Quantum Discord and Decoherence of inflationary perturbations

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Amaury Micheli [1] [2]

arXiv:2112.05037 AM, Jérôme Martin [1] , Vincent Vennin [1] [3]

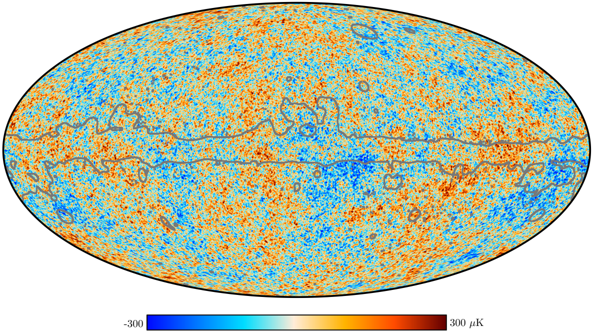
Institut d'Astrophysique de Paris [1]

Irene-Joliot Curie Laboratory, Orsay [2] APC, Paris [3]

INTRODUCTION : QUANTUM FEATURES IN THE EARLY UNIVERSE ?

CONTEXT I, INHOMOGENEITIES IN THE EARLY UNIVERSE

→ CMB [1] : isotropic temperature $T \sim 3K$ + **small anisotropies** $\Delta T/T \sim 10^{-4}$



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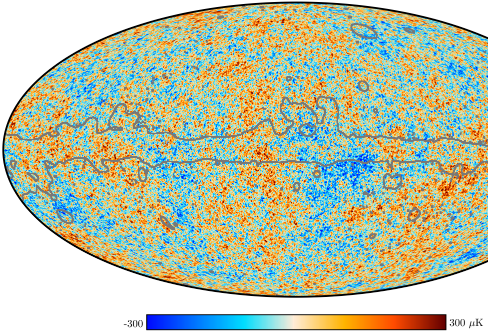

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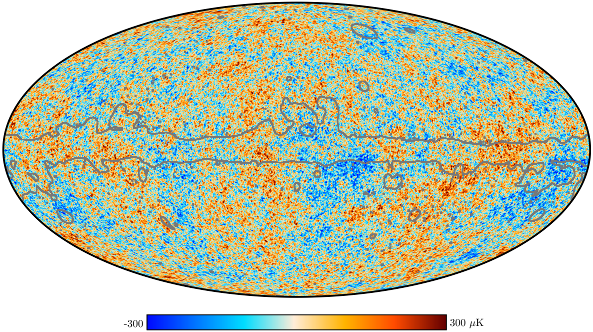

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→ Proposition ∼80s [2] : Inhomogeneities come from minimal (quantum) vacuum fluctuations at the beginning of inflation stretched to cosmological scales by expansion !

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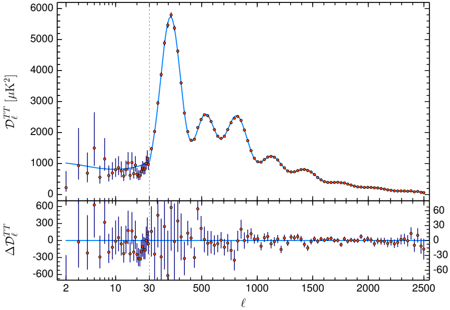

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CHARACTERIZING QUANTUMNESS OF INFLATIONNARY PERTURBATIONS

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$\mathcal{D}\left(\mathcal{S}_1, \mathcal{S}_2\right) > 0$ = **quantum state**

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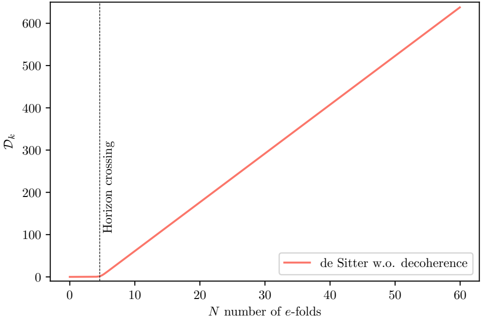

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Take-home message 1

Without decoherence Quantum Discord is strongly amplified by inflation and final state is very quantum in this sense. [2]

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Can this result be due to oversimplified models ?

