Lensing in cosmology

First case: lensing of a quasar by a galaxy

The Einstein cross: QSO 2337+0305

Distances in cosmology

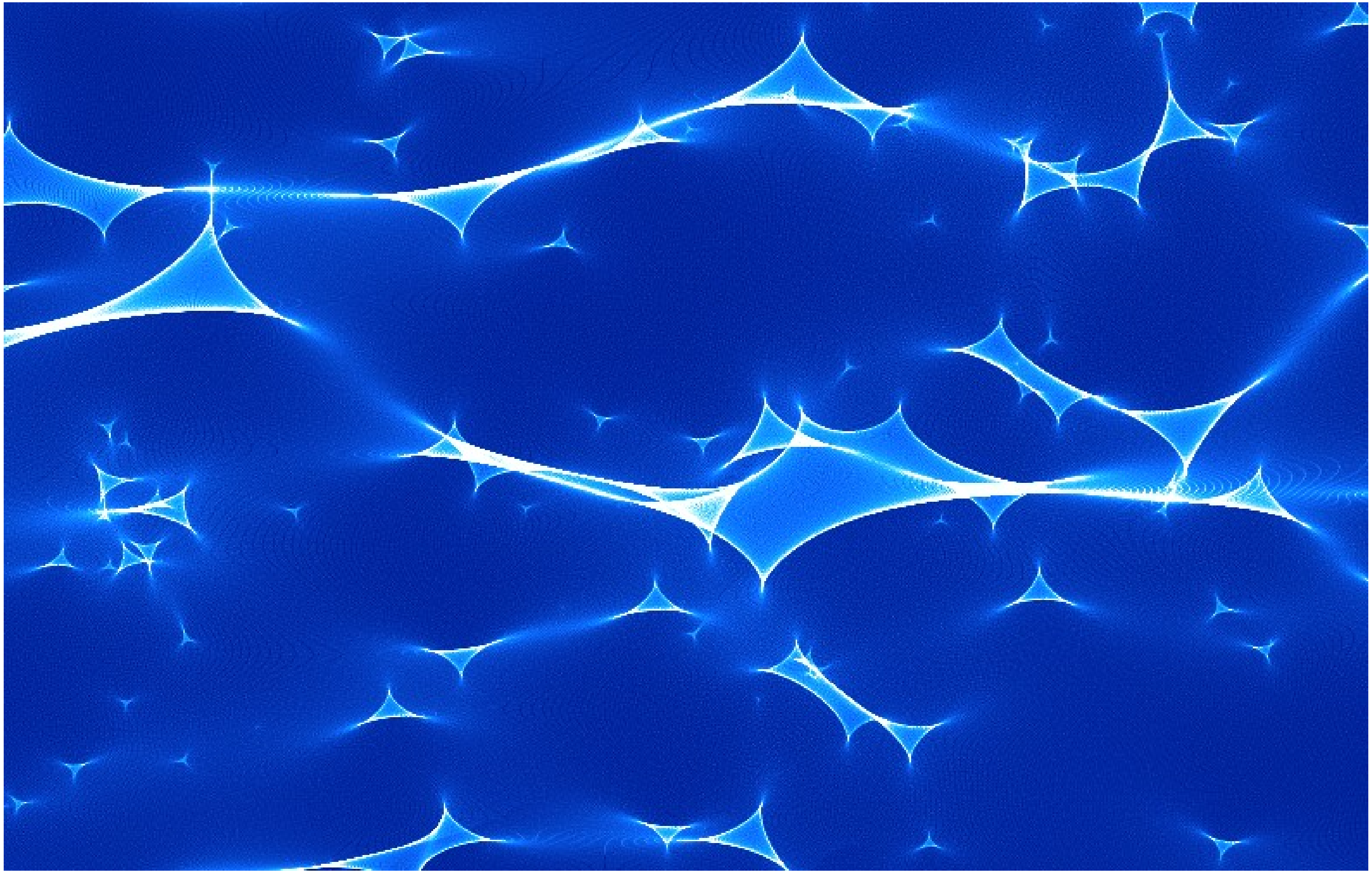
Images and caustics in the isothermal potential

The mass-sheet degeneracy

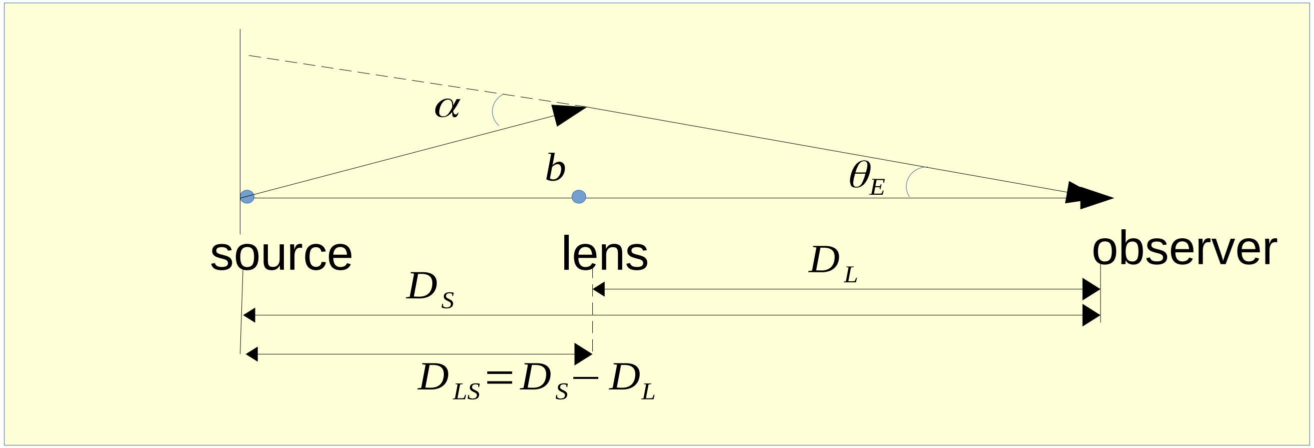
Time delays

Microlensing variability

This time the complexity will increase
We will see multiple caustics merging



The lens equation in cosmology



$$D_I \, \theta = d \quad \rightarrow \quad D_I = \frac{d}{\theta}$$ angular distances

In the weak field limit and for small deviations
The lens equation is still valid if we use the cosmological angular distances

See for instance Narayan & Bartelmann (2008)

Distances in cosmology

Comoving distance: $$D_C = \frac{c}{H_0} \int \frac{dz}{E(z)}$$ $$H(z) = H_0 E(z)$$

Comoving angular distance: $$D_M = \begin{cases} K^{\frac{-1}{2}} \sin\left(K^{\frac{1}{2}} D_C\right) & for \quad K>0 \\ D_C & for \quad K=0 \\ -K^{\frac{-1}{2}} \sinh\left(-K^{\frac{1}{2}} D_C\right) & for \quad K<0 \end{cases}$$

Curvature: K

curvature density parameter

$$\Omega_K = -\left(\frac{c}{H_0}\right)^2 K$$

Angular distance: $$D_A = \frac{D_M}{1+z}$$

Do not subtract angular distances: use comoving angular distance then normalize using redshift

An interesting cosmological situation
The Einstein cross: QSO 2337+0305

A distant quasar source: z=1.695

(light travel time: 9.846 Gyr)

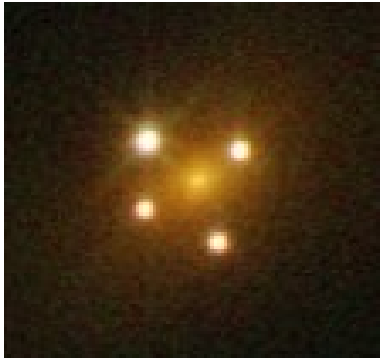
A nearby galactic lens: z=0.0395

(light travel time: 0.540 Gyr)

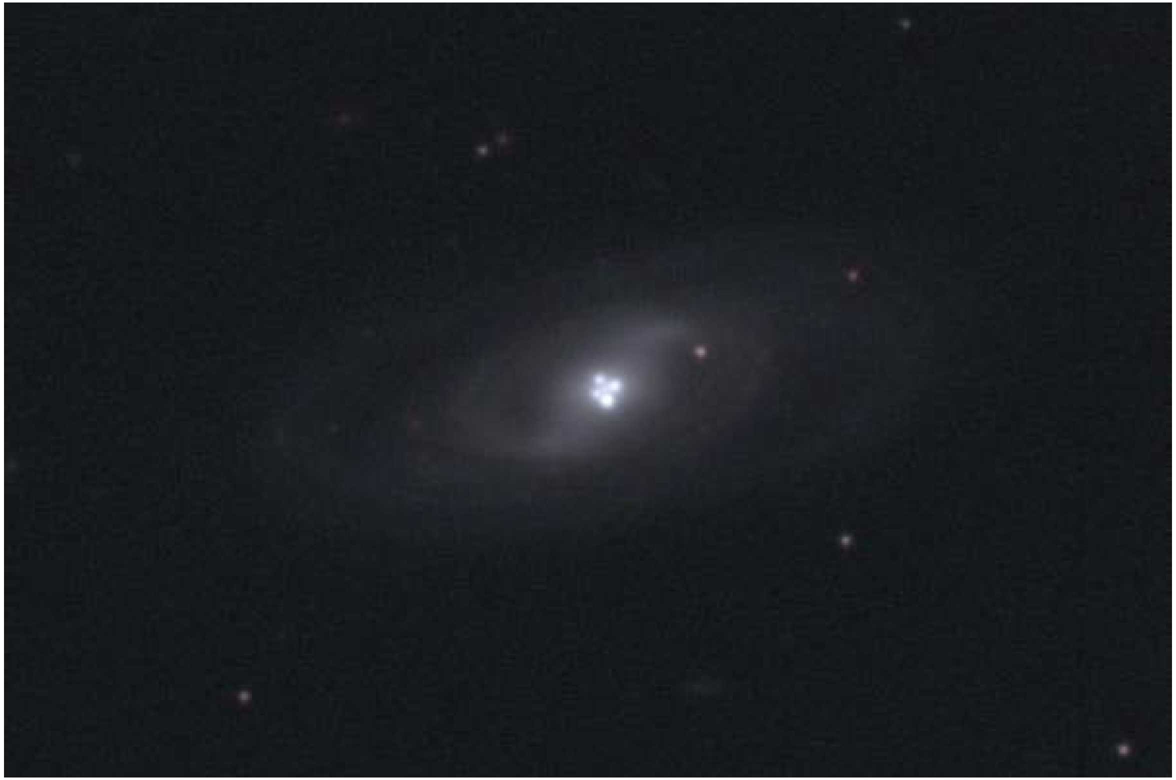
(discovered by John Huchra in 1985)

