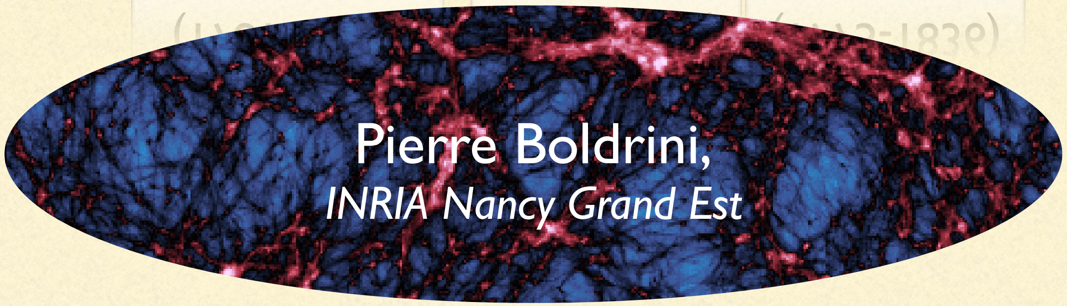
Monge-Ampère gravity at cosmological scales

Siméon Denis Poisson (1781-1840)	Gaspard Monge (1746-1818)	André-Marie Ampère (1775-1836)



Pierre Boldrini,
INRIA Nancy Grand Est

An interdisciplinary project
(*Mathematics, Computer science, Astrophysics*)

in collaboration with:

Yann Brenier, Roya Mohayaee, and Bruno Lévy

ENS | PSL | IAP | Inria

based on:

Boldrini et al. 2023, in prep.

Café-Club GECO, LAM, Marseille, April 2023

The nature of dark matter

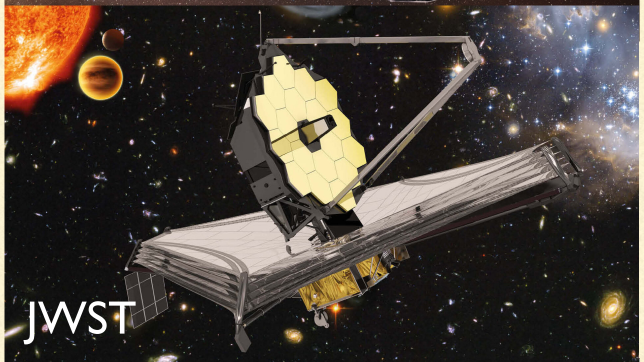
- Cold dark matter
- Warm dark matter
- Fuzzy dark matter
- Self-interacting dark matter
- Primordial black holes

- Monge-Ampère gravity

Current cosmological model

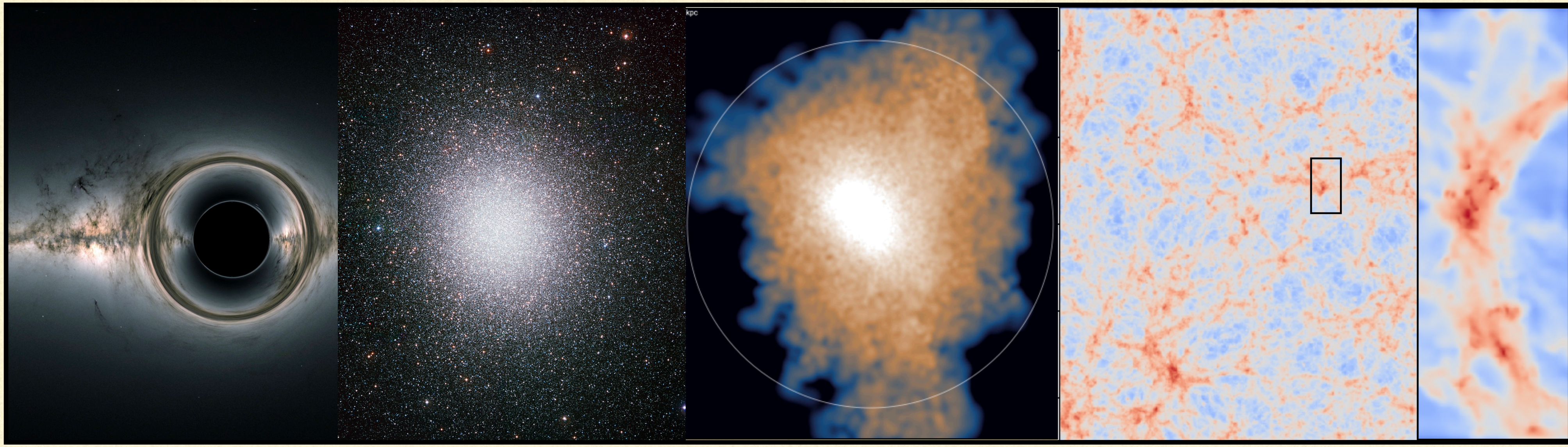
Alternative dark matter theories

Alternative gravity theory



The nature of dark matter

Black hole | Globular cluster | DM halo of galaxies | Large scale structures



Isolated simulations — Cosmological simulations
Orbital integrations

Cosmological simulations
Optimal transport

Boldrini+20c
Boldrini+20d
Chu+22

Boldrini+20b
Boldrini & Vitral +21
Boldrini & Bovy +21
Vitral+22

Boldrini+19
Boldrini+20a
Boldrini+20e
Boldrini+21

Boldrini+23 in prep.

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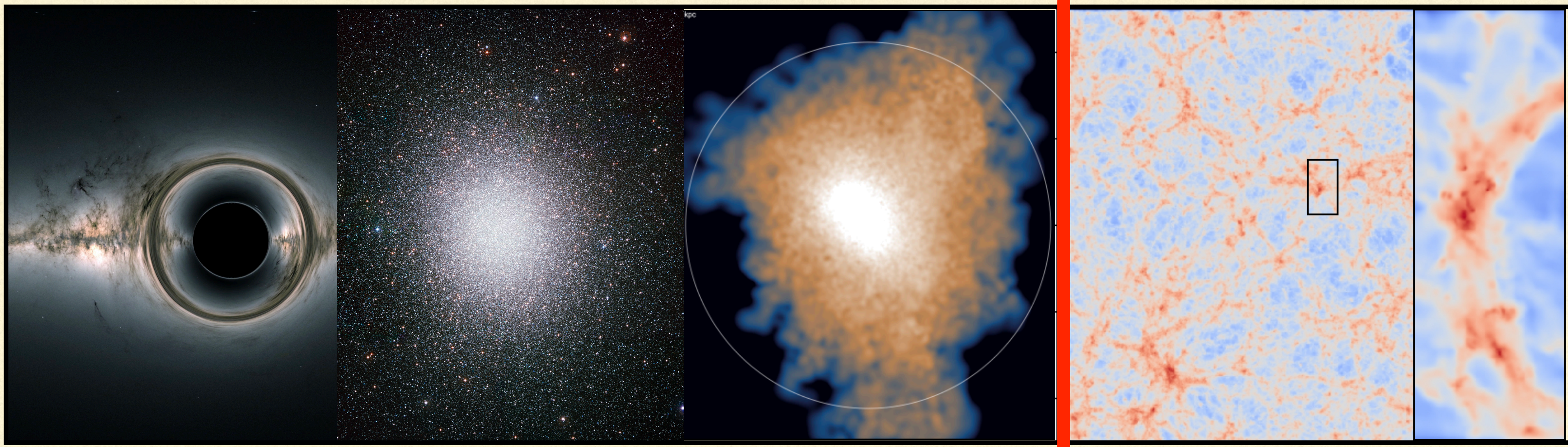

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From Poisson to Monge-Ampère

$$\frac{d^2x(t)}{dt^2} = -\nabla\phi(t)$$

$$\Delta\phi = 4\pi G(\rho - \bar{\rho})$$

Poisson equation

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$$det(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}} D^2\phi) = \frac{\rho}{\bar{\rho}}$$ Monge-Ampère equation

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$$D^2 = \left(\frac{d^2}{dx_i dx_j}\right)_{i,j}$$

From Poisson to Monge-Ampère

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In **one dimension**, Monge-Ampère is **equivalent** to Poisson

From Poisson to Monge-Ampère

A discret set of
N particles

Poisson

$$i = 0,1,\ldots,N$$

Monge-Ampère

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$$F_g = ?$$

From Poisson to Monge-Ampère

$$det(\mathbb{I} + \frac{1}{4\pi G\bar{\rho}}\, D^2\phi) = \frac{\rho}{\bar{\rho}} \qquad \& \qquad F_{\mathrm{g}} = -\, m\nabla_x \phi(x)$$

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With the following change of variable,

$$\psi(x) = \frac{|x|^2}{2} + \frac{1}{4\pi G\bar{\rho}}\phi(x)$$

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From Poisson to Monge-Ampère

Monge-Ampère equation

$$det(D^2\psi) = \frac{\rho}{\bar{\rho}} \quad \Leftrightarrow$$

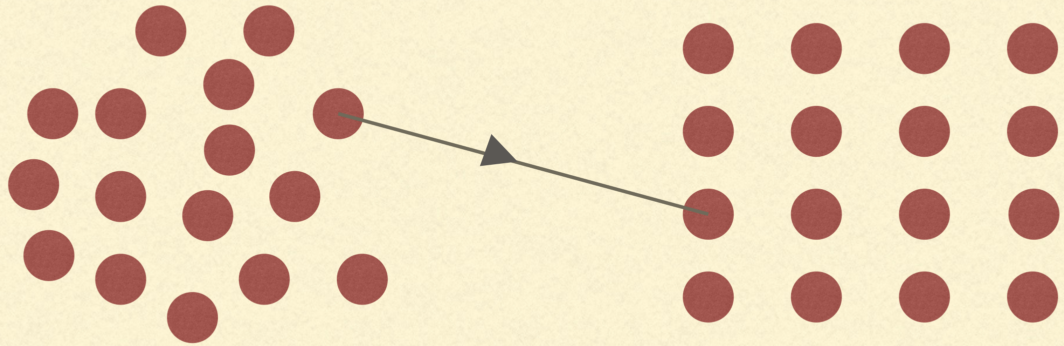
From Poisson to Monge-Ampère

Monge-Ampère equation

$$det(D^2\psi) = \frac{\rho}{\bar{\rho}}$$

$\Leftrightarrow$

Monge problem
or
What is the most efficient way of transporting one distribution of mass into another?



