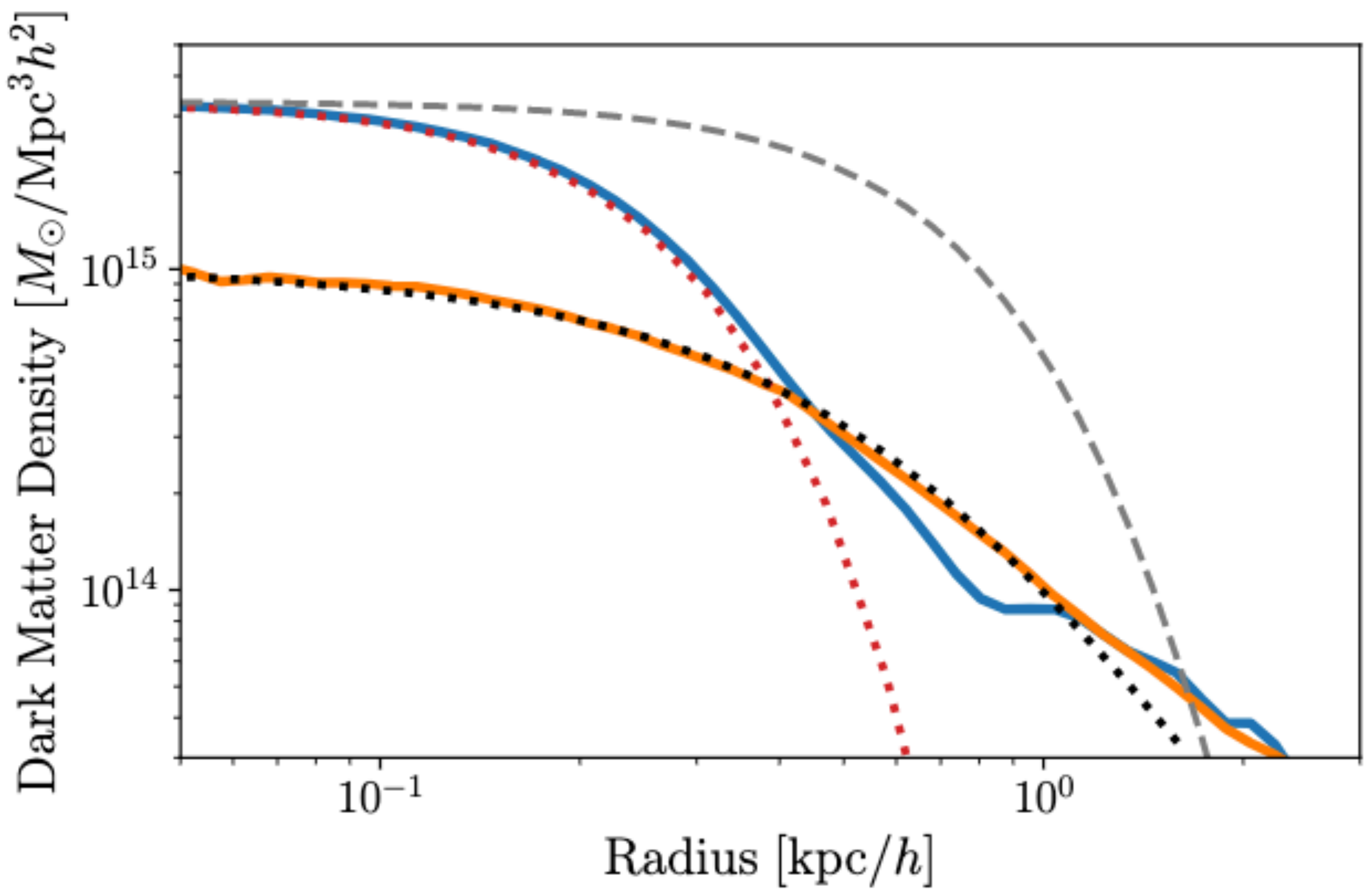

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Inverse Problem

$$\frac{\partial^2(r\phi)}{\partial r^2} = 2r\left(\frac{m_a^2}{\hbar^2}V - \frac{m_a}{\hbar^2}\gamma\right)\phi$$

Start with guess m_a, γ

Vary parameters until equation is (approximately) satisfied

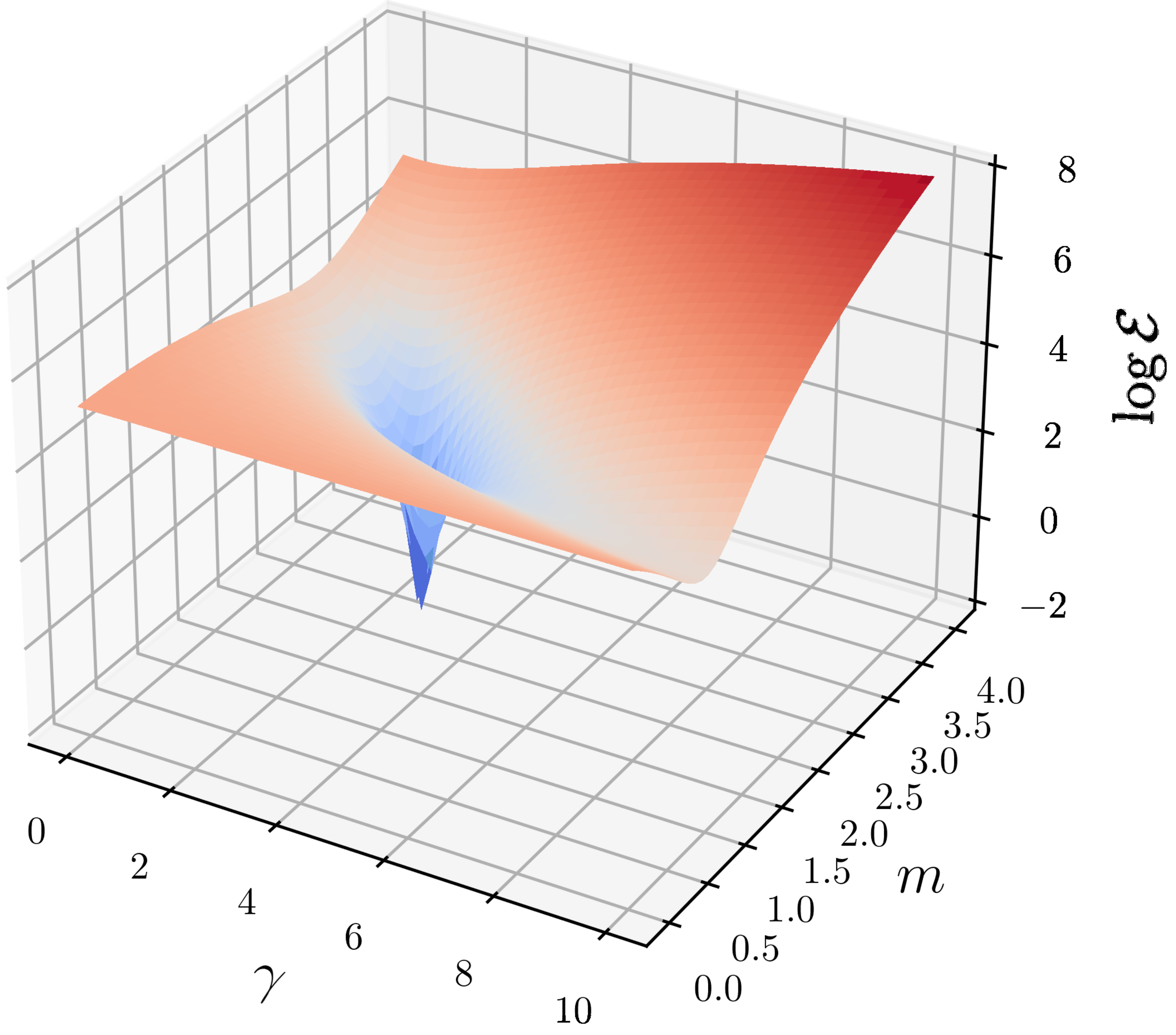
$$\mathscr{E}(m_a, \gamma) = \sum_i \left| \mathrm{LHS}\left(r_i\right) - \mathrm{RHS}\left(r_i\right) \right|^2$$

