Cosmic web in an alternative theory of gravity with Euclid

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INRIA
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An interdisciplinary project
(*Mathematics, Computer science, Astrophysics*)
in collaboration with: ***Yann Brenier*** *(ENS Paris-Saclay),* ***Clotilde Laigle*** *(IAP),*
Bruno Lévy *(INRIA) &* ***Roya Mohayaee*** *(IAP)*

Boldrini et al. 2023, in prep.

Credits: Illustris TNG

The nature of dark matter

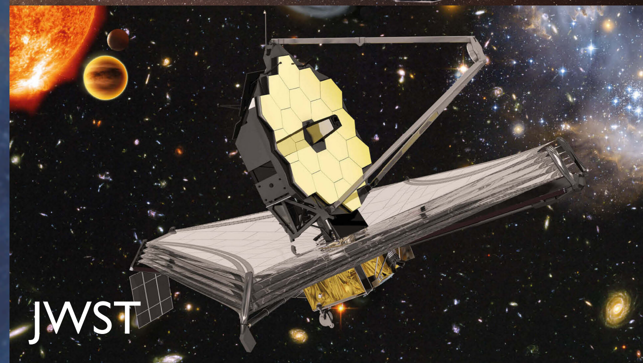
- Cold dark matter
- Warm dark matter
- Fuzzy dark matter
- Self-interacting dark matter
- Primordial black holes

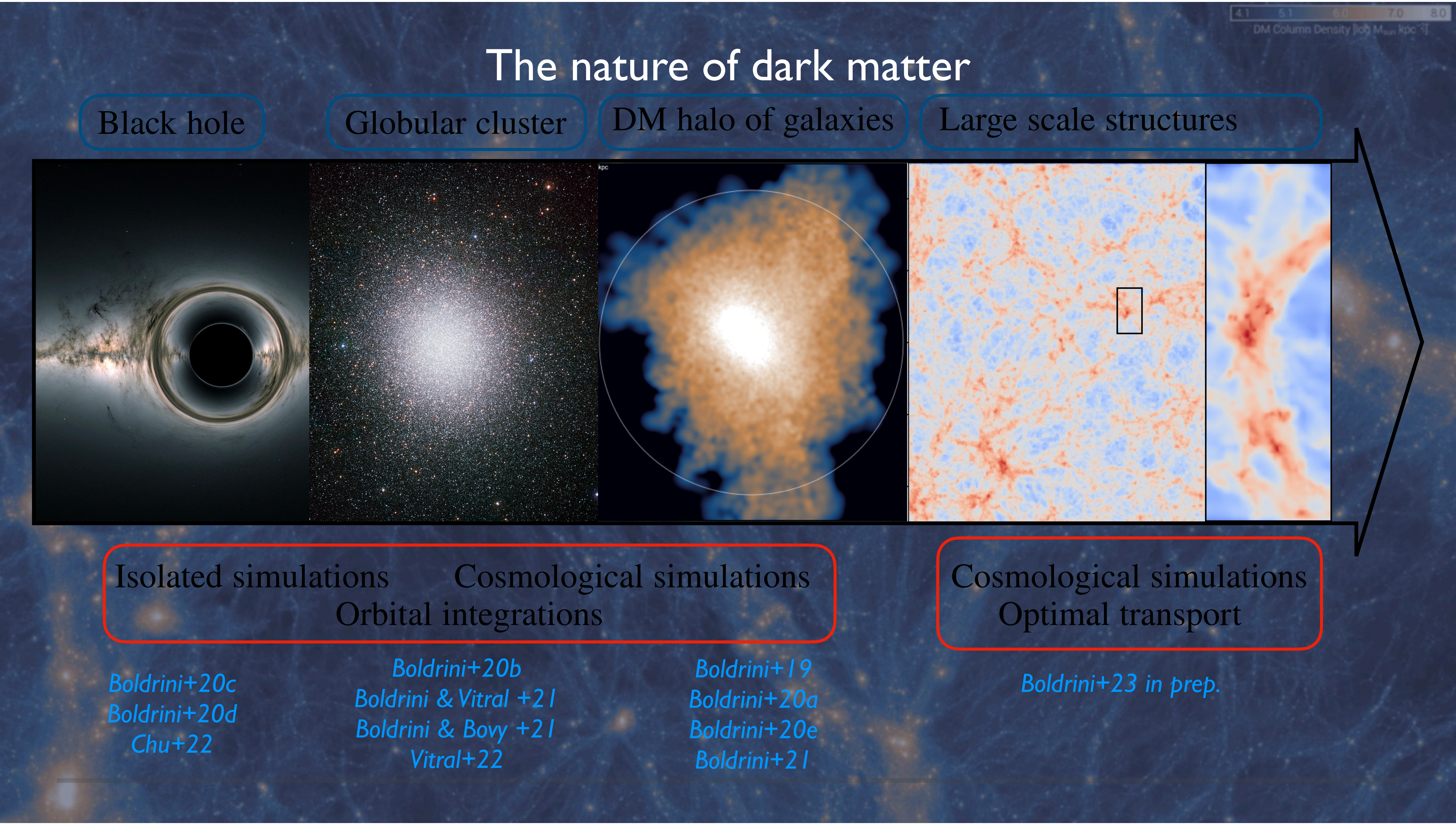
- Monge-Ampère gravity

Current cosmological model

Alternative dark matter theories

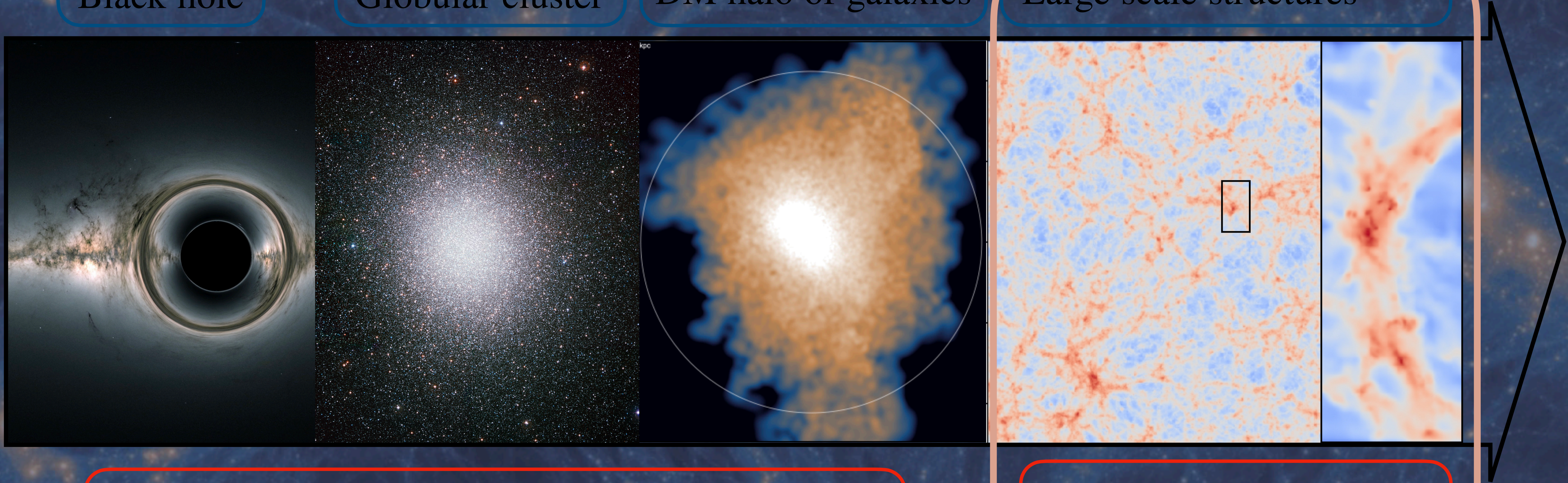
Alternative gravity theory




4.1 5.1 6.0 7.0 8.0
DM Column Density [log M_sun kpc^-2]
The nature of dark matter
Black hole
Globular cluster
DM halo of galaxies
Large scale structures
kpc
Isolated simulations
Cosmological simulations
Orbital integrations
Cosmological simulations
Optimal transport
Boldrini+20c
Boldrini+20d
Chu+22
Boldrini+20b
Boldrini & Vitral +21
Boldrini & Bovy +21
Vitral+22
Boldrini+19
Boldrini+20a
Boldrini+20e
Boldrini+21
Boldrini+23 in prep.

The nature of dark matter

Black hole | Globular cluster | DM halo of galaxies | Large scale structures



Isolated simulations   Cosmological simulations
Orbital integrations

Cosmological simulations
Optimal transport

Boldrini+20c
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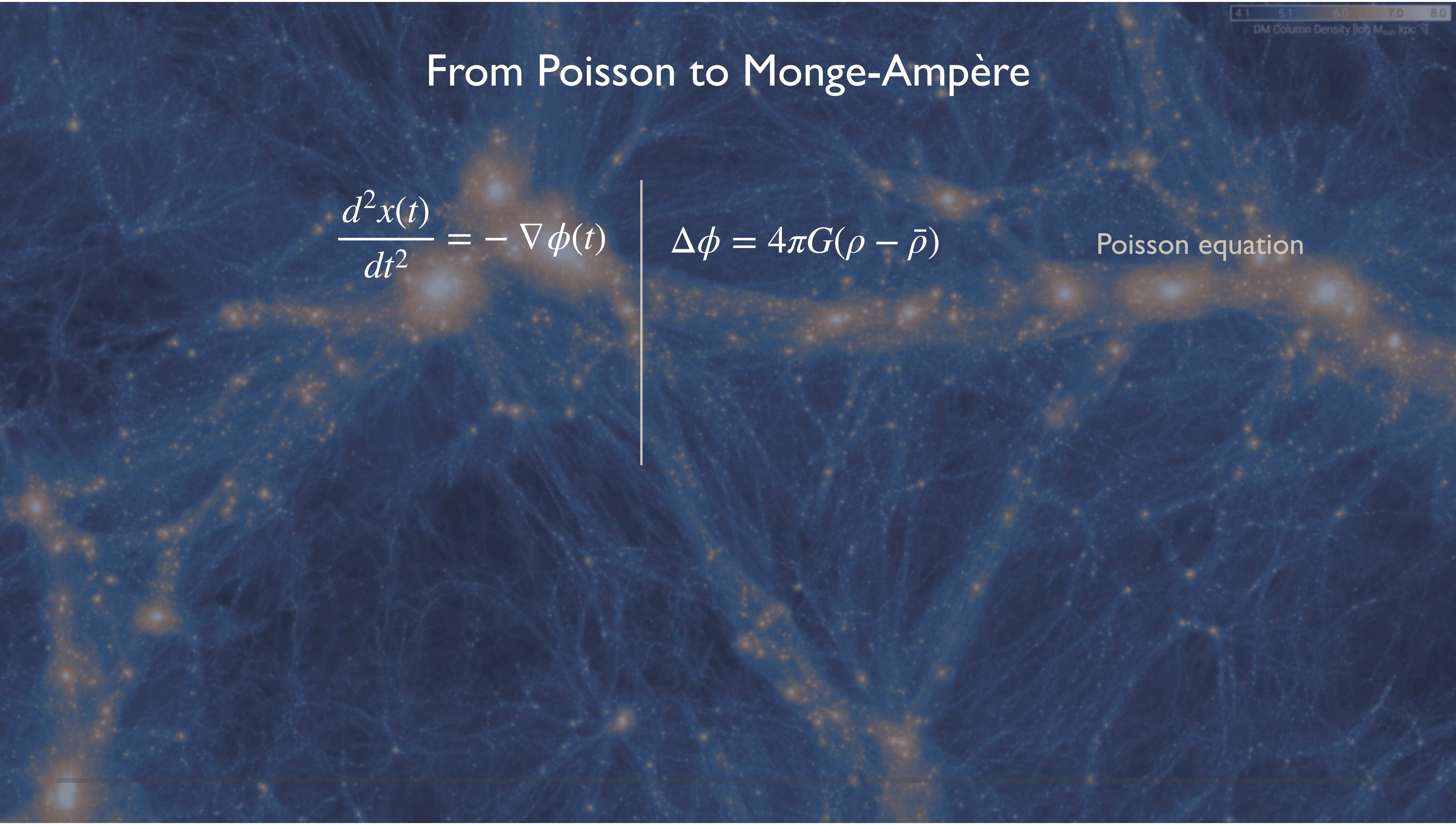
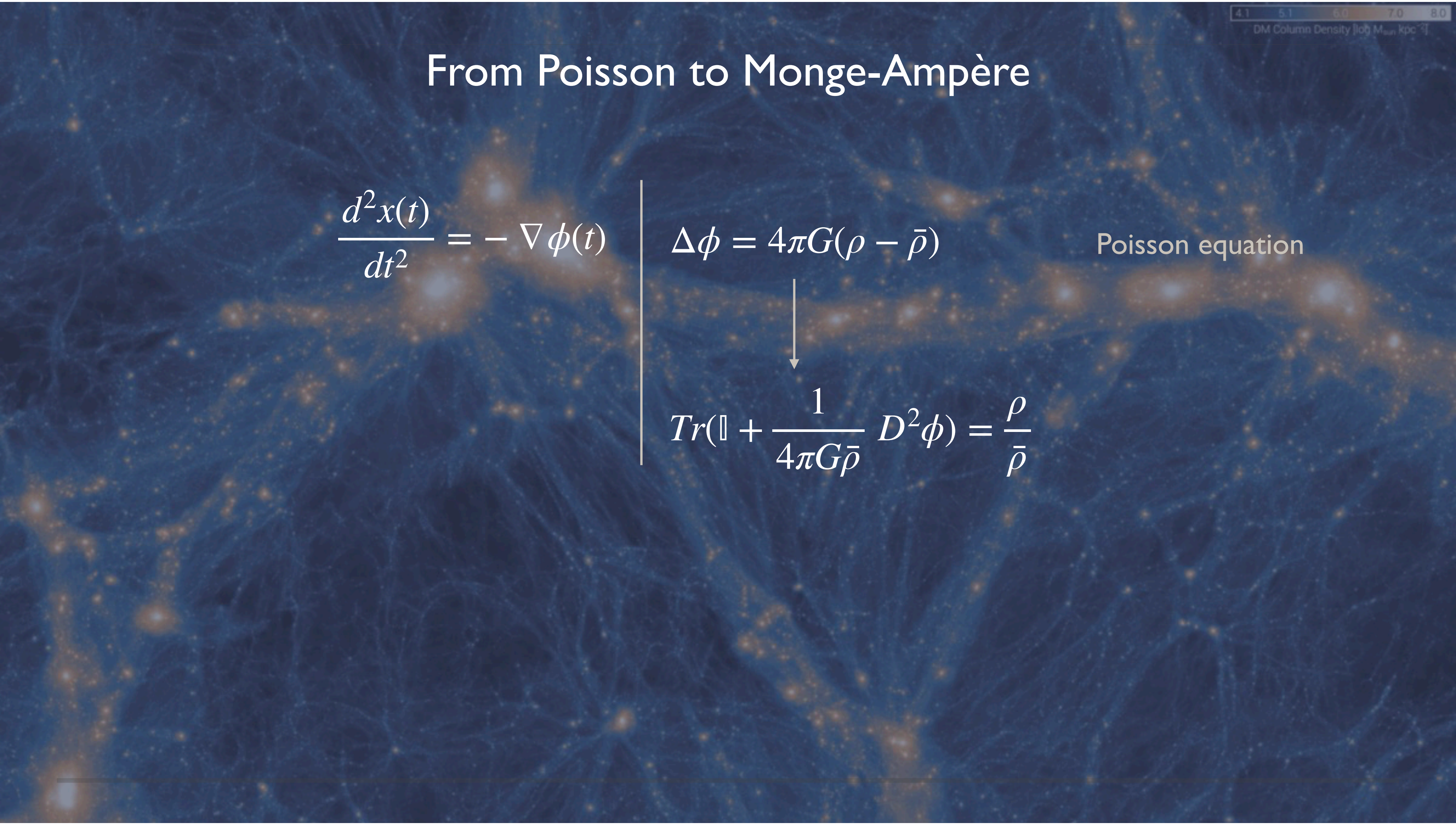
From Poisson to Monge-Ampère