The elliptical lens

The Einstein cross



QSO 2237+0305
(HST)



A simple model: elliptical isothermal potential

$$\phi=\sqrt{(1-\eta)\,x^2+(1+\eta)\,y^2}$$

for small ellipticity

$$\phi\approx r\left(1-\frac{\eta}{2}\cos 2\theta\right)$$

The lens equation: $\vec{r}_S=\vec{r}-\vec{\nabla}\phi$

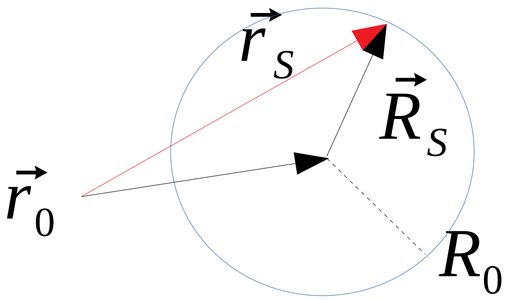
With: $dr=r-1$

$$\vec{r}_S=\left(dr+\frac{\eta}{2}\cos 2\theta\right)\vec{u}_r-\eta\sin 2\theta\;\vec{u}_\theta$$

(to first order in η)

A simple model: elliptical isothermal potential

Circular source with impact parameter $\vec{r}_0$ and radius R_0



$$\vec{r}_S = \left(dr + \frac{\eta}{2}\cos 2\theta\right)\ \vec{u}_r - \eta \sin 2\theta\ \vec{u}_\theta$$

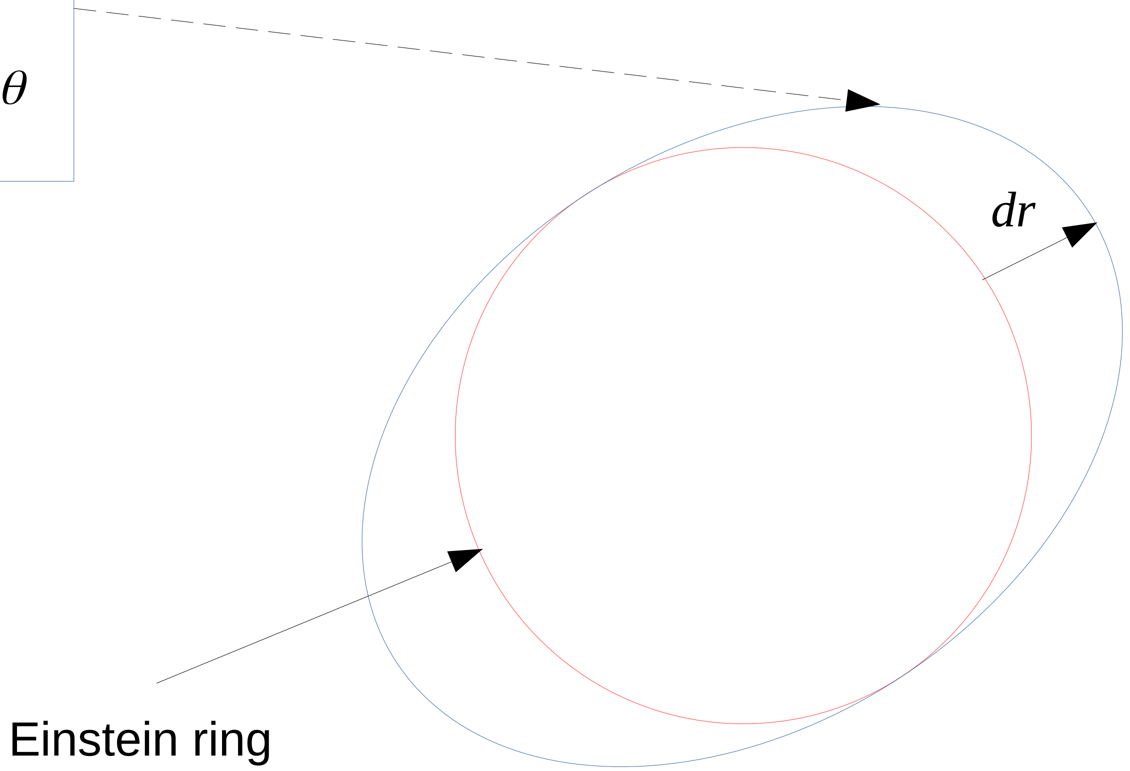
$$\vec{r}_s = \vec{R}_s + \vec{r}_0 \quad ; \quad |\vec{R}_s| = R_0$$

$$\vec{r}_0 = (x_0, y_0)$$

$$dr = \frac{\eta}{2}\cos 2\theta - x_0 \cos\theta - y_0 \sin\theta \pm \sqrt{R_0^2 - \left(\eta \sin 2\theta - x_0 \sin\theta + y_0 \cos\theta\right)^2}$$

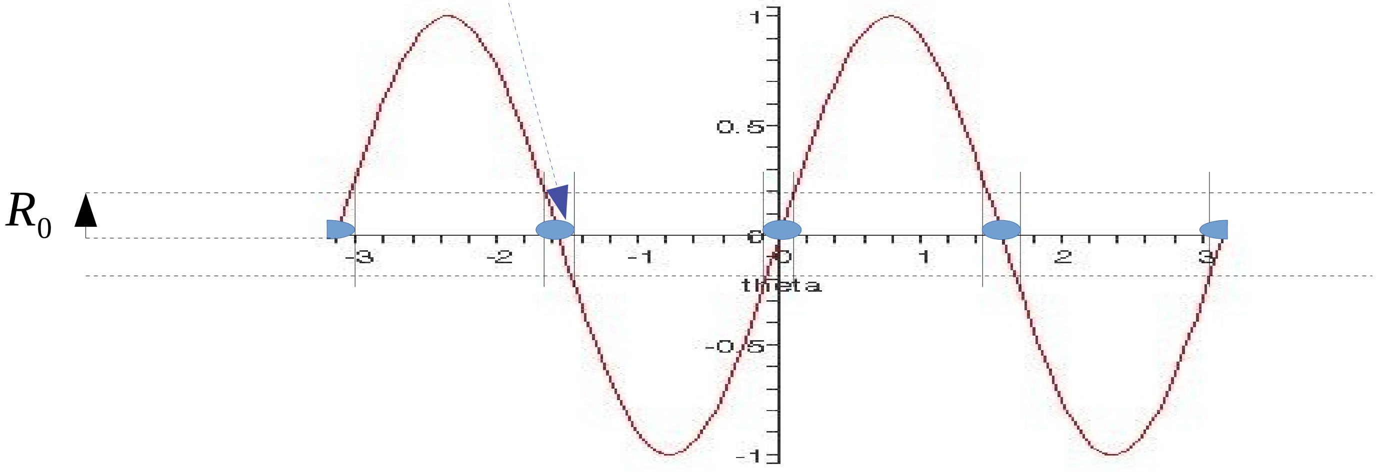
Radial position of the images

$$dr = -\frac{\eta}{2}\cos 2\theta - x_0 \cos\theta - y_0 \sin\theta$$



$$dr = \frac{\eta}{2}\cos 2\theta - x_0\cos\theta - y_0\sin\theta \pm \sqrt{R_0^2 - \left(\eta\sin 2\theta - x_0\sin\theta + y_0\cos\theta\right)^2}$$

Image forms if: $|df_0| = |\eta\sin 2\theta - x_0\sin\theta + y_0\cos\theta| < R_0$

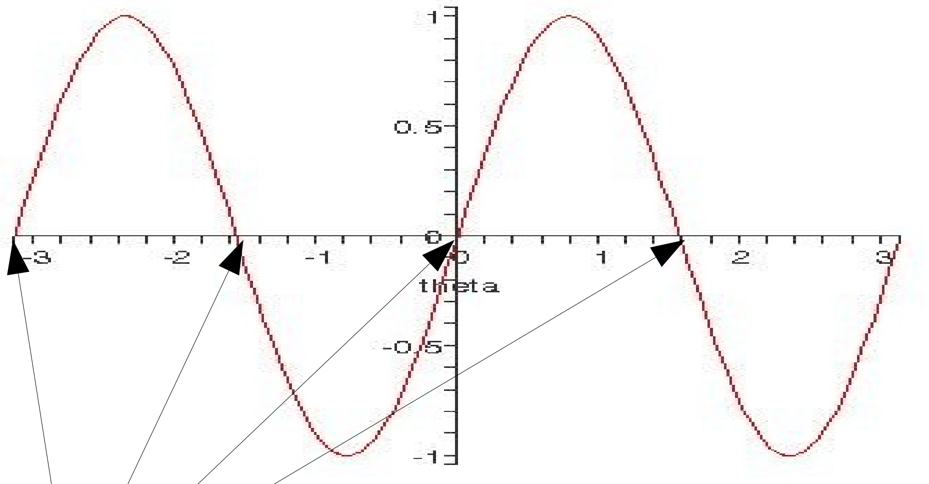


Here represented for: $x_0 = 0$; $y_0 = 0$; $df_0 = |\eta\sin 2\theta|$

Source at center of elliptical lens,

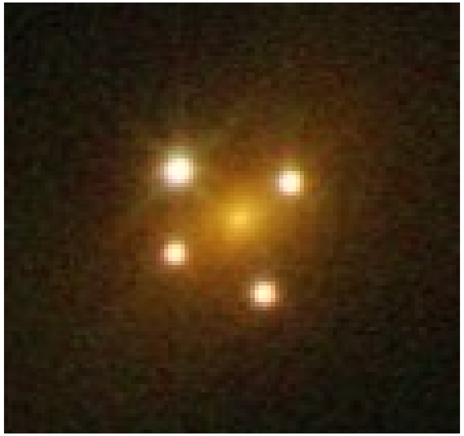
$$dr = \frac{\eta}{2}\cos 2\theta \pm \sqrt{R_0^2 - (\eta \sin 2\theta)^2}$$