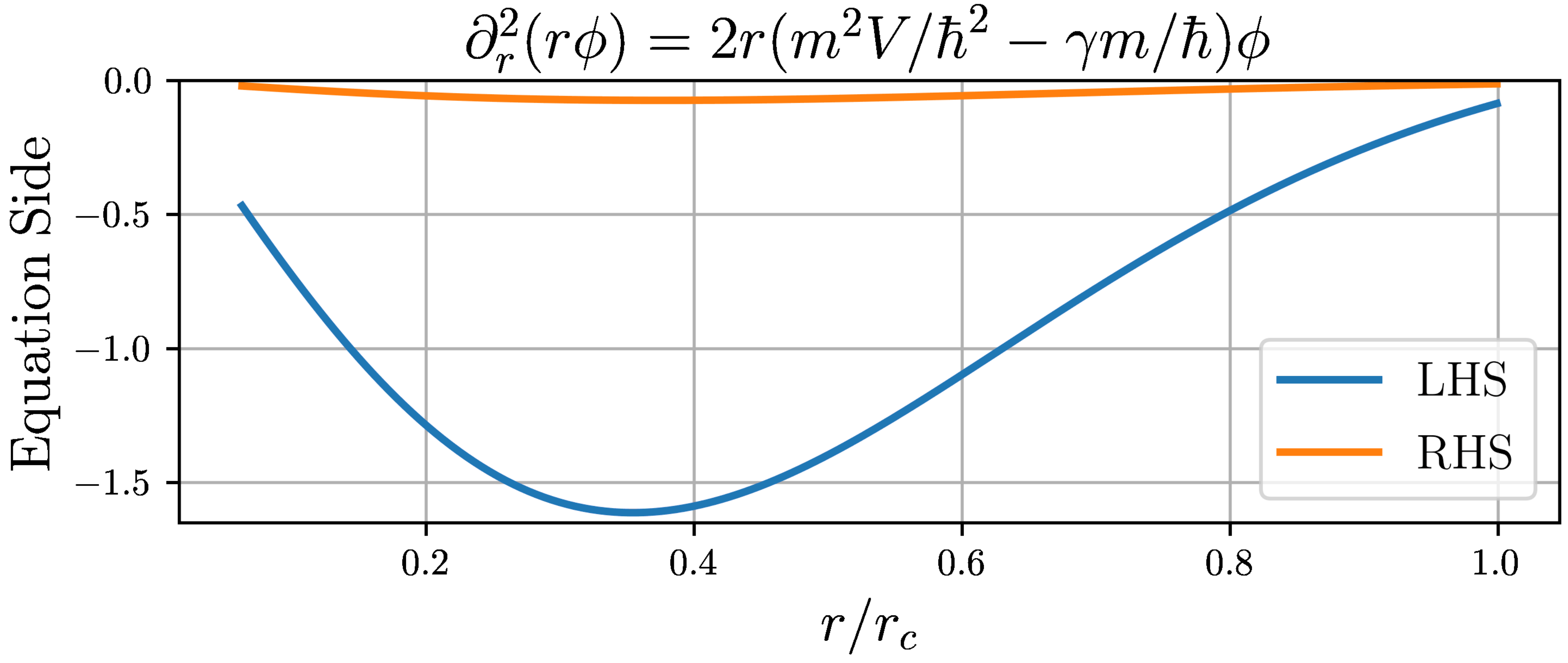
$$m_a^*, \gamma^* = \mathrm{argmin}(\mathscr{E})$$

Inverse Problem
Optimization

Goal: Solve inverse problem for m_a, γ

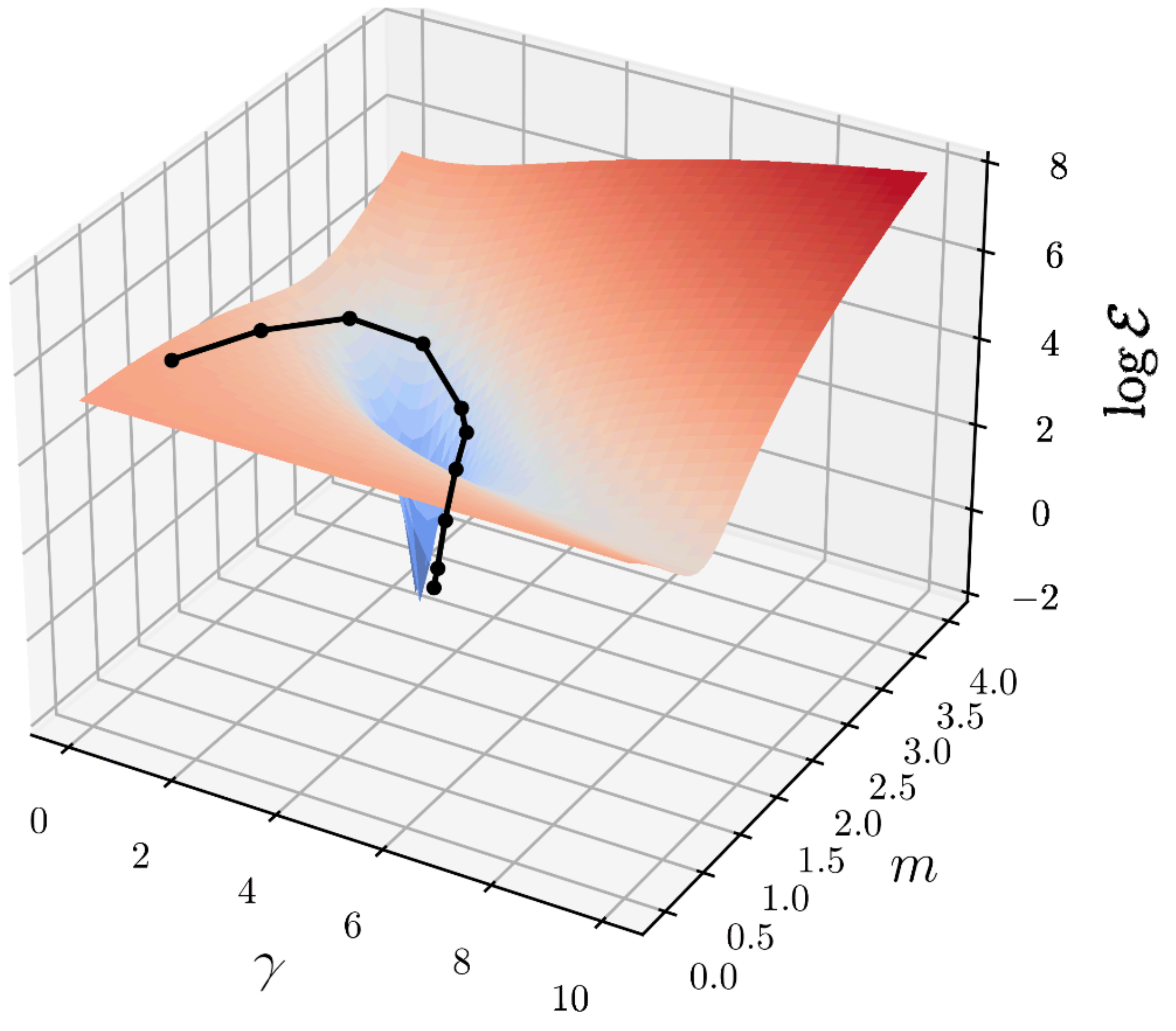
Use Neural Network Optimizers (PINN)





Inverse Problem

Optimization



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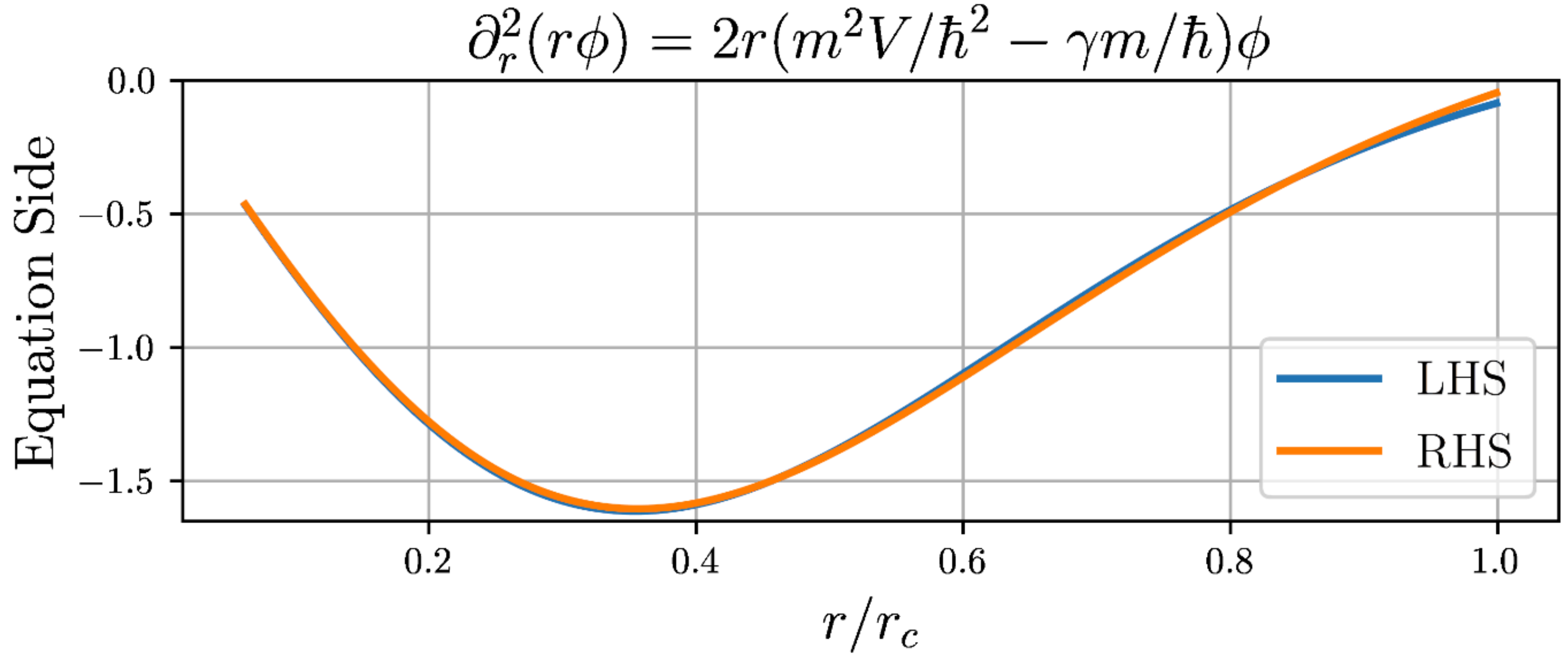
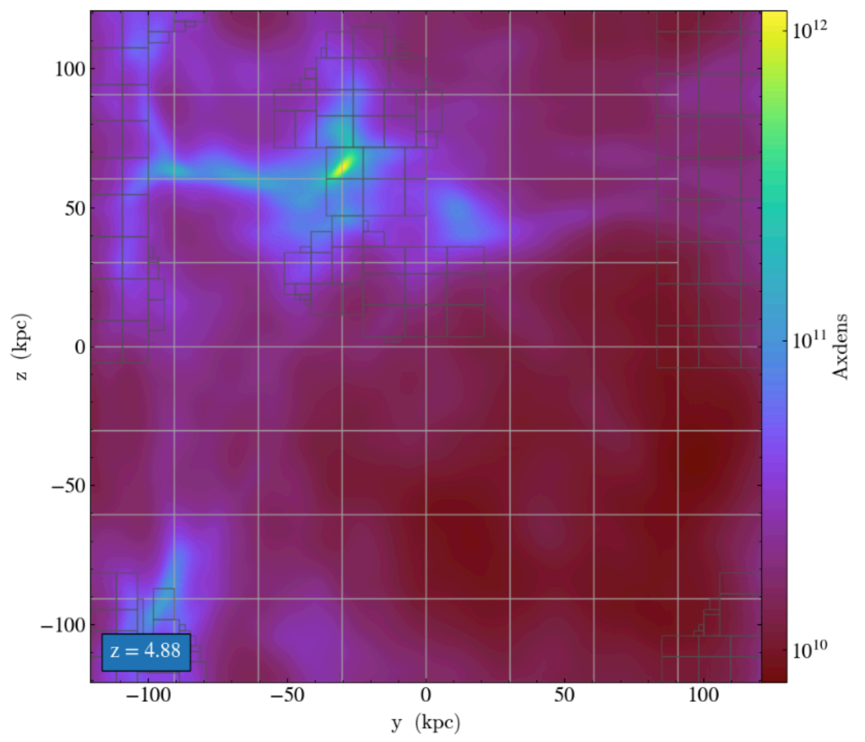


