Exoplanetary atmospheres: Observations and modelling

Benjamin Charnay
ARIEL-School, Biarritz, 2019
Introduction
A diversity of exoplanets

I) Observational techniques
- Transit
- Direct imaging
- Medium/high spectral resolution
- Lessons from observations of exoplanets

II) Modelling exoplanetary atmospheres
- Radiative transfer
- Thermal structure
- Clouds & aerosols
Radiative transfer + basics about physics/chemistry of exoplanetary atmospheres

Climate & habitability

Physics/chemistry/dynamics of planetary atmospheres & Solar System planets

Atmospheric evolution, habitability & early Earth
Not really a statistically significant sample
First exoplanet around main-sequence star
Hot Jupiters

Super-Earths & mini-Neptunes

Observational biases
Occurrence rate of planets

$$\frac{\text{Occ. Rate}}{\text{Completeness}} = \frac{\text{Num. of (real) Planet}}{\text{Number of stars for which a planet would be detected if it’s there}}$$

Planets around Sun-like stars are very common
High fraction of super-Earths and mini-Neptunes

Fressin et al. (2013)  
Petigura et al. (2013)
Planet formation: core accretion model

Core formation by solid accretion

Gas accretion beyond critical mass

$M_{\text{crit}} \sim 10$ Earth masses $\rightarrow$ For H-He envelopes

Planet formation: core accretion model

Core formation by solid accretion

Slow core accretion

Protoplanetary disk

99% gas 1% solids

$T_{\text{disk}} < 10^7 \text{yr}$

Gas giant
Ice giant
Planet formation: core accretion model

Core formation by solid accretion

Not enough planetesimals in feeding zone
Atmospheres as a probe of planetary interior and formation

Metallicity = fraction of heavy elements (heavier than H and He)

- For Solar System atmospheres, metallicity $\approx [\text{C}]/[\text{C}]_{\text{Solar}}$
- For exoplanetary atmospheres, metallicity $\approx [\text{O}]/[\text{O}]_{\text{Solar}}$

- Metallicity decreases with planetary mass in the Solar System
- Sub-Neptunes/Neptunes planets formed in-situ should have a relatively low metallicity

$\rightarrow$ Measuring the metallicity allows to test formation and migration mechanisms
Madhusudhan et al. (2014)
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I) Transit

Probability of transit

\[ p_{\text{tra}} = \left(\frac{R_* - R_p}{a}\right) \left(\frac{1+esin\omega}{1-e^2}\right) \]

Prob. of full occultation:

\[ p_{\text{occ}} = \left(\frac{R_* - R_p}{a}\right) \left(\frac{1-esin\omega}{1-e^2}\right) \]

If \( R_* \gg R_p \) and \( e \approx 0 \):

\[ p_{\text{tra}} = p_{\text{occ}} \approx 0.005 \left(\frac{R_*}{R_\odot}\right) \left(\frac{a}{1 \text{ AU}}\right)^{-1} \]

Seager et al. (2010)
I) Transit

Probability of transit

Interest of ultra-cool stars

Probability of transit of an Earth-size planet at Teff=255K

Figure from A. Triaud
I) Transit

Photometry

Transit depth:

\[ \delta_{tra} = \left( \frac{R_p}{R_*} \right)^2 \]

Occultation depth:

\[ \delta_{occ} = \frac{I_p}{I_*} \left( \frac{R_p}{R_*} \right)^2 \]
I) Transit

Effect of limb darkening

Transit of Venus
I) Transit

Effect of limb darkening
I) Transit

Atmospheric characterization with photometric transit lightcurves

- Measure of radius and density

![Graph](image)

Howard et al. (2013)

- Measure of thermal emission and reflected light during occultations
  \[\rightarrow\] effective temperature and geometric albedo
I) Transit

Spectroscopy

Variation of transit depth:

$$\Delta \delta_{\text{tra}} = \frac{\pi (R_p + N_H H)^2}{\pi R_*^2} - \frac{\pi R_p^2}{\pi R_*^2} \approx 2N_H \delta_{\text{tra}} \left( \frac{H}{R_p} \right)$$

Scale height: $$H = \frac{RT}{Mg}$$; Number of scale heights: $$N_H \approx 7$$ (for low resolution)

→ Transit spectroscopy easier for high scale height (e.g. hot giant planets)
I) Transit

Spectroscopy

Variation of transit depth:

\[ \Delta \delta_{tra} = \frac{\pi (R_p + N_H H)}{\pi R_*^2} - \frac{\pi R_p}{\pi R_*^2} \approx 2N_H \delta_{tra} \left( \frac{H}{R_p} \right) \]

Scale height: \( H = \frac{RT}{Mg} \); Number of scale heights: \( N_H \approx 7 \) (for low resolution)

For an Sun-like star:
- Hot Jupiter (\( T=1300 \) K, \( g=25 \) m s\(^{-2}\), \( M=2.3 \) g/mol): \( \delta_{tra} \approx 0.01 \), \( \Delta \delta_{tra} \approx 4.10^{-4} \)
- Earth-like planet (\( T=280 \) K, \( g=10 \) m s\(^{-2}\), \( M=28 \) g/mol): \( \delta_{tra} \approx 10^{-4} \), \( \Delta \delta_{tra} \approx 2.10^{-6} \)
I) Transit Spectroscopy

Variation of transit depth:

\[
\Delta \delta_{\text{tra}} = \frac{\pi (R_p + N_H H)^2}{\pi R_*^2} - \frac{\pi R_p^2}{\pi R_*^2} \approx 2 N_H \delta_{\text{tra}} \left( \frac{H}{R_p} \right)
\]

Scale height: \( H = \frac{RT}{Mg} \); Number of scale heights: \( N_H \approx 7 \) (for low resolution)

For Trappist-1 (0.015 \( R_\odot \)):
- Hot Jupiter (\( T=1300 \) K, \( g=25 \) m s\(^{-2}\), \( M=2.3 \) g/mol): \( \delta_{\text{tra}} \approx 0.7, \Delta \delta_{\text{tra}} \approx 2.10^{-2} \)
- Earth-like planet (\( T=280 \) K, \( g=10 \) m s\(^{-2}\), \( M=28 \) g/mol): \( \delta_{\text{tra}} \approx 6.10^{-3}, \Delta \delta_{\text{tra}} \approx 10^{-4} \)
I) Transit Spectroscopy

Assumptions: hydrostatic+isothermal

\[ p(z) = p(z_0) \exp \left( -\frac{z-z_0}{H} \right) \text{ with } H = \frac{RT}{Mg} \]