Images when: $\sin 2\theta < R_0$



4 images

Images centers: $\sin 2\theta = 0$



Source near center
Of elliptical lens

Caustics for the isothermal potential

$$\phi=\sqrt{(1-\eta)x^2+(1+\eta)y^2}\approx r\left(1-\frac{\eta}{2}\cos 2\theta\right)$$

$$x_S=x-\frac{\partial\phi}{\partial x}\qquad y_S=y-\frac{\partial\phi}{\partial y}$$

$$J=\frac{\partial x_s}{\partial x}\frac{\partial y_s}{\partial y}-\frac{\partial x_s}{\partial y}\frac{\partial y_s}{\partial x}\simeq\frac{r-1}{r}-\frac{3\cos 2\theta}{2r}\eta$$

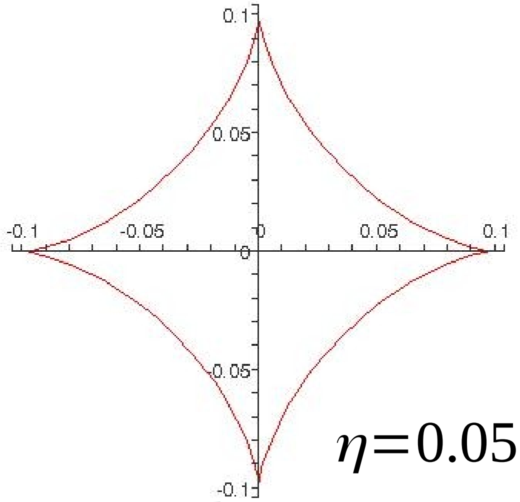
To first order in η

Critical lines: $J=0$ ➔ $r=1+\frac{3}{2}\eta\cos 2\theta$

We transform the equation for the critical lines to the source plane by using the lens equation

Caustics:

$$\begin{cases} xs = \left(\frac{3}{2}\cos\theta + \frac{1}{2}\cos 3\theta\right)\eta \\ ys = \left(-\frac{3}{2}\sin\theta + \frac{1}{2}\sin 3\theta\right)\eta \end{cases}$$



$\eta = 0.05$

The amplitude of the caustics diagram is: 2η

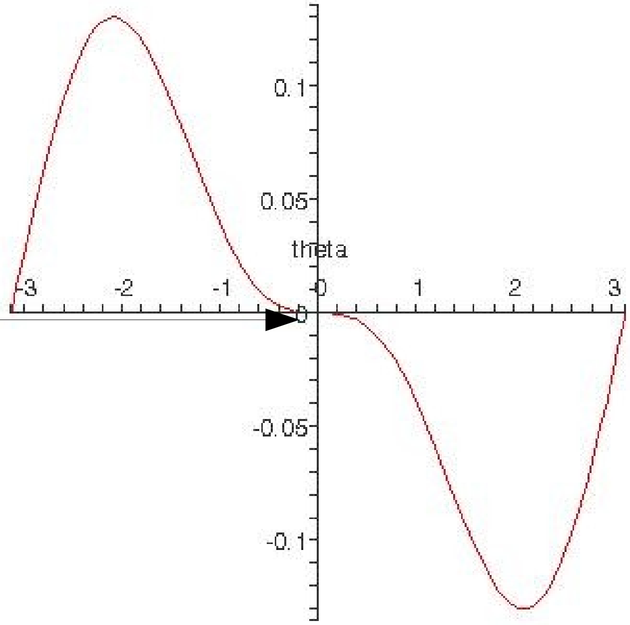
Image equation $$dr=\frac{\eta}{2}\cos 2\theta - x_0\cos\theta \pm \sqrt{R_0^2 - df_0^2}$$