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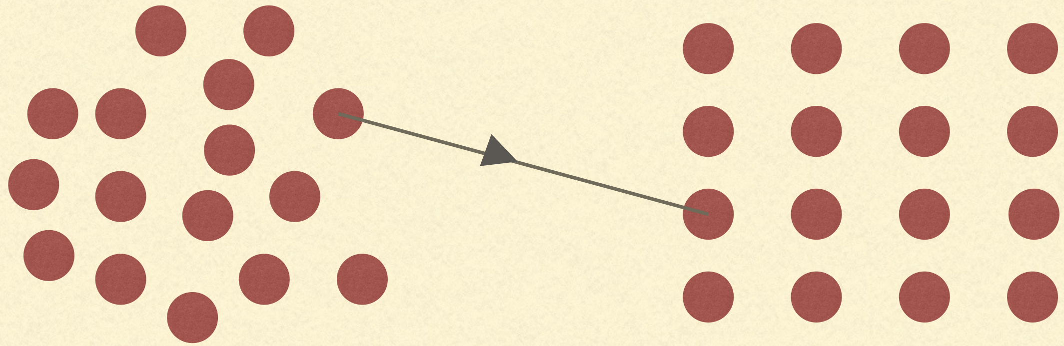

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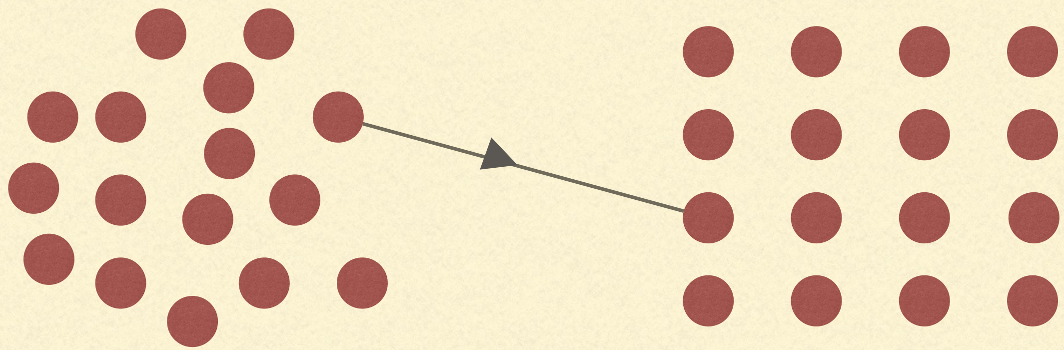

x_i $\qquad$ q_j

$$\inf \sum_i |x_i - q_j|^2$$

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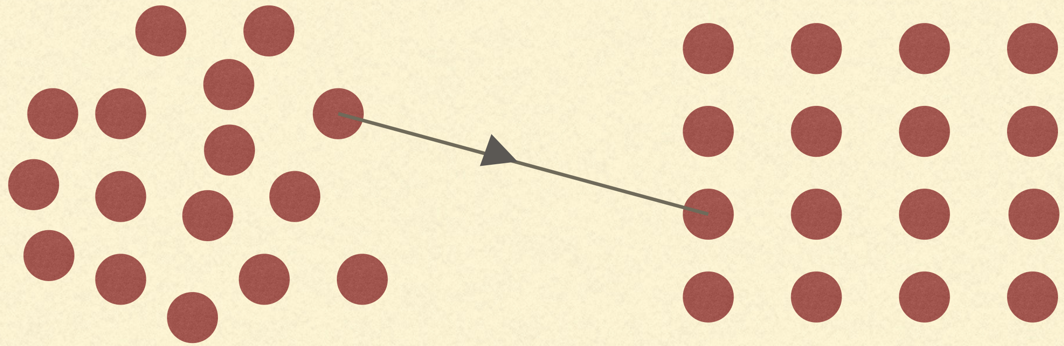

x_i **Purely combinatorial** q_j

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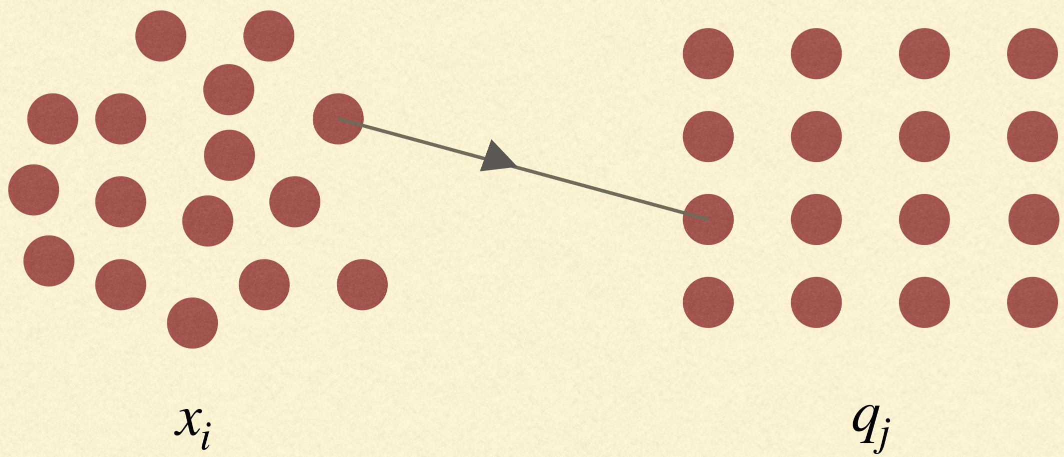

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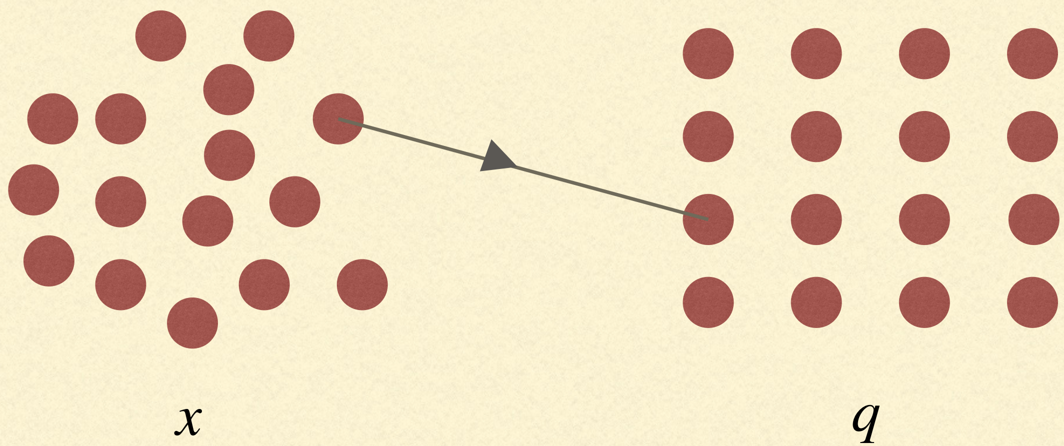

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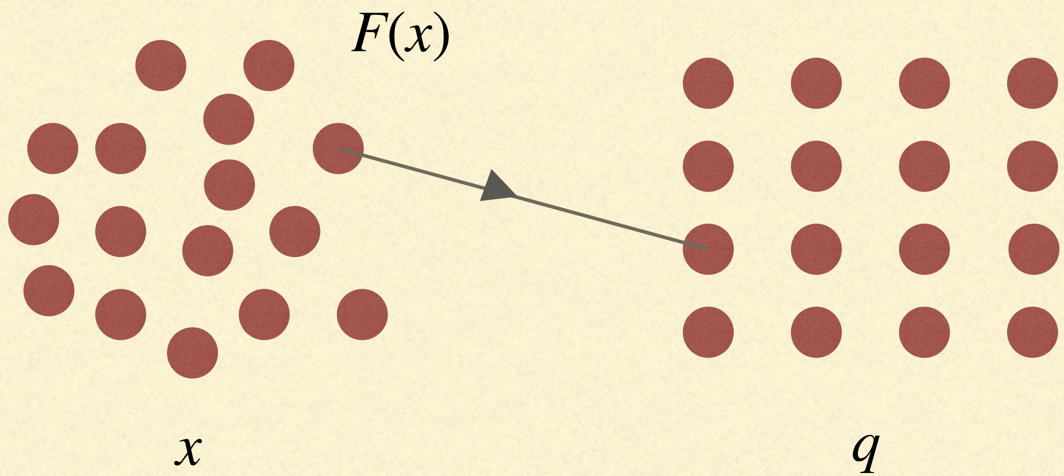

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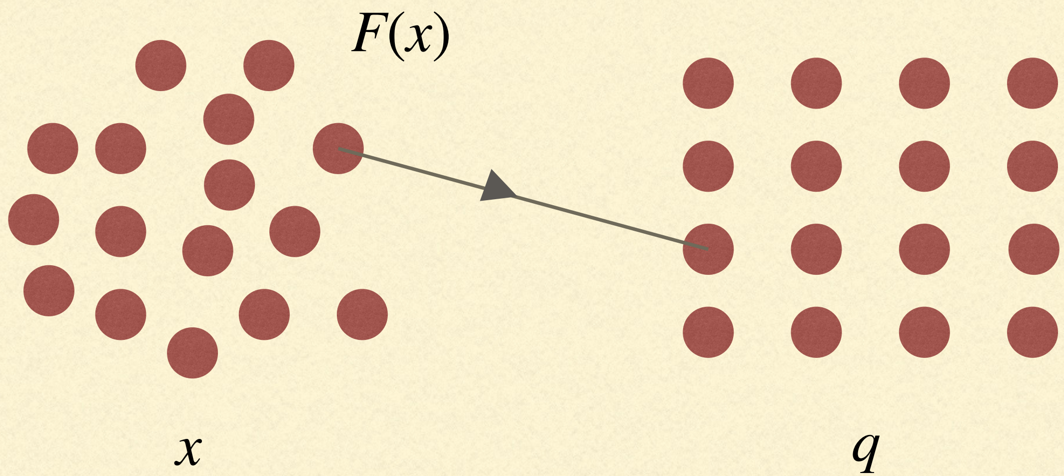
The mass conservation gives,

$$\bar{\rho} d^3 q = \rho(\mathbf{x}) d^3 x$$



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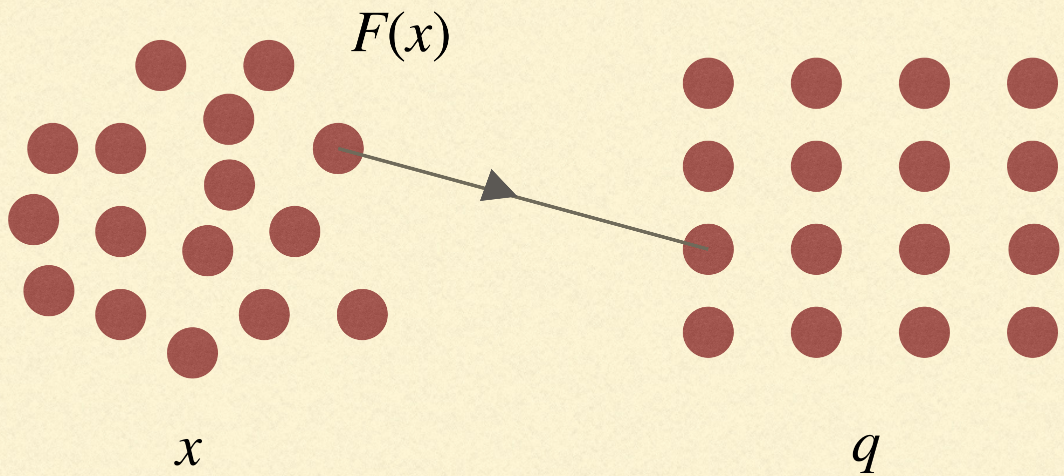