Optical depth (cross-section independent of P & T):

\[ \tau(b, \lambda) = \sum_i \int_{-\infty}^{+\infty} \sigma_i(\lambda)n_i(x) \, dx \]

\[ n_i(x) = n_{i0}e^{-z/H} \text{ with } z = \sqrt{b^2 + x^2} - R_p \approx b - R_p + \frac{x^2}{2b} \]

\[ \tau(b, \lambda) \approx \sum_i \sigma_i(\lambda)n_{i0}e^{-(b-Rp)/H} \int_{-\infty}^{+\infty} e^{-x^2/2RpH} \, dx = \sum_i \sigma_i(\lambda)n_{i0}e^{-(b-Rp)/H} \sqrt{2\pi bH} \]

Comparison with vertical optical depth:

\[ \eta = \frac{\tau_H}{\tau_V} = \sqrt{\frac{2\pi Rp}{H}} \]

Earth: \( \eta \approx 75 \)
Jupiter: \( \eta \approx 128 \)
HD209458b: \( \eta \approx 38 \)
I) Transit Spectroscopy

Assumptions: hydrostatic+isothermal

\[ p(z) = p(z_0) \exp \left( - \frac{z-z_0}{H} \right) \text{ with } H = \frac{RT}{Mg} \]

Optical depth (cross-section independent of P & T):

\[ \tau(b, \lambda) = \sum_i \sigma_i(\lambda)n_i \exp(-b/R_p) \sqrt{2\pi bH} \]

Transit depth:

\[ D(\lambda) = \left( \frac{R_p}{R_*} \right)^2 + \frac{2}{R_*^2} \int_{R_p}^{R_*} b(1 - e^{-\tau(b,\lambda)})db = \left( \frac{R_p + h_\lambda}{R_*} \right)^2 \]

Equivalent altitude:

\[ h_\lambda = -R_p + \sqrt{R_p^2 + 2 \int_{R_p}^{R_*} b(1 - e^{-\tau(b,\lambda)})db} \approx 0.577H + Hln \left( \sqrt{2\pi HR_p} \sum_i \sigma_i(\lambda)n_{i0} \right) \]

\[ h_\lambda \approx b(\tau = 0.56) - R_p \]

see De Wit & Seager (2013) and Macdonald & Cowan (2019)
I) Transit Spectroscopy

Synthetic Earth’s transit spectrum

Macdonald & Cowan (2019)
I) Transit

Phase curves

Courtesy Tom Louden
I) Transit

Phase curves

Photometric phase curve (ex: Spitzer)

Spectrally resolved phase curve (ex: HST)

Demory et al. (2016)

Cowan et al. (2014)

WASP 43b

55 Cancri e

Stevenson et al. (2014)
I) Transit

Open-access codes for lightcurve fitting

For transits:
Transit routines (IDL, FORTRAN): https://faculty.washington.edu/agol/transit.html
batman (Python): https://www.cfa.harvard.edu/~lkreidberg/code.html
STARRY (Python): https://github.com/rodluger/starry
ExoCTK (Python): https://exoctk.stsci.edu/lightcurve_fitting

For secondary eclipses & phase curves:
STARRY (Python): https://github.com/rodluger/starry
spiderman (Python): https://www.cfa.harvard.edu/~lkreidberg/code.html#spiderman
II) Direct imaging

Limitations
II) Direct imaging

Ingredients to overcome limitations
III) Medium/high spectral resolution

- **Low resolution:** \( R = \frac{\lambda}{\Delta \lambda} < 1000 \) (e.g. HST, ARIEL)
  - absorption bands

- **Medium resolution:** \( R = \frac{\lambda}{\Delta \lambda} \sim 1000 - 10000 \) (e.g. JWST, VLT/SINFONI)
  - strong molecular lines

- **High resolution:** \( R = \frac{\lambda}{\Delta \lambda} > 10000 \) (e.g. VLT/CRIRES, VLT/ESPRESSO)
  - resolve line shape and doppler shift
III) Medium/high spectral resolution

Medium resolution for direct imaging

- Distinguish planetary signal from stellar noise (speckles) thanks to intrinsic molecular lines
- Cross-correlation between the high-passing observed spectrum $S_{obs}$ and a model spectrum $S_{th}$

$$CCF(V_0) = \int S_{obs}(v) \times S_{th}(v + v \times V_0 / c) dv$$

with normalization: $\int S^2(v) dv = 1$

Wavelength-averaged image of beta Pic b with VLT-SINFONI

Hoeijmakers et al. (2018)
III) Medium/high spectral resolution

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$$CCF(V_0) = \int S_{\text{obs}}(\nu) \times S_{\text{th}}(\nu + \nu \times V_0/c) \, d\nu$$

with normalization: $\int S^2(\nu) \, d\nu = 1$

Hoeijmakers et al. (2018)
III) Medium/high spectral resolution

High resolution for transit spectroscopy

- Detection of CO during transit
- Doppler shift by orbital velocity
- Doppler shift by day-night winds

Snellen (2014)

Observation of HD209458 b

Snellen et al. (2010)

Observation of tau Böötis b

Brogi et al. (2012)

- Detection of CO in emission
- Stratospheric thermal inversion
Lessons from observations of exoplanet atmospheres

- **Radius & Interior**
  - Hot Jupiters are inflated
  - Gap in the occurrence rate between super-Earths and mini-Neptunes

- **Dynamics & Thermal structure**
  - Superrotation for strongly irradiated planets
  - Stratospheric thermal inversion for the hottest planets

- **Clouds/haze**
  - Most of exoplanets are cloudy/hazy
  - Inhomogeneous clouds distribution

- **Atmospheric composition**
  - Chemical disequilibrium (ex: CO/CH4)
  - Low-mass planets seem to have high-mean molecular weight

- **Atmospheric escape**
  - Atmospheric escape for strongly irradiated planets
IV) Lessons from observations

Inflated hot Jupiters

- Hot Jupiters are inflated compared to 1D models
- Correlation between inflated radii and stellar flux

Miller & Fortney (2011)
IV) Lessons from observations

Inflated hot Jupiters

Explanations for inflated hot Jupiters:

- Heat transfer to the adiabatic layer (10^{-4}\% - 1\% of the irradiation)

1) Ohmic dissipation

   Batygin & Stevenson (2010)

   Superrotation + magnetic field + ionization of H and alkali metals in hot Jupiters → Induced currents

   Heat production: \( P = \frac{j^2}{\sigma} \)

2) Advection of heat from global circulation

   Tremblin et al. (2017)
IV) Lessons from observations

A valley between super-Earths and mini-Neptunes

- Bimodal distribution with a gap at around 1.8 $R_E$
- Transition from mini-Neptunes to super-Earths with increasing instellation
  $\rightarrow$ Photoevaporation

*Fulton et al. (2018)*
IV) Lessons from observations

A valley between super-Earths and mini-Neptunes

Prediction of a photo-evaporation valley

If correct:
No water $\rightarrow$ formation inside the ice line

Photoevaporation can predict the 2 peaks. The location of the valley is very sensitive to the core composition $\rightarrow$ **cores seem to be Earth-like in composition.**