$$df_0=\eta\sin 2\theta - x_0\sin\theta + y_0\cos\theta$$

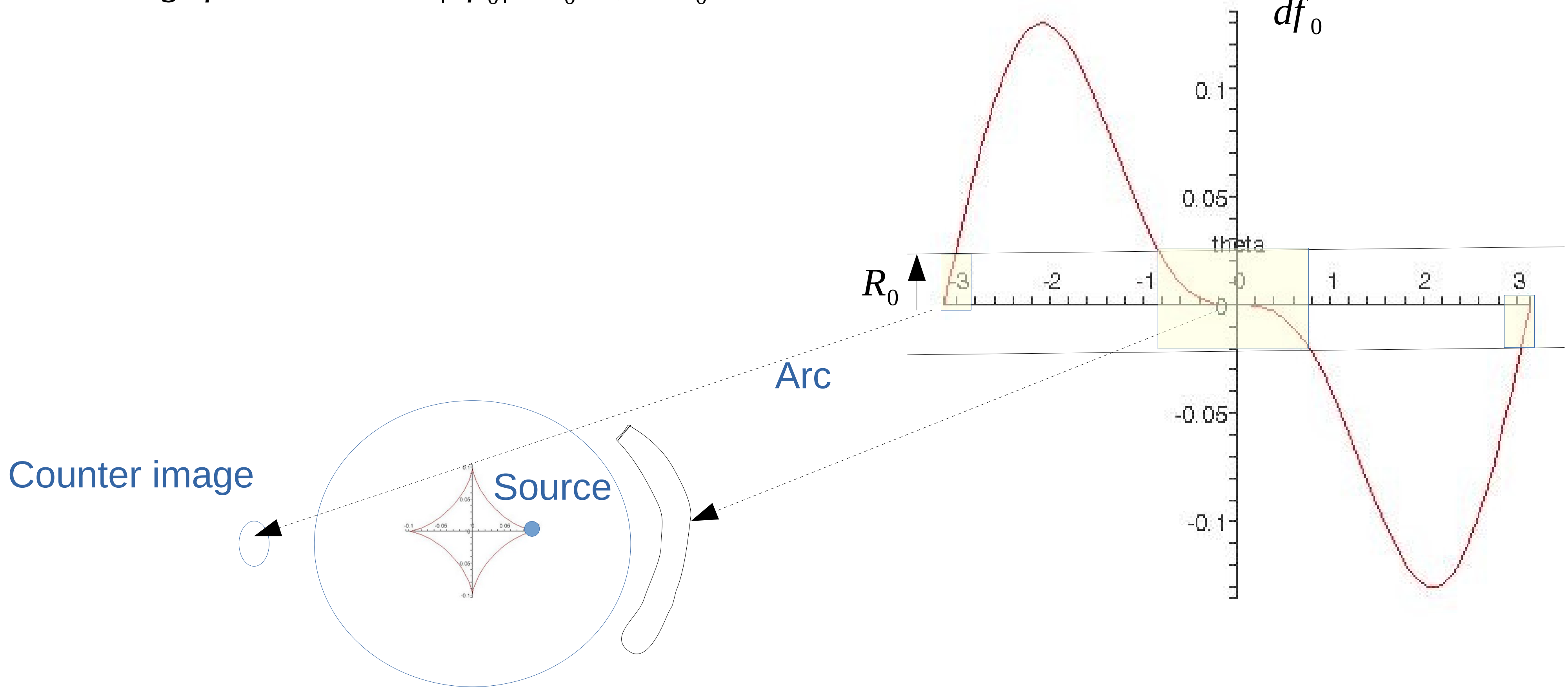
For $x_0=2\eta,\ y_0=0$

$(df_0)_{,\theta}=0$; $(df_0)_{,\theta,\theta}=0$

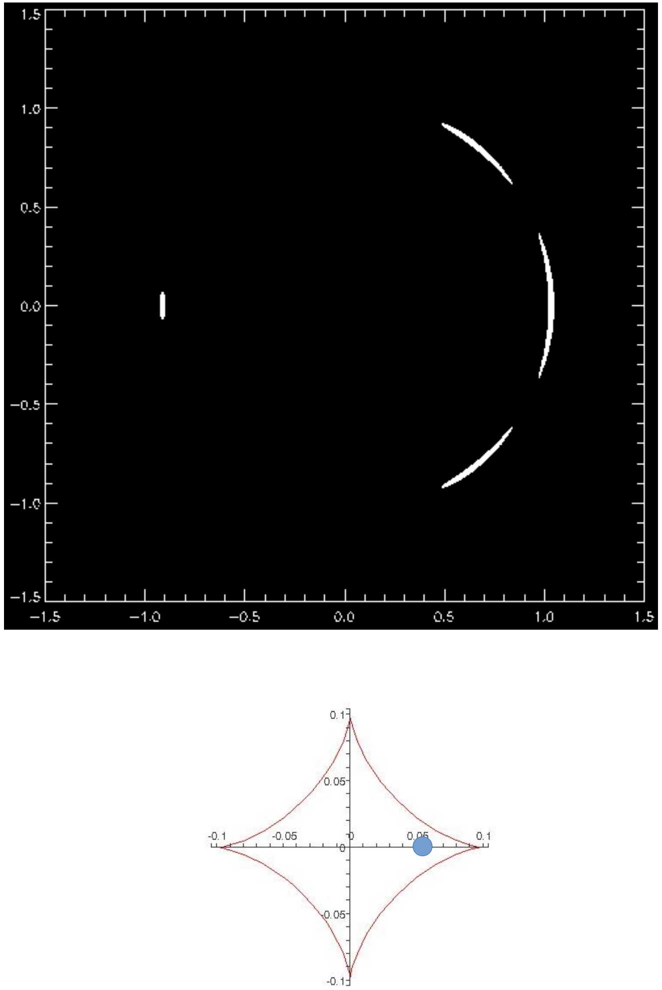
Cusp caustic=order 3



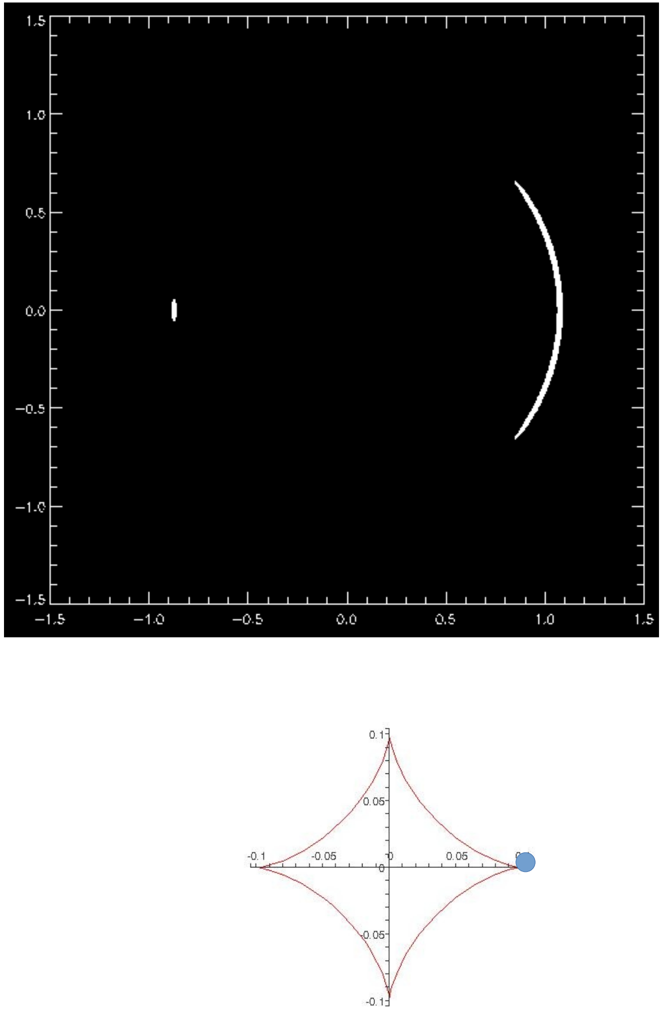
$$Image\,formation \rightarrow |df_0| < R_0 \quad ; \quad R_0\; source\, radius$$



sub-critical



Cusp



Beyond cusp

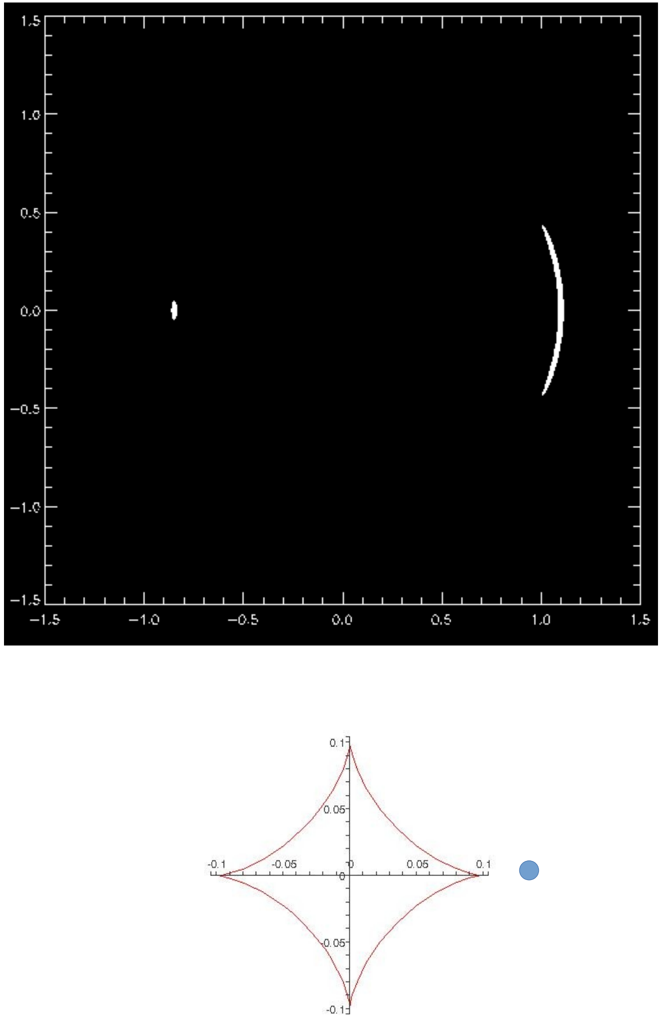


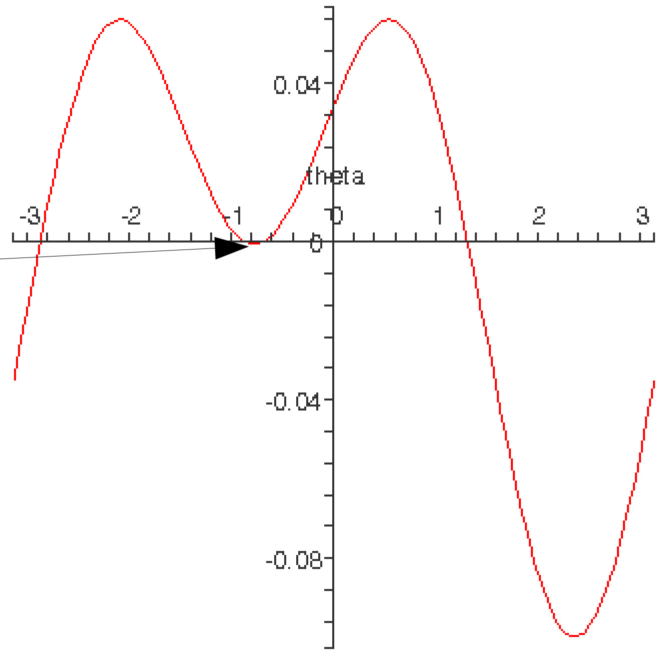
Image equation $$dr=\frac{\eta}{2}\cos 2\theta - x_0\cos\theta - y_0\sin(\theta) \pm \sqrt{R_0^2 - df_0^2}$$

$$df_0 = \eta \sin 2\theta - x_0 \sin\theta + y_0 \cos\theta$$

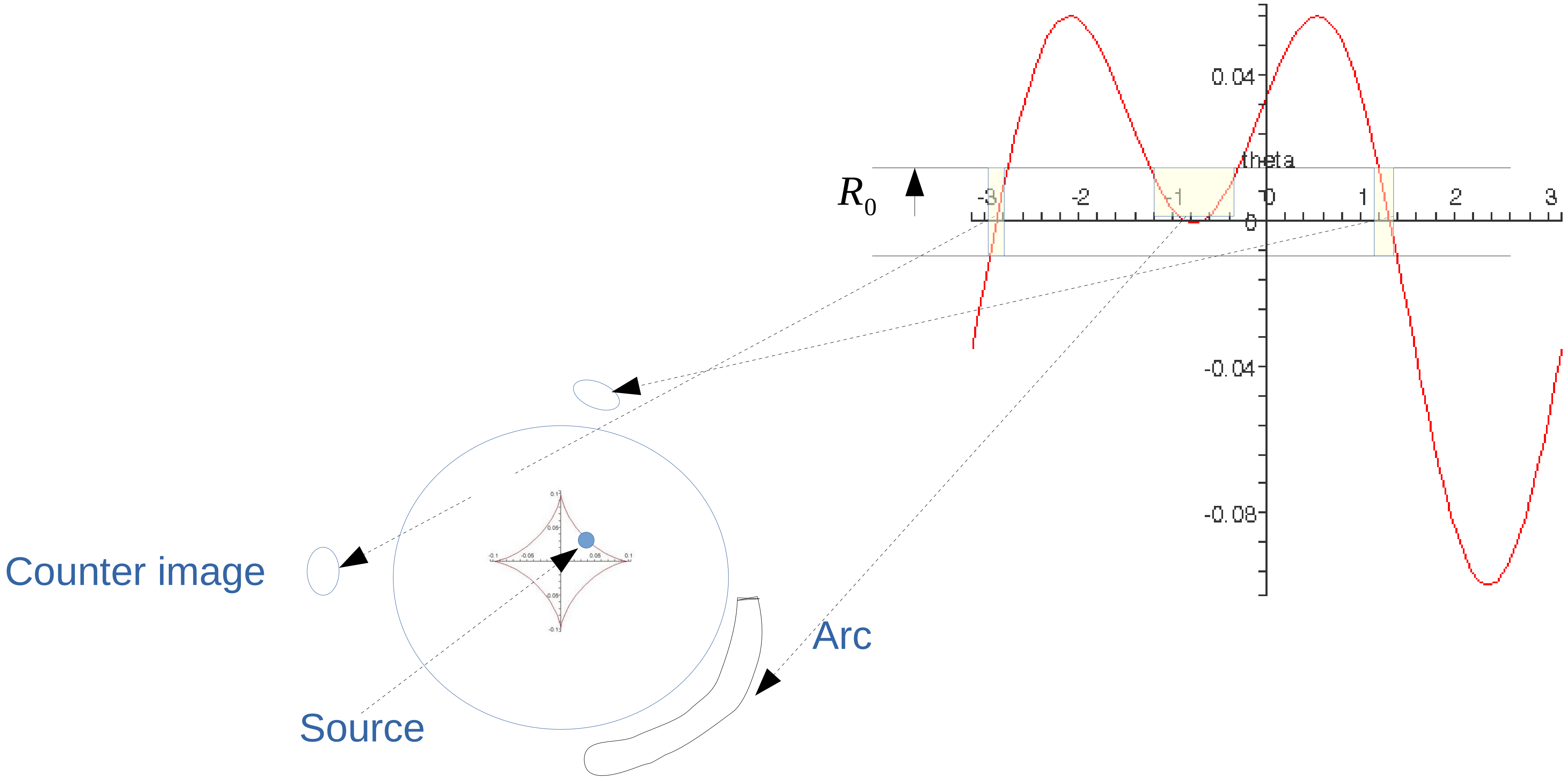
For $x_0 \simeq 0.7\,\eta,\quad y_0 \simeq 0.7\,\eta$