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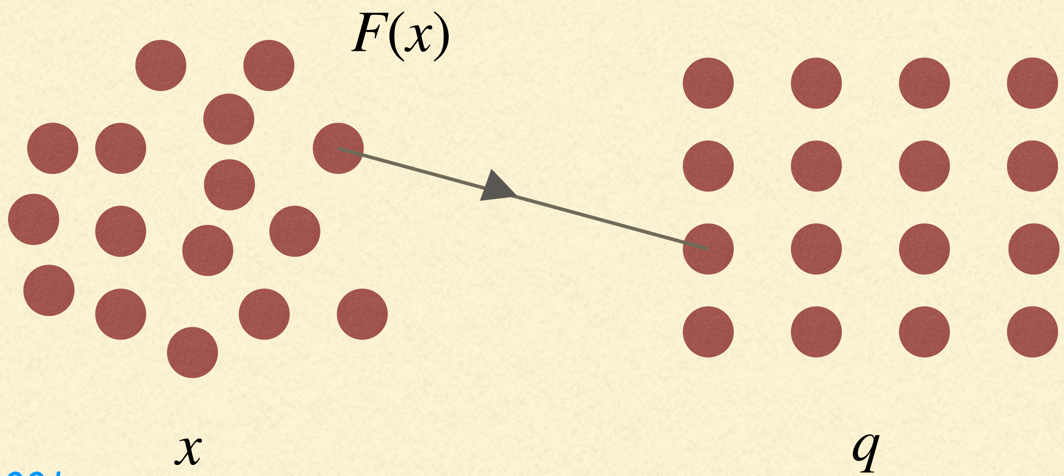

According to **Optimal Transport** theory, *Brenier 1991*

$$F = \nabla_x \psi(x)$$

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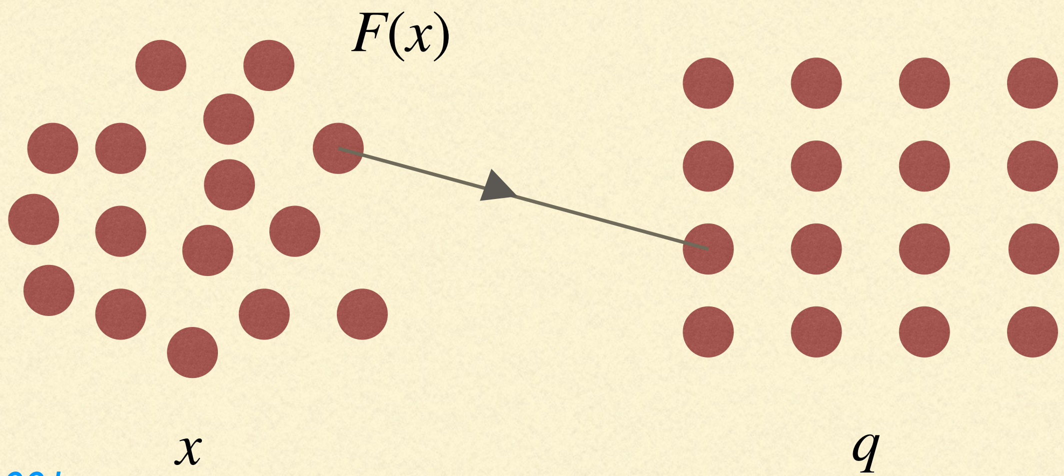

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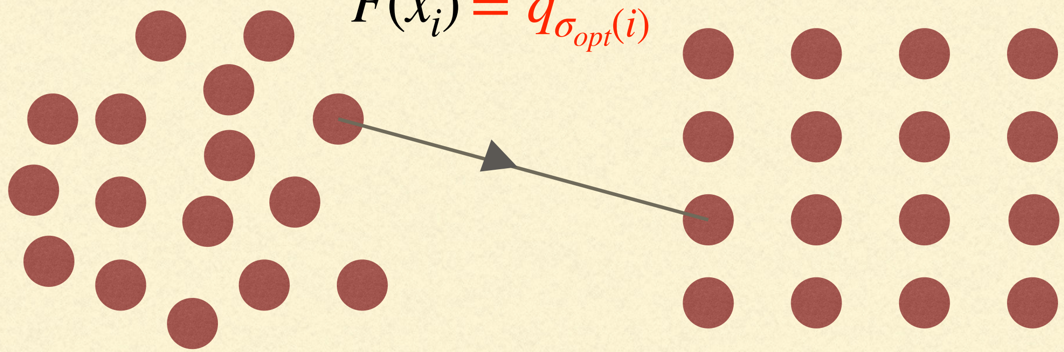

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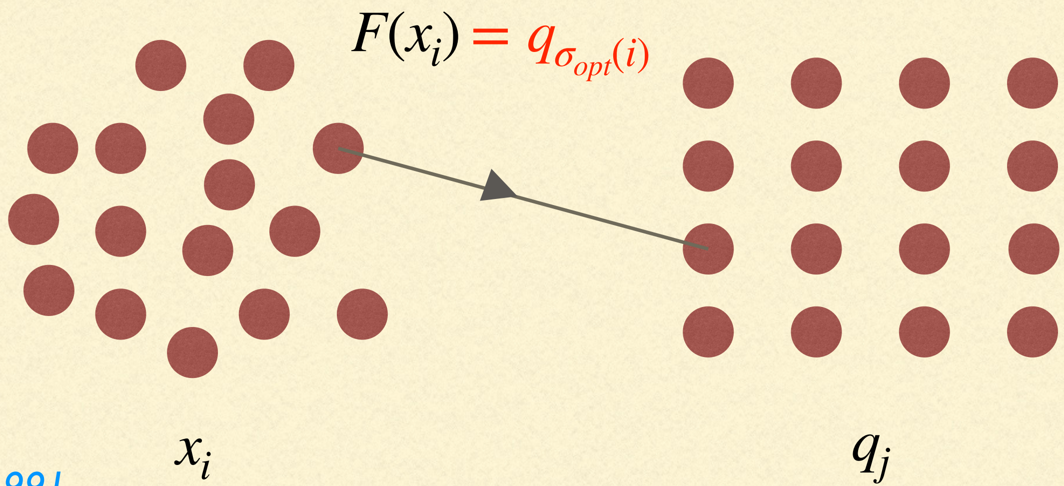

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Why Monge-Ampère gravity?

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1. Challenges to the ΛCDM Paradigm

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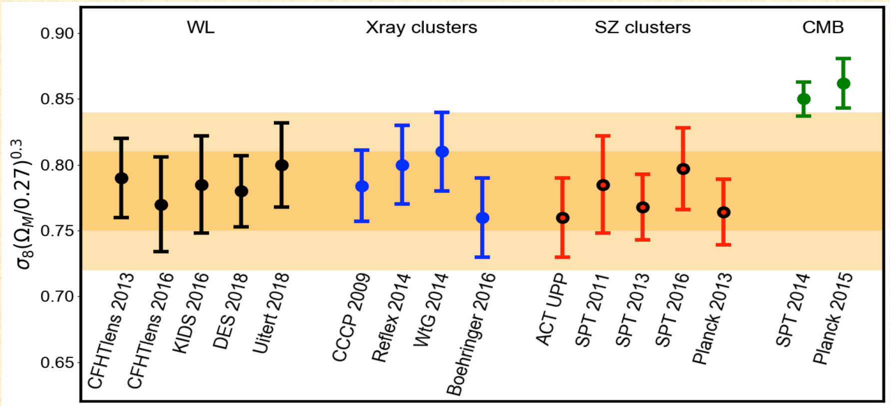
1 Challenges to the ΛCDM Paradigm

Tensions,

σ_8 tension



Douspis et al. 2018

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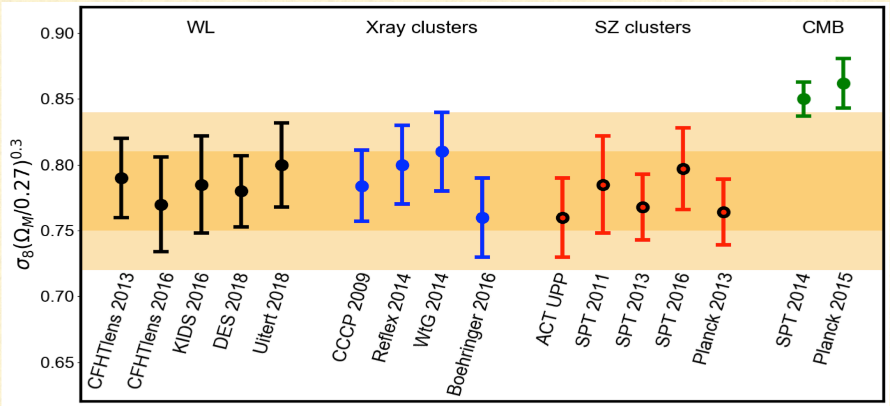

Douspis et al. 2018

- Neutrinos *Battye et al. 2014*
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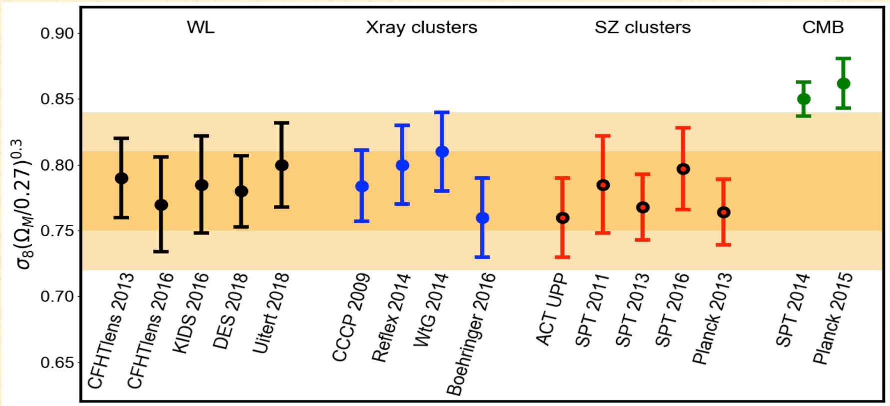

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An indication for a weaker gravity at low redshift?