*(Owen & Wu 2017; Jin & Mordasini 2017)*

*(Owen & Wu 2017)*

*(I)*

*(II)*

*(III)*

*(IV)*

*(V)*

*(VI)*

*(VII)*

*(VIII)*

*(IX)*

*(X)*

*(XI)*

*(XII)*
Lessons from observations of exoplanet atmospheres

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Superrotation for strongly irradiated planets

- Presence of an eastward super-rotating equatorial jet
- Maximum of temperature shifted east to the substellar point

Thermal phase curve and temperature map of HD189733b (Knutson et al. 2007)

Zonal-mean zonal winds for HD189733b

Phase offset due to competition between the radiative cooling and the speed of the equatorial jet.

\[
\tau_{rad} = \frac{c_p P}{g \sigma T^3}
\]

\[
\tau_{adv} = \frac{2\pi R}{U}
\]
IV) Lessons from observations

Stratospheric thermal inversion for hot planets

**Predictions:** Stratospheric thermal inversion due TiO and VO opacity in visible

**Observations:** fewer planets (ultra-hot) show stratospheric thermal inversion than expected

**Possible explanations:**
- Cold trapping of TiO/VO on the nightside?
- High C/O?
- Photodissociation of TiO/VO by high stellar activity?
Lessons from observations of exoplanet atmospheres

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Most of exoplanets are cloudy/hazy

A continuum from cloudy to cloud-free planets

Condensate clouds
(thermodynamic phase change)

Haze
(non-equilibrium chemistry)

→ Flat transit spectrum
→ Mie-scattering slope

Sing et al. (2015)
### IV) Lessons from observations

Most of exoplanets are cloudy/hazy

**Clouds are everywhere**

<table>
<thead>
<tr>
<th>Planet</th>
<th>Cloud Ingredients</th>
<th>Haze Ingredients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Venus</td>
<td>$\text{H}_2\text{SO}_4$</td>
<td>$\text{H}_2\text{SO}_4$ and other heavier products</td>
</tr>
<tr>
<td></td>
<td></td>
<td>like $S_8$ (?)</td>
</tr>
<tr>
<td>Earth</td>
<td>$\text{H}_2\text{O}$</td>
<td>$\text{Smog}$</td>
</tr>
<tr>
<td>Mars</td>
<td>$\text{H}_2\text{O}$, $\text{CO}_2$</td>
<td>$\text{No haze}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(but lots of dust)</td>
</tr>
<tr>
<td>Saturn</td>
<td>$\text{H}_2\text{O}$, $\text{NH}_3$, $\text{NH}_4\text{SH}$</td>
<td>$\text{Forms from}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{NH}_3$, $\text{CH}_4$, $\text{H}_2\text{S}$, etc.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{photochemistry}$</td>
</tr>
<tr>
<td>Jupiter</td>
<td>$\text{CH}_4$, $\text{HCN}$, $\text{C}_4\text{N}_2$, $\text{C}_2\text{H}_6$, other organics</td>
<td>$\text{Forms from}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{CH}_4$, $\text{N}_2$, $\text{CO}$, etc.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{photochemistry}$</td>
</tr>
<tr>
<td>Titan</td>
<td>$\text{H}_2\text{O}$, $\text{NH}_3$, $\text{NH}_4\text{SH}$</td>
<td>$\text{Forms from}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{NH}_3$, $\text{CH}_4$, $\text{H}_2\text{S}$, etc.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{photochemistry}$</td>
</tr>
<tr>
<td>Uranus</td>
<td>$\text{N}_2$</td>
<td>$\text{Forms from}$</td>
</tr>
<tr>
<td>Neptune</td>
<td></td>
<td>$\text{CH}_4$, $\text{N}_2$, $\text{CO}$, etc.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{photochemistry}$</td>
</tr>
<tr>
<td>Triton</td>
<td>$\text{N}_2$</td>
<td></td>
</tr>
<tr>
<td>Pluto</td>
<td>$\text{N}_2$</td>
<td></td>
</tr>
<tr>
<td>Exoplanets</td>
<td>$\text{CH}_4$, $\text{NH}_3$, $\text{H}_2\text{O}$, alkali metals, iron, silicates, other, etc.</td>
<td>$\text{Yes.}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>All the possible kinds</td>
</tr>
</tbody>
</table>

**Figure from Sarah Hörst**

IV) Lessons from observations

Inhomogeneous cloud distribution

→ Evaporation at hot spot
(Demory et al. 2013, Parmentier et al. 2016)
→ Probably thick clouds on nightside
(Keating et al. 2019)

Cloud mapping of brown dwarf

Crossfield et al. (2014)
Lessons from observations of exoplanet atmospheres

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IV) Lessons from observations

Chemical disequilibrium (*See Olivia’s course*)

Deviation from chemical equilibrium produced by mixing or photochemistry

Emission spectrum of HR8799e by VLT-GRAVITY

*Ex:* CO-CH4 conversion in young giant planets

CO + 3H$_3$ = CH$_4$ + H$_2$O

CO and CH4 abundances are quenched by vertical mixing

*Lacour et al. (2019)*
IV) Lessons from observations

Low-mass planets seem to have high-mean molecular weight

GJ1214b

K2-18b

Kreidberg et al. (2014)

Tsiaras et al. (2019)

Flat transit spectrum:
→ high mean molecular weight (i.e. high metallicity) + clouds
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IV) Lessons from observations

Atmospheric escape for strongly irradiated planets

- Hydrodynamic escape by strong EUV stellar flux
- Comet-like H cloud

Vidal-Madjar et al. (2003)

Ehrenreich et al. (2015)
Futur telescopes for the characterization of exoplanetary atmospheres
Futur telescopes for the characterization of exoplanetary atmospheres

Futur NASA Great Observatory (2035-2040)
HABEX

Telescope aperture diameter 4 m

Starshade diameter 52 m

Inner working angle (IWA)

75,600 km separation

Flux ratio to host star

Planet-to-Star Flux Ratio ($\times 10^{10}$)

Separation [arcsec]

Wavelength (\(\mu\)m)

Water

Oxygen

Ozone

Instrument curves are post-processed detection limits.
https://psg.gsfc.nasa.gov/

Exemple: GJ1214b with pure H₂O and HST as Kreidberg et al. 2014
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I) Radiative transfer

Definition intensity and flux

**Intensity** $I$ = amount of energy passing through a surface area $dS$, within a solid angle $d\Omega$, per frequency interval $dv$, per unit time ($I$ in J m$^{-2}$ sr$^{-1}$ Hz$^{-1}$):

$$dE = I(x, \hat{n}, \nu, t) \hat{n} \cdot \vec{k} \, d\Omega \, dS \, dv \, dt$$

**Moments:**

Mean intensity:

$$J = \int_{\Omega} I(x, \hat{n}, \nu, t) \, d\Omega$$

Flux:

$$F = \int_{\Omega} I(x, \hat{n}, \nu, t) \hat{n} \cdot \vec{k} \, d\Omega = \iint I(x, \theta, \varphi, \nu, t) \cos(\theta) \sin(\theta) \, d\theta \, d\varphi$$
I) Radiative transfer