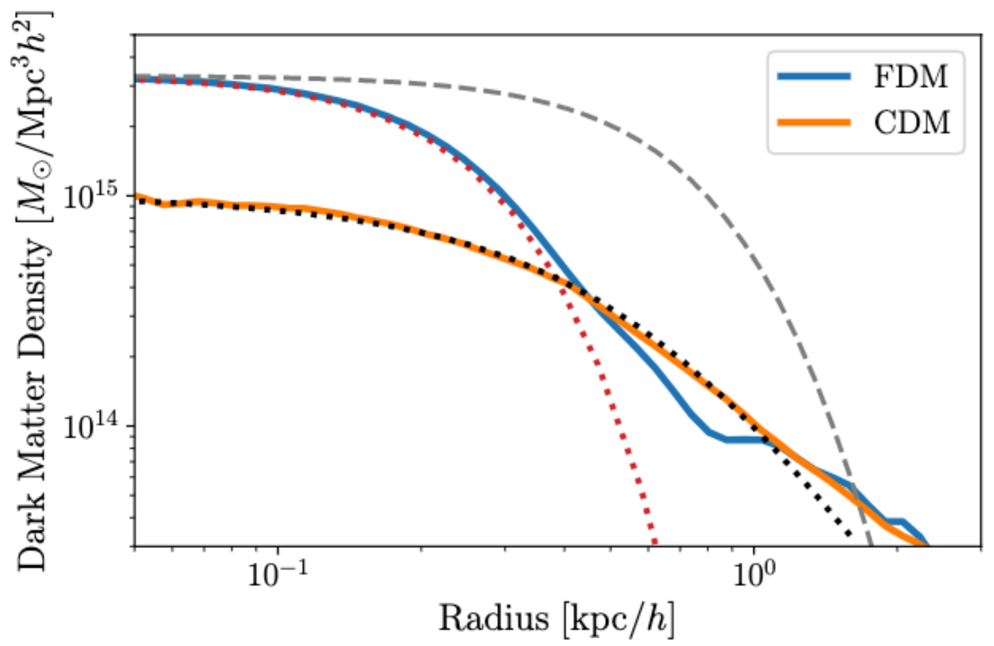
Use Neural Network Optimizers (PINN)