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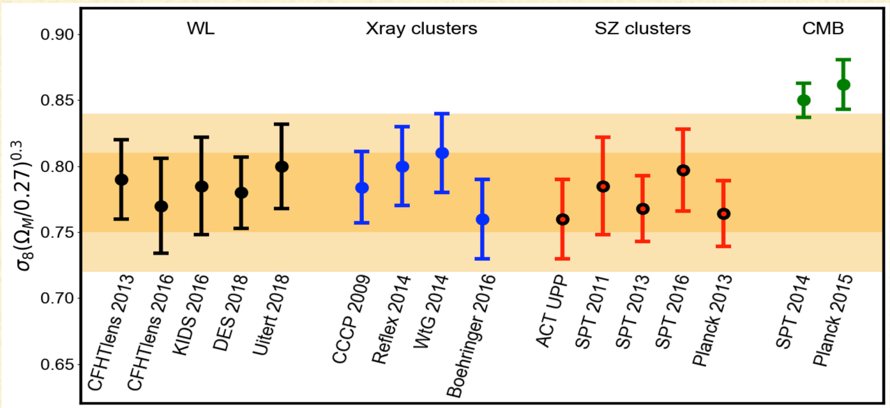
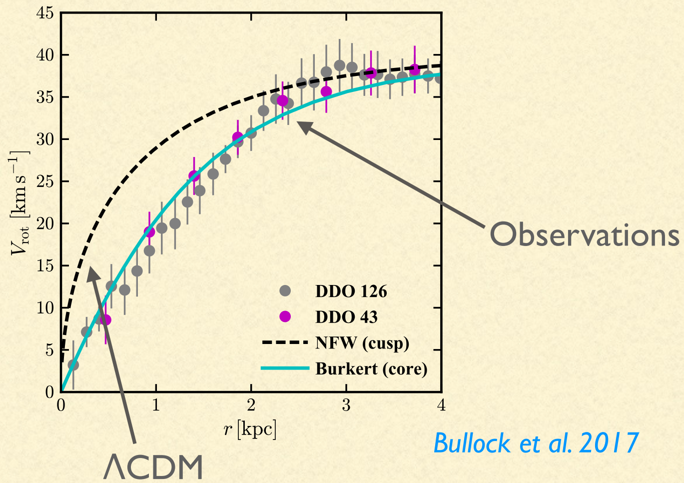


Douspis et al. 2018

At small scales,

Cusp-core problem



Bullock et al. 2017

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$$det(\mathbb{I} + \gamma D^2\phi) = \frac{\rho}{\bar{\rho}} \longrightarrow 1 + \gamma\Delta\phi + \mathcal{O}(\gamma^2) = \frac{\rho}{\bar{\rho}}$$

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Monge-Ampère gravitational force

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VS

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$$\begin{pmatrix} \partial_x^2\phi & \partial_x\partial_y\phi \\ \partial_x\partial_y\phi & \partial_y^2\phi \end{pmatrix} \quad \text{VS} \quad \begin{pmatrix} 1+\partial_x^2\phi & \partial_x\partial_y\phi \\ \partial_x\partial_y\phi & 1+\partial_y^2\phi \end{pmatrix}$$

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Cosmological simulation of Monge-Ampère gravity

Cosmological simulation of Monge-Ampère gravity

- Initial conditions
- Equations of motion in comoving coordinates
- How it works numerically?
- Comparing with Poisson *N*-body cosmological simulations
- Results

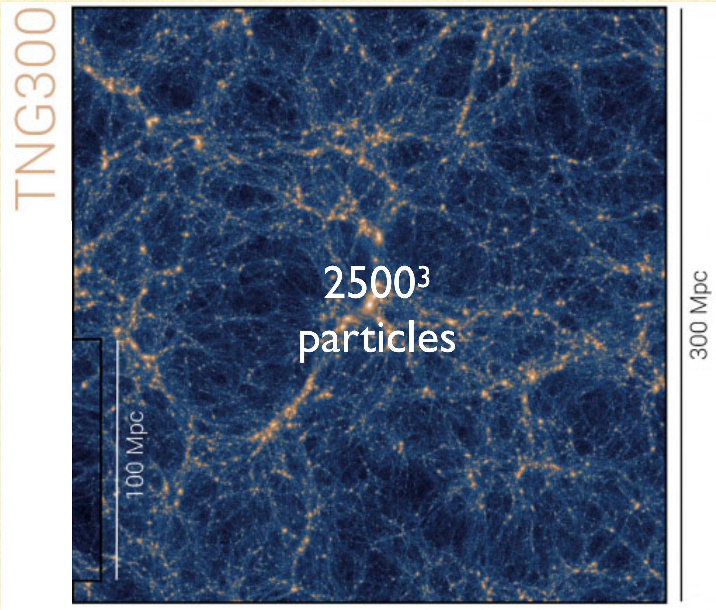
pyMAG 1.0

```
pip install pyMAG
```

Soon ….

Cosmological simulation of Monge-Ampère gravity

- Initial conditions



Springel et al. 2018

512^3 particles

205 Mpc/h ~ 300 Mpc

$z = 49 \longrightarrow z = 0$

$\Omega_\mathrm{m} = 0.3089$

$\Omega_\Lambda = 0.6911$

$H_0 = 67.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$

$m_{DM} \sim 10^{10} M_\odot$

Cosmological simulation of Monge-Ampère gravity

- Comparing with standard N-body cosmological simulation

Poisson

Monge-Ampère

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Poisson

GADGET - 2

A code for cosmological simulations of structure formation

Springel et al. 2018

Monge-Ampère

pyMAG 1.0

Boldrini et al. 2022, in prep

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Monge-Ampère

pyMAG 1.0

Boldrini et al. 2022,
in prep

$$\mathbf{F}_{\mathrm{g}} = \frac{3 H_0^2 \Omega_{\mathrm{m}} m}{2a^2} (\mathbf{x_i} - \mathbf{g_i})$$

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Tree-Code

Barnes and Hut, 1986

$$\mathcal{O}(N \log N)$$

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Cosmological simulation of Monge-Ampère gravity

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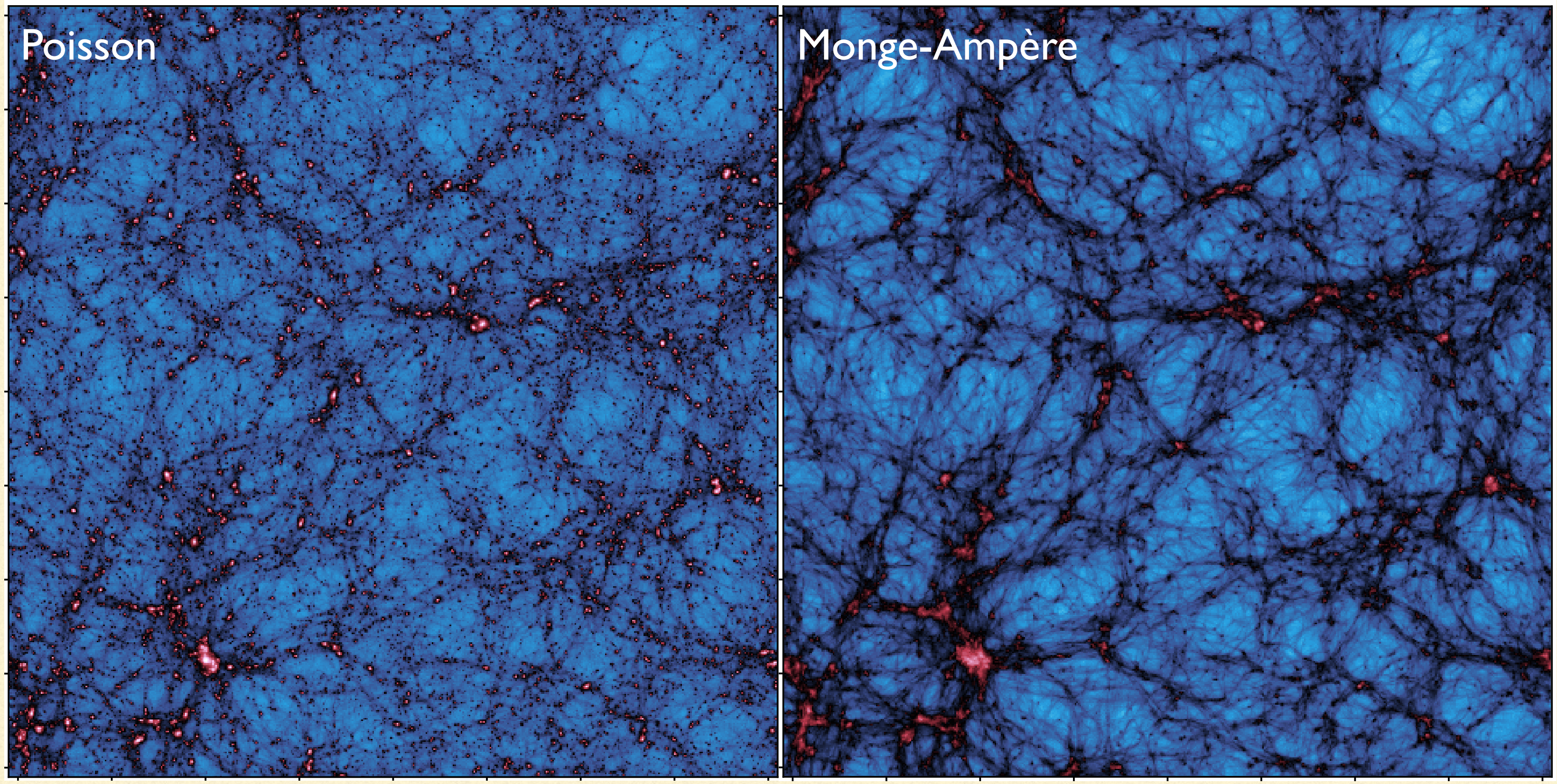
Tree-Code	Optimal transport algorithm
Barnes and Hut, 1986	*Lévy 2022*
$\mathcal{O}(N \log N)$	$\mathcal{O}(N \log N)$