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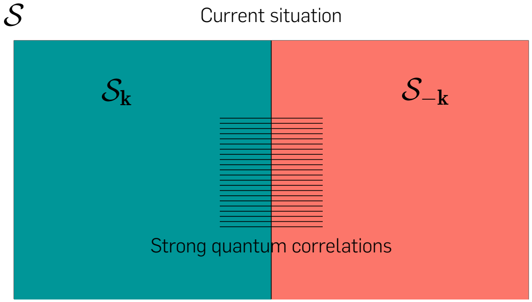
DECOHERENCE AND LOSS OF QUANTUMNESS

NON-LINEARITIES, INTERACTIONS : DECOHERENCE



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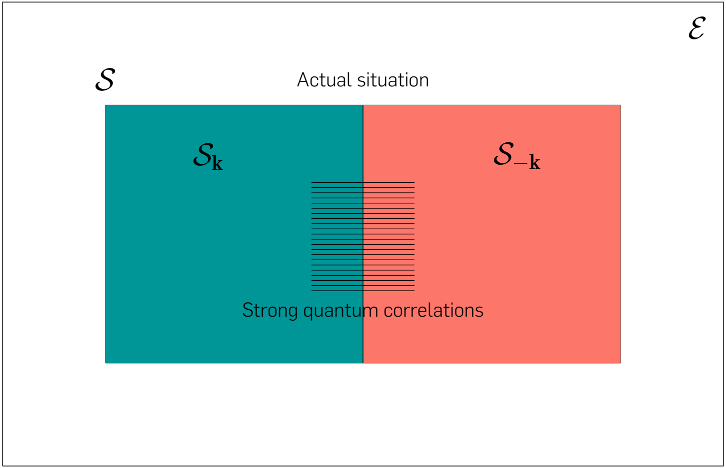

In fact $\mathcal{S}$ has an environment $\mathcal{E}$ (e.g. $\mathcal{S}_{\pm\mathbf{k}'}$ with $\mathbf{k}' \neq \mathbf{k}$) or other fields.

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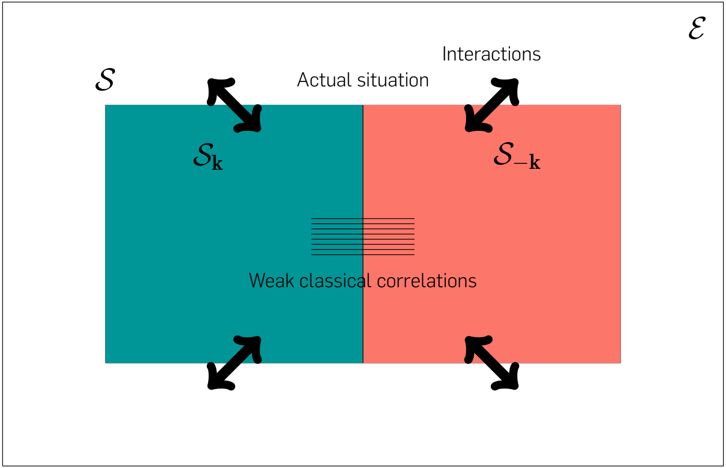

Interactions $\mathcal{S}$ / $\mathcal{E}$ destroy correlations $\mathcal{S}_{\mathbf{k}}$ / $\mathcal{S}_{-\mathbf{k}}$: decoherence.

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Assumptions :

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→ Further computations [3] decoherence term $\propto k_\Gamma{}^2 a^{p-3}$.

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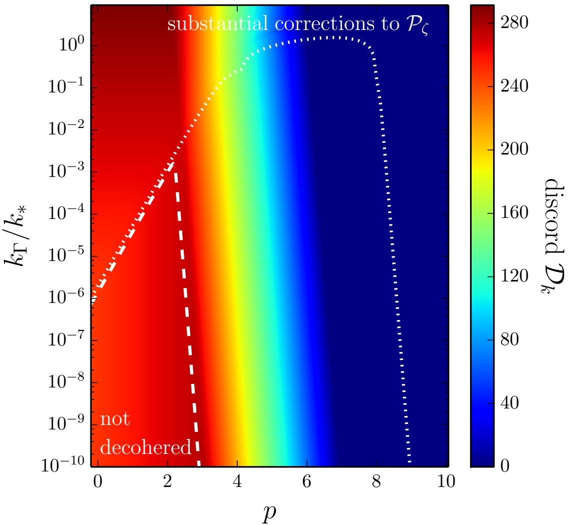
Is Quantum Discord spoiled by decoherence?

COMPETITION OF ENTANGLEMENT AND DECOHERENCE



[4]

4. [arXiv:2112.05037 Martin et al., 2021]

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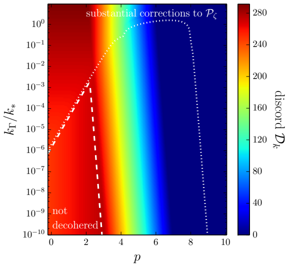

Take-home message 2 [4]

Decoherence does not always destroy Quantum Discord. Its fate is the result of a competition with generation of quantum correlations by inflation.

4. [arXiv:2112.05037 Martin et al., 2021]

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- → Compare the effect of decoherence on different criteria (Bell Inequalities, non-separability etc.).
- → Use a more realistic interaction for decoherence, for instance non-linearities of pure gravity and see whether quantum discord is destroyed or not.

Thank you for your attention !