$$\frac{d^2x(t)}{dt^2} = -\nabla\phi(t)$$

$$\Delta\phi = 4\pi G(\rho - \bar{\rho})$$

Poisson equation



From Poisson to Monge-Ampère

$$\frac{d^2x(t)}{dt^2} = -\nabla\phi(t)$$

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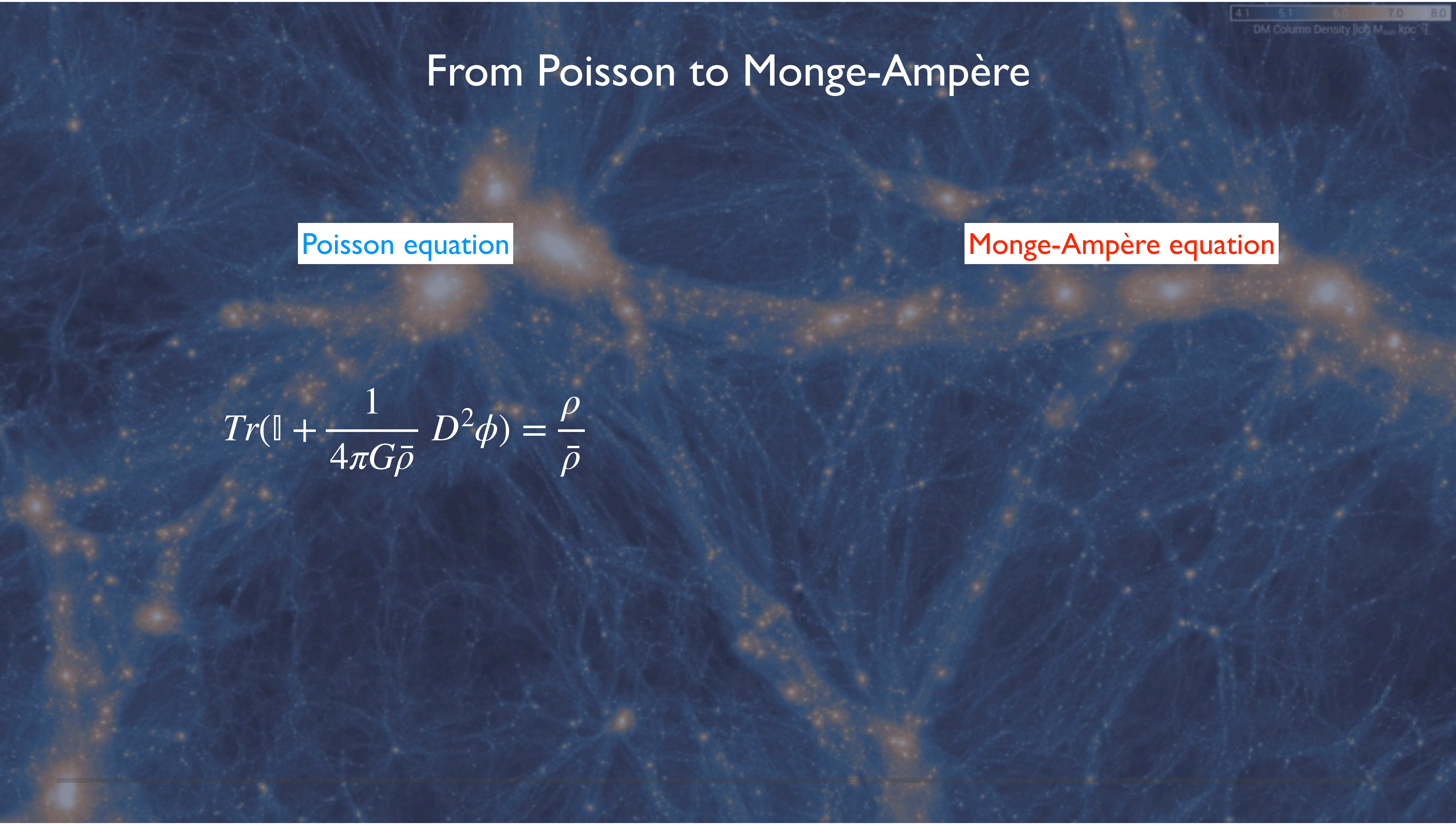
Poisson equation

$$Tr(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$



From Poisson to Monge-Ampère

Poisson equation

Monge-Ampère equation

$$Tr(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$

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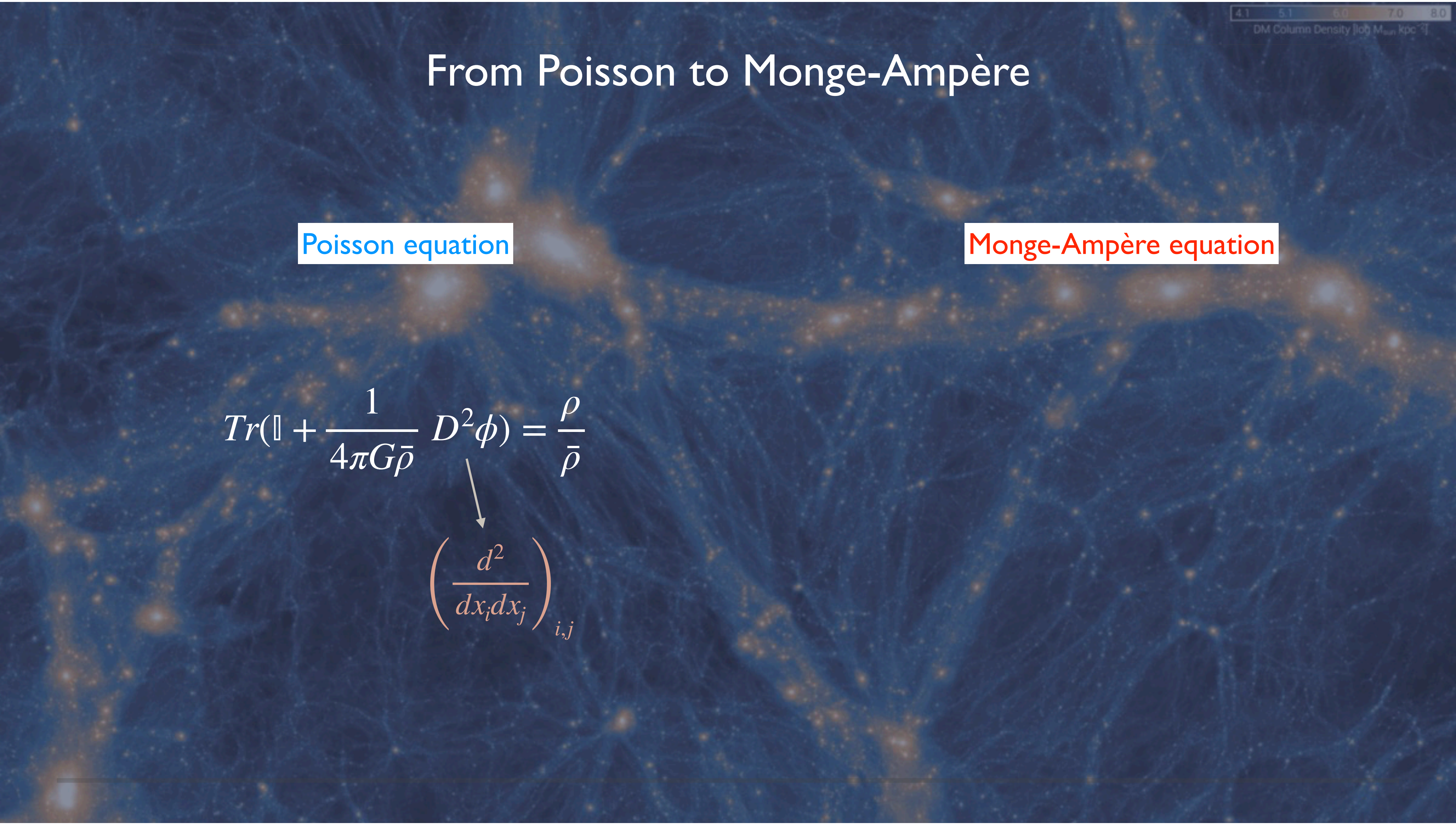


Poisson equation

Monge-Ampère equation

$$Tr(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$

$$\left(\frac{d^2}{dx_i dx_j}\right)_{i,j}$$

From Poisson to Monge-Ampère

Poisson equation

$$Tr(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$

→

Monge-Ampère equation

$$det(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$

From Poisson to Monge-Ampère

Poisson equation

$$Tr(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$

$\longrightarrow$

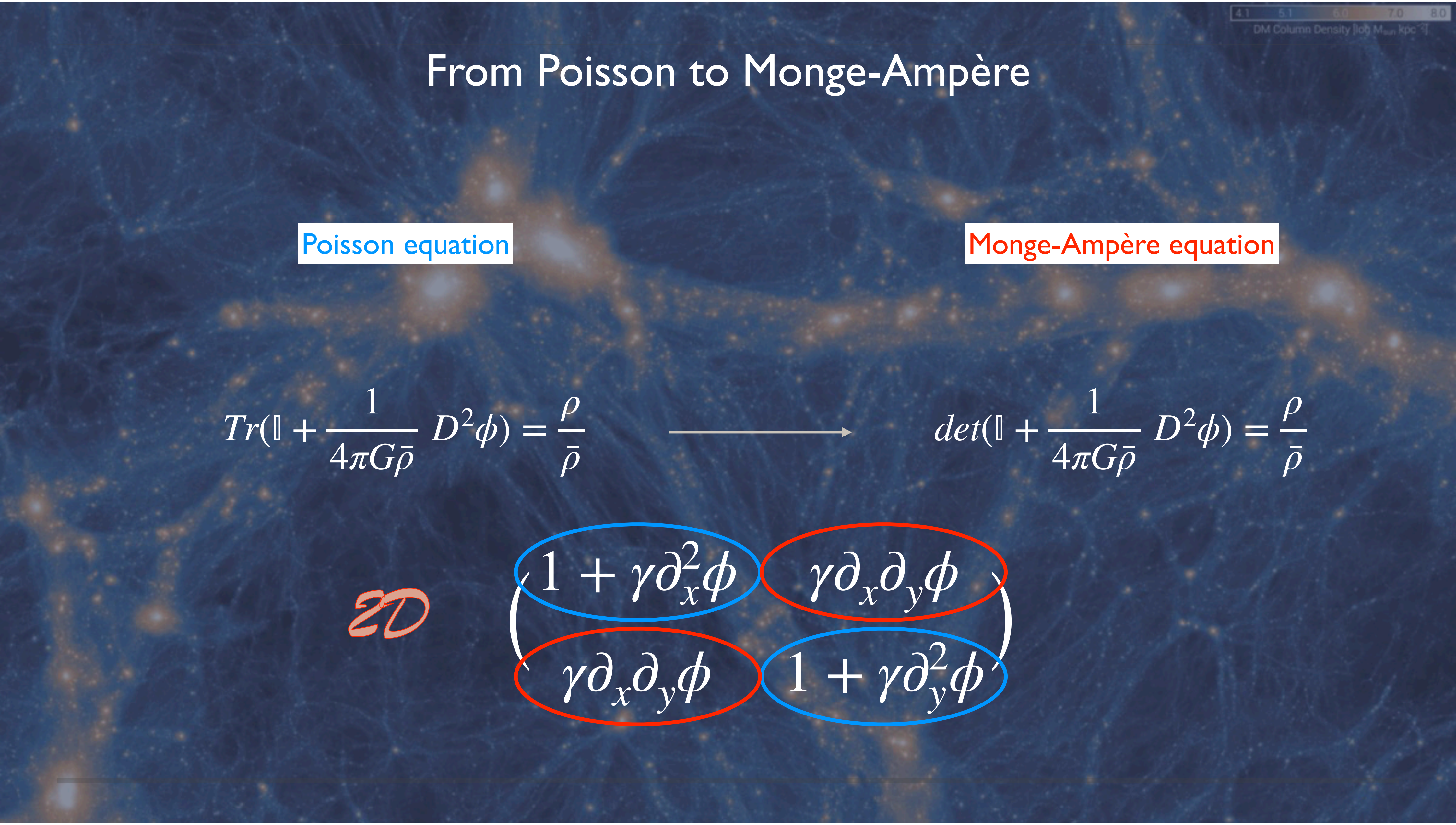
Monge-Ampère equation

$$det(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$

2D

$$\begin{pmatrix} 1 + \gamma\partial_x^2\phi & \gamma\partial_x\partial_y\phi \\ \gamma\partial_x\partial_y\phi & 1 + \gamma\partial_y^2\phi \end{pmatrix}$$

From Poisson to Monge-Ampère



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Poisson equation

Monge-Ampère equation

$$Tr\left(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi\right) = \frac{\rho}{\bar{\rho}} \longrightarrow det\left(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi\right) = \frac{\rho}{\bar{\rho}}$$

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$$\begin{pmatrix} 1 + \gamma\partial_x^2\phi & \gamma\partial_x\partial_y\phi \\ \gamma\partial_x\partial_y\phi & 1 + \gamma\partial_y^2\phi \end{pmatrix}$$

In **one dimension**, Monge-Ampère is **equivalent** to Poisson

From Poisson to Monge-Ampère

Poisson equation

$$Tr(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$

$$SO(3)$$

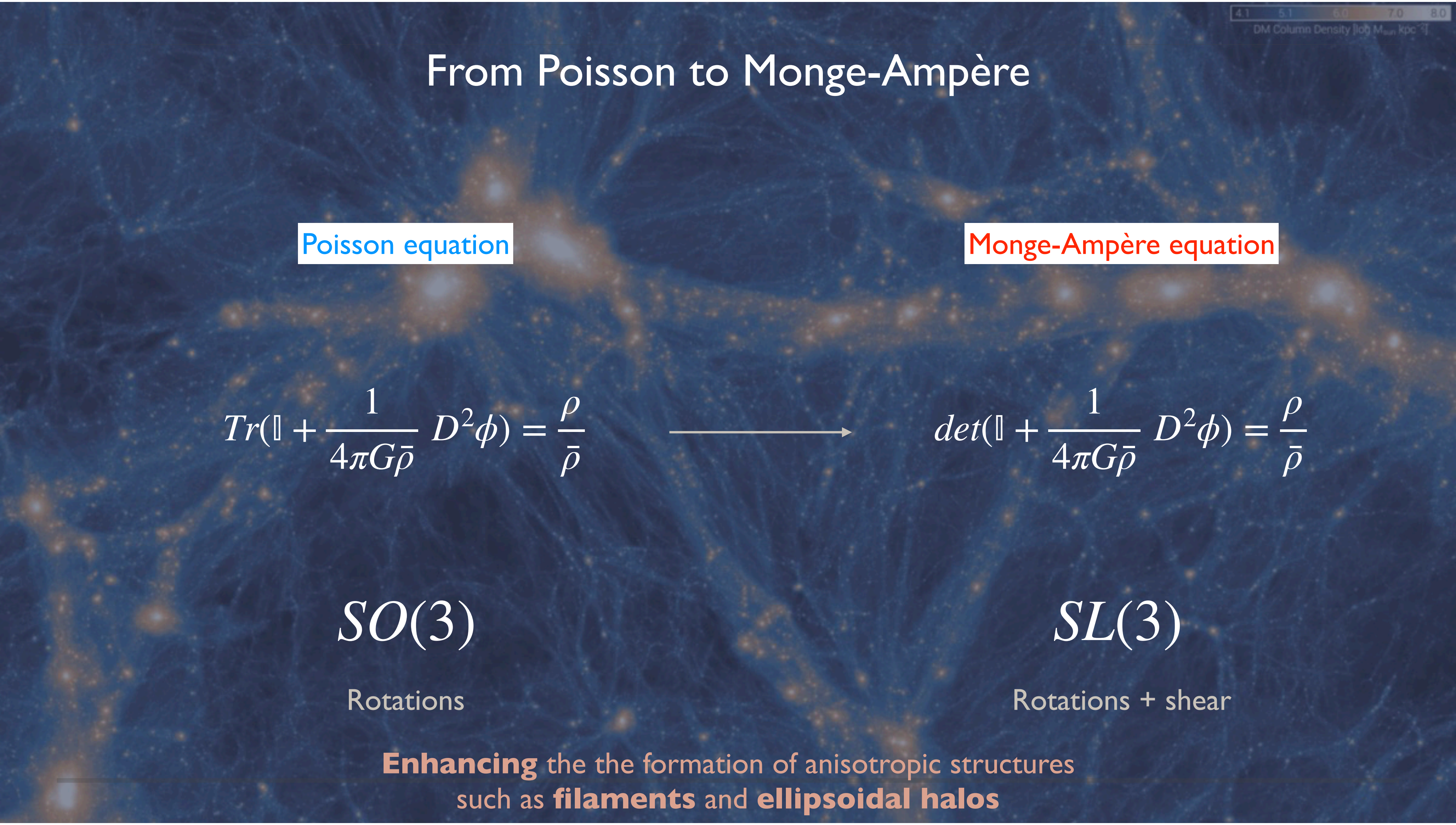
Rotations

Monge-Ampère equation

$$det(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$

$$SL(3)$$

Rotations + shear



From Poisson to Monge-Ampère

Poisson equation

$$Tr\left(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi\right) = \frac{\rho}{\bar{\rho}}$$