Validation

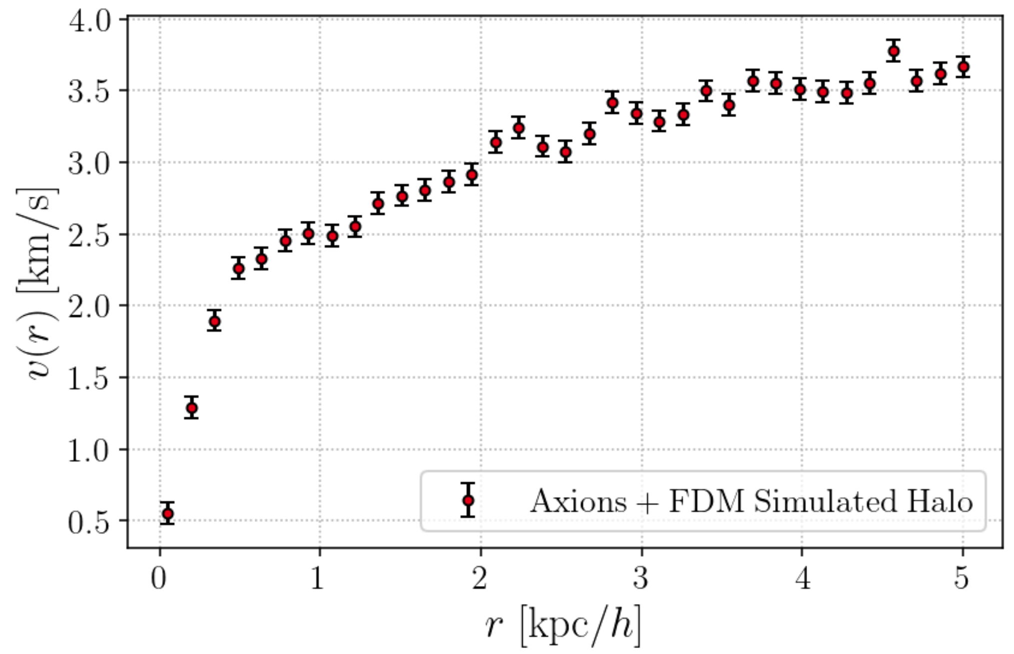
Simulation



Density



Simulated Rotation Curve



m_a

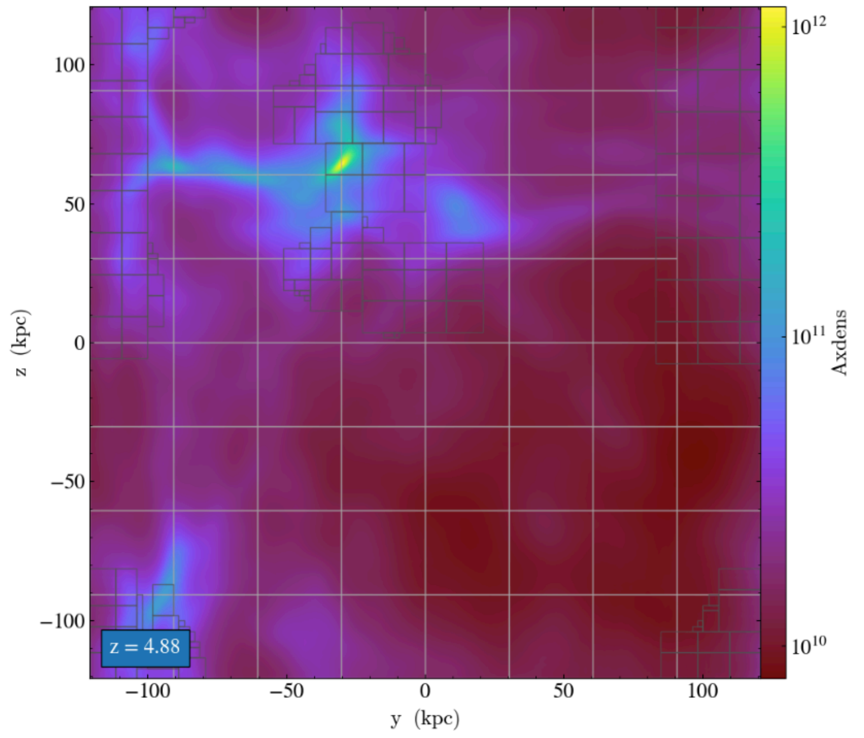
true

Simulate Observation

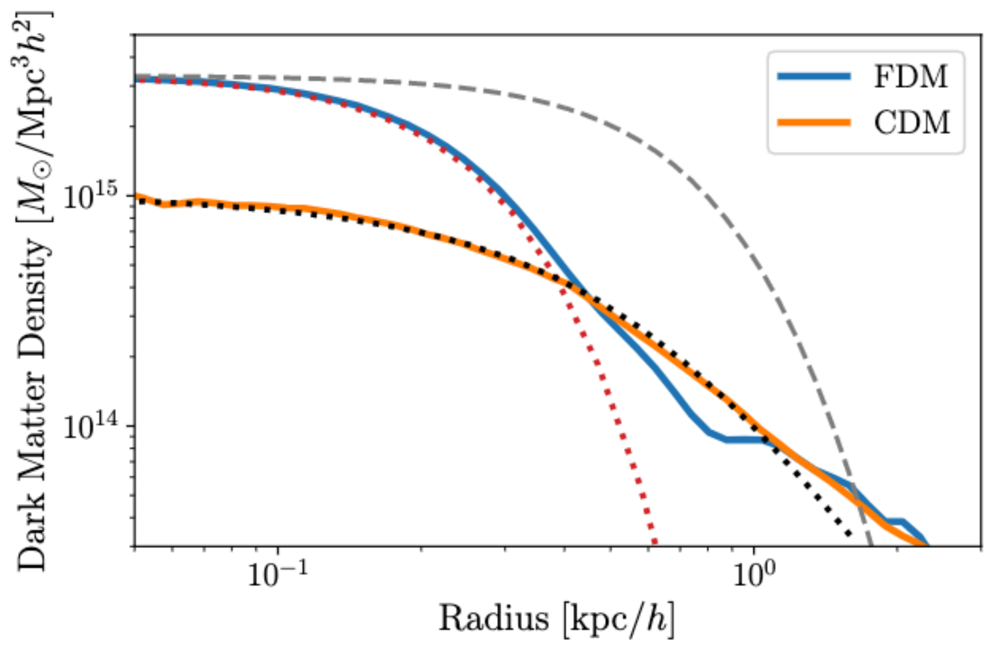
Validation

m_a
true

Simulation



Density



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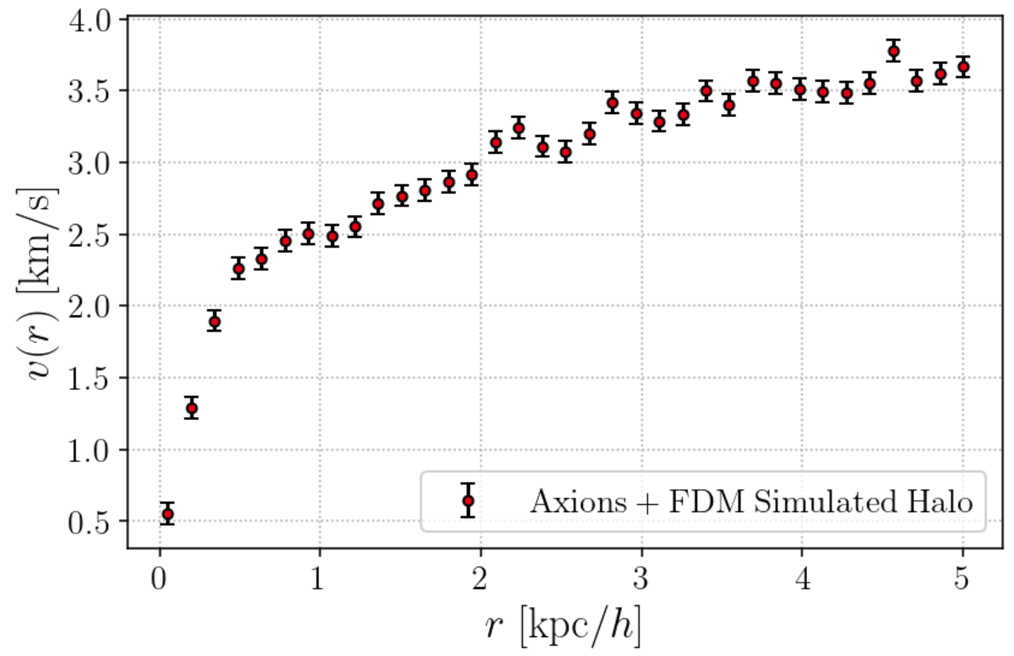
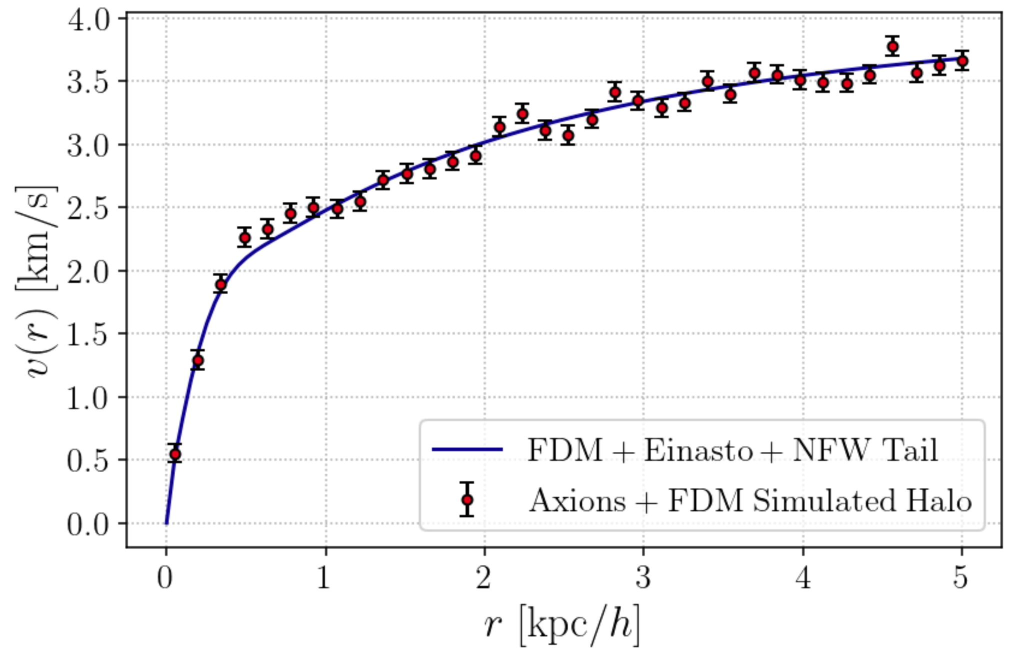