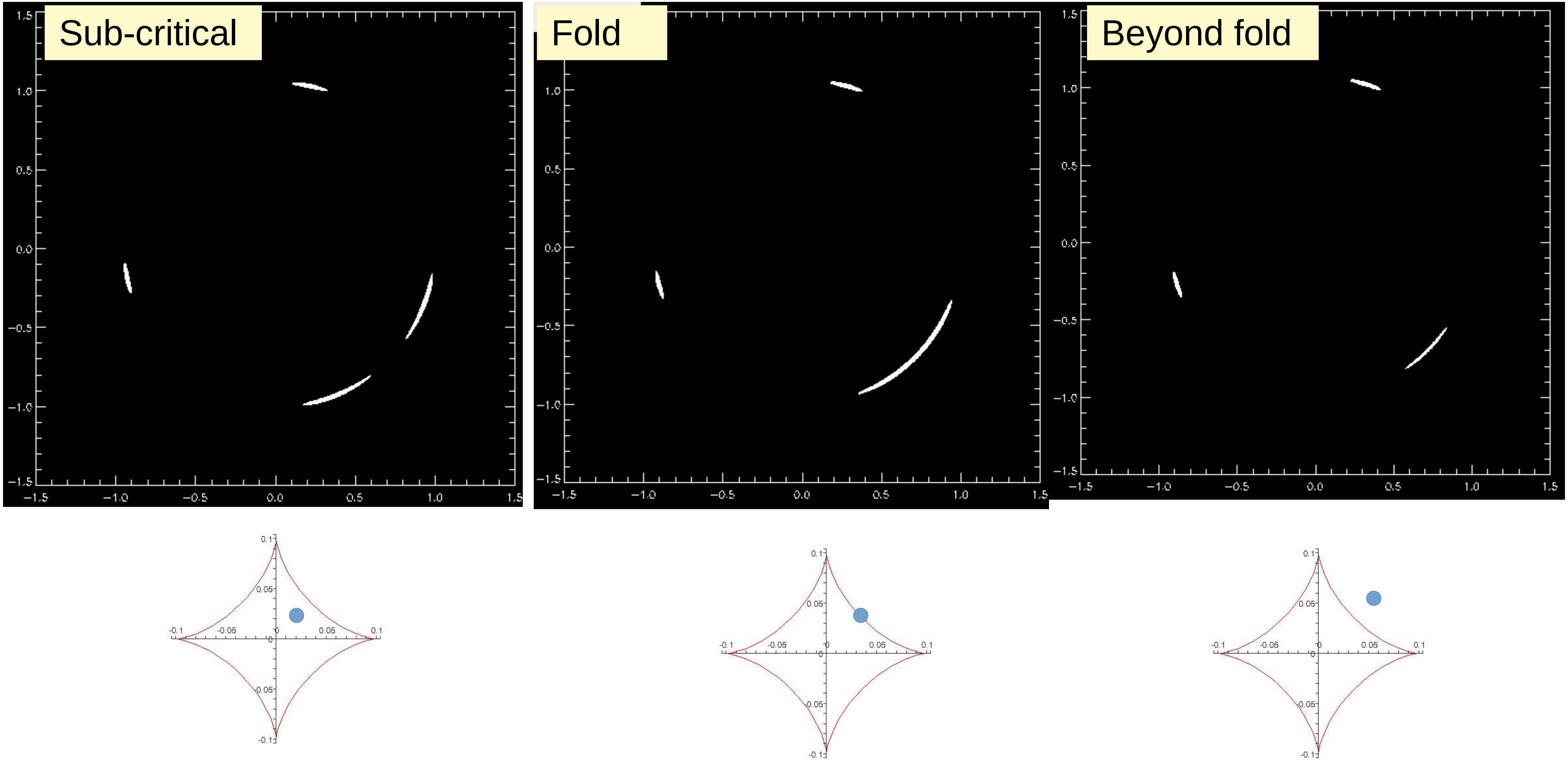
$$(df_0)_{,\theta} = 0$$

Fold caustic=order 2



$$Image\ formation \rightarrow |df_0| < R_0 \quad ; \quad R_0\ source\ radius$$




Sub-critical
Fold
Beyond fold

The mass sheet degeneracy

Let introduce a new surface density $\tilde{\kappa}$

It relates to the initial surface density κ by:

$$\kappa = (1-\lambda)\tilde{\kappa} + \lambda$$

With: $\kappa = \frac{1}{2}\Delta\phi$ and: $\tilde{\kappa} = \frac{1}{2}\Delta\tilde{\phi}$ → $\phi = (1-\lambda)\tilde{\phi} + \frac{1}{2}\lambda(x^2+y^2)$

↓

(take laplacian and check it is working)

$$\kappa=(1-\lambda)\tilde{\kappa}+\lambda \longrightarrow \phi=(1-\lambda)\tilde{\phi}+\frac{1}{2}\lambda(x^2+y^2)$$

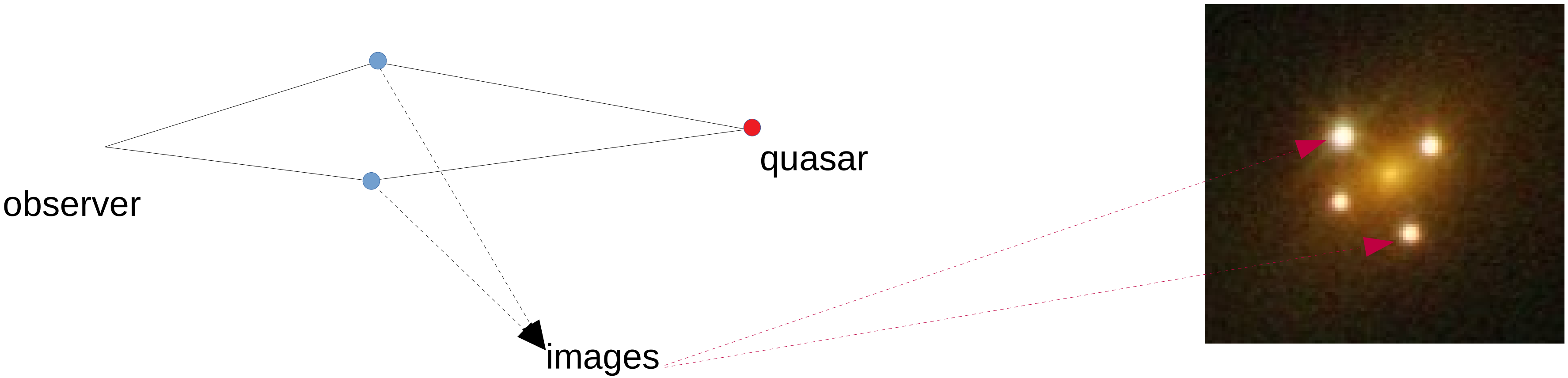
The lens equation:

$$\begin{cases} x_s=x-\frac{\partial\phi}{\partial x}=(1-\lambda)\left(x-\frac{\partial\tilde{\phi}}{\partial x}\right)=(1-\lambda)\tilde{x}_s \\ y_s=y-\frac{\partial\phi}{\partial y}=(1-\lambda)\left(y-\frac{\partial\tilde{\phi}}{\partial y}\right)=(1-\lambda)\tilde{y}_s \end{cases}$$

The lens equation with the new surface density $\tilde{\kappa}$
Is equivalent to the former lens equation
If we re-scale the source coordinates the two equations are equivalent

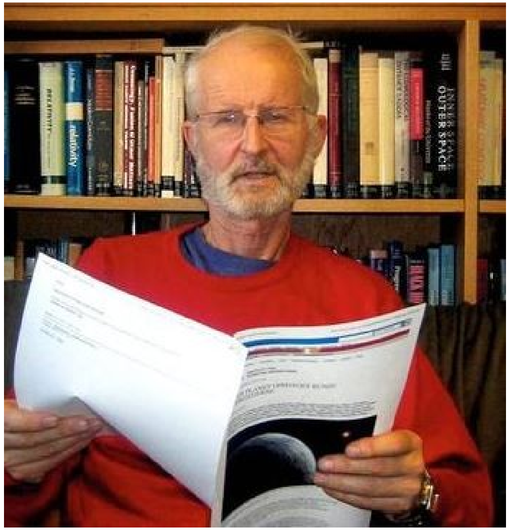
This is known as the mass-sheet degeneracy (adding a constant density)
Leads to a re-scaling of both lens and source coordinates

Time delays



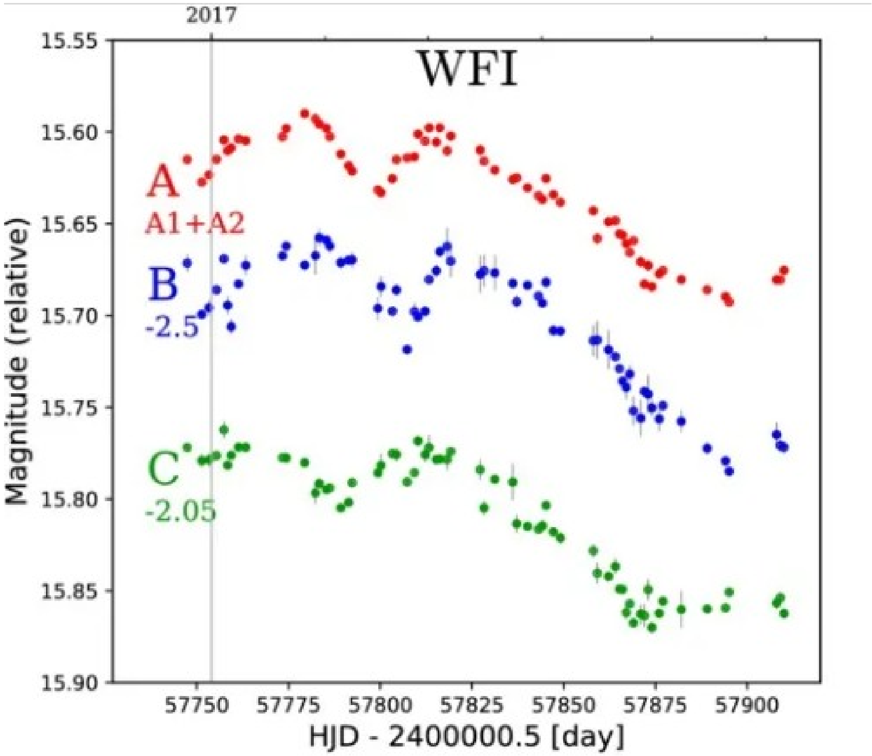
Basic idea: The path of light for each image is different
Consequence: a time delay between the images

Refsdal (1964)