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Rotations

→

Monge-Ampère equation

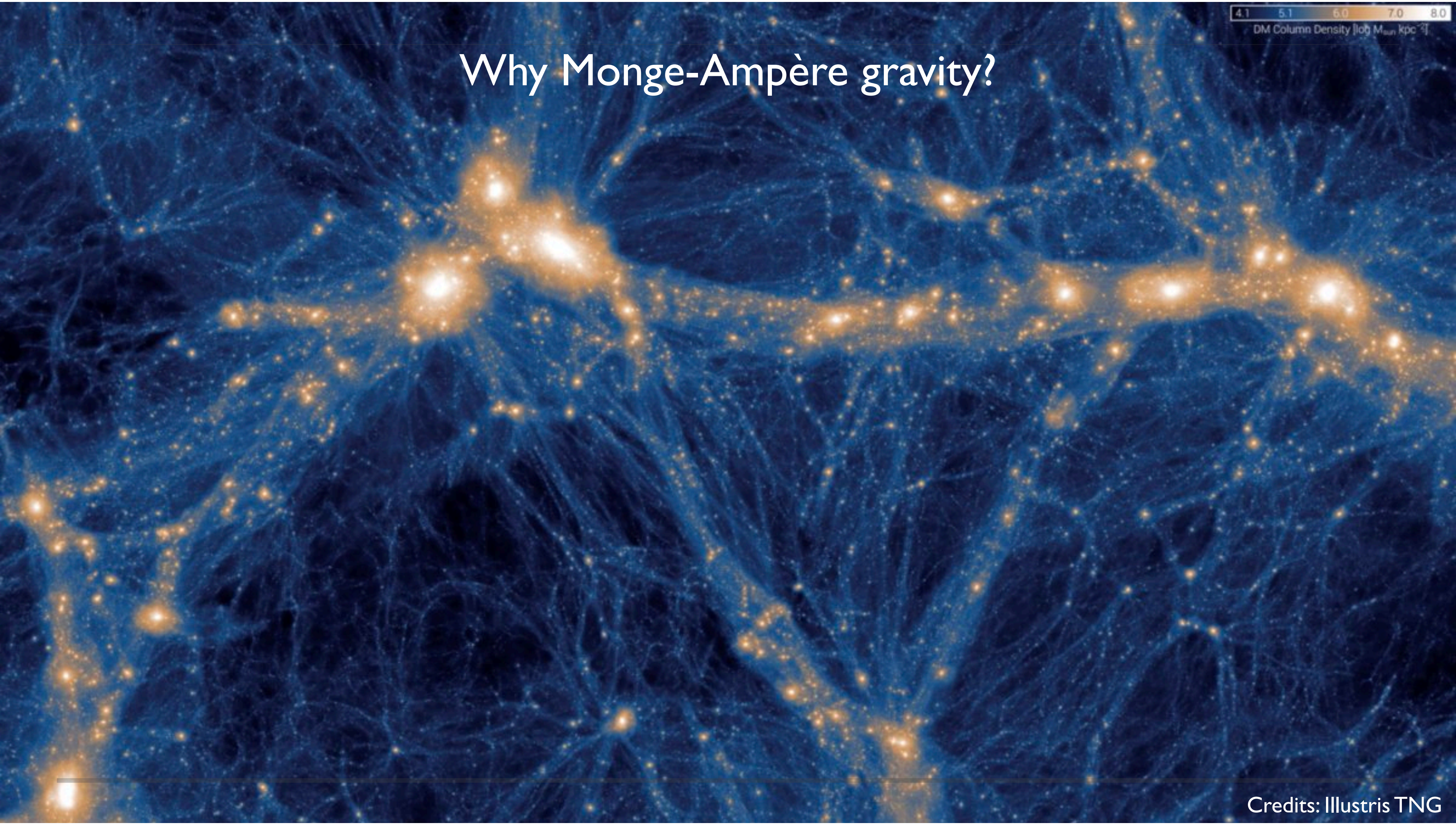
$$det\left(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi\right) = \frac{\rho}{\bar{\rho}}$$

$$SL(3)$$

Rotations + shear

Enhancing the the formation of anisotropic structures such as **filaments** and **ellipsoidal halos**

Why Monge-Ampère gravity?



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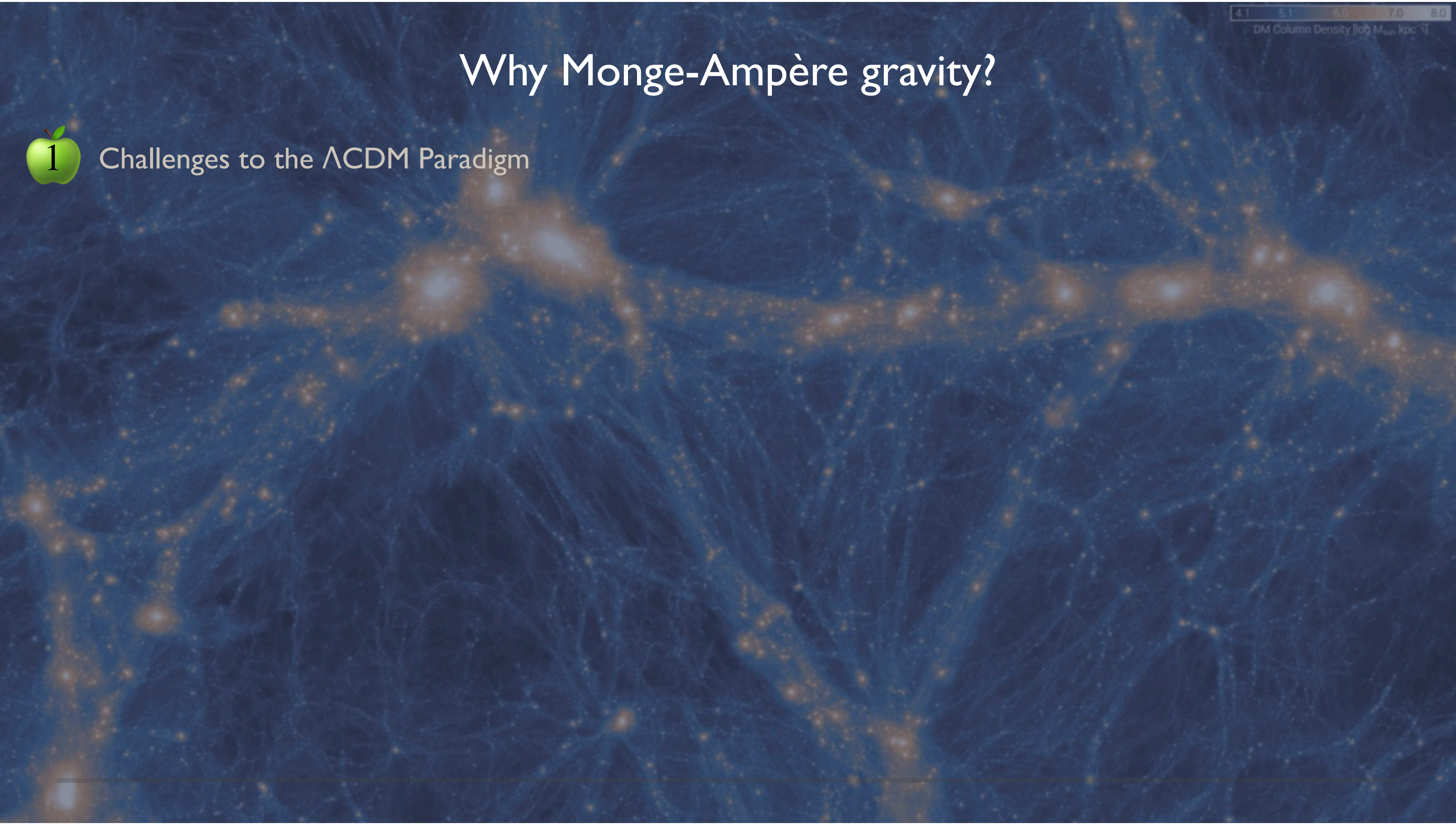
Why Monge-Ampère gravity?

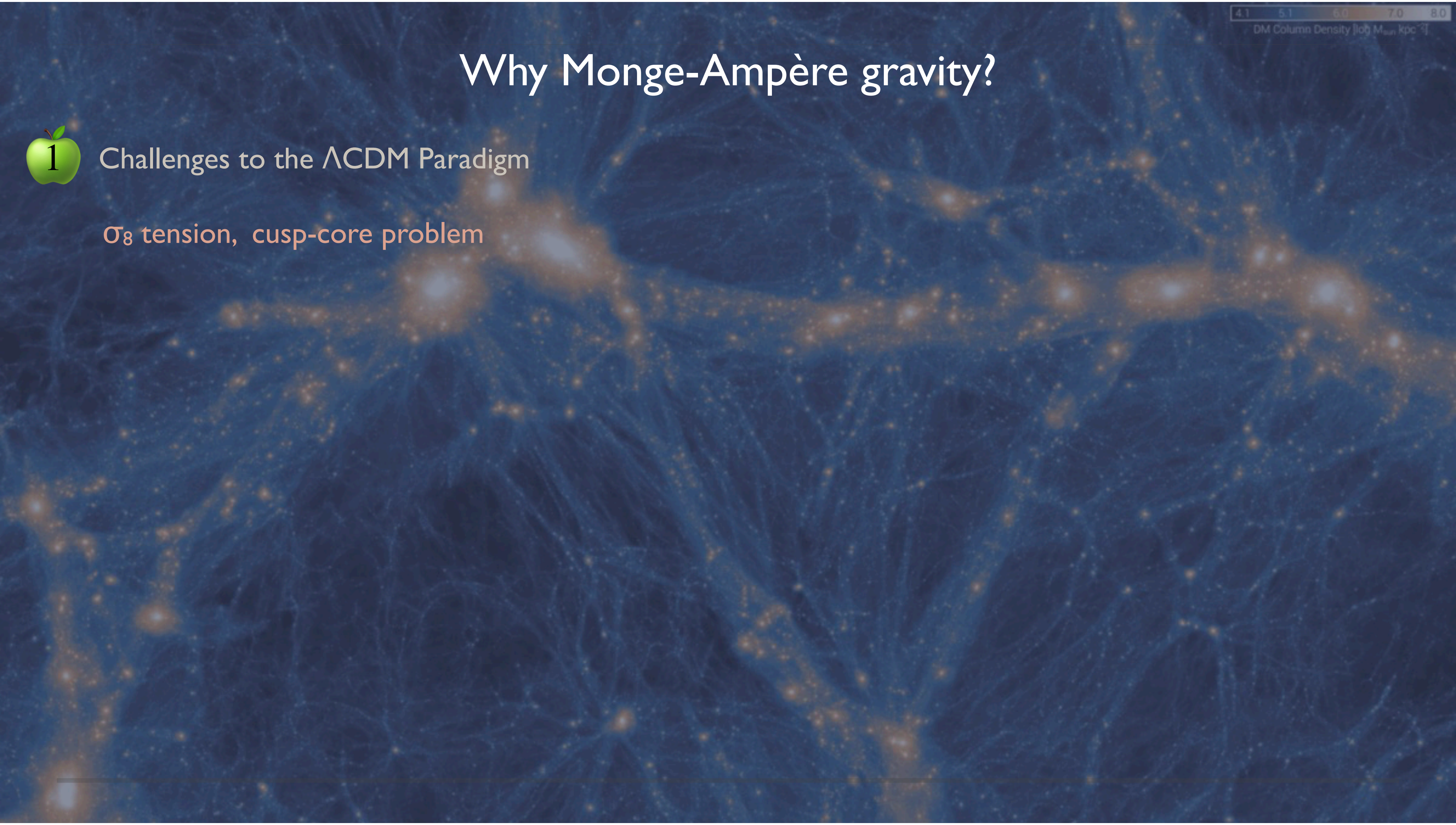
1 Challenges to the ΛCDM Paradigm



Why Monge-Ampère gravity?

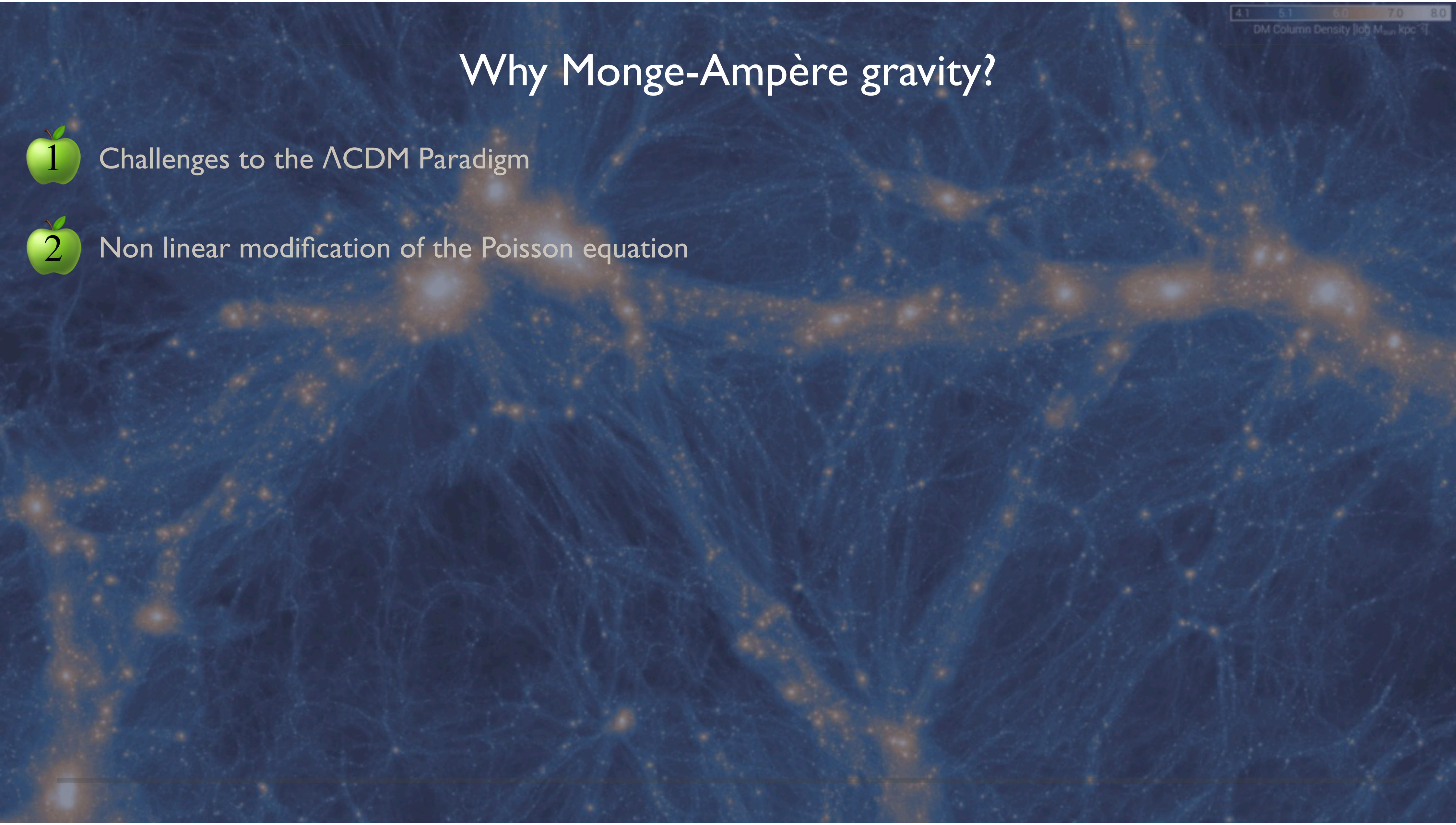
1 Challenges to the ΛCDM Paradigm

σ_8 tension, cusp-core problem



Why Monge-Ampère gravity?