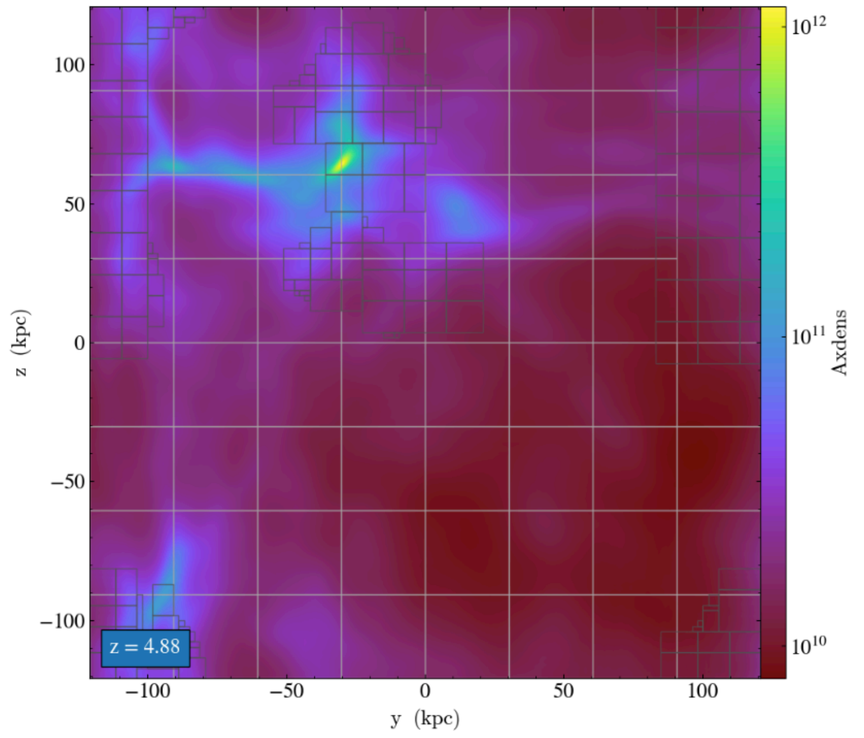


Fit Rotation Curve

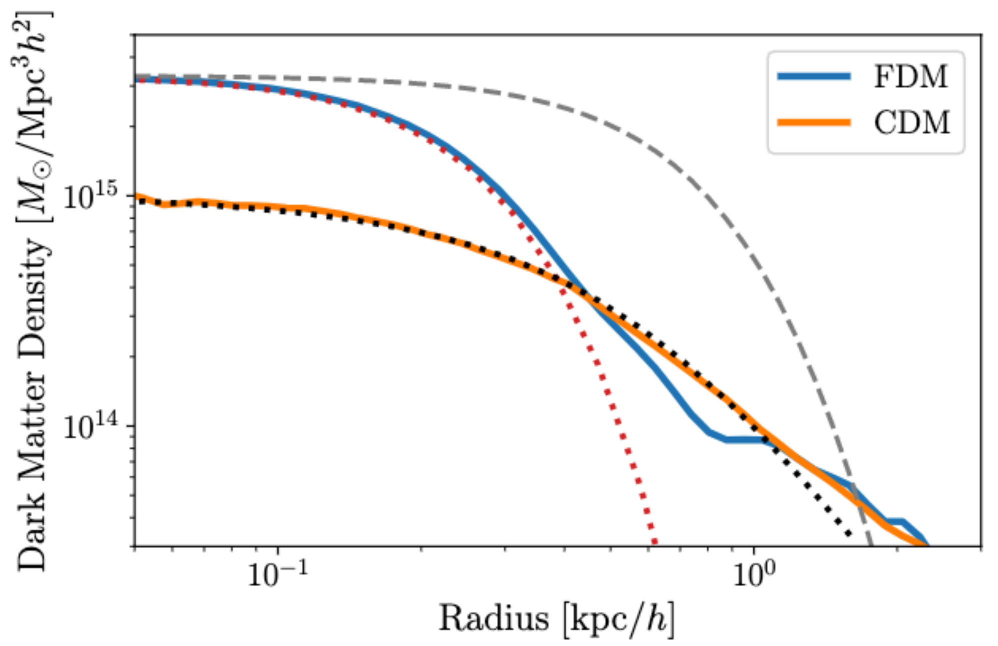
Validation

m_a
true

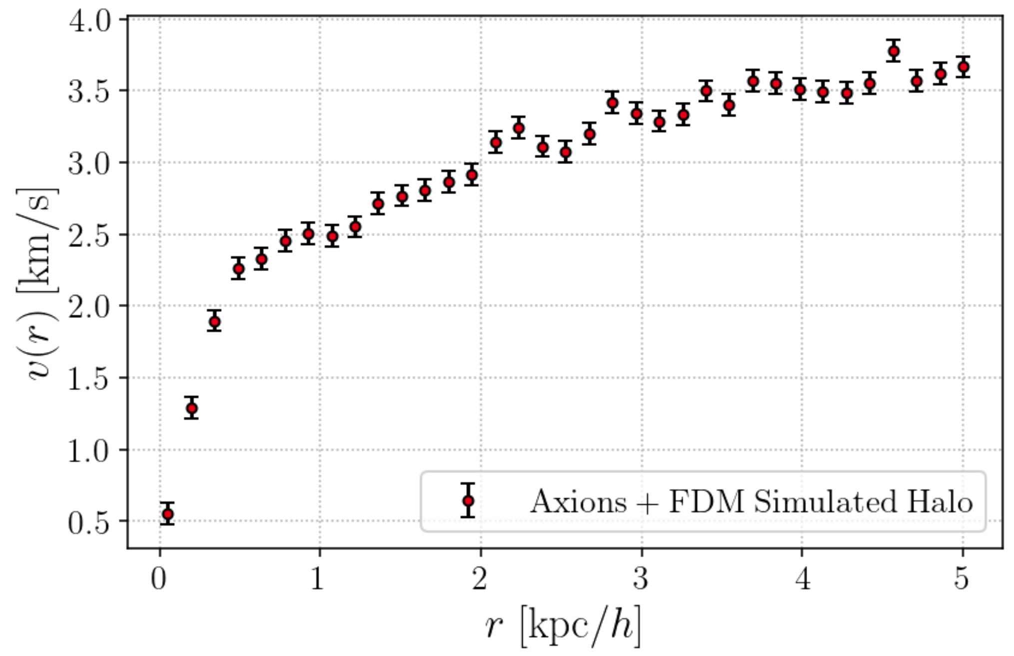
Simulation



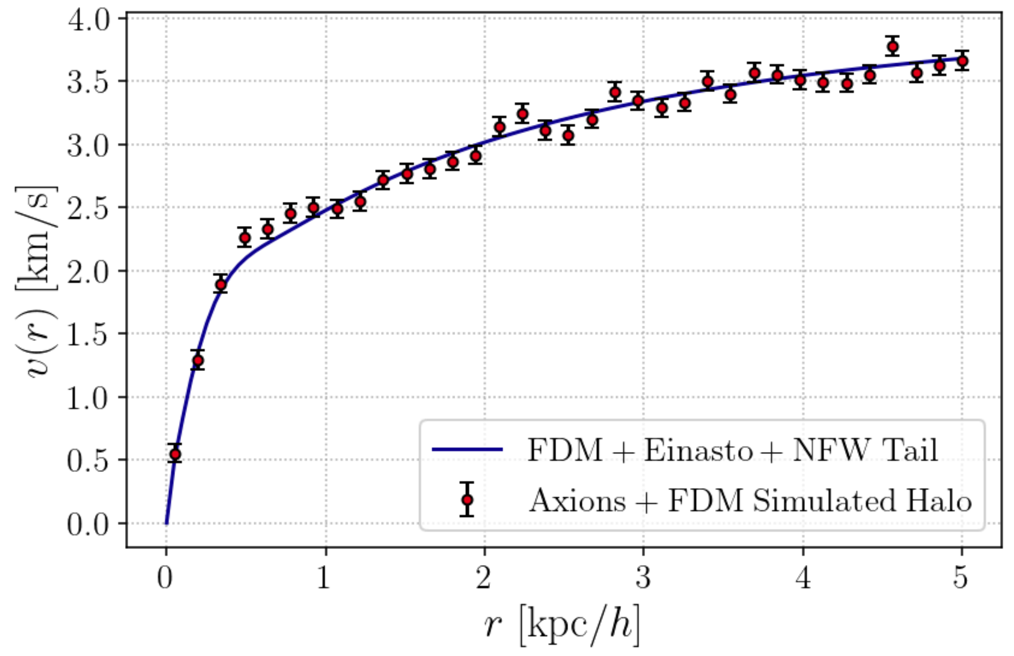
Density



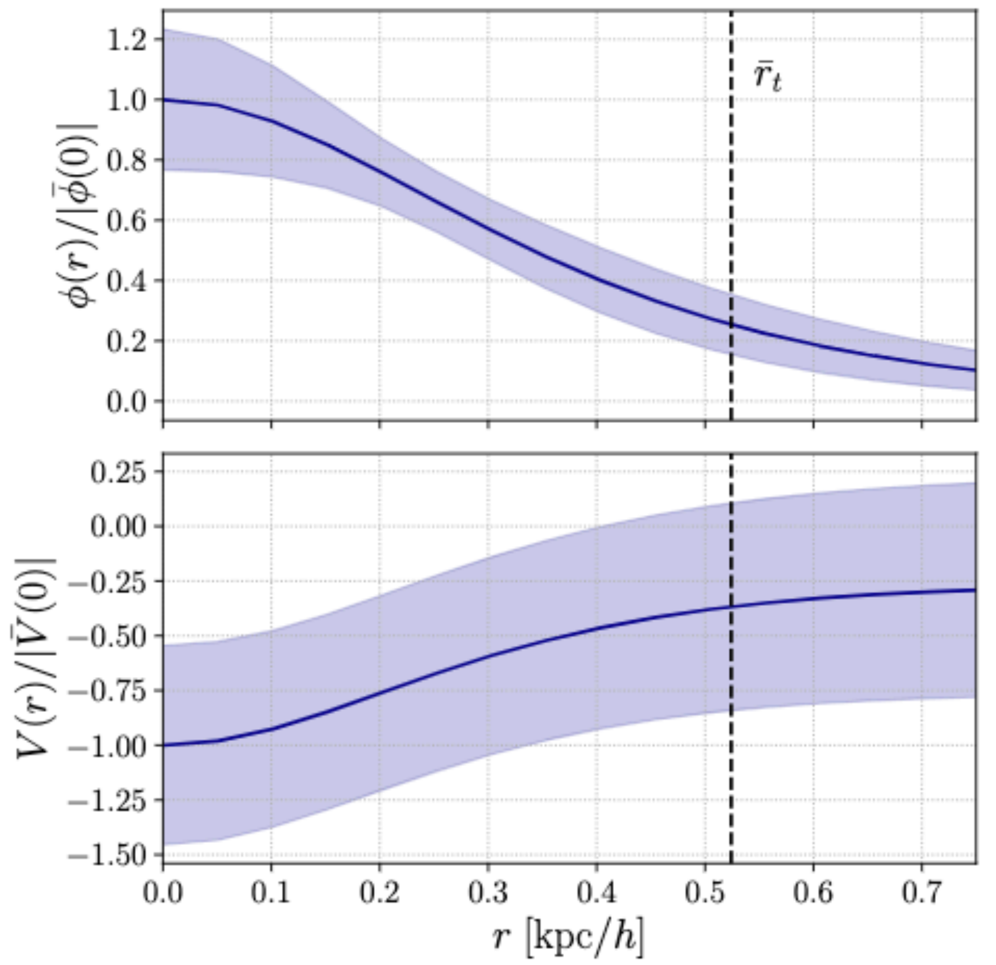
Simulated Rotation Curve



Fit Rotation Curve

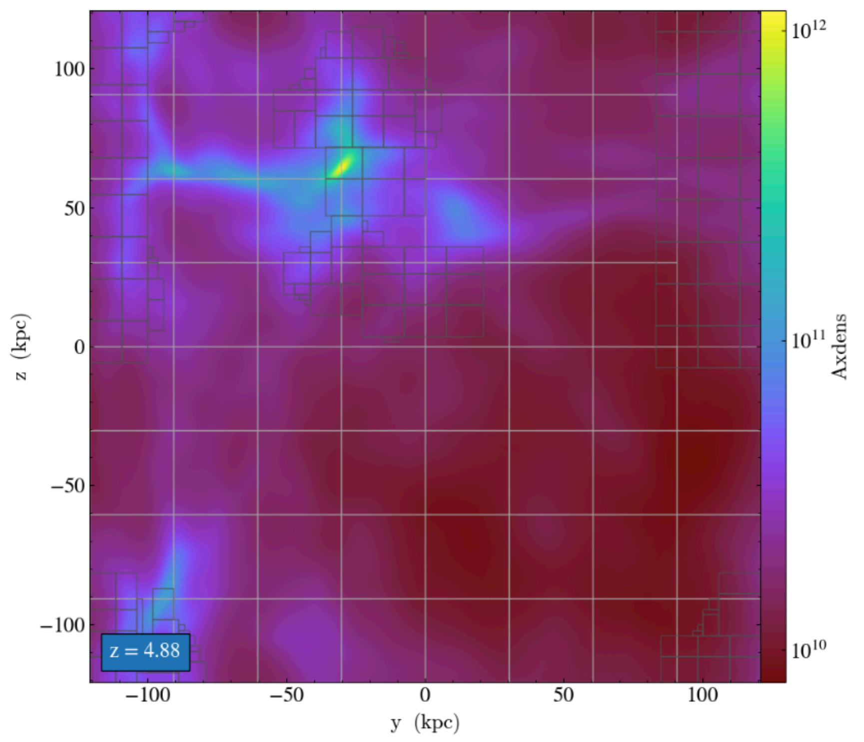


Estimated Density Profile (MCMC)