Definition intensity and flux

Blackbody radiation:

\[ B(T, \nu) = \frac{2hv}{c^2} \frac{1}{e^{hv/kT} - 1} \]

\[ B(T, \lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \]

Flux from one hemisphere (isotropic radiation):

\[ F_s(T, \nu) = \pi B(T, \nu) \]

Total flux from one hemisphere (Stefan–Boltzmann law):

\[ F_s(T) = \sigma T^4, \sigma = 5.67 \times 10^{-8} \text{ J K}^{-4} \text{ m}^{-2} \text{ s}^{-1} \]

Brightness temperature:

\[ T_b = \frac{hv}{k} \frac{1}{\ln \left( 1 + \frac{2\pi hv}{c^2 F_s} \right)} \]

with \( F_{obs} = F_s \left( \frac{R_p}{\text{Dist}} \right)^2 \)
I) Radiative transfer

Radiative transfer equation for plane-parallel

Optical depth & extinction coefficient:

\[ d\tau = -k(T, P, \nu) \mu \, ds \]

\[ k(T, P, \nu) = \sum_i n_i (\sigma_{i,abs} + \sigma_{i,scat}) \]

Optical mean free path: \( l = \frac{1}{k} \)

Radiative transfer equation:

\[ \mu \frac{dI}{d\tau} = I - S \]

Local thermodynamic Equilibrium (LTE):

\[ T_{\text{radiation}} = T_{\text{kinetics}} \]

Condition: mean free path of photons \( \ll \) length scale of \( T \) variations (for non-LTE see Pierre’s talk)
I) Radiative transfer

Source function

$$S_v(\mu, \phi) = (1 - \omega_0) B(T, v) + \frac{\omega_0}{4} \iint P(\mu, \mu', \phi, \phi') I(v, \mu', \phi') d\mu' d\phi'$$

Thermal emission

$$\omega_0: \text{single scattering albedo} = \frac{k_{scat}}{k_{scat} + k_{abs}}$$

Scattering

$$P = \text{scattering phase function}$$

$$\frac{1}{4\pi} \int_{\Omega} P(\Theta) d\Omega = 1$$

Rayleigh scattering: $$P(\Theta) = \frac{3}{4} (1 + \cos^2 \Theta)$$

$$g: \text{asymmetry factor} = \frac{1}{4\pi} \int_{\Omega} \cos \Theta P(\Theta) d\Omega , -1 \leq g \leq 1$$

$$g = 0 \text{ for isotropic or symmetric scattering (e.g. Rayleigh scattering)}$$
I) Radiative transfer

The two-stream approximation

\[
\mu \frac{dI}{d\tau} = I - S
\]

Goal: to compute the total upward and downward flux

\[
J_{\uparrow} = \int_0^{2\pi} \int_0^1 I \, d\mu \, d\phi,
\]
\[
J_{\downarrow} = \int_0^{2\pi} \int_{-1}^0 I \, d\mu \, d\phi,
\]
\[
F_{\uparrow} = \int_0^{2\pi} \int_0^1 \mu I \, d\mu \, d\phi,
\]
\[
F_{\downarrow} = \int_0^{2\pi} \int_{-1}^0 \mu I \, d\mu \, d\phi,
\]

Case of stellar radiation with no scattering:

\[
F_{\uparrow} = 0
\]
\[
F_{\downarrow} = F_s e^{-\tau/\mu_*}
\]

\(\mu_*\) is related to the angle of stellar irradiation. For 1D, we use a mean value, generally \(\mu_* = 1/\sqrt{3}\) or \(\cos(60^\circ)\)
I) Radiative transfer

The two-stream approximation

\[
\mu \frac{dI}{d\tau} = I - S
\]

Goal: to compute the total upward and downward flux

\[
J^\uparrow = \int_0^{2\pi} \int_0^1 I \, d\mu \, d\phi,
\]

\[
J^\downarrow = \int_0^{2\pi} \int_{-1}^0 I \, d\mu \, d\phi,
\]

\[
F^\uparrow = \int_0^{2\pi} \int_0^1 \mu I \, d\mu \, d\phi,
\]

\[
F^\downarrow = \int_0^{2\pi} \int_{-1}^0 \mu I \, d\mu \, d\phi,
\]

Case of a purely emitting atmosphere:

\[
\frac{\partial F^\uparrow}{\partial \tau} = J^\uparrow - 2\pi B
\]

\[
\frac{\partial F^\downarrow}{\partial \tau} = -J^\downarrow + 2\pi B
\]

The two-stream solution consists in approximating \( I \) so that it is related to \( F \).

We assume \( \frac{F^\uparrow}{J^\uparrow} = \frac{F^\downarrow}{J^\downarrow} = \frac{1}{\gamma} \) (generally \( \gamma = \sqrt{3} \))

\[
\frac{\partial F^\uparrow}{\partial \tau} = \gamma F^\uparrow - 2\pi B
\]

\[
\frac{\partial F^\downarrow}{\partial \tau} = -\gamma F^\downarrow + 2\pi B
\]
I) Radiative transfer

The two-stream approximation

Case of a purely emitting atmosphere:

\[
\begin{align*}
\frac{\partial F^\uparrow}{\partial \tau} &= \gamma F^\uparrow - 2\pi B \\
\frac{\partial F^\downarrow}{\partial \tau} &= -\gamma F^\downarrow + 2\pi B 
\end{align*}
\]

Resolution:

\[
\begin{align*}
F^\uparrow(\tau) &= F_{surf}^\uparrow e^{-\gamma(\tau_0 - \tau)} + \int_\tau^{\tau_0} 2\pi B e^{-\gamma(\tau' - \tau)} \, d\tau' \\
F^\downarrow(\tau) &= F^\downarrow(\tau = 0) e^{-\gamma\tau} + \int_0^\tau 2\pi B e^{-\gamma(\tau - \tau')} \, d\tau' 
\end{align*}
\]

Outgoing radiation:

\[
OLR = \int_0^\infty \left[ F_{surf}^\uparrow e^{-\gamma\tau_0} + \int_0^{\tau_0} 2\pi B e^{-\gamma\tau} \, d\tau \right] \, dv 
\]
I) Radiative transfer

The two-stream approximation

Case of a purely emitting atmosphere:

Outgoing radiation:

\[ OLR = F^\uparrow(\tau_0)e^{-\gamma\tau_0} + \int_0^{\tau_0} 2\pi Be^{-\gamma\tau} d\tau \]

Contribution function:

\[ cf(P) = B(v, T) \frac{de^{-\gamma\tau}}{d\log(P)} \]

Peak of contribution:

at \( \tau \sim 2/3 \) also called the photosphere

Fortney 2018
I) Radiative transfer

The two-stream approximation

Case of a purely emitting atmosphere:

\[
\frac{\partial F^\uparrow}{\partial \tau} = \gamma F^\uparrow - 2\pi B \\
\frac{\partial F^\downarrow}{\partial \tau} = -\gamma F^\downarrow + 2\pi B
\]