1. Challenges to the ΛCDM Paradigm
2. Non linear modification of the Poisson equation



Why Monge-Ampère gravity?

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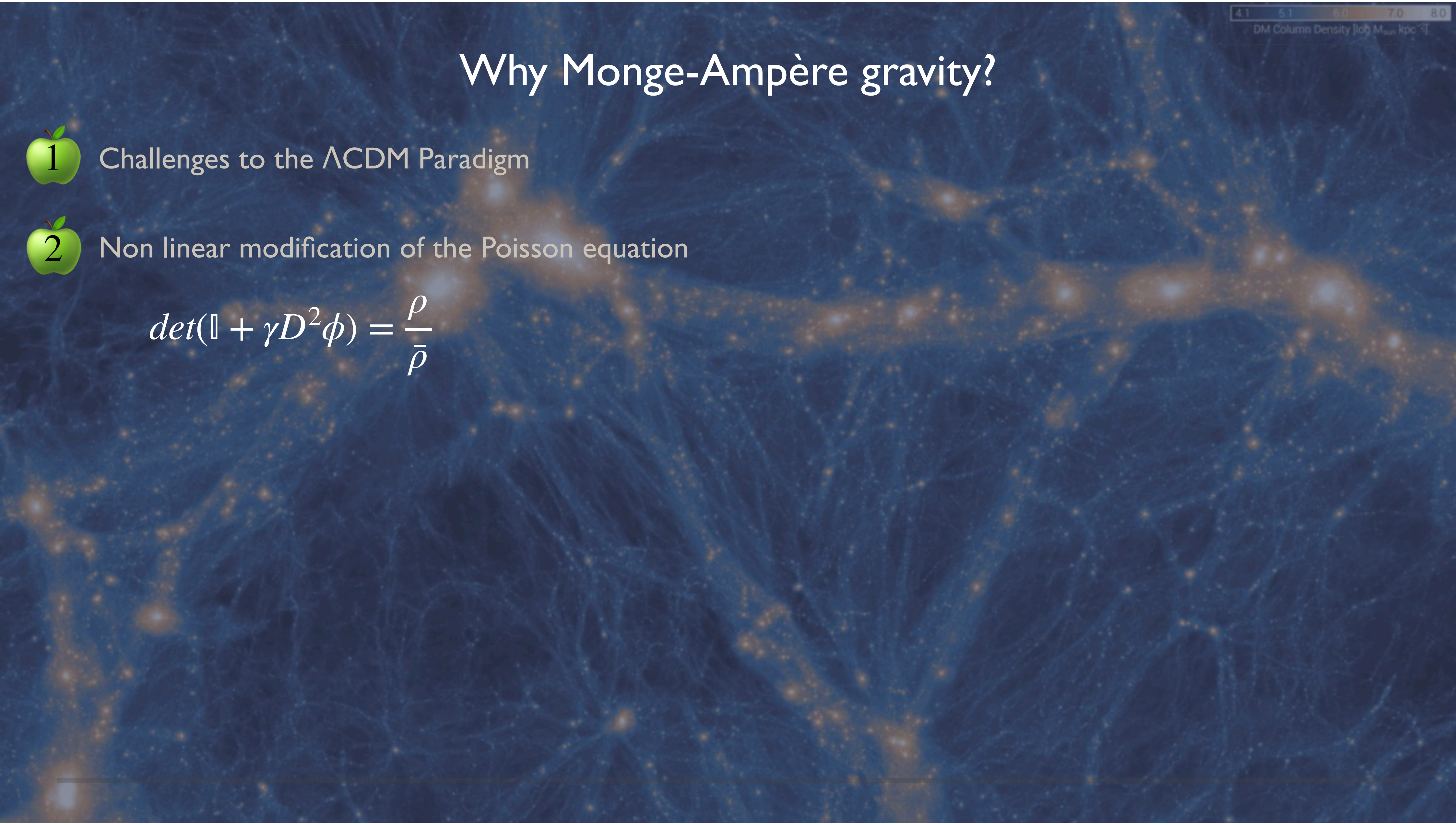
$$det(\mathbb{I} + \gamma D^2\phi) = \frac{\rho}{\bar{\rho}}$$



Why Monge-Ampère gravity?

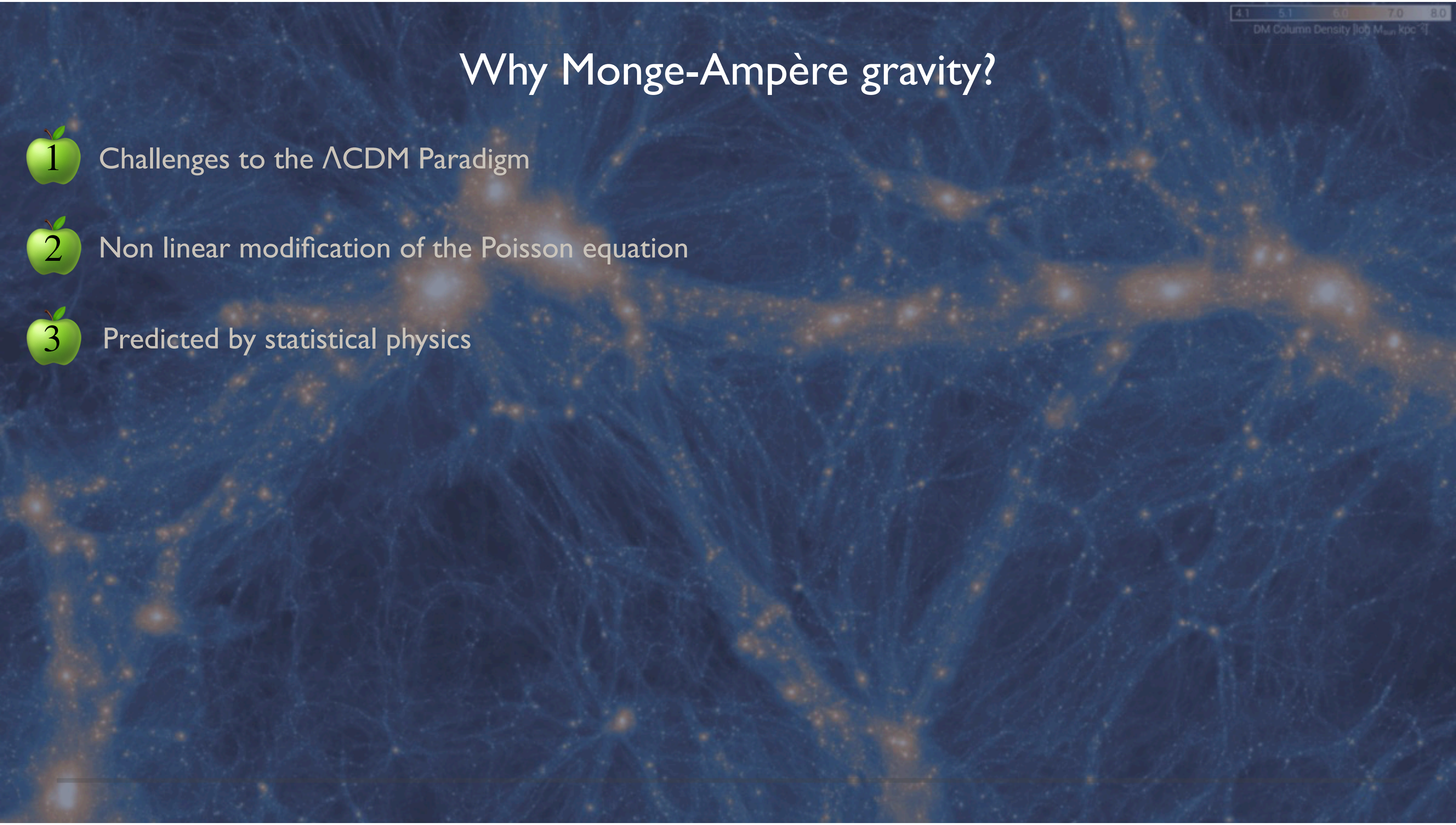
1. Challenges to the ΛCDM Paradigm

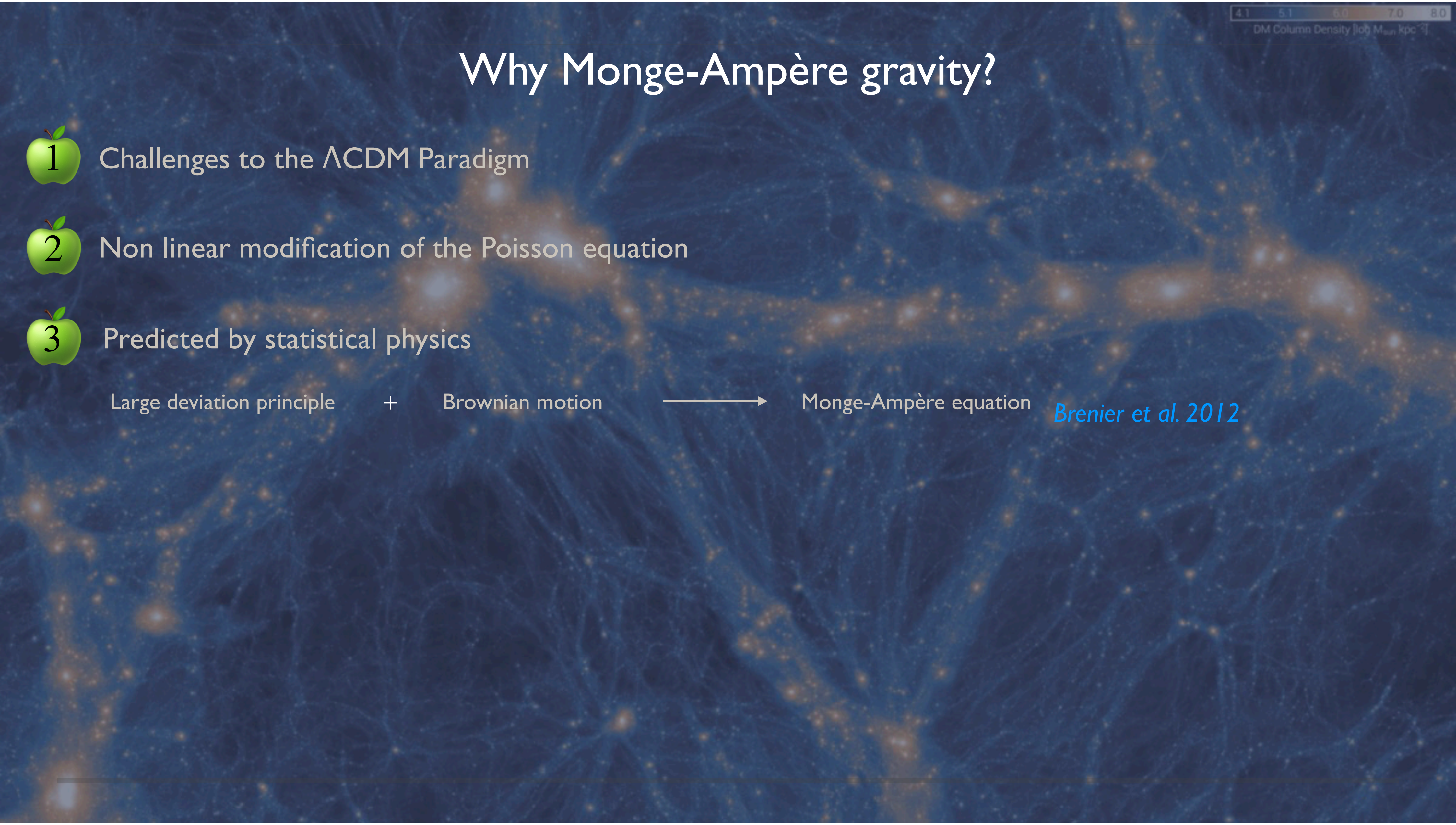
2. Non linear modification of the Poisson equation

$$det(\mathbb{I} + \gamma D^2\phi) = \frac{\rho}{\bar{\rho}} \longrightarrow 1 + \gamma\Delta\phi + \mathcal{O}(\phi^2) = \frac{\rho}{\bar{\rho}}$$

Why Monge-Ampère gravity?

1. Challenges to the ΛCDM Paradigm
2. Non linear modification of the Poisson equation
3. Predicted by statistical physics





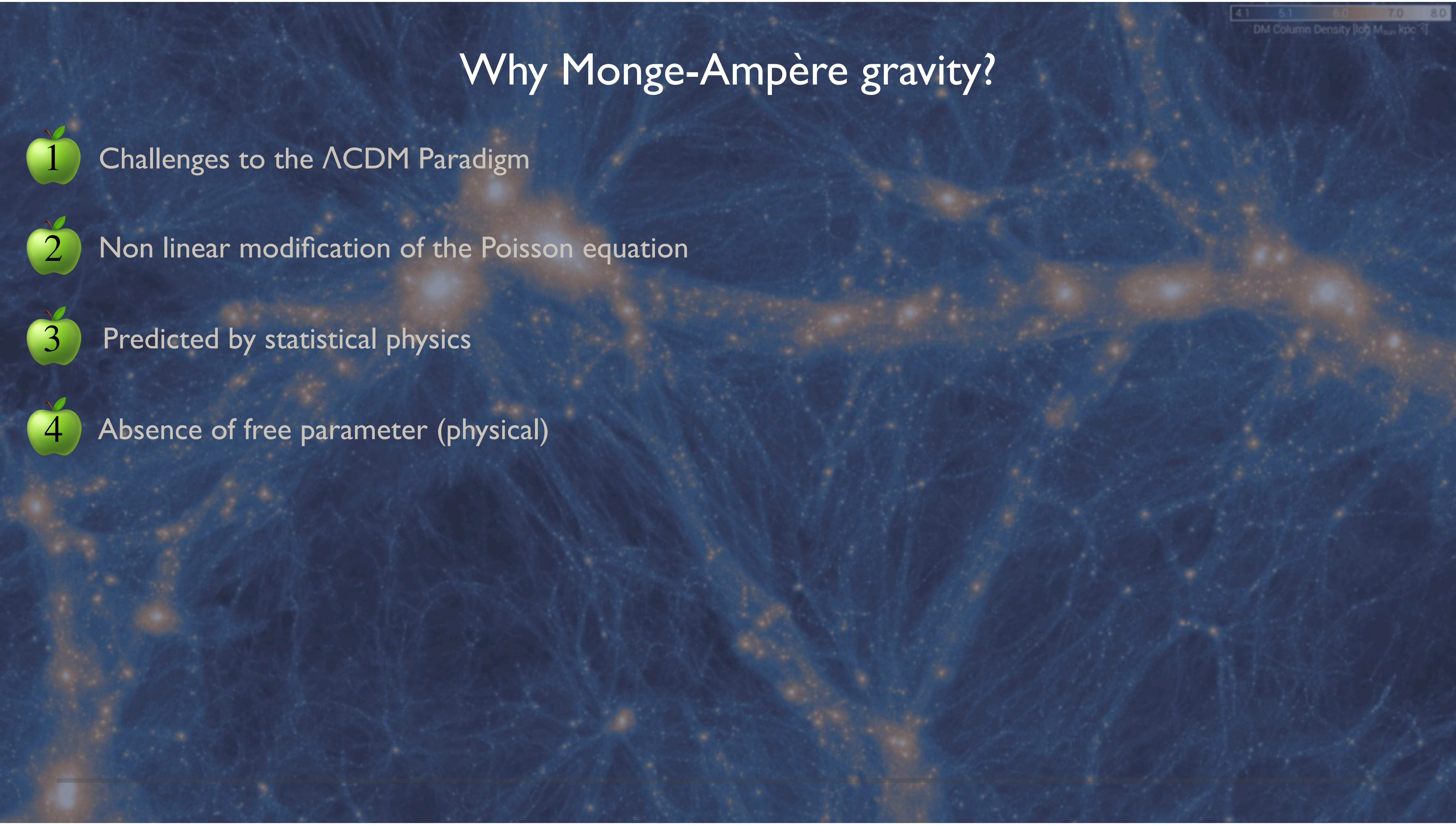
Why Monge-Ampère gravity?