Cosmological simulation of Monge-Ampère gravity

- Large scale-structures $z = 0$



Cosmological simulation of Monge-Ampère gravity

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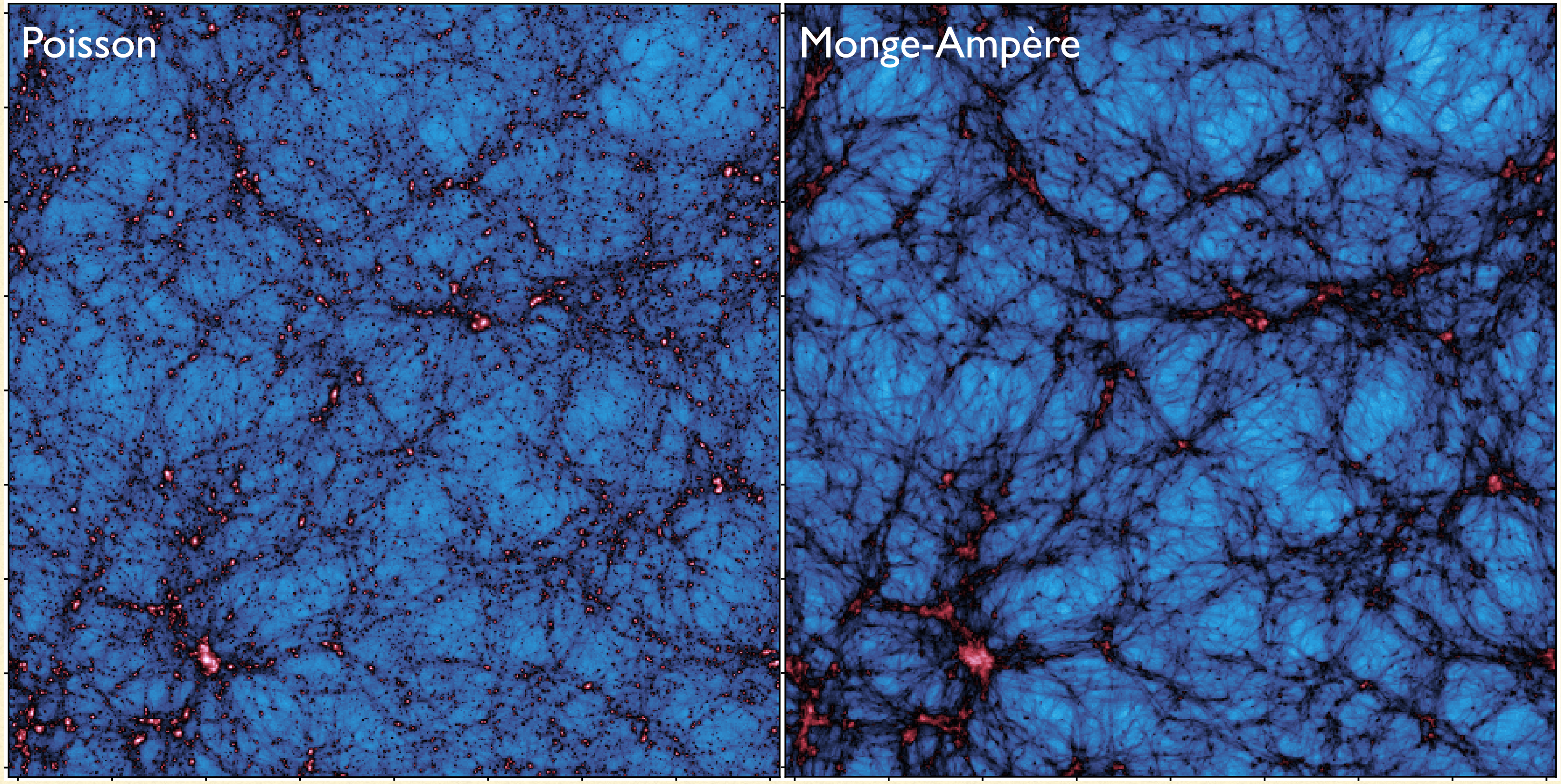
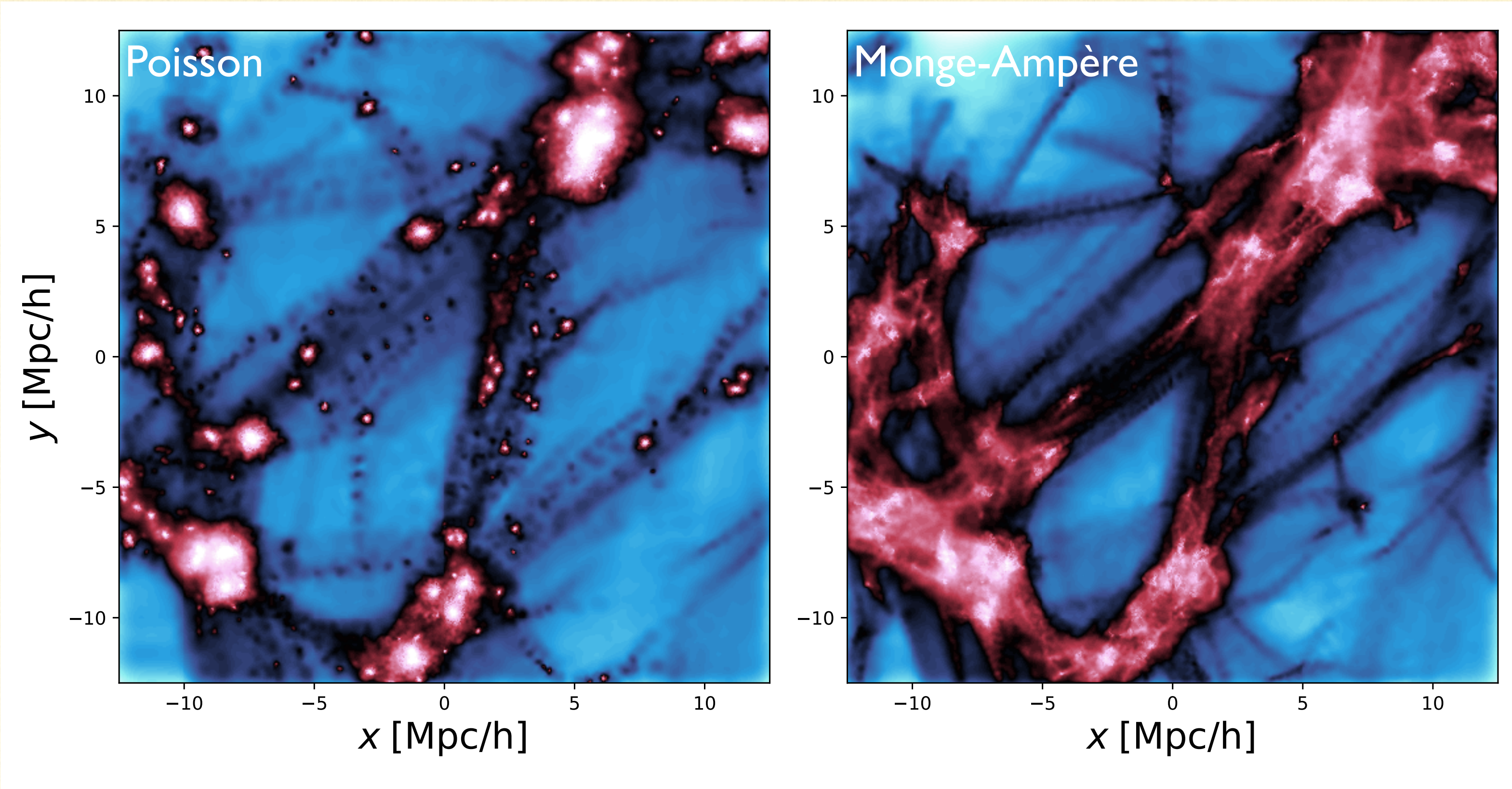


A **weaker gravitational** clustering

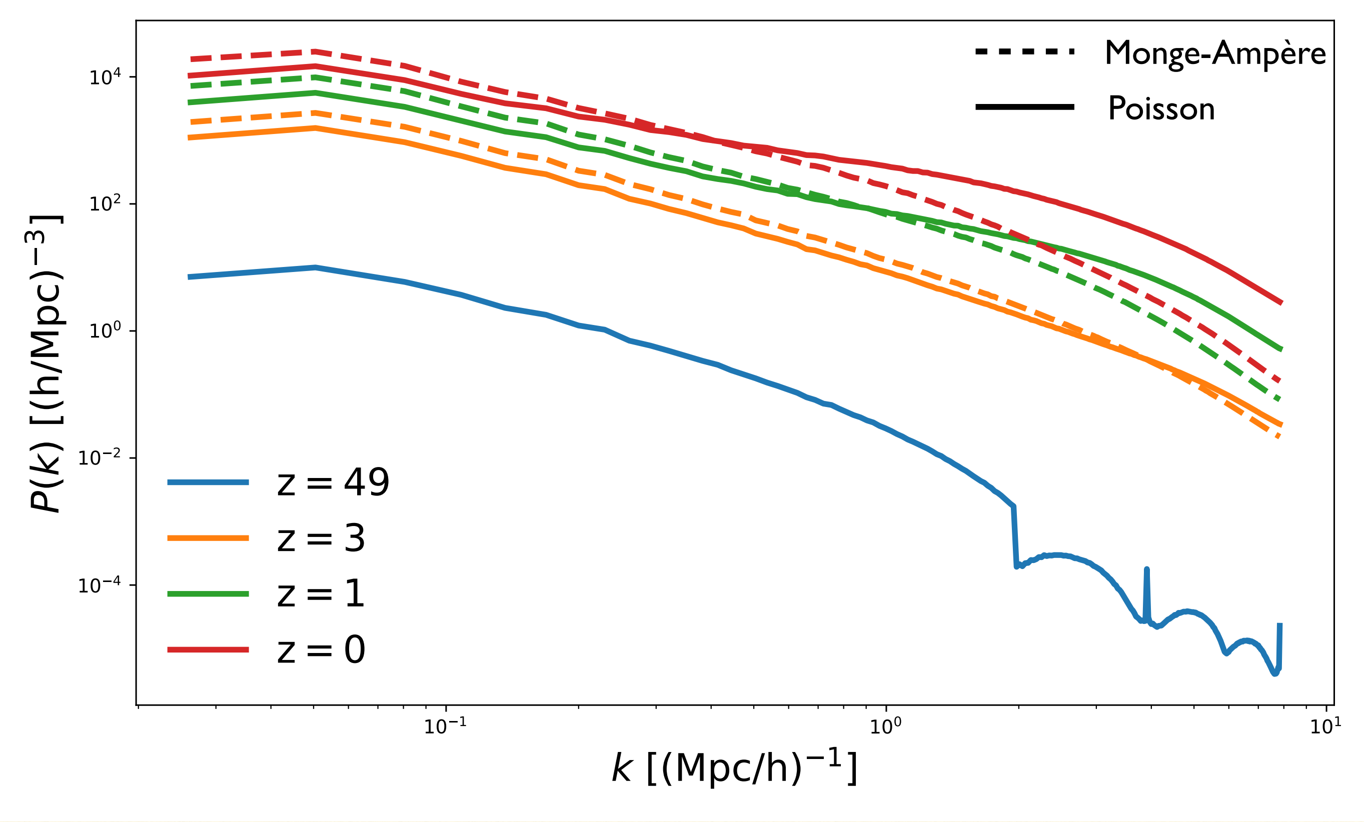
Cosmological simulation of Monge-Ampère gravity

- Zoom $z = 0$



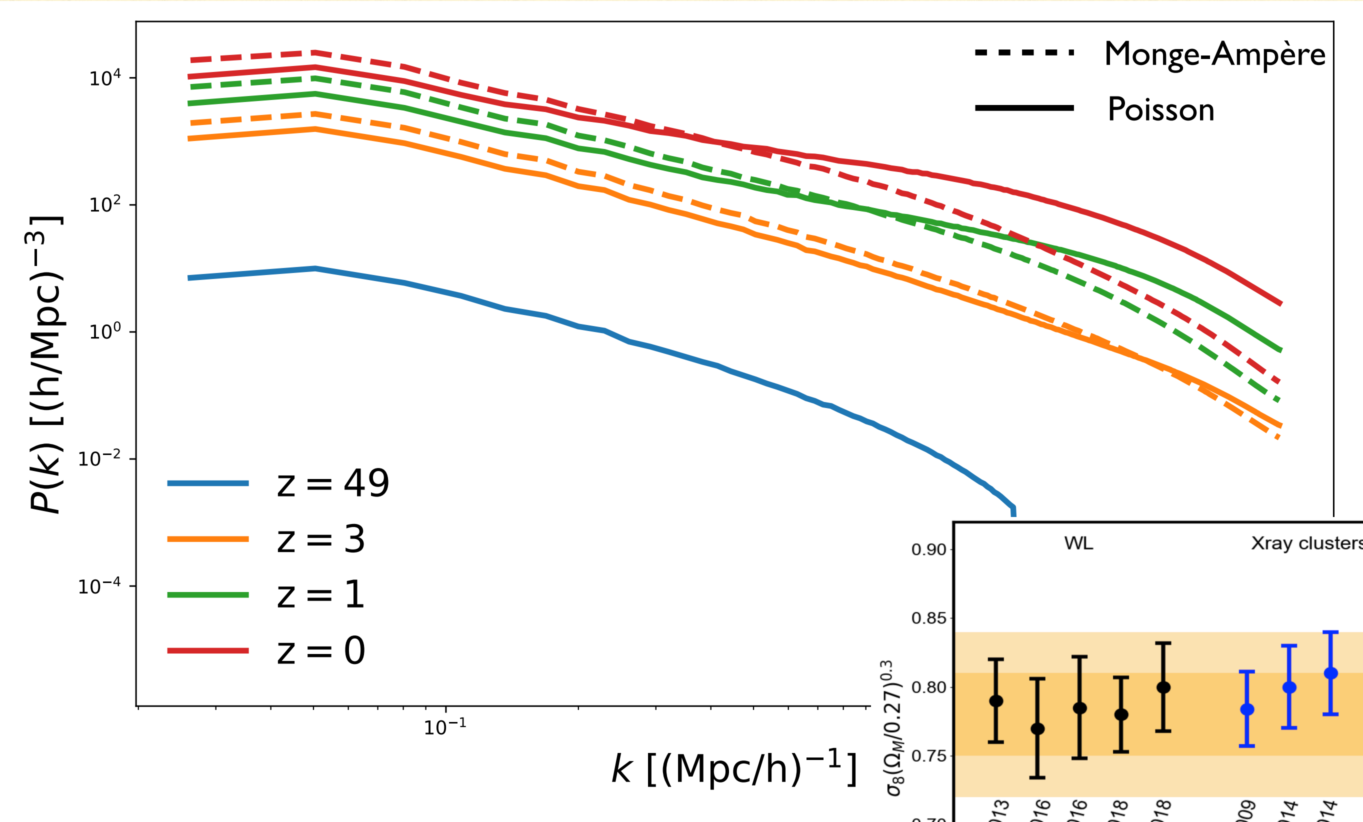
Cosmological simulation of Monge-Ampère gravity