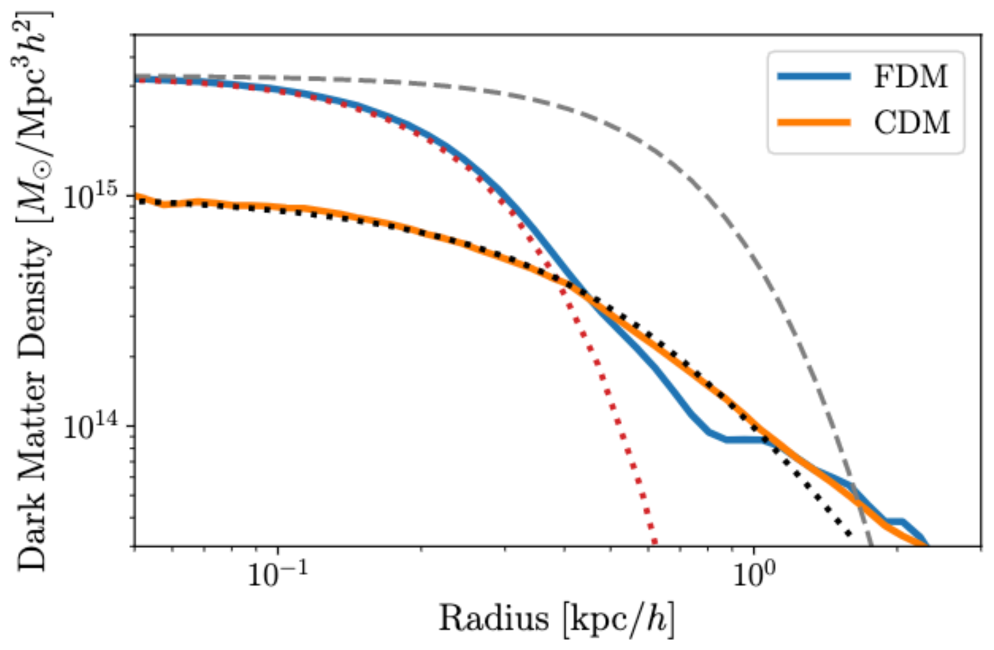


Validation

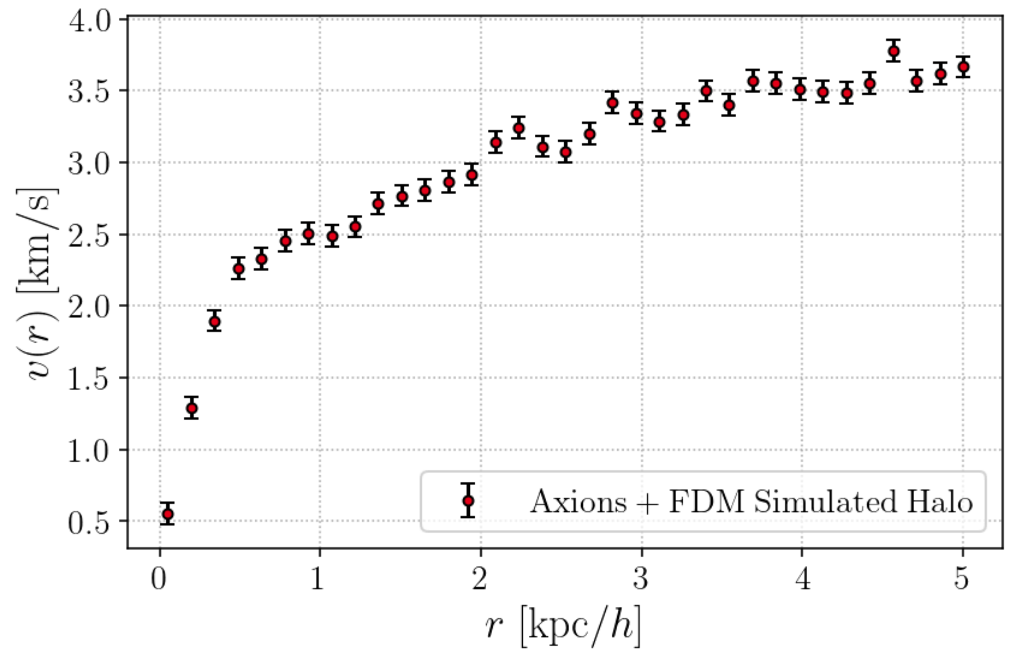
Simulation



Density



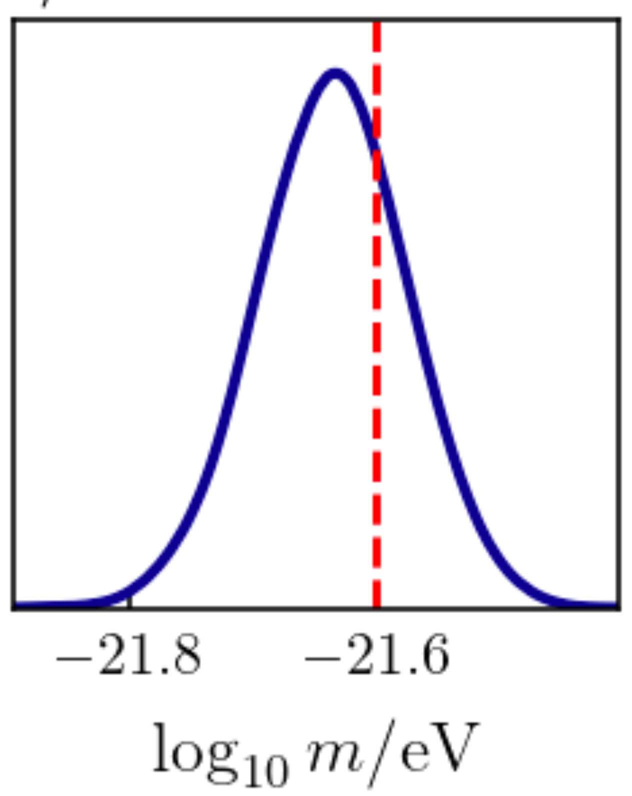
Simulated Rotation Curve



m_a
true

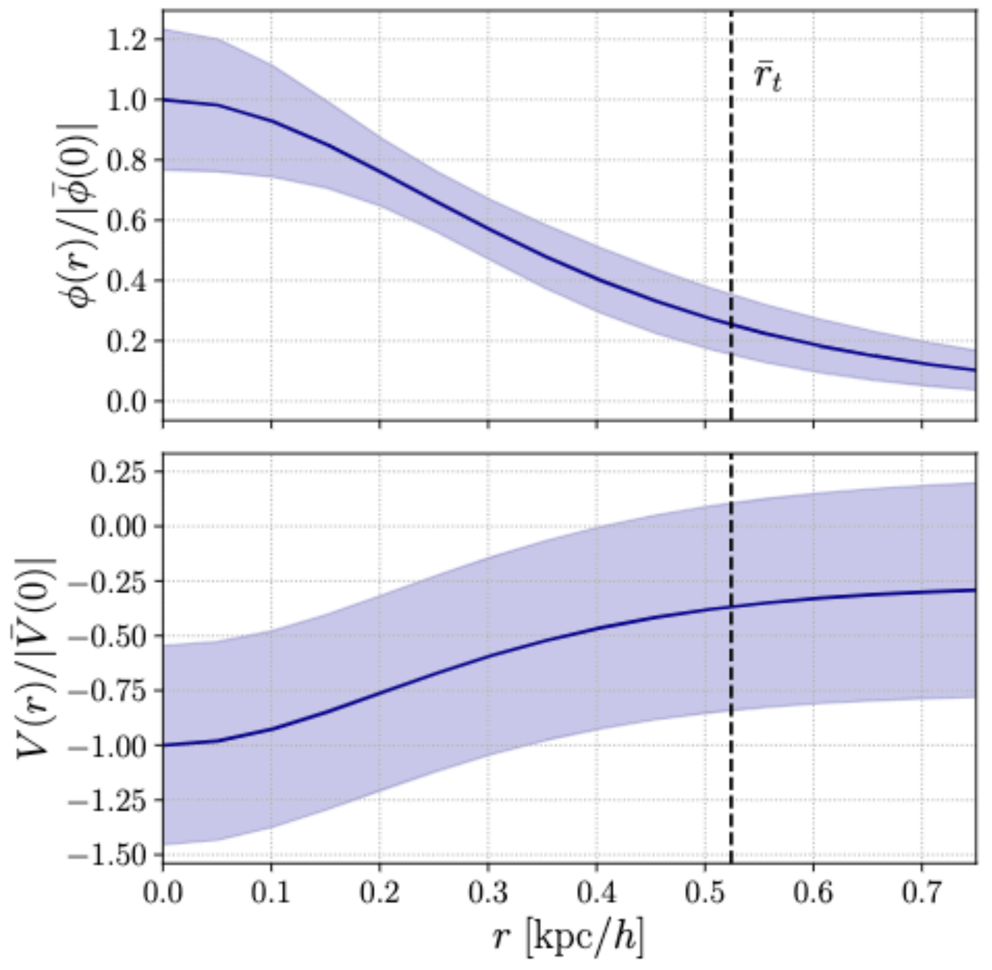
m_a
estimated

$\log_{10} m/\mathrm{eV} = -21.637 \pm 0.061$



Particle Mass Probability

PINN !