1. Challenges to the ΛCDM Paradigm
2. Non linear modification of the Poisson equation
3. Predicted by statistical physics

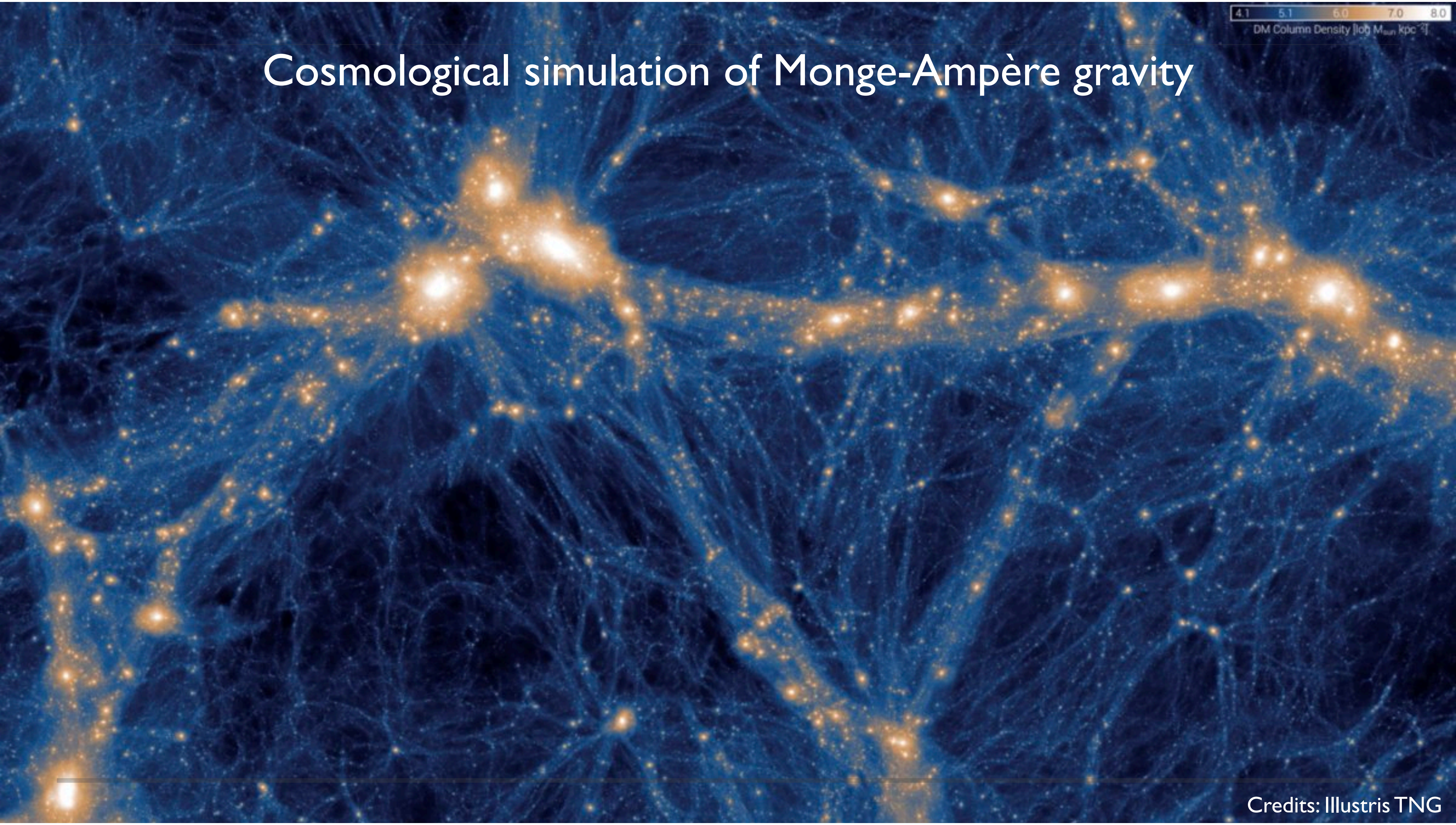
Large deviation principle + Brownian motion ⟶ Monge-Ampère equation *Brenier et al. 2012*

Why Monge-Ampère gravity?

1. Challenges to the ΛCDM Paradigm
2. Non linear modification of the Poisson equation
3. Predicted by statistical physics
4. Absence of free parameter (physical)



Cosmological simulation of Monge-Ampère gravity



Cosmological simulation of Monge-Ampère gravity

- Initial conditions
- Equations of motion in comoving coordinates
- How it works numerically?
- Comparing with Poisson *N*-body cosmological simulations
- Results

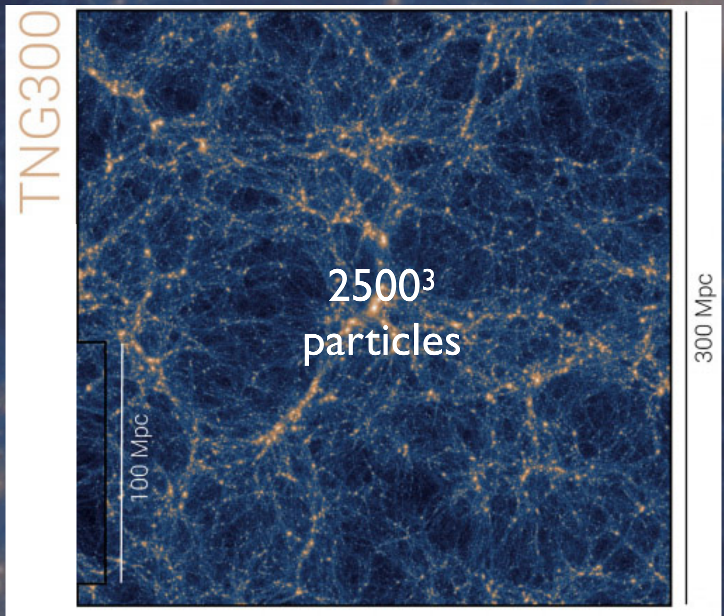
pyMAG 1.0

```
pip install pyMAG
```

Soon ….

Cosmological simulation of Monge-Ampère gravity

🍋 Initial conditions



2500^3 particles

Springel et al. 2018

512^3 particles

205 Mpc/h $\sim$ 300 Mpc

$z = 49 \longrightarrow z = 0$

$\Omega_{\mathrm{m}} = 0.3089$

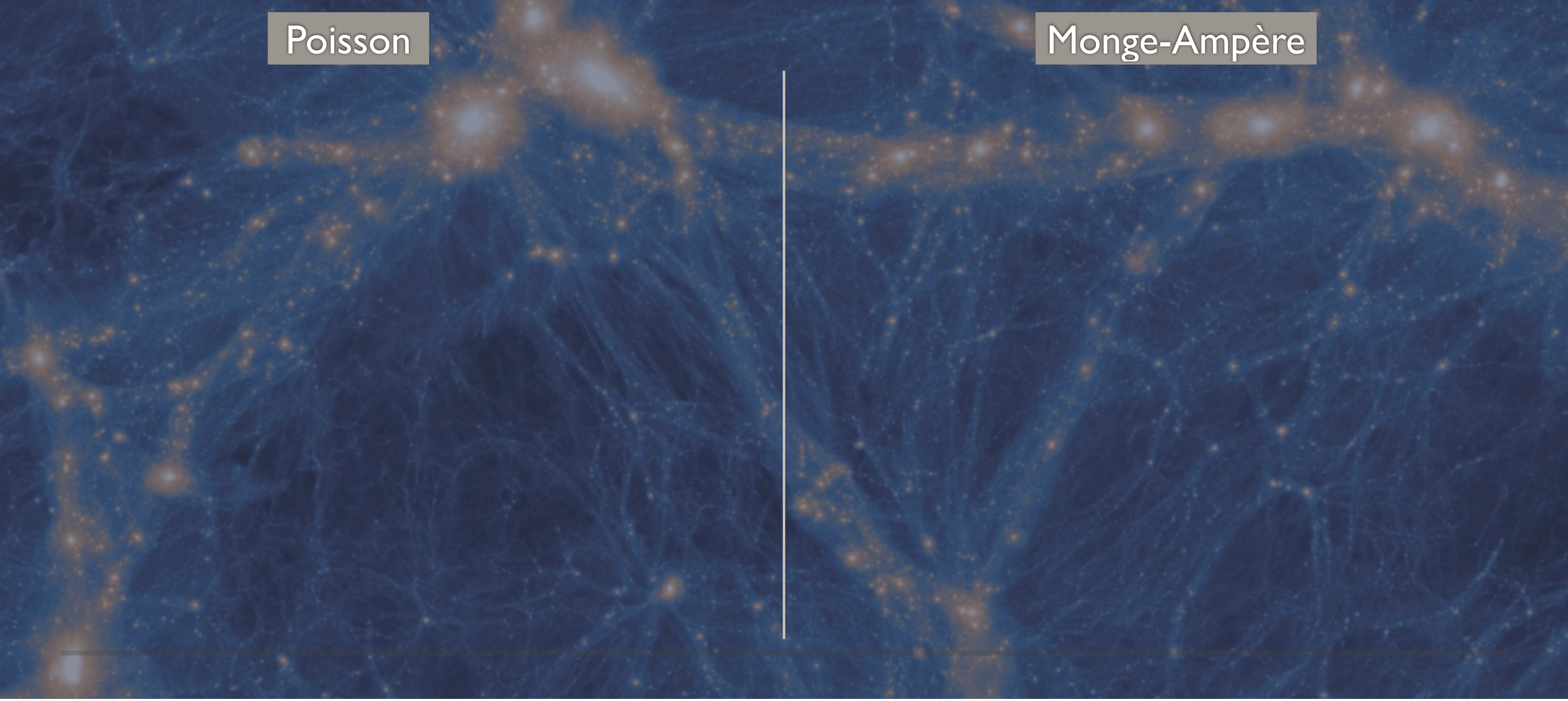
$\Omega_{\Lambda} = 0.6911$

$H_0 = 67.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$

$m_{DM} \sim 10^{10} M_{\odot}$

Cosmological simulation of Monge-Ampère gravity

- Comparing with standard N-body cosmological simulation



Cosmological simulation of Monge-Ampère gravity

- Comparing with standard N-body cosmological simulation

Poisson

GADGET - 2

A code for cosmological simulations of structure formation

Springel et al. 2018

Monge-Ampère

pyMAG 1.0

Boldrini et al. 2022, in prep

Cosmological simulation of Monge-Ampère gravity

- Comparing with standard N-body cosmological simulation

Poisson

GADGET - 2

A code for cosmological simulations of structure formation

Springel et al. 2018

Tree-Code

Barnes and Hut, 1986

$\mathcal{O}(N \log N)$

Monge-Ampère

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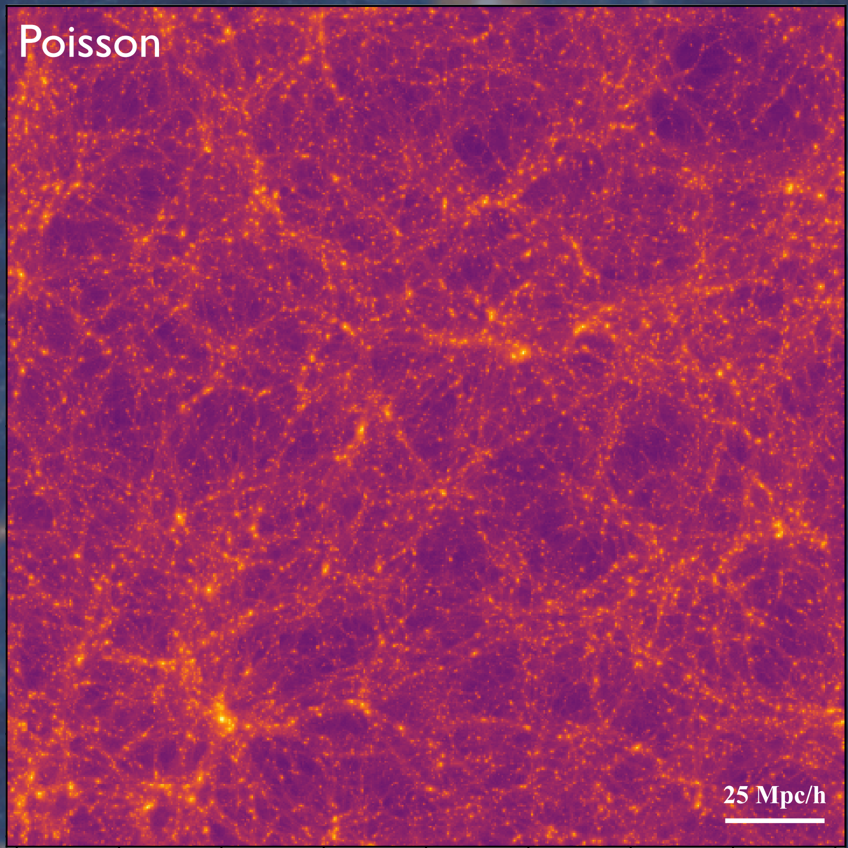
Cosmological simulation of Monge-Ampère gravity

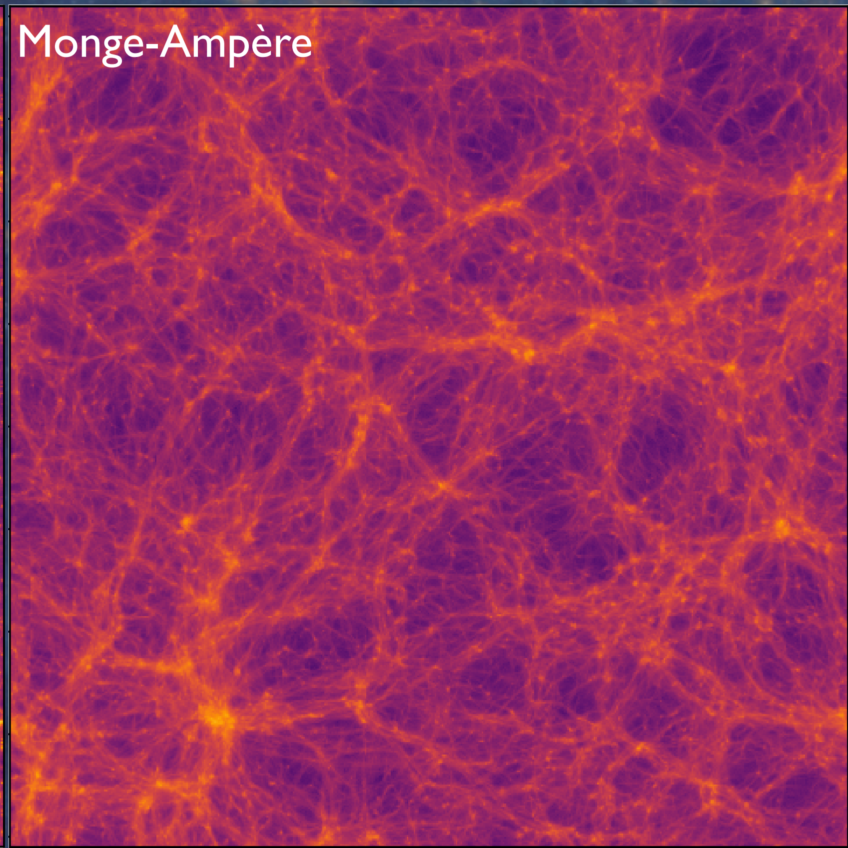
- Comparing with standard N-body cosmological simulation

Poisson	Monge-Ampère
GADGET - 2 A code for cosmological simulations of structure formation	pyMAG 1.0
Springel et al. 2018	*Boldrini et al. 2022, in prep*
Tree-Code	Optimal transport algorithm
Barnes and Hut, 1986	*Lévy 2022*
$\mathcal{O}(N \log N)$	$\mathcal{O}(N \log N)$