- Power spectra



Cosmological simulation of Monge-Ampère gravity

- Power spectra

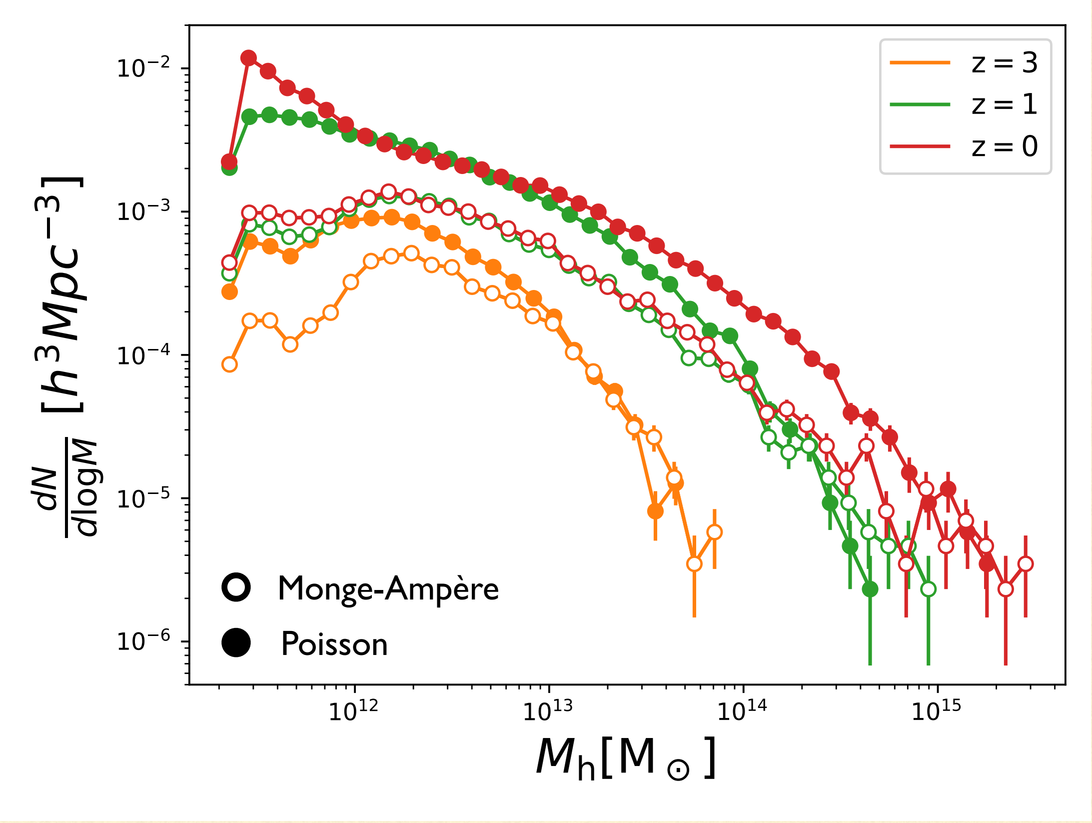


σ8 tension

Gravity is getting **weaker** at low z?

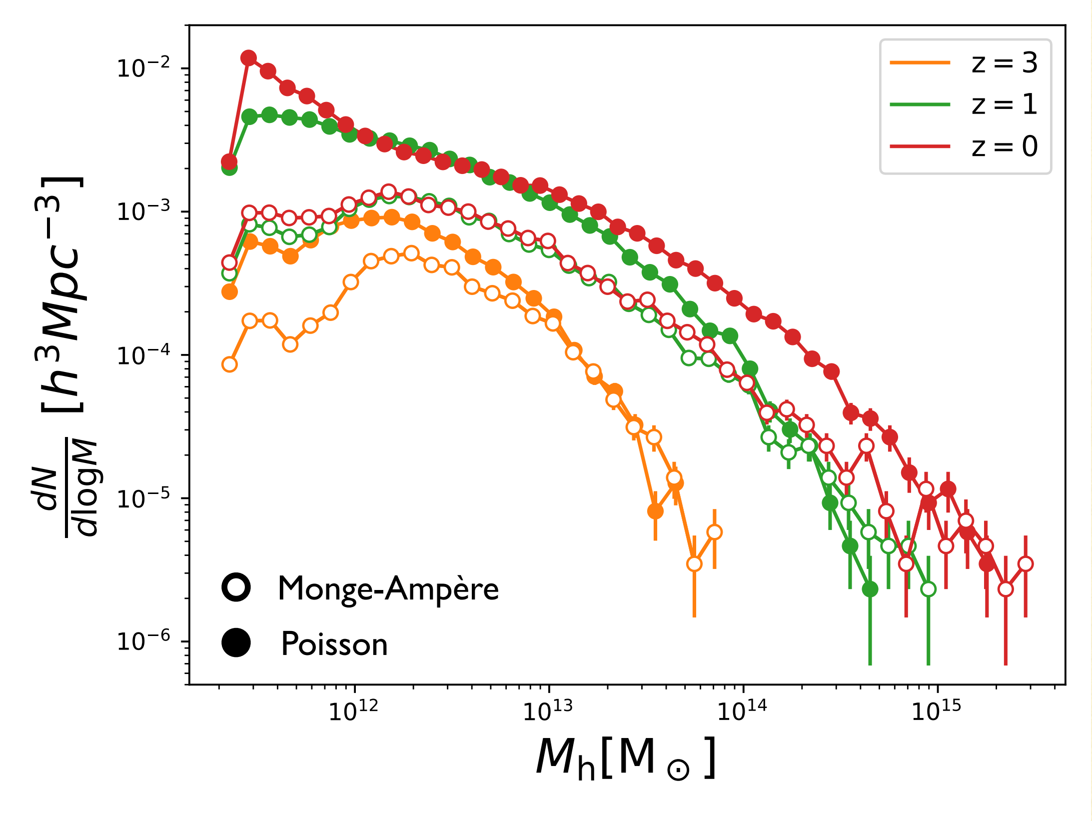
Cosmological simulation of Monge-Ampère gravity