With \(\gamma = \sqrt{3}\) and \(d\tau = k_\nu \, dz\):

Radiative equilibrium:

\[
\frac{\partial F_\nu}{\partial \tau} = \frac{\partial (F^\uparrow - F^\downarrow)}{\partial \tau} = 0
\]

\[\Rightarrow \gamma F^\uparrow + \gamma F^\downarrow = 4\pi B\]

\[\Rightarrow F_\nu(\tau) = \frac{4\pi \, dB}{\gamma^2 \, d\tau}\]

\[\Rightarrow F_\nu(z) = -\frac{4\pi \, dB \, dT}{3k_\nu \, dT \, dz}\]

Total flux:

\[F(z) = \int_0^\infty F_\nu(z) \, dv = -\frac{4\pi \, dT}{3} \int_0^\infty \frac{1}{k_\nu} \, dB \, d\nu\]

Diffusive form:

\[F(z) = -\frac{16 \, \sigma T^3 \, dT}{3 \, k_R \, dz} = -D_R \frac{dT}{dz}\]

\[\frac{1}{k_R} = \frac{\int_0^\infty \left(\frac{1}{k_\nu}\right) \left(\frac{dB_\nu}{dT}\right) \, dv}{\int_0^\infty \left(\frac{dB_\nu}{dT}\right) \, dv}\]

\(k_R\) is the Rosseland opacity
I) Radiative transfer

The two-stream approximation

$$\mu \frac{dI}{d\tau} = I - S$$

General case for thermal emission with scattering

$$\frac{\partial F^\uparrow}{\partial \tau} = \gamma_1 F^\uparrow - \gamma_2 F^\downarrow - 2\pi (1 - \omega_0)B$$

$$\frac{\partial F^\downarrow}{\partial \tau} = \gamma_2 F^\uparrow - \gamma_1 F^\downarrow + 2\pi (1 - \omega_0)B$$

<table>
<thead>
<tr>
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<th>$\mu_*$</th>
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<td>$-[1 - \omega_0(4 - 3g)]/4$</td>
<td>1/2</td>
</tr>
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<td>$\sqrt{3}\omega_0(1 + g)/2$</td>
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<td>$2 - \omega_0(1 + g)$</td>
<td>$\omega_0(1 - g)$</td>
<td>1/2</td>
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Quadrature for deep atmosphere & Hemispheric mean for the upper atmosphere

See Toon et al. (1989) for the complete solution with multi-layers
I) Radiative transfer

The two-stream approximation

\[ \mu \frac{dI}{d\tau} = I - S \]

General case for thermal emission with scattering

\[ \frac{\partial F^\uparrow}{\partial \tau} = \gamma_1 F^\uparrow - \gamma_2 F^\downarrow - 2\pi (1 - \omega_0)B \]

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Quadrature for deep atmosphere & Hemispheric mean for the upper atmosphere

See Toon et al. (1989) for the complete solution with multi-layers
I) Radiative transfer

Methods for solving RT

1) Semi-grey analytical model

Optically thin ($\tau < 1$)
Planck mean opacity

$$k_p(T, P, \nu) = \frac{\int_0^\infty k_\nu(T, P, \nu)B_\nu d\nu}{\int_0^\infty B_\nu(T, \nu)d\nu}$$

Optically thick ($\tau > 1$)
Rosseland mean opacity

$$\frac{1}{k_R} = \frac{\int_0^\infty \frac{1}{k_\nu(T, P, \nu)}\frac{dB_\nu}{dT}d\nu}{\int_0^\infty \frac{dB_\nu}{dT}d\nu}$$

with $k_1 \gg k_2$

$k_p = k_1(\Delta \nu - \delta \nu_2)/\Delta \nu$

$k_R = k_2\delta \nu_2 / \Delta \nu$

Transmittance of a layer $\Delta z$:

$$T = \int e^{-k\Delta z}d\nu$$

If $k_1\Delta z \gg 1$ & $k_2\Delta z \ll 1$:

$$T \approx k_2\delta \nu_2\Delta z / \Delta \nu = k_R \Delta z$$
I) Radiative transfer

Methods for solving RT

1) Semi-grey analytical model

Optically thin ($\tau < 1$)
Planck mean opacity

$$k_p(T, P, \nu) = \frac{\int_0^\infty k_v(T, P, \nu)B_\nu d\nu}{\int_0^\infty B_\nu(T, \nu) d\nu}$$

Optically thick ($\tau > 1$)
Rosseland mean opacity

$$\frac{1}{k_R} = \frac{\int_0^\infty \frac{1}{k_v(T, P, \nu)} dB_\nu(T) d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}$$

Mean opacity for H\textsubscript{2}-dominated atmosphere:

Table in Freedman et al. (2008)
I) Radiative transfer

Methods for solving RT

1) Semi-grey analytical model

Optically thin ($\tau < 1$)  
Planck mean opacity

$$k_p(T, P, \nu) = \frac{\int_0^\infty k_v(T, P, \nu)B_v dv}{\int_0^\infty B_v(T, \nu)dv}$$

Optically thick ($\tau > 1$)  
Rosseland mean opacity

$$\frac{1}{k_R} = \frac{\int_0^\infty \frac{1}{k_v(T, P, \nu)}dB_v dv}{\int_0^\infty \frac{dB_v}{dT}dv}$$

Mean opacity for H$_2$-dominated atmosphere:

Parameterization in Freedman et al. (2018)
I) Radiative transfer

Methods for solving RT

1) Semi-grey analytical model

Only for computing the thermal structure (e.g. for retrieval or thermal evolution)

- Model of Guillot et al. (2010):
  Two parameters ($k_{vis}$ and $k_{ir}$) for visible (stellar) and infrared (planetary) radiation

- Models with sub-bands:
  e.g. Parmentier et al. (2014) and Robinson & Catling (2012):
  One parameter for visible ($k_{vis}$) and three parameters for infrared ($k_{ir1}$, $k_{ir2}$, $\beta = \frac{\delta \nu_2}{\Delta \nu}$)
I) Radiative transfer

Methods for solving RT

2) Correlated-k method

\[ T = \int_{v}^{v+\Delta v} \exp[-k(v)\Delta z] \frac{dv}{\Delta v} \]

Going from frequency space to g-space, where \( g \) is the cumulative opacity distribution function: \( dg = f(k)dk \)

\[ T \approx \sum_{i}^{N_{a}} \exp[-ki\Delta z] \Delta g_{i} \]

Fast method, excellent for low and medium resolution
Widely used for atmospheric models and 3D GCM
I) Radiative transfer

Methods for solving RT

2) Correlated-k method

Possibility to combine multiple species

\[ T = \int_{v}^{v+\Delta v} \exp[-X_1k_1(v) + X_2k_2(v)\Delta z] \frac{dv}{\Delta v} \]