Cosmological simulation of Monge-Ampère gravity

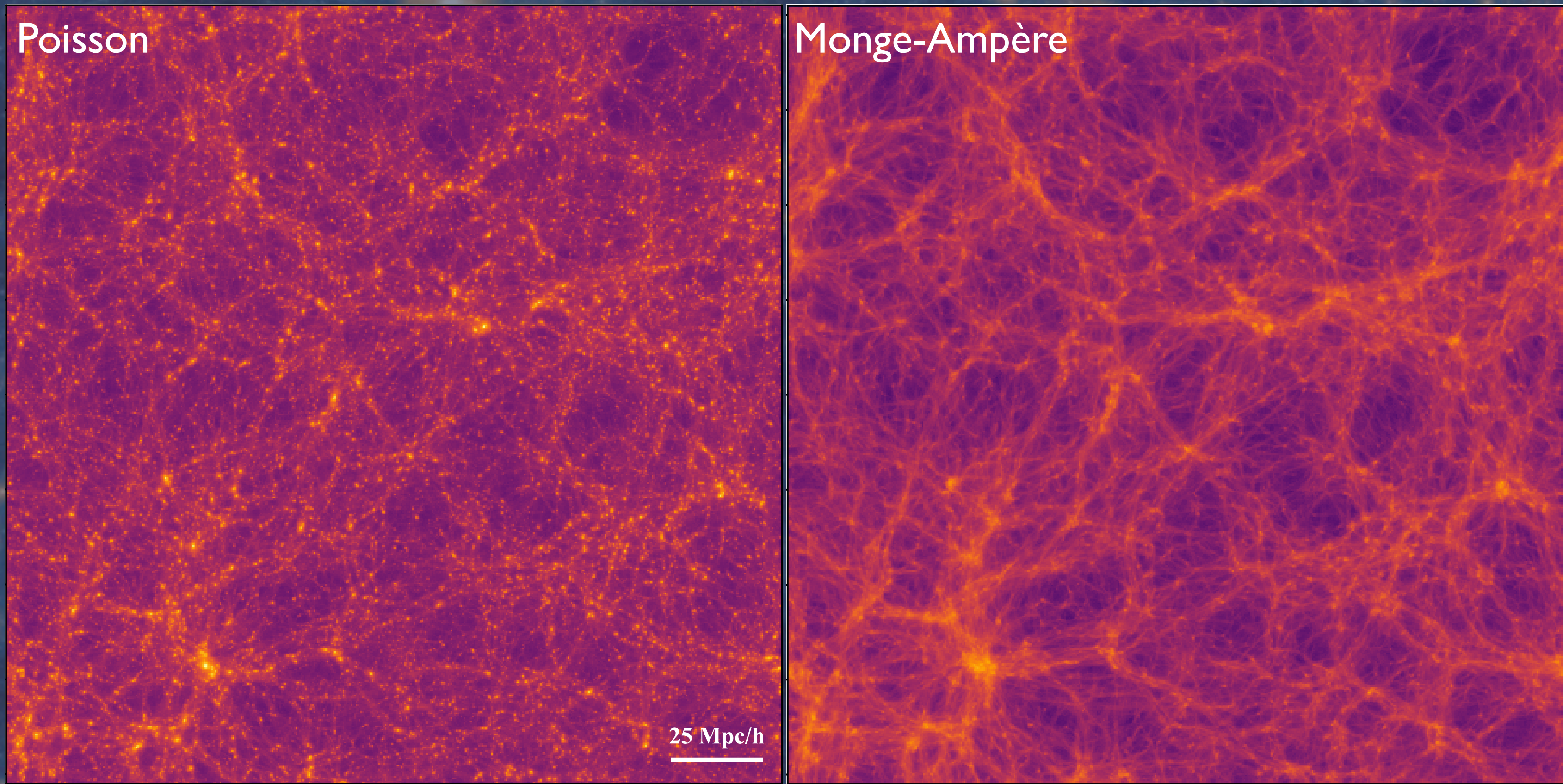
🍋 Large scale-structures $z = 0$





Cosmological simulation of Monge-Ampère gravity

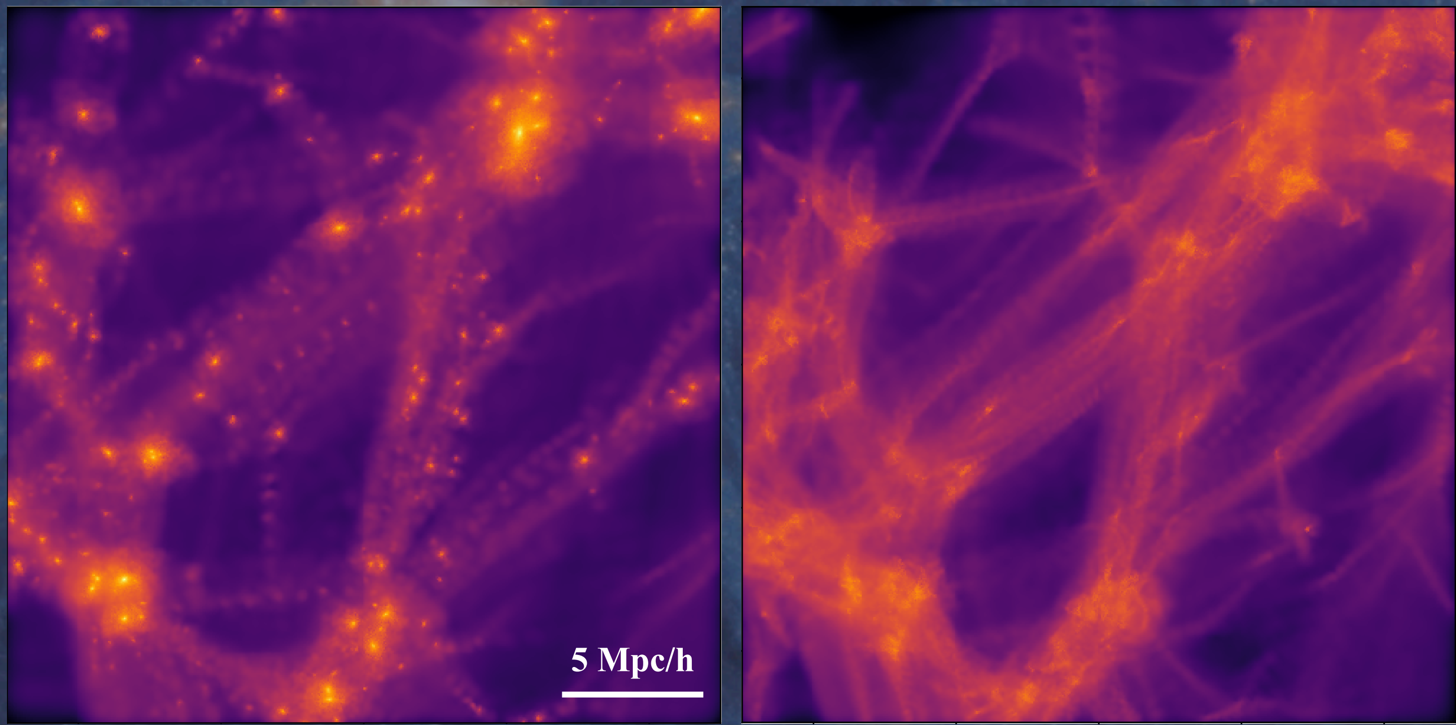
- Large scale-structures $z = 0$



A **weaker gravitational** clustering

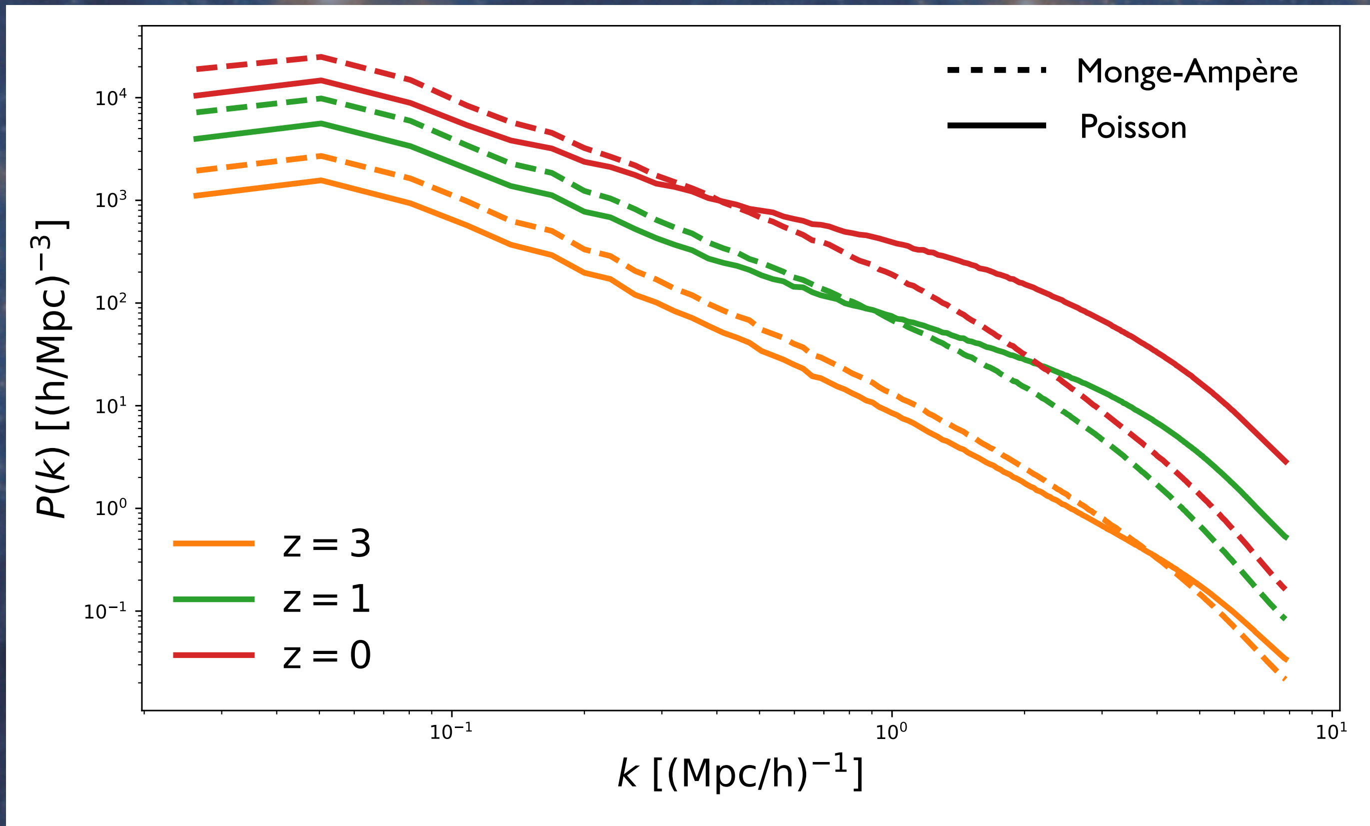
Cosmological simulation of Monge-Ampère gravity

🍋 Zoom $z = 0$



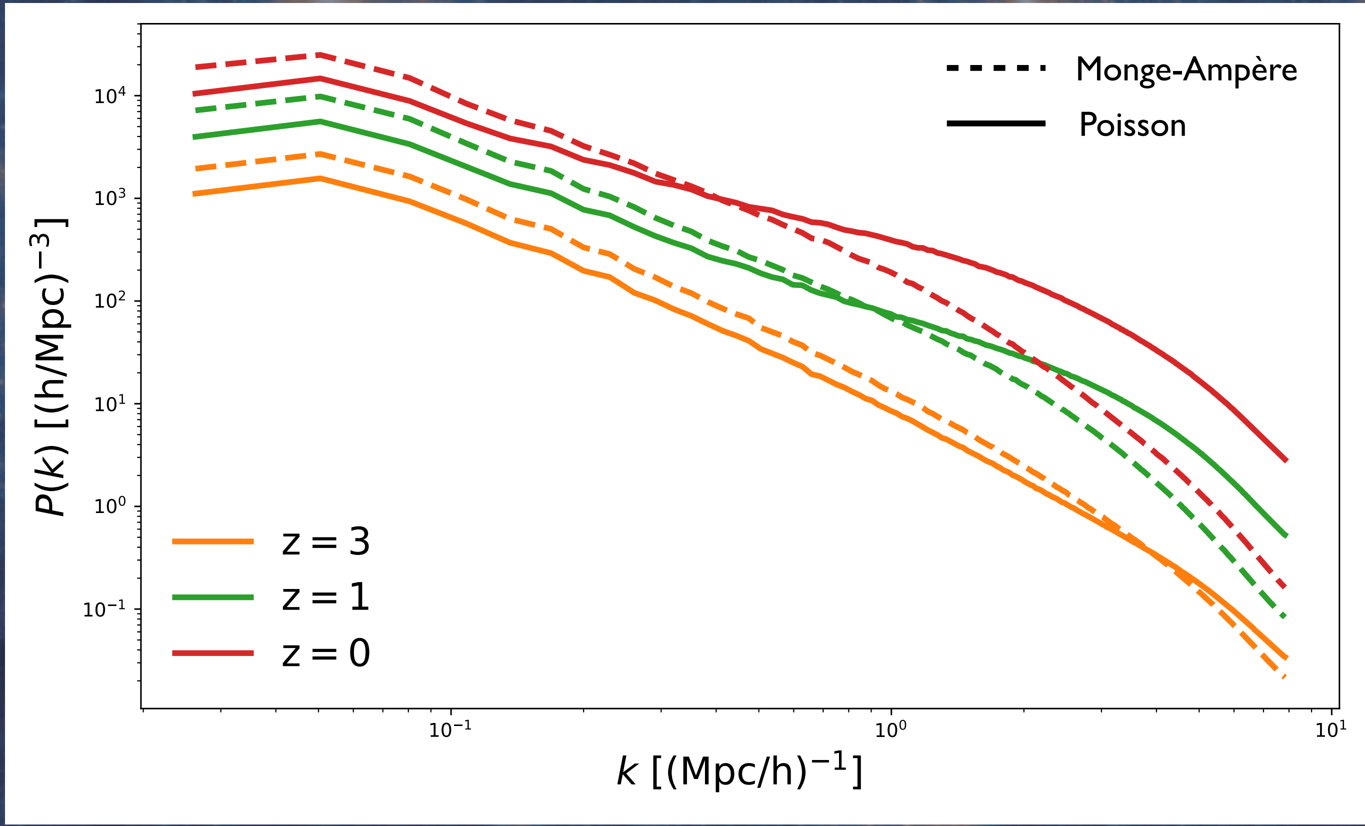
Cosmological simulation of Monge-Ampère gravity

- Power spectra



Cosmological simulation of Monge-Ampère gravity

🍋 Power spectra

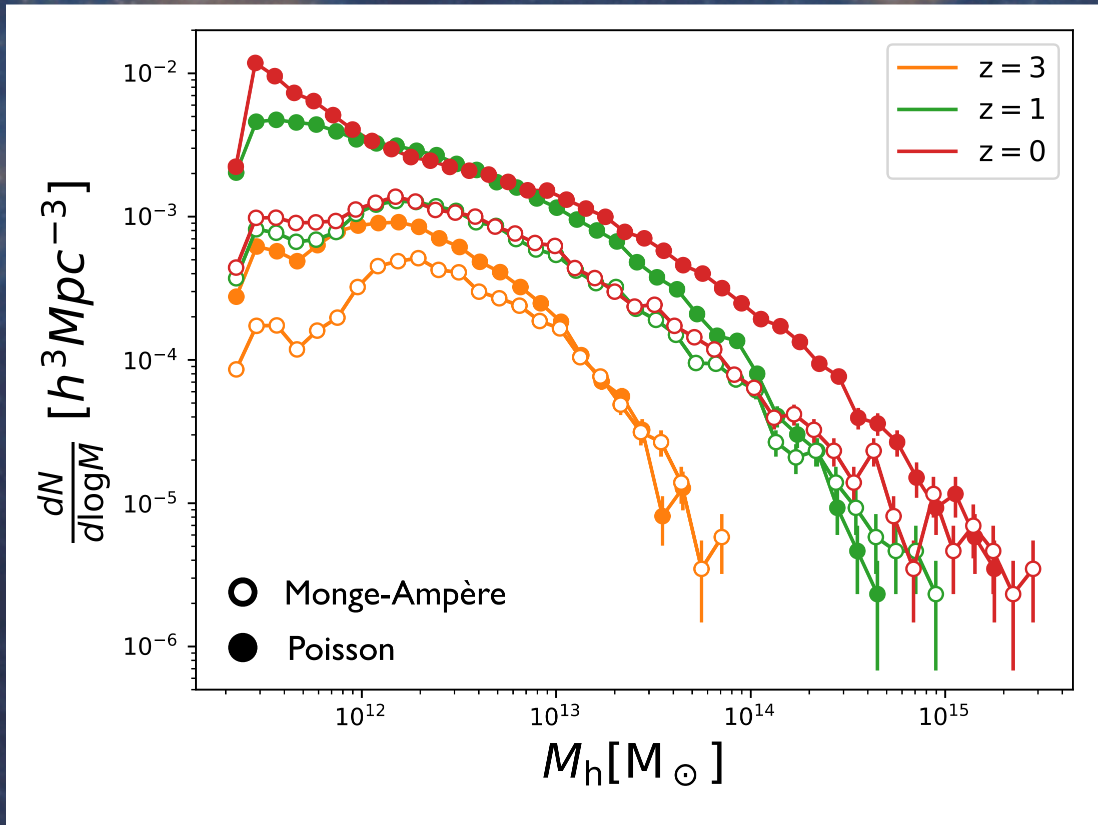


Gravity is getting **weaker** at low z?