- Halo mass function



Cosmological simulation of Monge-Ampère gravity

- Halo mass function



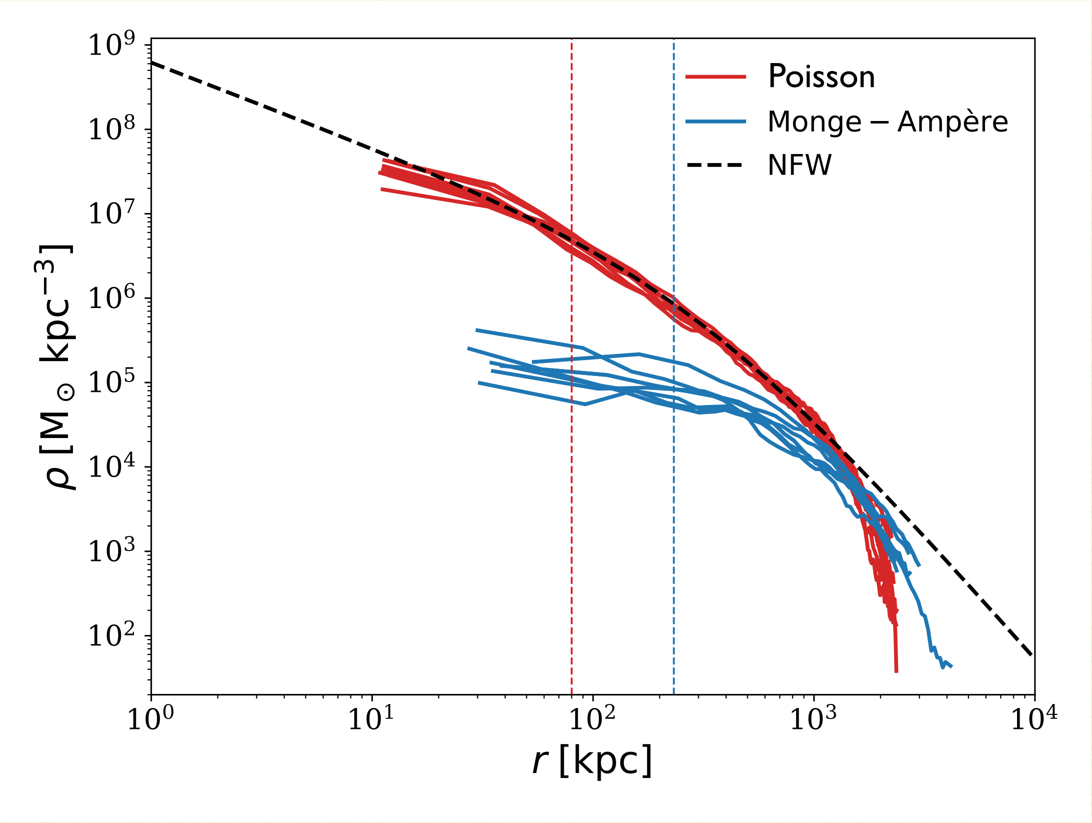
Poisson | Monge-Ampère

$N_h = 66091 \,|\, 16057$

4 times less halos at $z = 0$

Cosmological simulation of Monge-Ampère gravity

- Dark matter density profiles



Cosmological simulation of Monge-Ampère gravity

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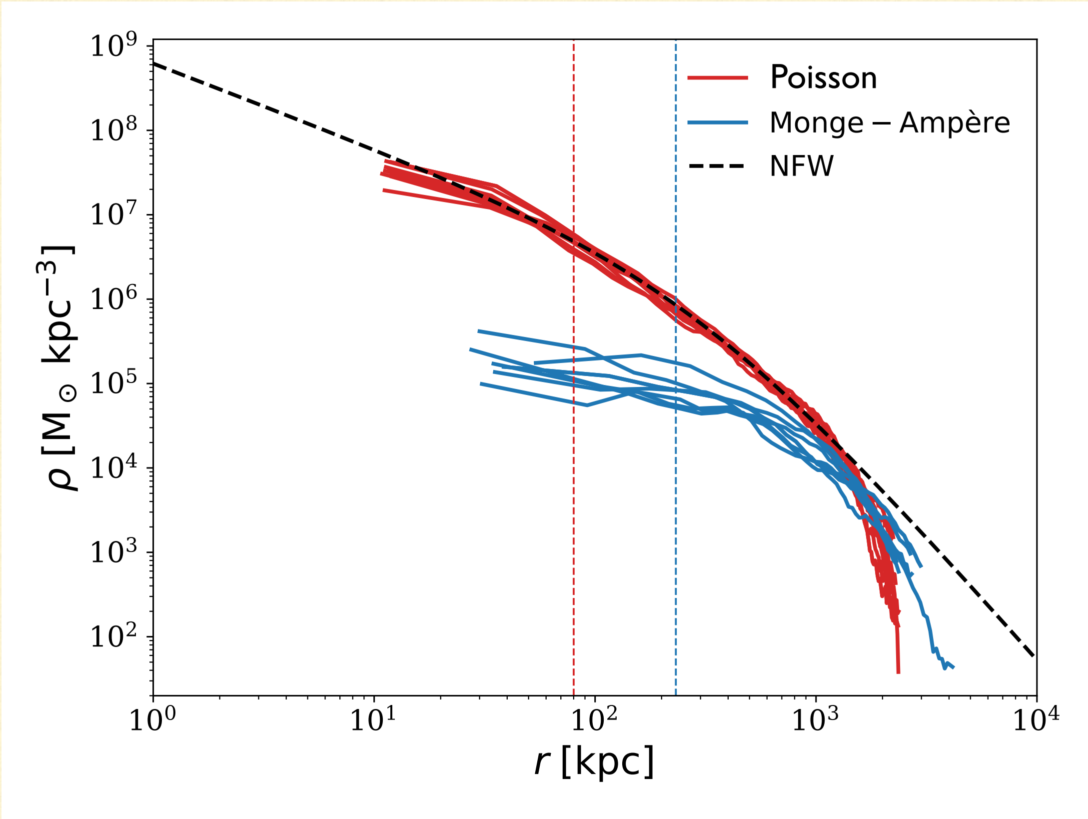
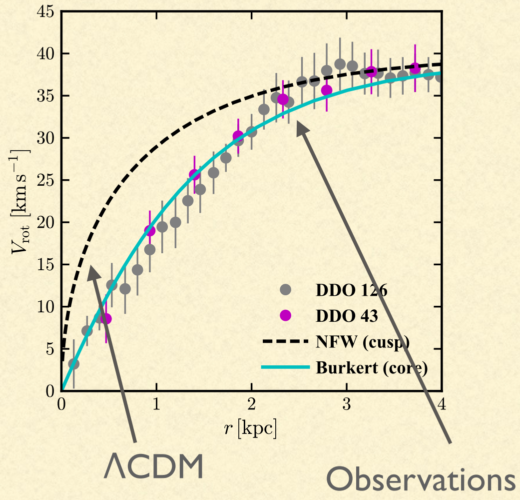


Cusp-core problem



A potential solution to the cusp-core problem

Why Monge-Ampère is getting weaker at low redshift?

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- Monge-Ampère equation in spherical symmetry

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Why Monge-Ampère is getting weaker at low redshift?

- Monge-Ampère equation in spherical symmetry

$$\det\left(D^2\Psi\right) = \frac{\rho}{\bar{\rho}},$$

$$F_{\mathrm{g}} = m4\pi G\bar{\rho}\left(x - \nabla\Psi\right),$$

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↓

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🍋 Monge-Ampère equation in spherical symmetry

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For a point mass,

$$\rho(r) = m\delta(r)$$

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$$F_g = \frac{-3mH_0^2\Omega_m r}{2}\left(\left[1 + \frac{R_{MA}^3}{r^3}\right]^{1/3} - 1\right),$$

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If $r \gg R_{\mathrm{MA}}$,

If $r \ll R_{\mathrm{MA}}$,

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If $r \gg R_{\rm MA}$,

$$F_{\rm g} \sim \frac{-1}{r^2}, \longrightarrow \text{Poisson regime}$$

If $r \ll R_{\rm MA}$,

$$F_{\rm g} \sim r - R_{\rm MA}, \longrightarrow \text{Monge-Ampère regime}$$

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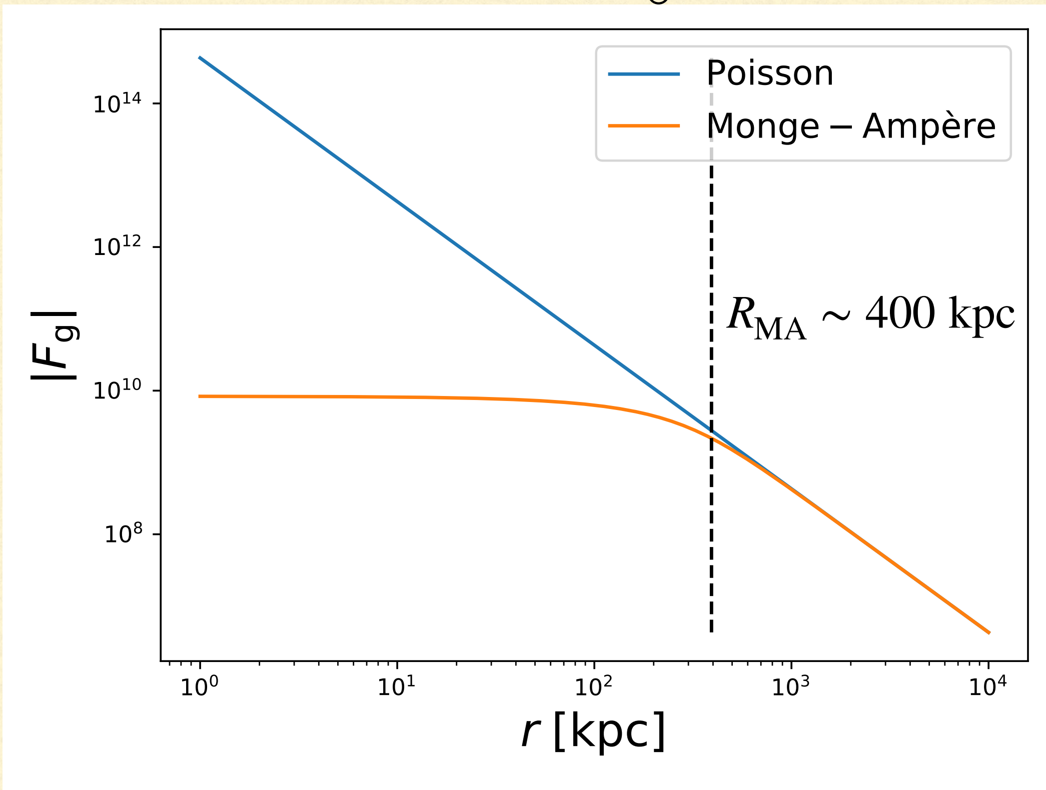

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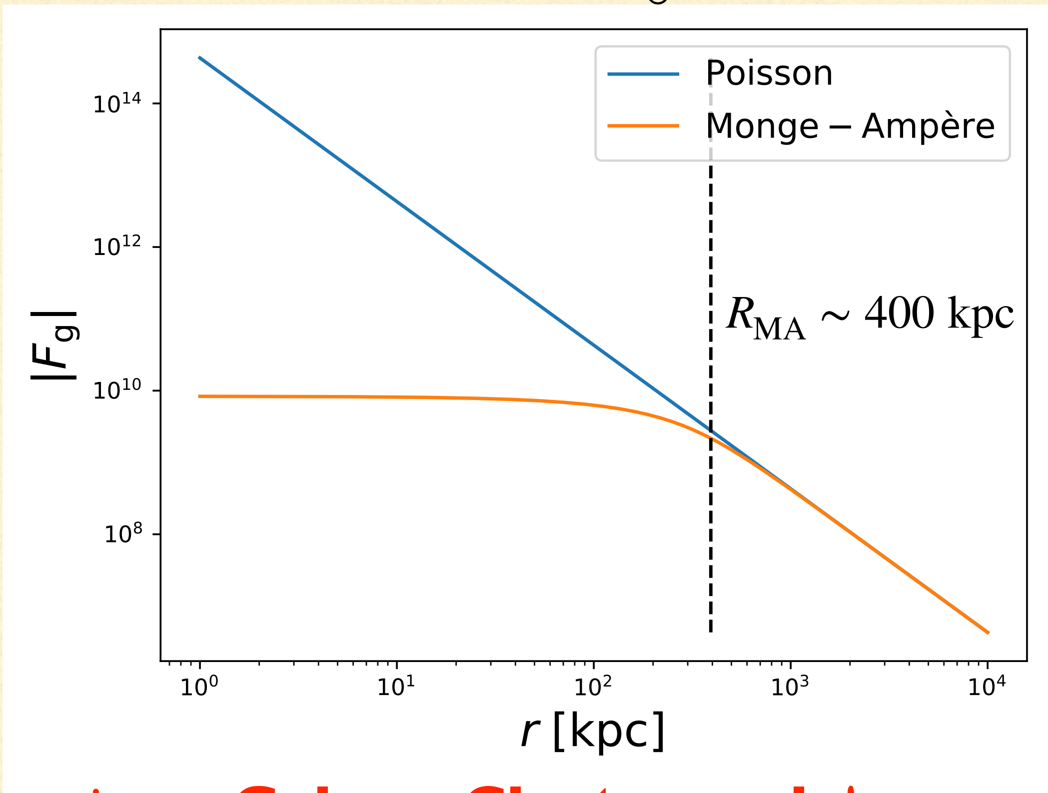

Monge-ampère gravity is acting at **Galaxy-Cluster scale**!

Why Monge-Ampère is getting weaker at low redshift?

🍋 Structure formation scenarios

⟶ Bottom-up hierarchical formation

⟶ Top-down hierarchical formation

Why Monge-Ampère is getting weaker at low redshift?

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→ Bottom-up hierarchical formation (ΛCDM)

Mergers of both early formed and later formed halos.