We assume that \( k_1 \) and \( k_2 \) are uncorrelated

\[ T = \left[ \int_{v}^{v+\Delta v} e^{-X_1k_1(v)\Delta z} \frac{dv}{\Delta v} \right] \left[ \int_{v}^{v+\Delta v} e^{-X_2k_2(v)\Delta z} \frac{dv}{\Delta v} \right] \]

Going from frequency space to g-space:

\[ T \approx \sum_{i}^{N_g} \sum_{j}^{N_g} \exp[-X_1k_{1i} + X_2k_{2j}\Delta z] \Delta g_i \Delta g_j \]

Equivalent to a single gas with: \( k_{ij} = X_1k_{1i} + X_2k_{2j} \) and \( \Delta g_{ij} = \Delta g_i \Delta g_j \)

→ ordering of increasing \( k_{ij} \) → interpolate on g-space → iterate with another specie
I) Radiative transfer

Methods for solving RT

3) Line-by-line models

For computing accurate transmittance & spectra at medium/high resolution

- Ex: LBLRTM
  [http://rtweb.aer.com/lblrtm_frame.html](http://rtweb.aer.com/lblrtm_frame.html)

Earth atmospheric transmittance at Mauna Kea & Dome C (computed with LBLRTM)

*Burton et al. (2004)*
I) Radiative transfer

A first look at the greenhouse effect

Outgoing radiation:

\[
OLR = \int_0^\infty \left[ F_{surf} e^{-\gamma_0} + \int_0^{\tau_0} 2\pi B e^{-\gamma_\tau} d\tau \right] d\nu
\]

Hanel et al. (1972)
I) Radiative transfer

A first look at the greenhouse effect

The efficiency of a greenhouse gas is related to how much it reduces spectral windows

**Question:** What is the strongest greenhouse gas between CO₂ and CH₄?

1) For current Earth’s atmosphere
2) For a pure N₂ atmosphere
I) Radiative transfer

A first look at the greenhouse effect

The efficiency of a greenhouse gas is related to how much it reduces spectral windows

Question: What is the strongest greenhouse gas between CO\textsubscript{2} and CH\textsubscript{4}? 
1) For current Earth's atmosphere $\rightarrow$ CH\textsubscript{4} $\approx$ 20$\times$CO\textsubscript{2} 
2) For a pure N\textsubscript{2} atmosphere $\rightarrow$ CO\textsubscript{2} $\approx$ 6$\times$CH\textsubscript{4}
I) Radiative transfer

A first look at the greenhouse effect

Radiative forcing of a greenhouse gas:

\[ \Delta F = ASR - OLR(C) = OLR(C_0) - OLR(C) \]

\[ \Delta F_{CO_2} = 5.35 \times \ln \left( \frac{C}{C_0} \right) \] (CO\(_2\) concentration \(C\) in ppm)

\[ \Delta F_{CH_4} = 0.036 \times (\sqrt{C} - \sqrt{C_0}) \] (CH\(_4\) concentration \(C\) in ppb)  \(\textit{Mhyre et al.} (1998)\)

Climate sensitivity: \( S = \Delta T \) for 2\(\times\)CO\(_2\)

IPPC report: \( S = 1.5 - 4.5 \) K  \(\rightarrow\) \[ \frac{S}{\Delta F_{2\times CO_2}} \approx 0.8 \) K W\(^{-1}\) m\(^2\)
I) Radiative transfer

A first look at the greenhouse effect

Radiative forcing of a greenhouse gas:

\[
\Delta F = ASR - OLR(C) = OLR(C_0) - OLR(C)
\]

\[
\Delta F_{CO_2} = 5.35 \times \ln \left( \frac{C}{C_0} \right) \quad \text{(CO}_2 \text{ concentration } C \text{ in ppm)}
\]

Mhyre et al. (1998)

Width of the optically thick band \( \propto \ln(C) \)
II) Thermal structure

Skin temperature (radiative)

Stratosphere (radiative)

Troposphere (convective)

Marley & Robinson (2014)
II) Thermal structure

Resolution of the two-stream for semi-grey case with no scattering

1) Grey atmosphere heated from below

Thermal emission
\[
\frac{\partial F^\uparrow}{\partial \tau} = \gamma F^\uparrow - 2\pi B \\
\frac{\partial F^\downarrow}{\partial \tau} = -\gamma F^\downarrow + 2\pi B
\]

- Internal flux: \( F_{int} = \sigma T_{int}^4 = \sigma T_{eff}^4 \)
- We choose \( \gamma = \sqrt{3} \) in the deep atmosphere
- \( T(\tau = 0) = T_{skin} = 2^{-1/4} T_{int} \)

Heating:
\[
\rho c_v \frac{\partial T}{\partial t} = -\frac{dF}{dz} = 0 \text{ at equilibrium}
\]

Beautiful excercise: show that
\[
T^4 = \frac{3}{4} T_{int}^4 \left( \tau + \frac{2}{3} \right)
\]
II) Thermal structure

Resolution of the two-stream for semi-grey case with no scattering

1) Grey atmosphere heated from below

Thermal emission

\[
\begin{align*}
\frac{\partial F^\uparrow}{\partial \tau} &= \gamma F^\uparrow - 2\pi B \\
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\]

- Internal flux: \( F_{int} = \sigma T_{int}^4 = \sigma T_{eff}^4 \)
- We choose \( \gamma = \sqrt{3} \) in the deep atmosphere
- \( T(\tau = 0) = T_{skin} = 2^{-1/4}T_{int} \)

\[
(1)-(2) \Rightarrow \frac{\partial F}{\partial \tau} = \gamma (F^\uparrow + F^\downarrow) - 4\pi B
\]

\[
\int d\nu \rightarrow \frac{\partial F_{tot}}{\partial \tau} = \gamma (F_{tot}^\uparrow + F_{tot}^\downarrow) - 4\sigma T^4 = 0
\]

Derivative \(
3(F_{tot}^\uparrow - F_{tot}^\downarrow) - 4\sigma \frac{\partial T^4}{\partial \tau} = 0
\)

\( \rightarrow \quad 3T_{int}^4 - 4 \frac{\partial T^4}{\partial \tau} = 0 \)

Integration \( T^4 = \frac{3}{4} T_{int}^4 \left( \tau + \frac{2}{3} \right) \)

At the ground: \( T_g^4 - T_{int}^4(\tau_g + 2) = T_{int}^4 \)

\( T_g^4 = \frac{3}{4} T_{int}^4(\tau + 2) \)
II) Thermal structure

Resolution of the two-stream for semi-grey case with no scattering

2) Grey atmosphere heated from above and below

Model of Guillot et al. (2010)