In practice the source: quasar
Is variable

Thus time delays can be observed



The time delay

(for spatially flat universe or small curvature)

$$\tau=\frac{(1+z_L)}{c}\frac{D_L D_S}{D_{LS}}\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2-\psi(\vec{\theta})\right)$$

$$d_I=\left(\frac{c}{H_0}\right)^{-1} D_I \longrightarrow D_I \propto D_C=\frac{c}{H_0}\int\frac{dz}{E(z)}$$

$$\tau=\frac{(1+z_L)}{H_0}\frac{d_L d_S}{d_{LS}}\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2-\psi(\vec{\theta})\right)$$

d_I : *dimensionless distances*

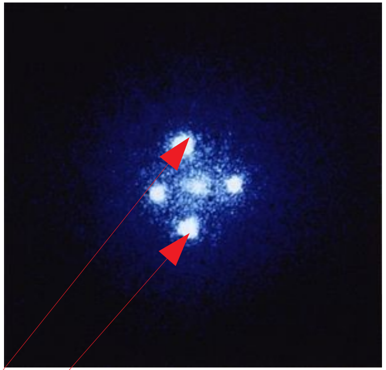
The first thing to note is that the time delay is proportional to: H_0^{-1}

Thus measuring the time delay is direct measurement of H_0

In practice what we measure is the differential time delay between the images

$$\tau = \frac{(1+z_L)}{H_0}\frac{d_L d_S}{d_{LS}}\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi(\vec{\theta})\right) \longrightarrow \tau(\theta,\beta) = T_d\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi(\vec{\theta})\right)$$

$$\Delta\,\tau_{A,B} = \tau(\theta_A,\beta) - \tau(\theta_B,\beta) = T_d\left(\frac{1}{2}(\vec{\theta}_A-\vec{\beta})^2 - \psi(\vec{\theta}_A) - \frac{1}{2}(\vec{\theta}_B-\vec{\beta})^2 + \psi(\vec{\theta}_B)\right)$$



This is clearly model dependent: one needs to estimate the potential

A,B

For a singular isothermal sphere: $\Delta\,\tau_{A,B} \propto \left(R_A^2 - R_B^2\right)$ → Images positions

Kochaneck & Schechter (2004)

How to interpret the time delay

First it is nothing really new...

If we minimize the time delay with respect to $\vec{\theta}$

We obtain the lens equation:

$$\tau = \frac{(1+z_L)}{H_0}\frac{d_L d_S}{d_{LS}}\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi(\vec{\theta})\right)$$

$$\vec{\beta} = \vec{\theta} - \vec{\nabla}\phi$$

The formulation: time delay or lens equation
Are seen as equivalent

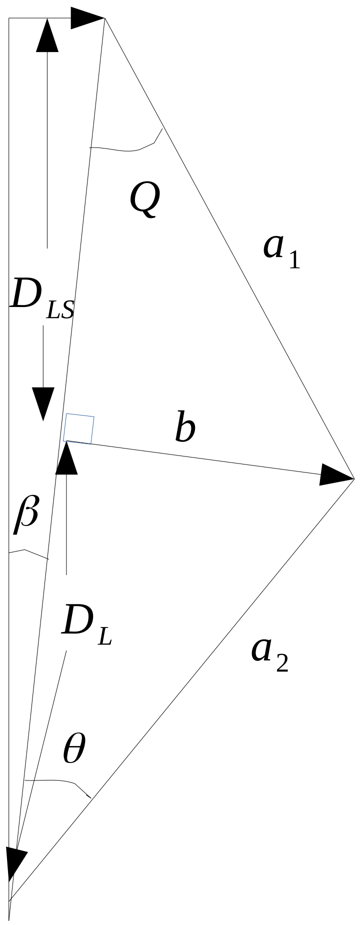
The physical interpretation of the time delay

$$\tau = \frac{(1+z_L)}{H_0}\frac{d_L d_S}{d_{LS}}\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi(\vec{\theta})\right)$$

Geometric delay

Gravitational delay

(the Shapiro delay),



$$\delta_1 = a_1 - D_{LS} \simeq \frac{1}{2}\frac{b^2}{D_{LS}} \qquad \delta_2 = a_2 - D_L \simeq \frac{1}{2}\frac{b^2}{D_L}$$

$$b = D_L(\theta - \beta)$$

$$\delta = \delta_1 + \delta_2 = \frac{D_L D_S}{D_{LS}}(\theta - \beta)^2$$

$$d_I = \left(\frac{c}{H_0}\right)^{-1} D_I \qquad \tau \equiv \frac{\delta}{c} \qquad \tau = \frac{(1+z_L)}{H_0}\frac{d_L d_S}{d_{LS}}\left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta})\right)$$

With the appropriate scale factor we recover the geometric time delay

The Shapiro time delay

First predicted in 1964 by Irwin Shapiro

For a nearly static and weak field

The time delay due to the gravitational field

is directly proportional to the Newtonian potential

$$\tau = \frac{(1+z_L)}{H_0} \frac{d_L d_S}{d_{LS}} \left(\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right)$$

How do time delay look like in practice ?

Problem with time delay estimations

Some practical examples of light curves of images for a variety of lenses

Problem with time delay estimations

The time delay is model dependent

Any model of the potential or surface density is affected
by the mass-sheet degeneracy

It is essential to find a method to deal with the mass-sheet degeneracy

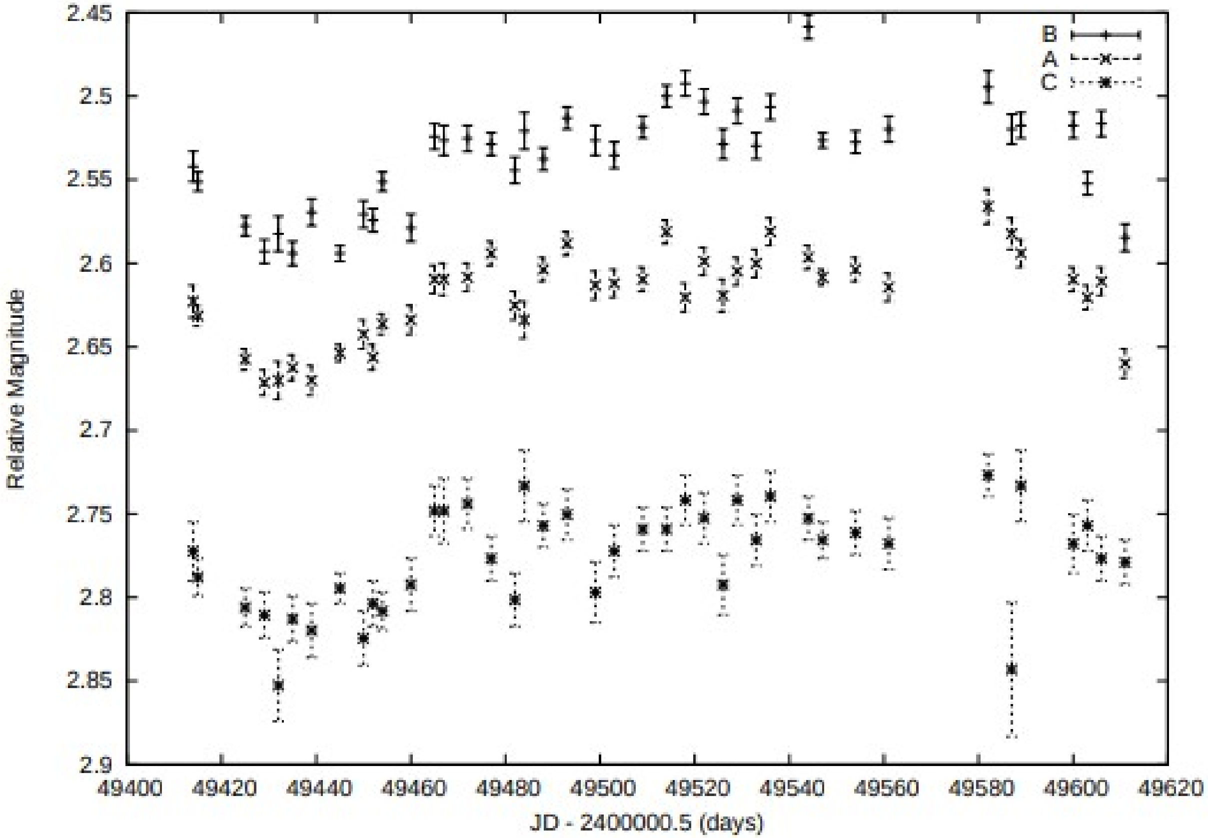
Treu & Koopmans (2002) propose to use stellar kinematics

Keeton & Zabludoff (2004) use the environment of the
lens (galaxy counts, weak lensing)

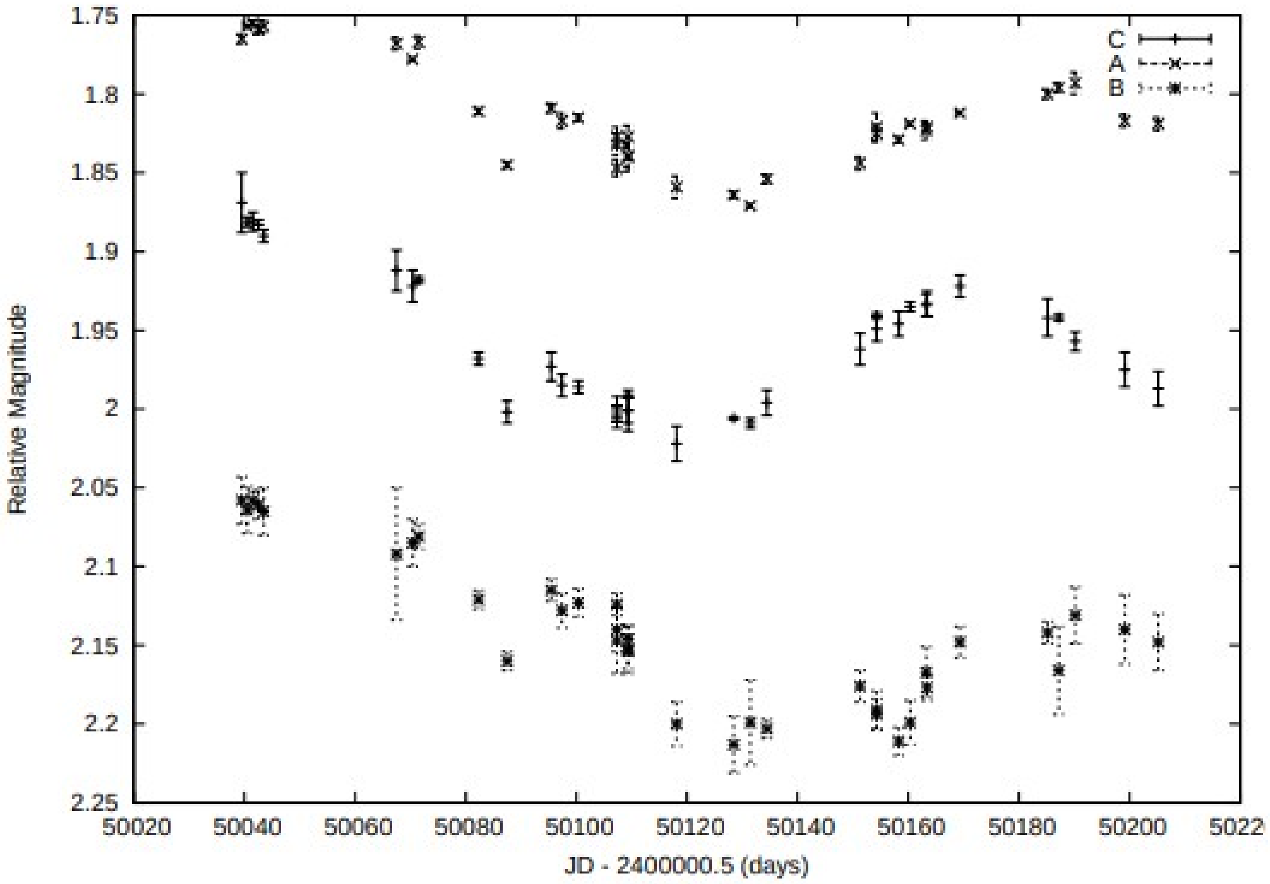
Some practical examples of light curves of images for a variety of lenses