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EXTRA

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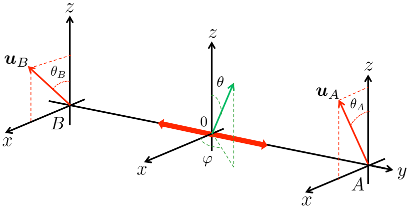

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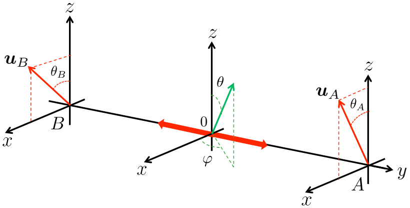

- → Smart combination of measurements $\mathcal{O}$.

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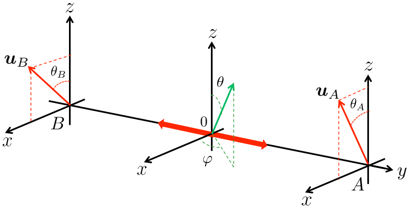

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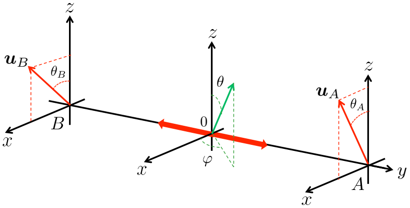

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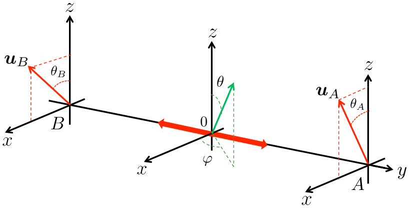

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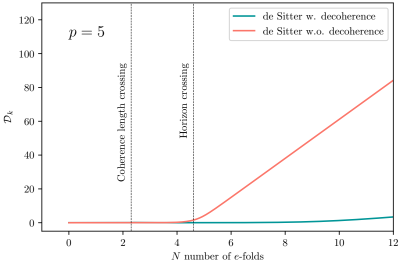
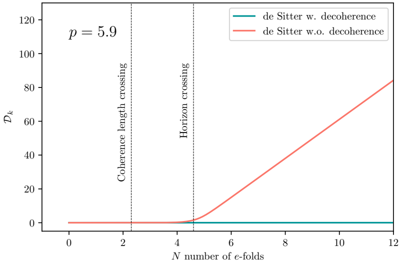
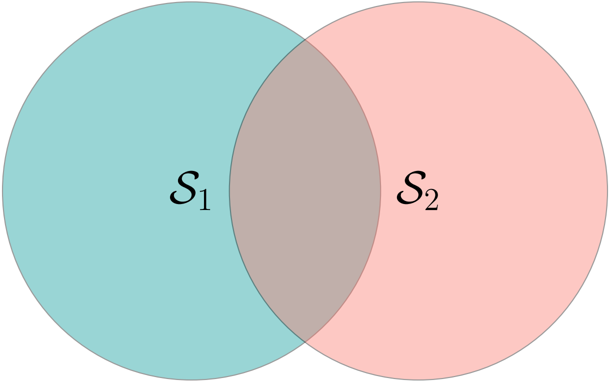
If measure $\langle \hat{\mathcal{O}} \rangle > 2$, correlations stronger than classical ones $\rightarrow$ quantum state.

GROWTH OF $\mathcal{D}_K$ AND VALUES p





MUTUAL INFORMATION



$$\mathcal{I}\left(\mathcal{S}_1, \mathcal{S}_2\right) = H\left(\mathcal{S}_1\right) + H\left(\mathcal{S}_2\right) - H\left(\mathcal{S}\right)$$

DECOHERED INFLATIONNARY FLUCTUATIONS

→ Environment for $\mathcal{S}$? Modeled by Lindblad equation + linear interaction with strength

$$k_\Gamma^2 \left(\frac{a}{a_\star}\right)^{p-3} H \left(1 - \frac{k\ell_E}{a}\right) . \tag{1}$$

DECOHERENCE, LINDBLAD EQUATION

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$$\frac{\mathrm{d}\hat{\rho}_{\mathcal{S}}}{\mathrm{d}\eta} = -g^2\eta_{\rm C} \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\, C_{\mathcal{E}}\left(\mathbf{x},\mathbf{y}\right) \left[\hat{O}_{\mathcal{S}}(\mathbf{x}), \left[\hat{O}_{\mathcal{S}}(\mathbf{y}), \hat{\rho}_{\mathcal{S}}\right]\right] \ .$$

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Assumptions :

→ Perturbation $g \ll 1$.

→ $\mathcal{E}$ stationnary and not perturbed by $\mathcal{S}$.

→ Consider evolution of $\mathcal{S}$ for $\eta \gg \eta_C$ auto-correlation time of $\mathcal{E}$.

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→ Free-parameters : k_Γ and p [5].

5. [Martin and Vennin, 2016]