- Internal flux: $F_{\text{int}} = \sigma T_{\text{int}}^4$
- Stellar flux: $F_{\text{ext}} = \sigma T_{\text{irr}}^4$
- Effective temperature: $T_{\text{eff}}^4 = f T_{\text{irr}}^4 + T_{\text{int}}^4$
- $\tau = \tau_{\text{ir}}$ ; $\gamma = k_{\text{vis}} / k_{\text{ir}}$

\[
T^4 = \frac{3}{4} T_{\text{int}}^4 \left( \tau + \frac{2}{3} \right) + \frac{3}{4} T_{\text{irr}}^4 f \left[ \frac{2}{3} + \frac{1}{\gamma \sqrt{3}} + \left( \frac{\gamma}{\sqrt{3}} - \frac{1}{\gamma \sqrt{3}} \right) e^{-\gamma \tau \sqrt{3}} \right]
\]

$f = 1$ at substellar point,
$f = 1/2$ for a day-side average
$f = 1/4$ for an average over the whole planet

\[
T_{\text{skin}}^4 = \frac{1}{2} T_{\text{int}}^4 + \frac{3}{4} T_{\text{irr}}^4 f \left[ \frac{2}{3} + \frac{\gamma}{\sqrt{3}} \right]
\]
II) Thermal structure

Resolution of the two-stream for semi-grey case with no scattering

2) Grey atmosphere heated from above and below

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- Internal flux: \( F_{\text{int}} = \sigma T_{\text{int}}^4 \)
- Stellar flux: \( F_{\text{ext}} = \sigma T_{\text{irr}}^4 \)
- Effective temperature: \( T_{\text{eff}}^4 = fT_{\text{irr}}^4 + T_{\text{int}}^4 \)
- \( \tau = \tau_{\text{ir}} ; \gamma = k_{\text{vis}}/k_{\text{ir}} \)

\[
T^4 = \frac{3}{4} T_{\text{int}}^4 \left( \tau + \frac{2}{3} \right) + \frac{3}{4} T_{\text{irr}}^4 f \left[ \frac{2}{3} + \frac{1}{\gamma \sqrt{3}} + \left( \frac{\gamma}{\sqrt{3}} - \frac{1}{\gamma \sqrt{3}} \right) e^{-\gamma \tau \sqrt{3}} \right]
\]

For \( \tau \gg 1 \):
\[
T_{\text{deep}}^4 = \frac{3}{4} \tau T_{\text{int}}^4 + \frac{3}{4} T_{\text{irr}}^4 f \left[ \frac{2}{3} + \frac{1}{\gamma \sqrt{3}} \right]
\]

For inflated hot Jupiters:
Heat transfer to the deep atmosphere (10^{-4}% - 1% \( F_{\text{ext}} \) into \( F_{\text{int}} \))

Huge change for the temperature in the deep atmosphere
II) Thermal structure

Resolution of the two-stream for semi-grey case with no scattering

2) Grey atmosphere heated from above and below

Model of Guillot et al. (2010)

- Internal flux: $F_{\text{int}} = \sigma T_{\text{int}}^4$
- Stellar flux: $F_{\text{ext}} = \sigma T_{\text{irr}}^4$
- Effective temperature: $T_{\text{eff}}^4 = f T_{\text{irr}}^4 + T_{\text{int}}^4$
- $\tau = \tau_{\text{irr}} ; \gamma = k_{\text{vis}} / k_{\text{ir}}$

T profiles for $\gamma =$0.01-100

Stratospheric thermal inversion for $\gamma > 1$

$$T_{\text{deep}}^4 = \frac{3}{4} T_{\text{irr}}^4 f \left[ \frac{2}{3} + \frac{1}{\gamma \sqrt{3}} \right]$$

Parmentier et al. (2014)
II) Thermal structure

Troposphere and convective instability

Adiabatic lapse rate

For an air parcel with no heat transfer:

\[
\Gamma_d = -\frac{dT}{dz} = \frac{g}{C_p}
\]

\[
\frac{d\ln T}{d\ln P} = \frac{R}{C_p} \leftrightarrow T = T_0 \left(\frac{P_0}{P}\right)^{R/C_p}
\]

Stability of an air parcel

\[\Gamma > \Gamma_d\] Unstable

\[\Gamma < \Gamma_d\] Stable
II) Thermal structure

Troposphere and convective instability

Question: For a grey atmosphere heated from below, where is the air unstable?
1) For \( \tau \propto P \) (constant absorption)
2) For \( \tau \propto P^2 \) (opacity controlled by pressure-broadening or CIA)

\[
T^4 = \frac{3}{4} T_{int}^4 \left( \tau + \frac{2}{3} \right)
\]

We assume \( \frac{R}{C_p} = \frac{2}{7} \) (e.g. \( \text{N}_2 \)) or \( \frac{R}{C_p} = \frac{2}{9} \) (e.g. \( \text{CO}_2 \))
II) Thermal structure

Troposphere and convective instability

**Question:** For a grey atmosphere heated from below, where is the air unstable?

1) For $\tau \propto P$ (constant absorption)
2) For $\tau \propto P^2$ (opacity controlled by pressure-broadening or CIA)

$$T^4 = \frac{3}{4} T_{int}^4 \left( \tau + \frac{2}{3} \right)$$

We assume $\frac{R}{C_p} = \frac{2}{7}$ (e.g. N$_2$) or $\frac{R}{C_p} = \frac{2}{9}$ (e.g. CO$_2$)

$$\frac{d \ln T}{d \ln P} = \frac{P}{4(\tau + \frac{2}{3})} \frac{d \tau}{d P}$$

1) $$\frac{d \ln T}{d \ln P} = \frac{\tau}{4(\tau + \frac{2}{3})} \Rightarrow \left[ \frac{d \ln T}{d \ln P} \right]_{(max)} = \frac{1}{4}$$

$\frac{d \ln T}{d \ln P} (max) < \frac{R}{C_p}$ for N$_2$

Always stable

$\frac{d \ln T}{d \ln P} (max) > \frac{R}{C_p}$ for CO$_2$

Potentially unstable

2) $$\frac{d \ln T}{d \ln P} = \frac{\tau}{2(\tau + \frac{2}{3})} \Rightarrow \left[ \frac{d \ln T}{d \ln P} \right]_{(max)} = \frac{1}{2}$$

$\frac{d \ln T}{d \ln P} (max) < \frac{R}{C_p}$ for N$_2$ & CO$_2$

Convection for $\tau = \frac{8}{9}$ (N$_2$) & $\frac{8}{15}$ (CO$_2$)

$\tau \approx 1$
II) Thermal structure

Tropopause & stratospheric thermal inversion

Robinson & Catling (2013)

No analytical general solution to the radiative-convective TP profile determination doing iterations

Tropopause generally at 0.1 bar
III) Clouds & Aerosols

Cloud impact on planetary atmospheres

- Atmospheric composition/chemistry
- Radiative transfert (scattering & absorption)
- Atmospheric dynamics
- Temperature and climate

Atmospheric retrieval
III) Clouds & Aerosols

Cloud impact on planetary atmospheres

- Atmospheric composition/chemistry
- Radiative transfert (scattering & absorption)
- Atmospheric dynamics
- Temperature and climate

Atmospheric retrieval

Clouds seen by astronomers

Clouds seen by atmospheric scientists
III) Clouds & Aerosols

Cloud impact on planetary atmospheres

- Atmospheric composition/chemistry
- Radiative transfert (scattering & absorption)
- Atmospheric dynamics
- Temperature and climate

Need for atmospheric models with clouds simulated properly and self-consistently
Phase diagram

Likely liquid water clouds on K2-18b

Benneke et al. 2019
III) Clouds & Aerosols

Phase diagram

Hum not so sure...