A short review from the literature

JVAS B1422+231

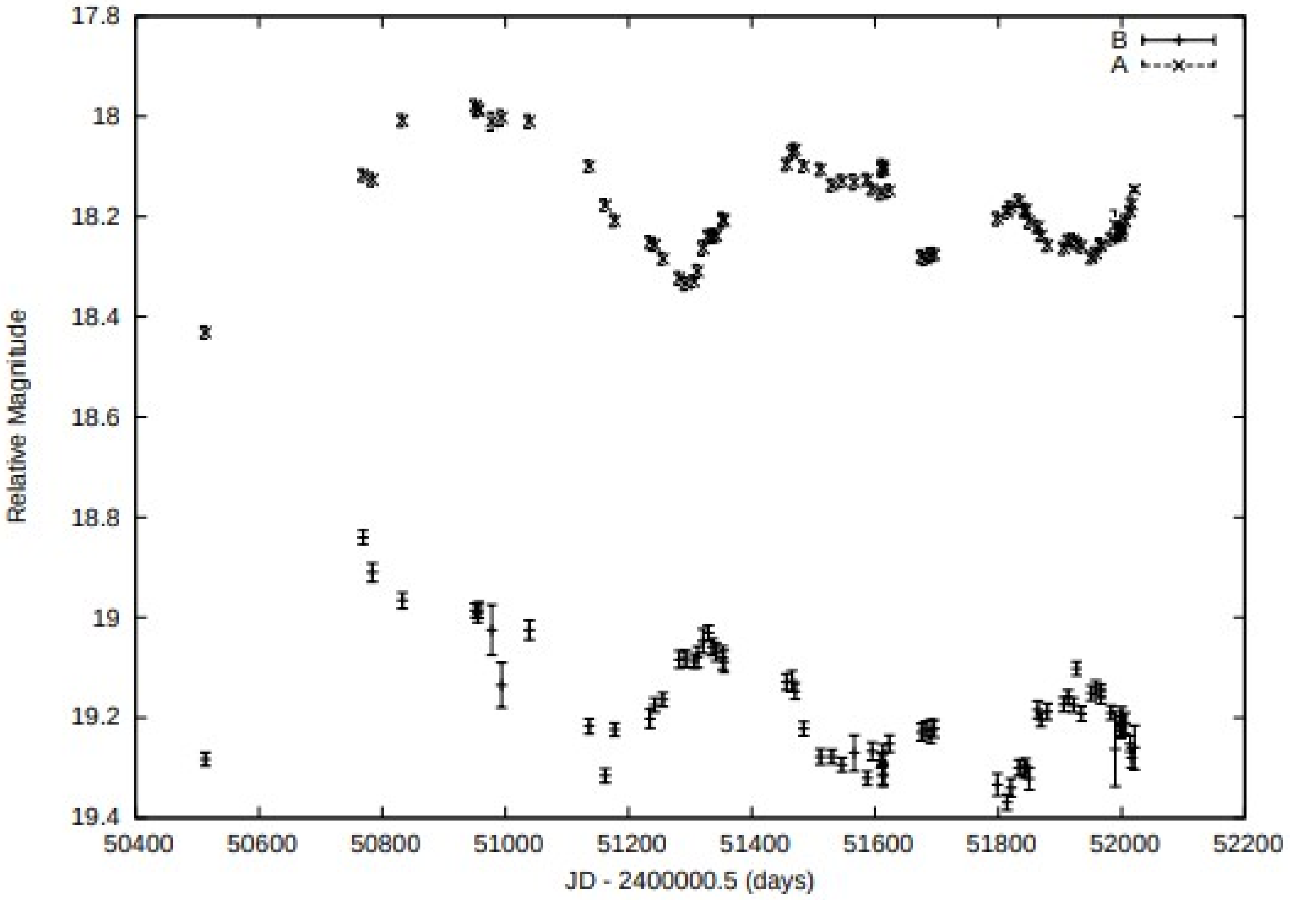


PG 1115+080



C
A
B
Relative Magnitude
1.75
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
2.2
2.25
50020
50040
50060
50080
50100
50120
50140
50160
50180
50200
5022
JD - 2400000.5 (days)

RX J0911+0551



Compilation
For 11 systems

Eulaers
(2012)

System	Our Results	Published Values	Reference
JVAS B0218+357	$\Delta t_{AB} = 9.9^{+4.0}_{-0.9}$	$\Delta t_{AB} = 10.1^{+1.5}_{-1.6}$	Cohen et al. (2000)
	or	$\Delta t_{AB} = 12 \pm 3$	Corbett et al. (1996)
	$\Delta t_{AB} = 11.8 \pm 2.3$	$\Delta t_{AB} = 10.5 \pm 0.4$	Biggs et al. (1999)
SBS 0909+523	unreliable	$\Delta t_{BA} = 49 \pm 6$	Goicoechea et al. (2008)
		$\Delta t_{BA} = 45^{+11}_{-1}$	Ullán et al. (2006)
RX J0911+0551	2 solutions:	$\Delta t_{BA} = 150 \pm 6$	Burud (2001)
	$\Delta t_{BA} \sim 146$ or ~ 157	$\Delta t_{BA} = 146 \pm 4$	Hjorth et al. (2002)
FBQS J0951+2635	unreliable	$\Delta t_{AB} = 16 \pm 2$	Jakobsson et al. (2005)
HE 1104-1805		$\Delta t_{BA} = 152^{+2.8}_{-3.0}$	Poindexter et al. (2007)
	impossible to distinguish	$\Delta t_{BA} = 161 \pm 7$	Ofek & Maoz (2003)
	but identical	$\Delta t_{BA} = 157 \pm 10$	Wyrzykowski et al. (2003)
	within error bars	$\Delta t_{BA} = 162.2^{+6.3}_{-5.9}$	Morgan et al. (2008a)
PG 1115+080	dependent on method	$\Delta t_{CA} \sim 9.4$	Schechter et al. (1997)
		$\Delta t_{CB} = 23.7 \pm 3.4$	Schechter et al. (1997)
		$\Delta t_{CB} = 25.0^{+3.3}_{-3.8}$	Barkana (1997)
JVAS B1422+231	contradictory results	$\Delta t_{BA} = 1.5 \pm 1.4$	Patnaik & Narasimha (2001)
	between methods:	$\Delta t_{AC} = 7.6 \pm 2.5$	
	BAC or CAB?	$\Delta t_{BC} = 8.2 \pm 2.0$	
SBS 1520+530	$\Delta t_{AB} = 125.8 \pm 2.1$	$\Delta t_{AB} = 130 \pm 3$	Burud et al. (2002c)
		$\Delta t_{AB} = 130.5 \pm 2.9$	Gaynullina et al. (2005b)
CLASS B1600+434	$\Delta t_{AB} = 47.8 \pm 1.2$	$\Delta t_{AB} = 51 \pm 4$	Burud et al. (2000)
CLASS B1608+656	$\Delta t_{BA} = 31.6 \pm 1.5$	$\Delta t_{BA} = 31.5^{+2}_{-1}$	Fassnacht et al. (2002)
	$\Delta t_{BC} = 35.7 \pm 1.4$	$\Delta t_{BC} = 36.0 \pm 1.5$	
	$\Delta t_{BD} = 77.5 \pm 2.2$	$\Delta t_{BD} = 77.0^{+2}_{-1}$	
HE 2149-2745	unreliable	$\Delta t_{AB} = 103 \pm 12$	Burud et al. (2002a)