Douspis et al. 2018

Cosmological simulation of Monge-Ampère gravity

- Halo mass function



Cosmological simulation of Monge-Ampère gravity

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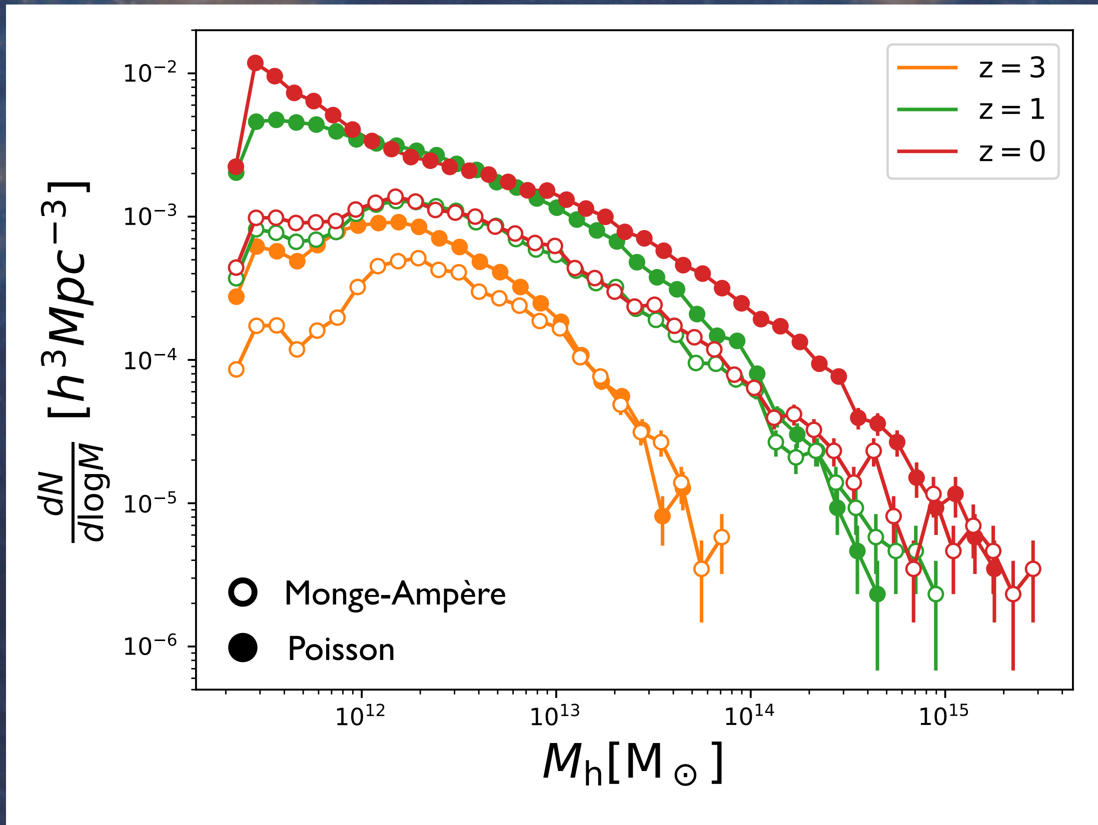


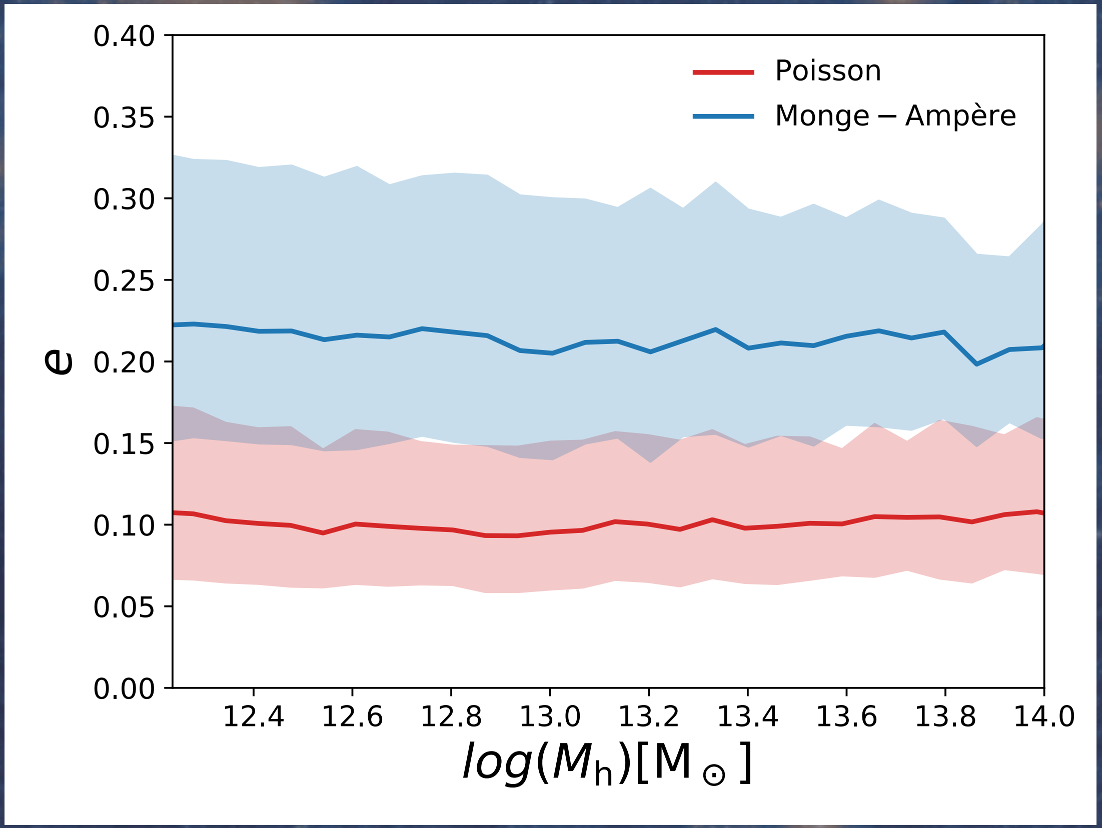
Poisson | Monge-Ampère

$N_h = 66091 \mid 16057$

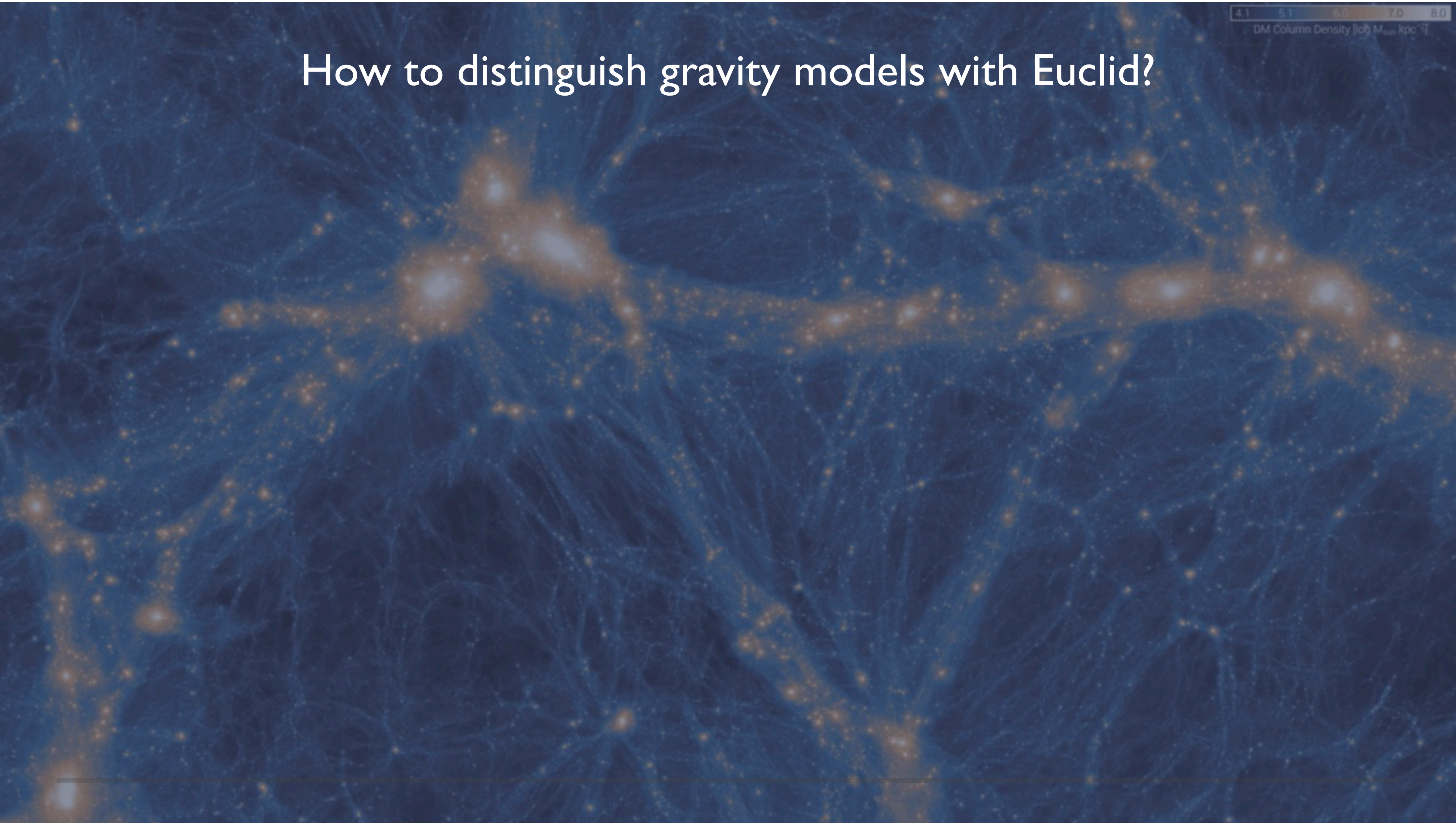
4 times less halos at $z = 0$

Cosmological simulation of Monge-Ampère gravity

- Ellipticity

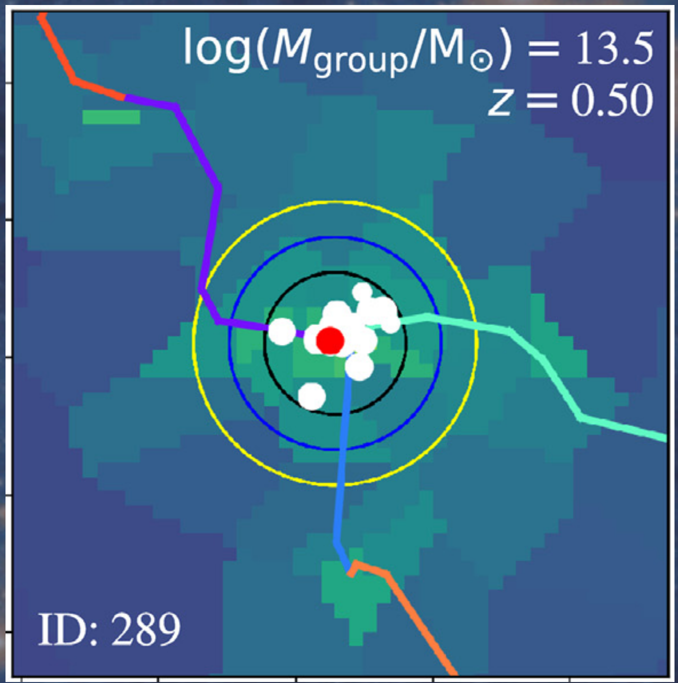


How to distinguish gravity models with Euclid?



How to distinguish gravity models with Euclid?

Connectivity of filaments



Darragh Ford et al. 2019

Already used as **alternatives probes** of cosmologies

Codis et al. 2018, L'Hullier et al. 2017

How to distinguish gravity models with Euclid?

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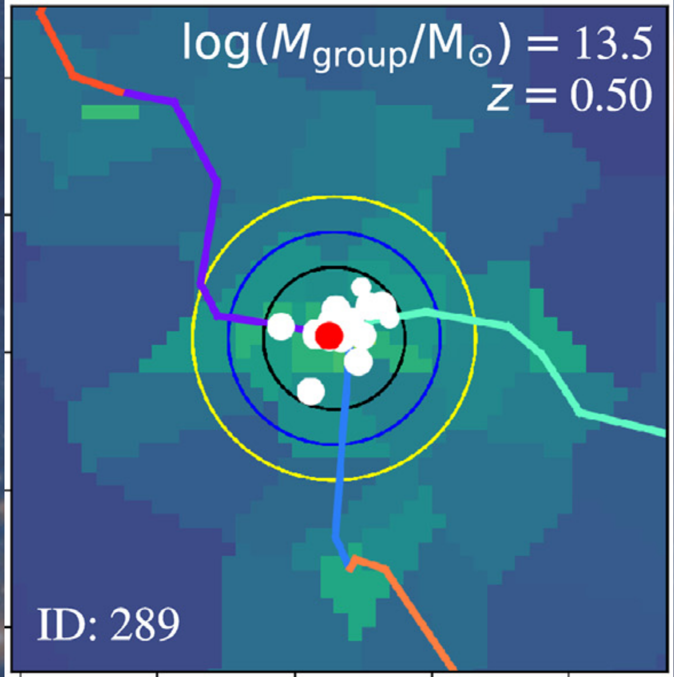

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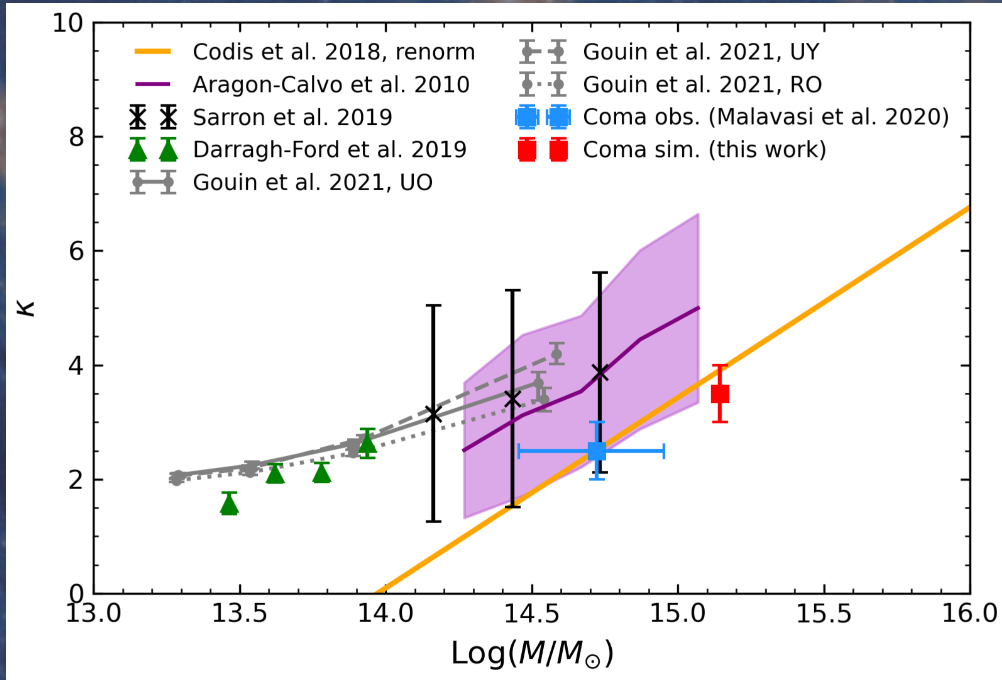