Transition gas $\rightarrow$ condensed phase
what matters is the partial pressure not the total pressure

Likely liquid water clouds on K2-18b !!!

\[
\text{1xsolar: } P_{\text{H}_2\text{O}} \approx 10^{-3} P_{\text{tot}} \\
100\text{xsolar: } P_{\text{H}_2\text{O}} \approx 0.1 P_{\text{tot}}
\]

No condensation or just water ice for these TP profiles!

Benneke et al. 2019
III) Clouds & Aerosols

Condensation curves

Temperature condensation curves for solar metallicity

Elemental abundances from Lodders et al. (2003)
Temperature condensation curves from Visscher et al. (2006, 2010)

Clausius-Clapeyron relation:

\[ P_{sat} = P_{sat}(T_0)e^{\frac{L}{R}(\frac{1}{T} - \frac{1}{T_0})} \]
III) Clouds & Aerosols

1D Cloud models

1) Model with $f_{sed}$ from Ackerman & Marley 2001

At equilibrium:

$$\frac{\partial q_c}{\partial z} = - \frac{\partial q_s}{\partial z} - \frac{V_{sed}}{K_{zz}} q_c$$

- $q_c =$ mass mixing ratio of condensate
- $q_s =$ mass mixing ratio of vapor at saturation
- $V_{sed}$ = sedimentation speed
- $K_{zz}$ = eddy diffusion coefficient

Mixing length theory:

$$K_{zz} = \frac{H}{3} \left( \frac{L}{H} \right)^{4/3} \left( \frac{r F_{conv}}{c_p \rho_a} \right)^{1/3}$$

* Ackerman & Marley 2001
  - Mixing length: $L=H$
  - $F_{conv} = \sigma T_{eff}^4$

Assumption: $f_{sed} = \frac{HV_{sed}^2}{K_{zz}} = \text{constant}$ (generally $f_{sed} =$ 1-5)

Above condensation:

$$q_c = q_{c0} \left( \frac{P}{P_0} \right)^{f_{sed}}$$
III) Clouds & Aerosols

1D Cloud models

2) Model with simple microphysics using timescales from Rossow 1978

\[ \tau_{\text{mixing}} = \frac{H^2}{K_{zz}} \]
\[ \tau_{\text{sed}} = \frac{H}{v_{\text{sed}}} \]
\[ \tau_{\text{cond}} = \frac{r^2}{(2 f \eta q_s S)} \rho_c \]
\[ \tau_{\text{coal}} = \frac{4r \rho_c}{3 \alpha v_{\text{sed}} \rho_{\text{air}} q_c} \]

\( S = 10^{-3} - 10^{-1} \)

E.g. BT-Settl (Allard et al. 2001) and Exo-REM (Charnay et al. 2018)
III) Clouds & Aerosols

1D Cloud models

3) Models with full microphysics

e.g. Drift-Phoenix (Woitke & Helling 2003)
III) Clouds & Aerosols

Opacity

We usually compute aerosol optical properties ($Q_{\text{ext}} = \sigma_{\text{ext}}/\pi r^2$, $\omega_0$, $g$) from Mie Theory with optical indexes and assuming spherical particules.

\[
\frac{Q_{\text{ext}}}{\omega_0} = \frac{4}{3} \left( \frac{r}{\lambda} \right)^2 w \rho \frac{1}{r_e}
\]

where:
- $w$: mass column
- $\rho$: volumic mass
- $r_e$: effective radius

Cloud optical depth: $\tau_c = \frac{3}{4} \frac{Q_{\text{ext}} W}{\rho r_e}$

Optical regime $(r \gg \lambda)$

Rayleigh regime $(r \ll \lambda)$

Mie regime

Hansen & Travis (1974)

Optical indexes (Kitzmann et al. 2017)
III) Clouds & Aerosols

Radiative effects

1) **Scattering of stellar radiation**
   - surface cooling by albedo effect

Cloud albedo for pure scattering cloud ($\omega_0 \rightarrow 1$) using the two-stream approximation:

$$A_c = \frac{\sqrt{3}(1 - gc)\tau_c}{2 + \sqrt{3}(1 - gc)\tau_c}$$

_Hansen & Lacis (1974)_

2) **Absorption of stellar flux**
   - local warming & surface cooling (anti-greenhouse effect, e.g. Titan’s haze)

3) **Absorption/emission of thermal radiation**
   - surface warming by greenhouse effect
   - Stronger effect for upper clouds (e.g. cirrus)
   - Same effect for back-scattering of thermal flux

On Earth, clouds globally have a net cooling effect
III) Clouds & Aerosols

Radiative effects: absorption/emission of thermal radiation

Emission spectra and brightness temperature (Teff=1300K, log(g)=5)

- Clouds produce a decrease of flux in spectral windows and an increase in spectral bands (greenhouse warming).
- With thick clouds, spectrum close to a blackbody

Charnay et al. (2018)
III) Clouds & Aerosols

LT transition for brown dwarfs

Color-magnitude diagram of brown dwarfs

L dwarfs
Silicate and iron clouds visible

T dwarfs
Silicate and iron clouds below photosphere

~1300 K
III) Clouds & Aerosols

LT transition for brown dwarfs
III) Clouds & Aerosols

LT transition for brown dwarfs

$T_{\text{eff}} = 700/900/1300/1600 \text{ K, } \log(g) = 5 \text{ (g in cm/s}^2\text{)}$

Cloud below photosphere for $T_{\text{eff}} < 1300 \text{ K}$

$\Rightarrow$ LT transition

Charnay et al. (2018)
What’s next

For strongly irradiated exoplanets, we need 3D GCM!
Current and future space telescopes for exoplanets

- **Spitzer**
  Photometry (3.6, 4.5 µm)

- **HST**
  Spectroscopy (1.1 - 1.7 µm)

- **Kepler**
  Photometry (0.4 - 0.9 µm)

- **TESS**
  Photometry (0.6 - 1 µm)

- **CHEOPS** (2019)
  Photometry (0.4 - 1 µm)

- **JWST** (2021)
  Spectroscopy (NIRISS, NIRSpect & MIRI)

- **PLATO** (2026)
  Photometry (0.5 - 1 µm)

- **ARIEL** (2028)
  Photometry (0.5 - 0.95 µm)
  Spectroscopy (0.95 - 7.95 µm)