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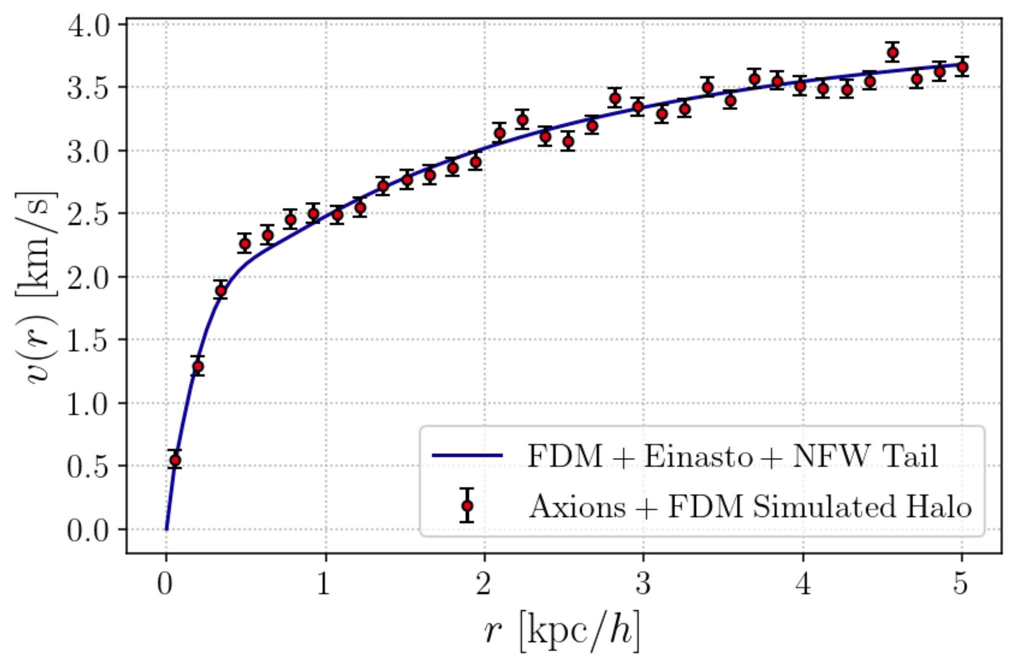
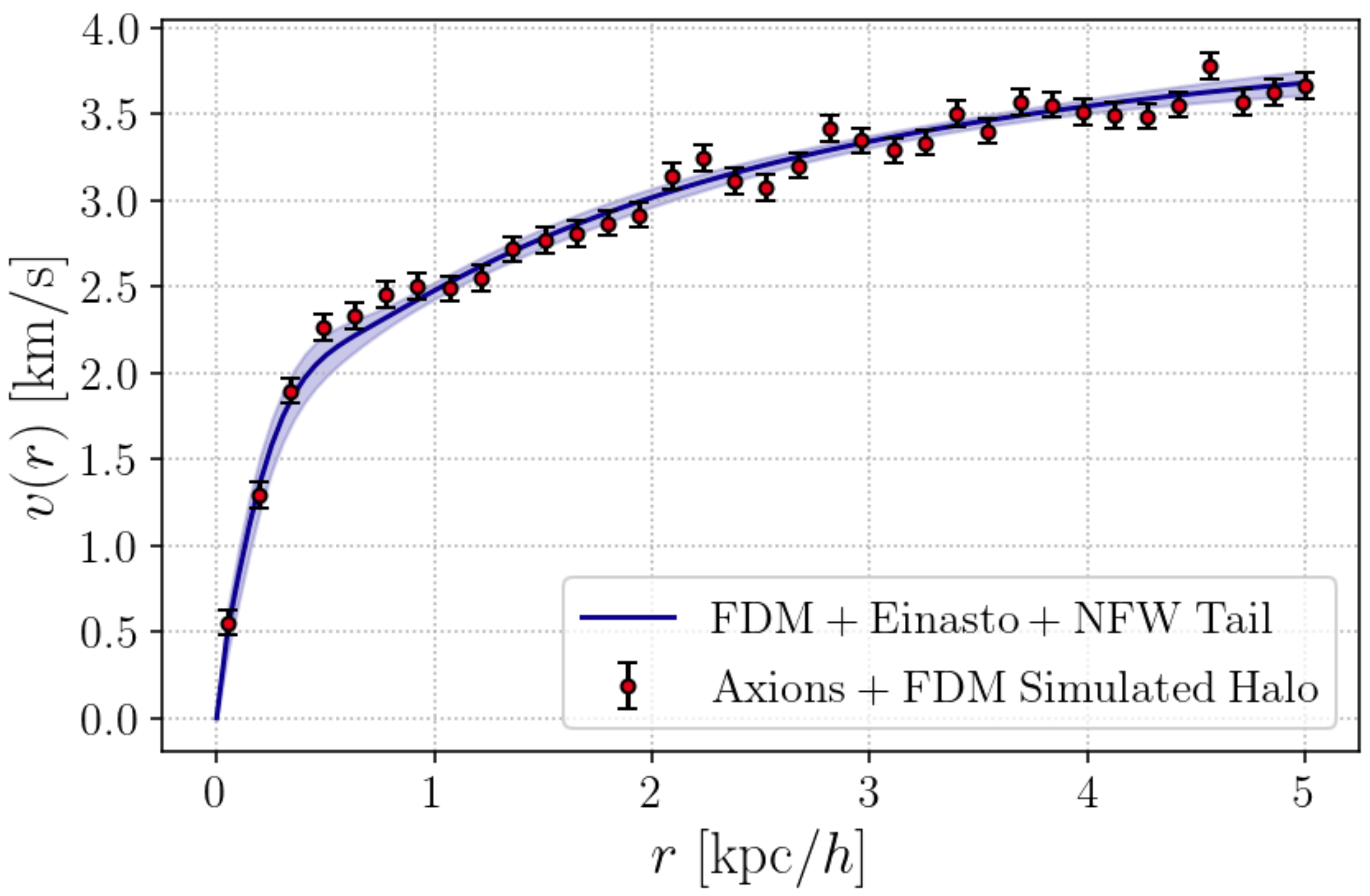


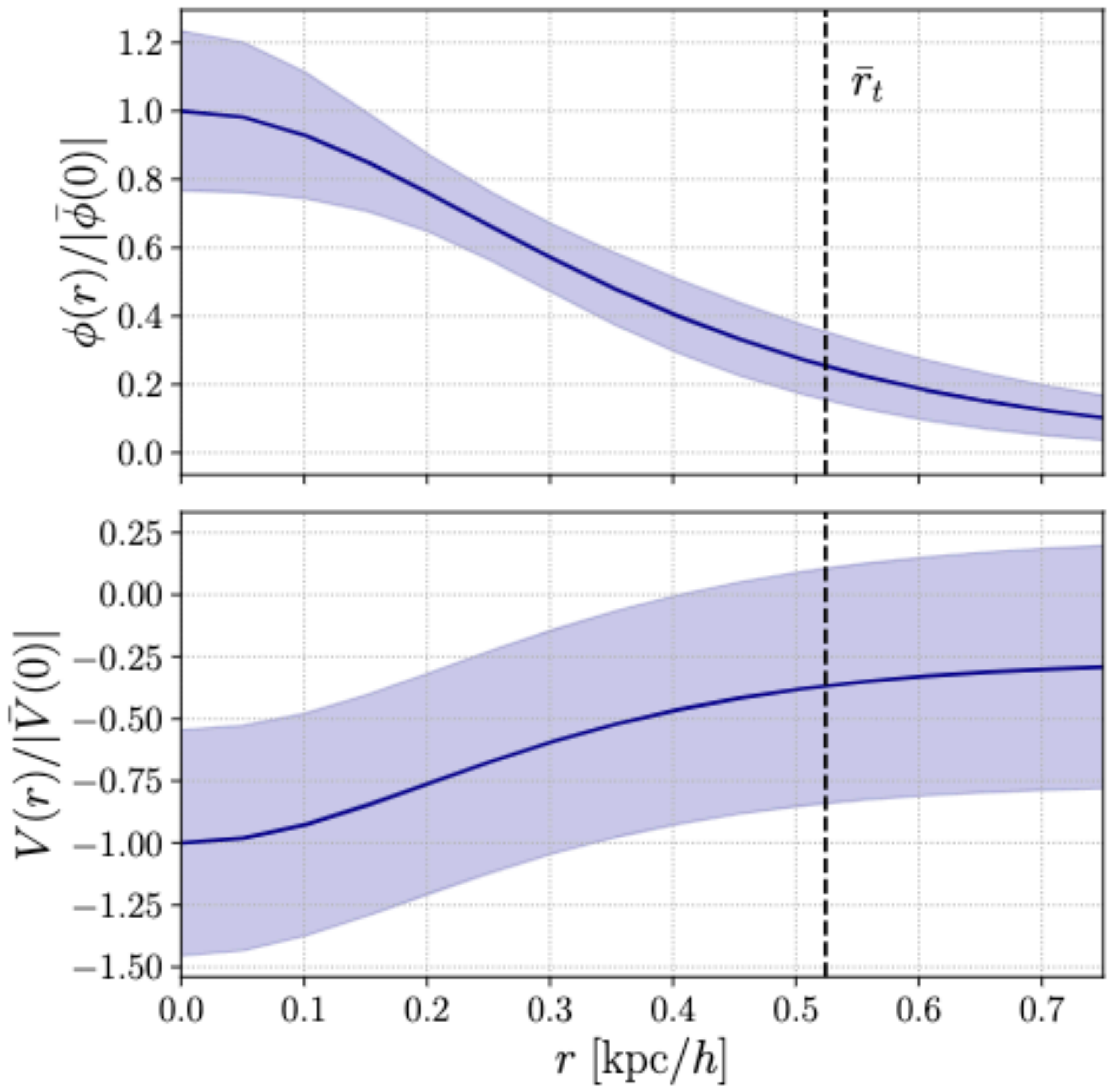
Fit Rotation Curve

Inverse Problem (Validation on Sims)