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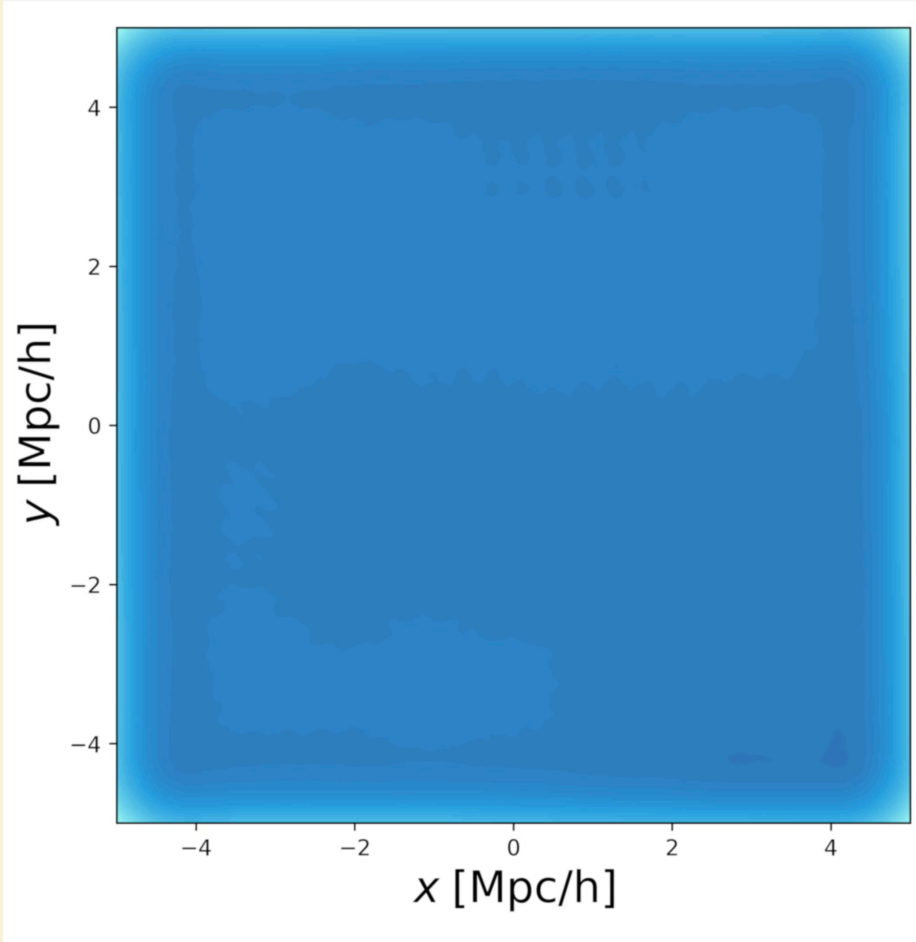

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→ Bottom-up hierarchical formation (ΛCDM)

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Sheets collapsing into filaments, collapsing into halos.

Paduroiu et al. 2015

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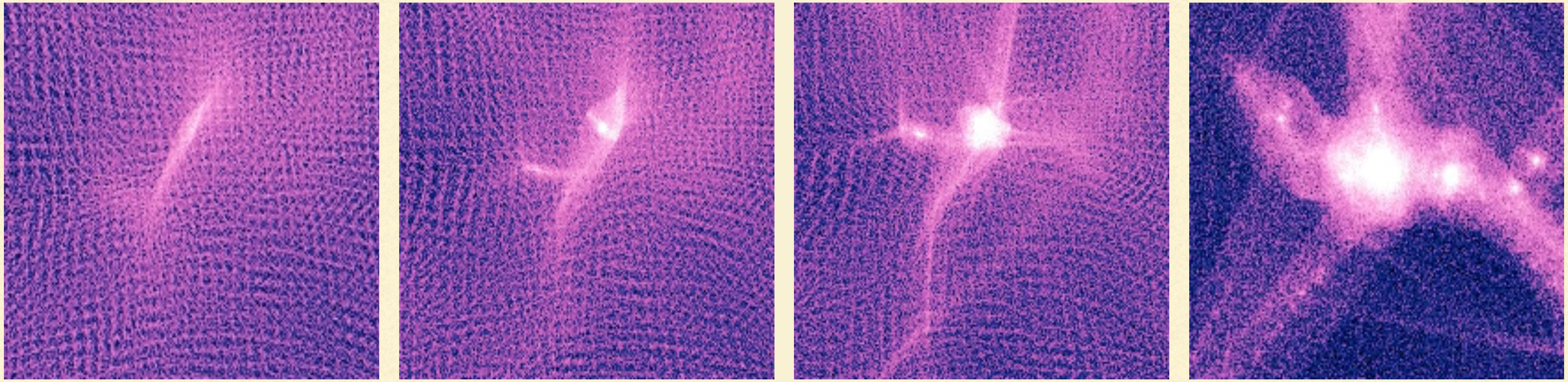

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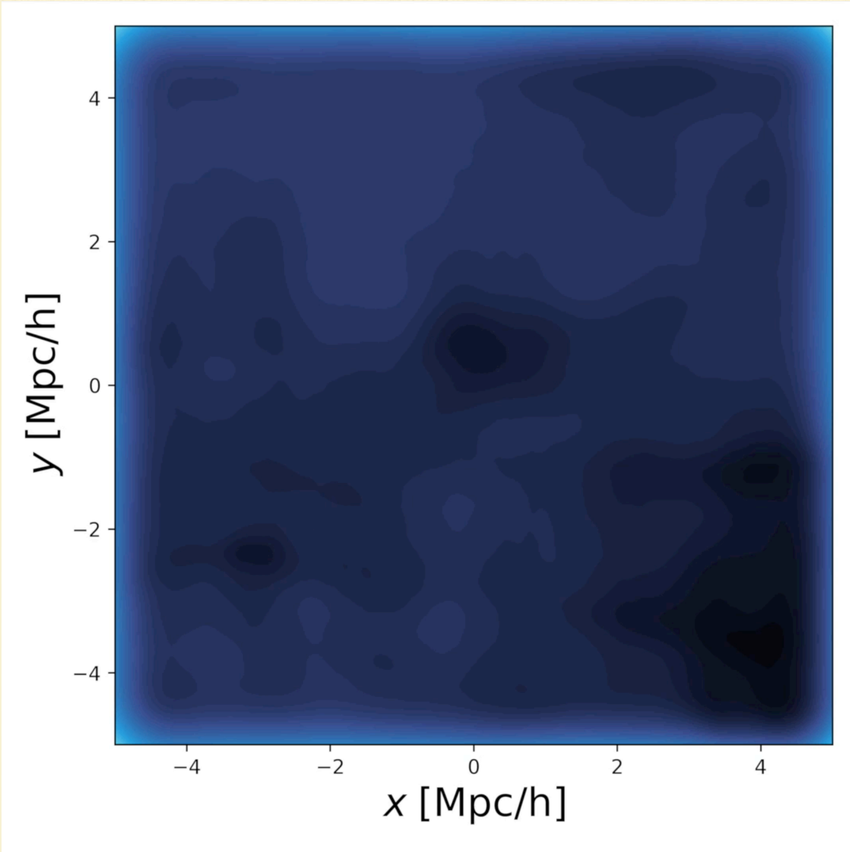

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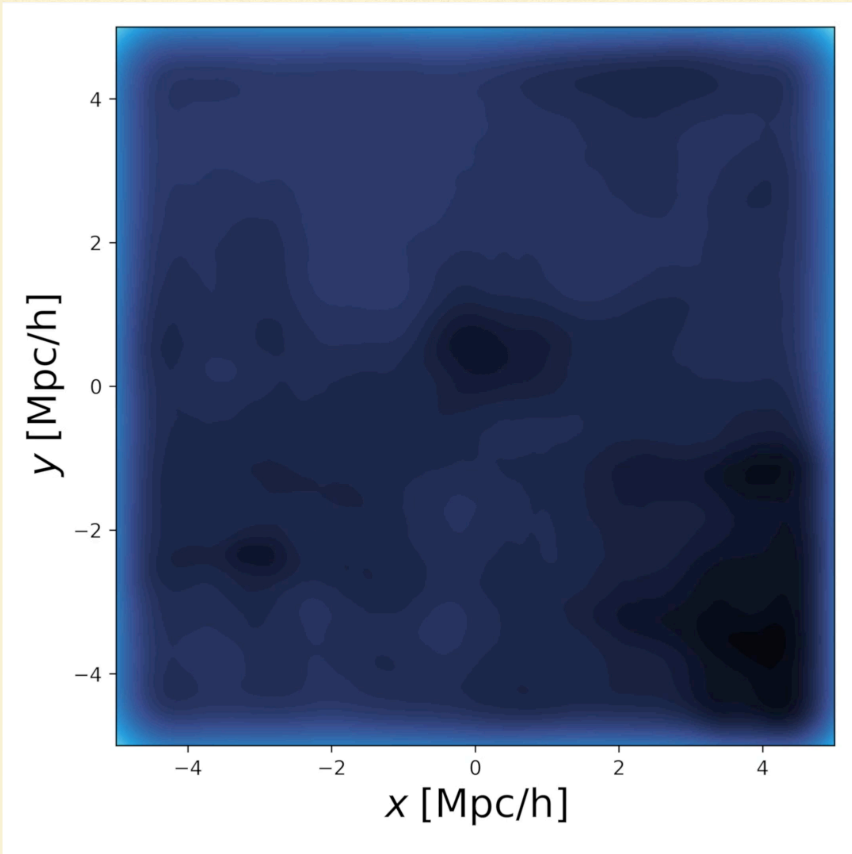
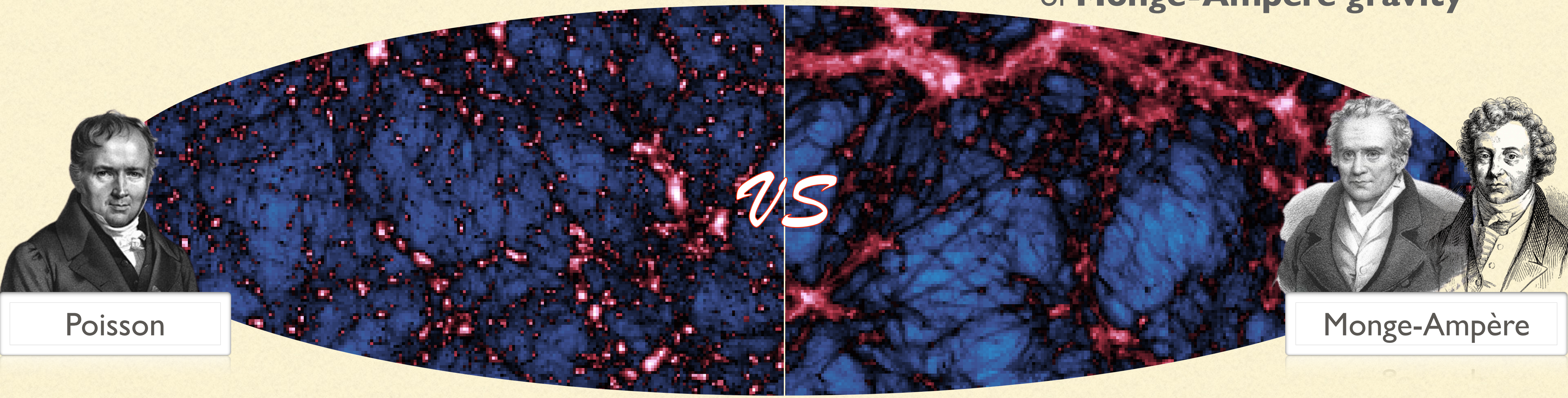


Monge-ampère exhibits **top-down structure formation**!

Conclusion

A **motivated** gravity theory

First N-body cosmological simulation
of **Monge-Ampère gravity**



State-of-the-art algorithms
from computer science for **Optimal Transport** problem

A **weaker gravitational** clustering
for dark matter particles

Promising alternative theory of gravity

Future projects

- σ_8 tension

  Evaluating the constraints from CMB and weak lensing measurements

- Cusp-core problem

  Running smaller cosmological boxes and zoom simulations

- Cosmic filaments

  Extracting the cosmic web with ***DisPerSE****, as alternative cosmological probe*

  Confirming the structure formation scenario!

Thanks for your attention!