Malavasi et al. 2023

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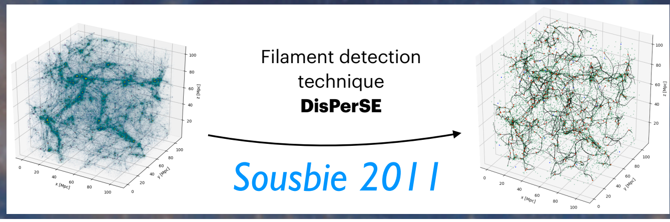
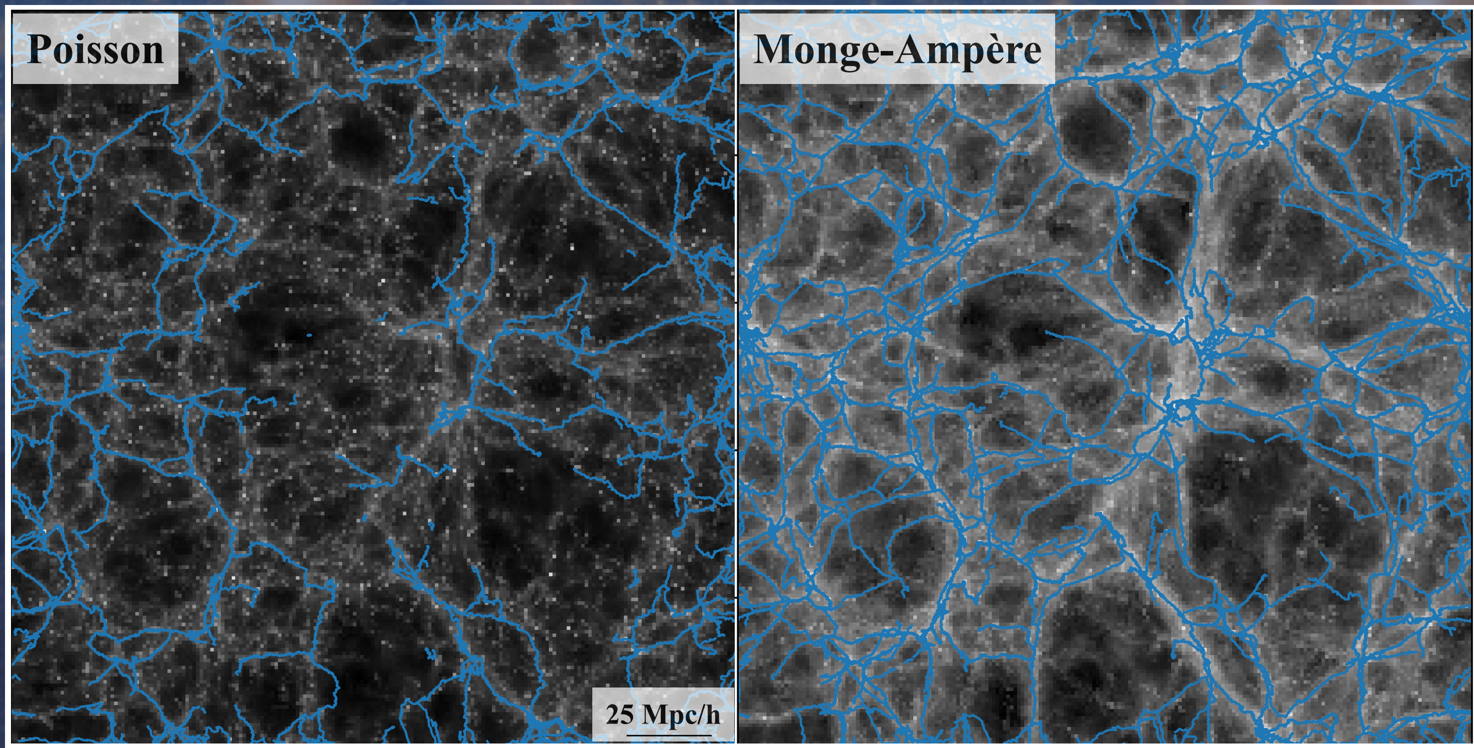
How to distinguish gravity models with Euclid?

- Filaments

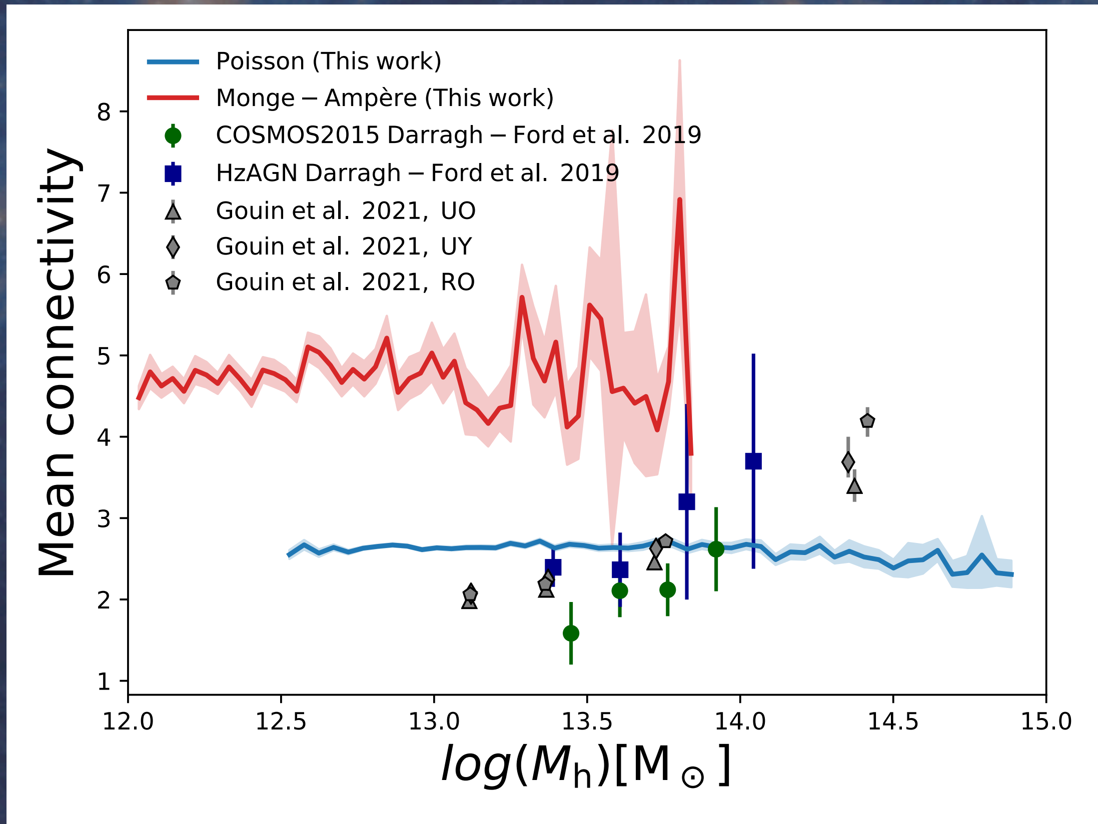


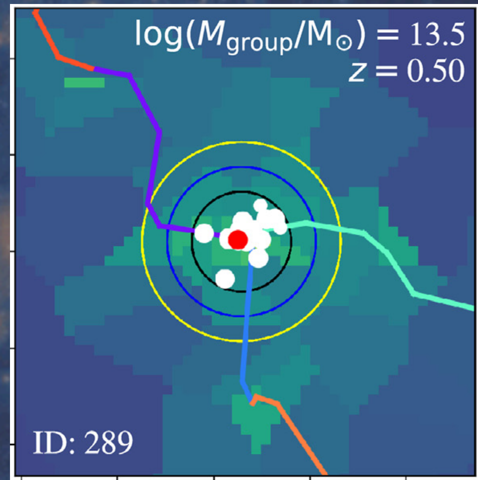


How to distinguish gravity models with Euclid?

- Connectivity of filaments

This is a preliminary result!





Darragh Ford et al. 2019

How to distinguish gravity models with Euclid?

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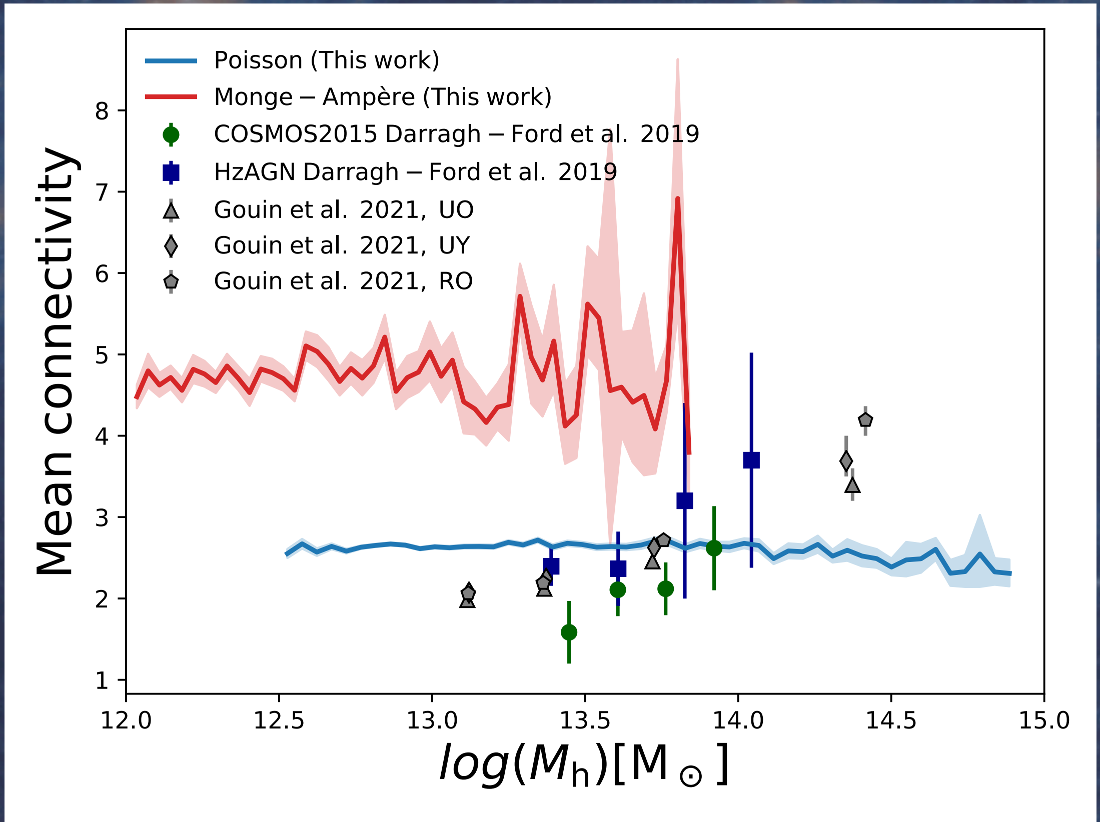
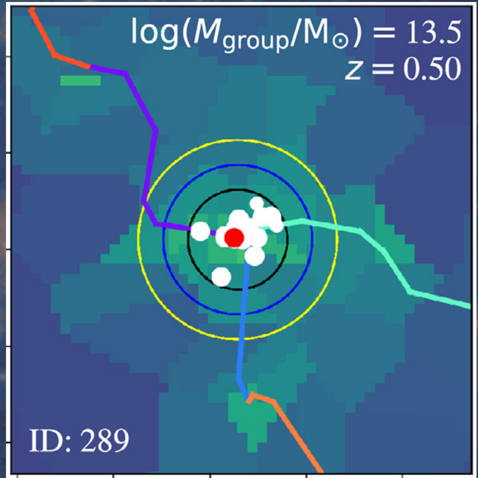
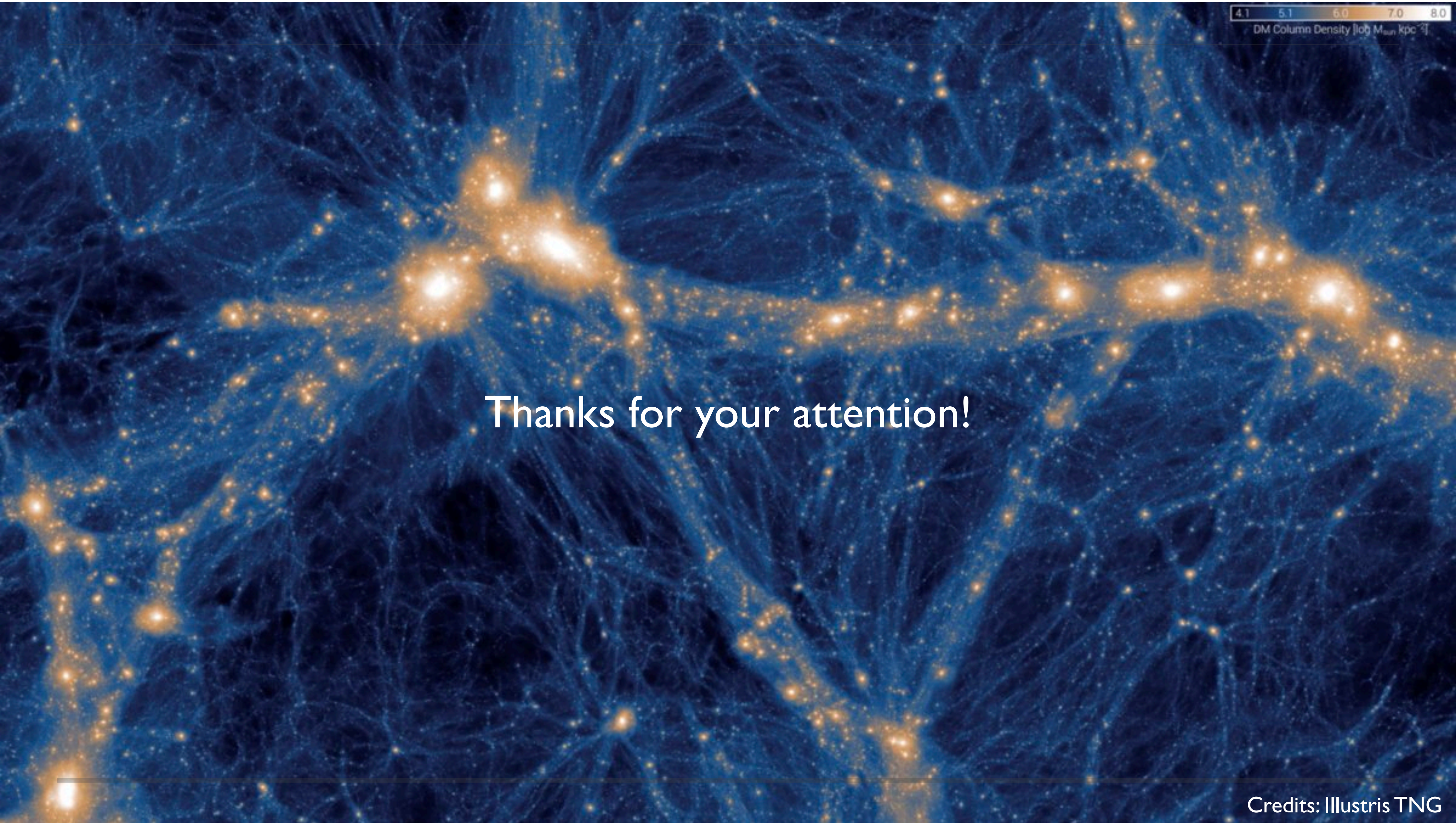




Darragh Ford et al. 2019

The **filament connectivity** could be used as a **probe of our gravity model** at cosmological scales with the **Euclid mission**



Thanks for your attention!

Credits: Illustris TNG