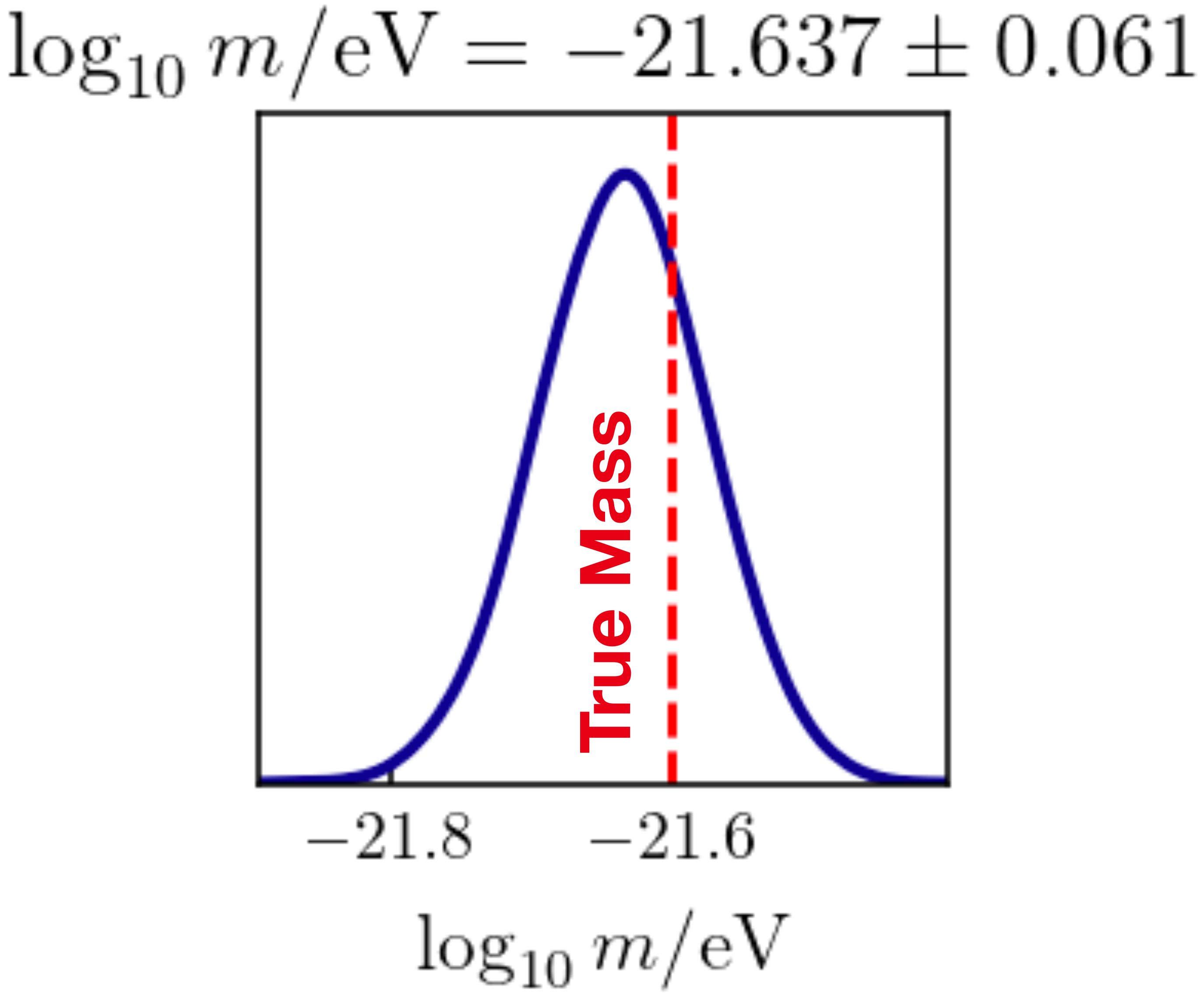
Run PINN for each possible ϕ & V



Distribution of Rotation Curves



Distribution of ϕ & V

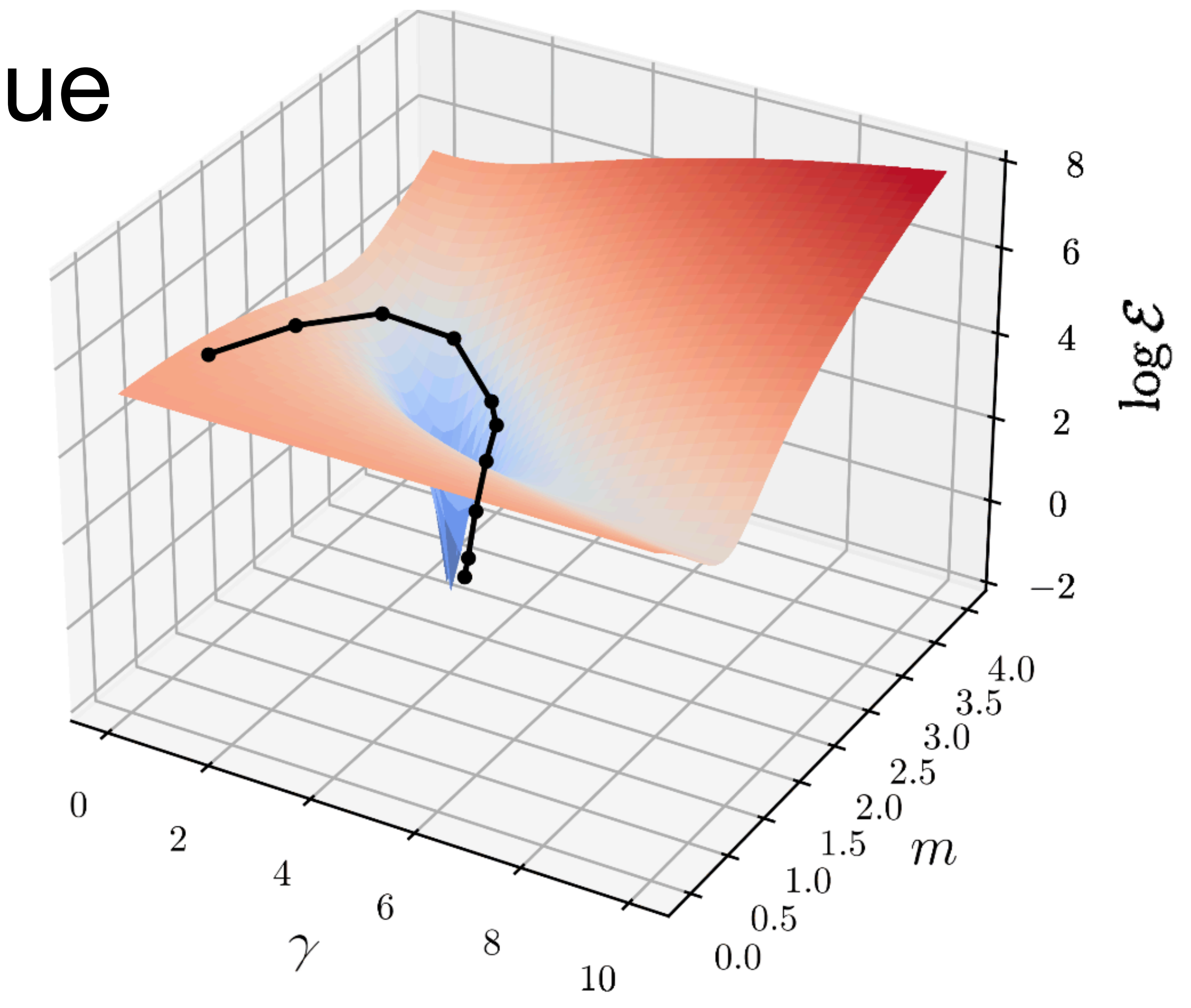


Distribution of m_a

Conclusions

PINNs in Cosmology

1. Inverse problems approach useful when fitting a PDE to data
2. Cheaper than solving the forward problem (expensive sims)
3. Can be combined with standard MCMC techniques
4. Excellent for finding maximum likelihood parameter value



Thank you! Questions?