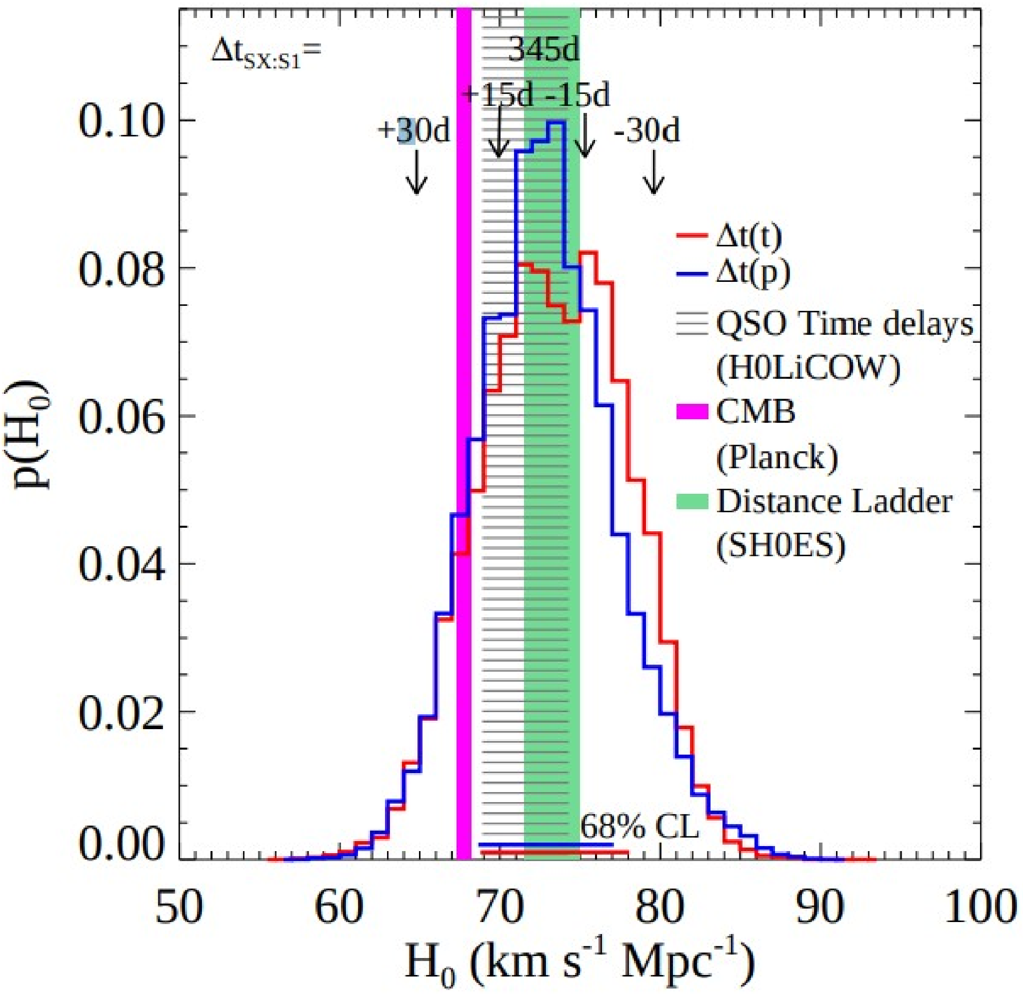
Grillo etal. (2018)

Grillo etal. (2020)

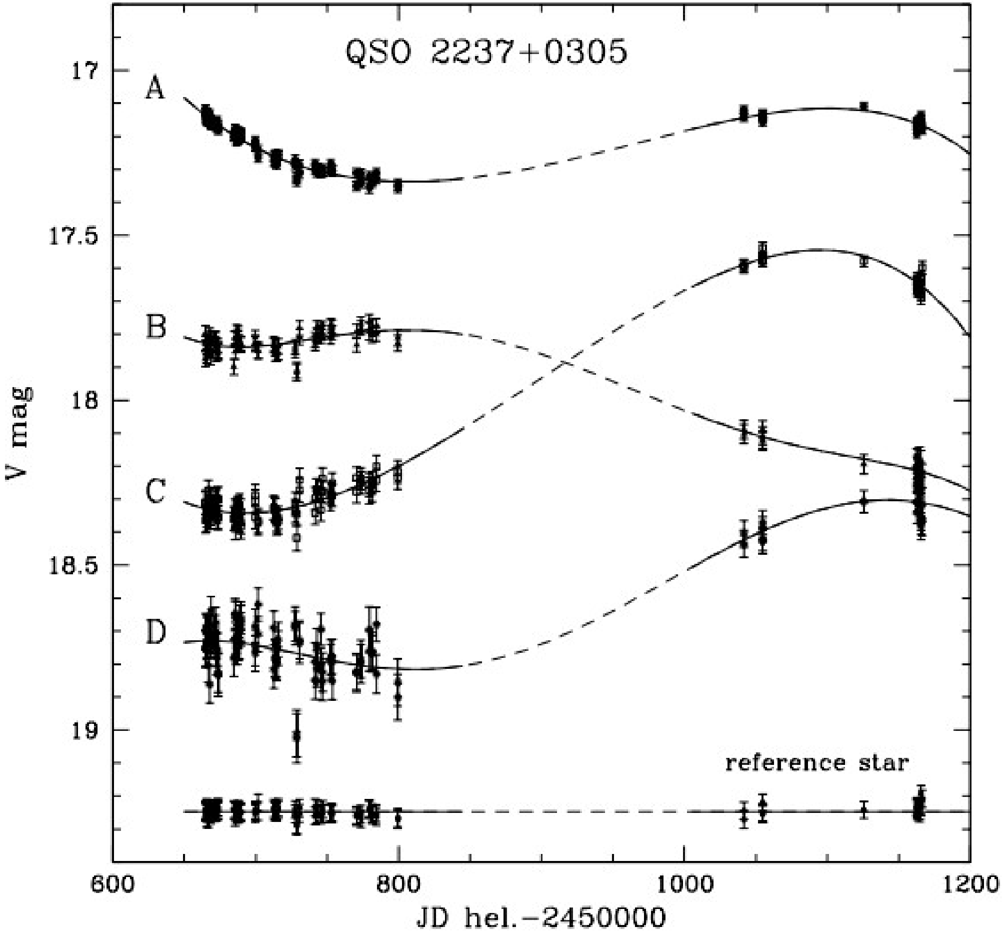
$H_0 \simeq 73\, km/s/Mpc$

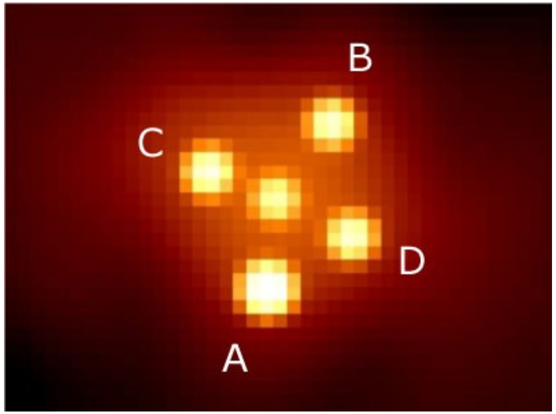
Planck estimate

$H_0 \simeq 67.4\, km/s/Mpc$



Why do we observe un-correlated variability of the images of QSO 2337+0305 ?



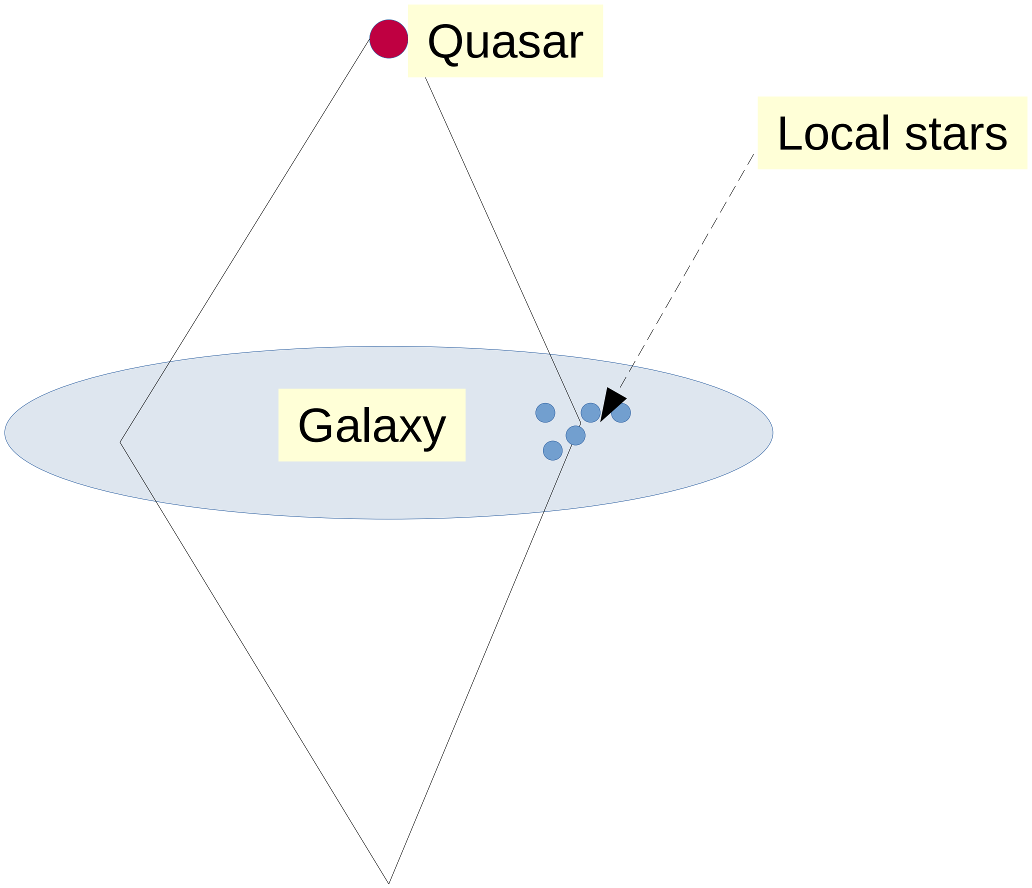


Typical time scale of variations
~ a few months to years

Typical Einstein radius crossing time
For a solar mass star in the galaxy
A few hundred days

Wozniak *etal.* (2000)

What is going on ?



The deflection angle is perturbated
By the field of the local stars

Typical Numbers

The main galaxy: $M \simeq 10^{10}\, solar\ mass$; $R_E \propto \sqrt{M}$ ➜ $R_E \simeq 30\ Kpc$

Solar mass star: $R_E \simeq \sqrt{10^{-10}} \times 30\, kpc \simeq 0.3\ pc$

Density in the solar neighborhood: $0.08\ \ solar\, mass / pc^3$

Projected density in the solar neighborhood: $0.08 \times scale\, height \simeq 0.08 \times 150 \simeq 12\, solar\ mass / pc^2$

Mean distance between stars: $\sqrt{\frac{1}{12}} \simeq 0.29\, pc$

Perturbation by stars very likely

Use ray tracing to reconstruction the amplification map
And the local caustics due to the stars

Local equations: total field=field of the galaxy+sum of the field of the local stars

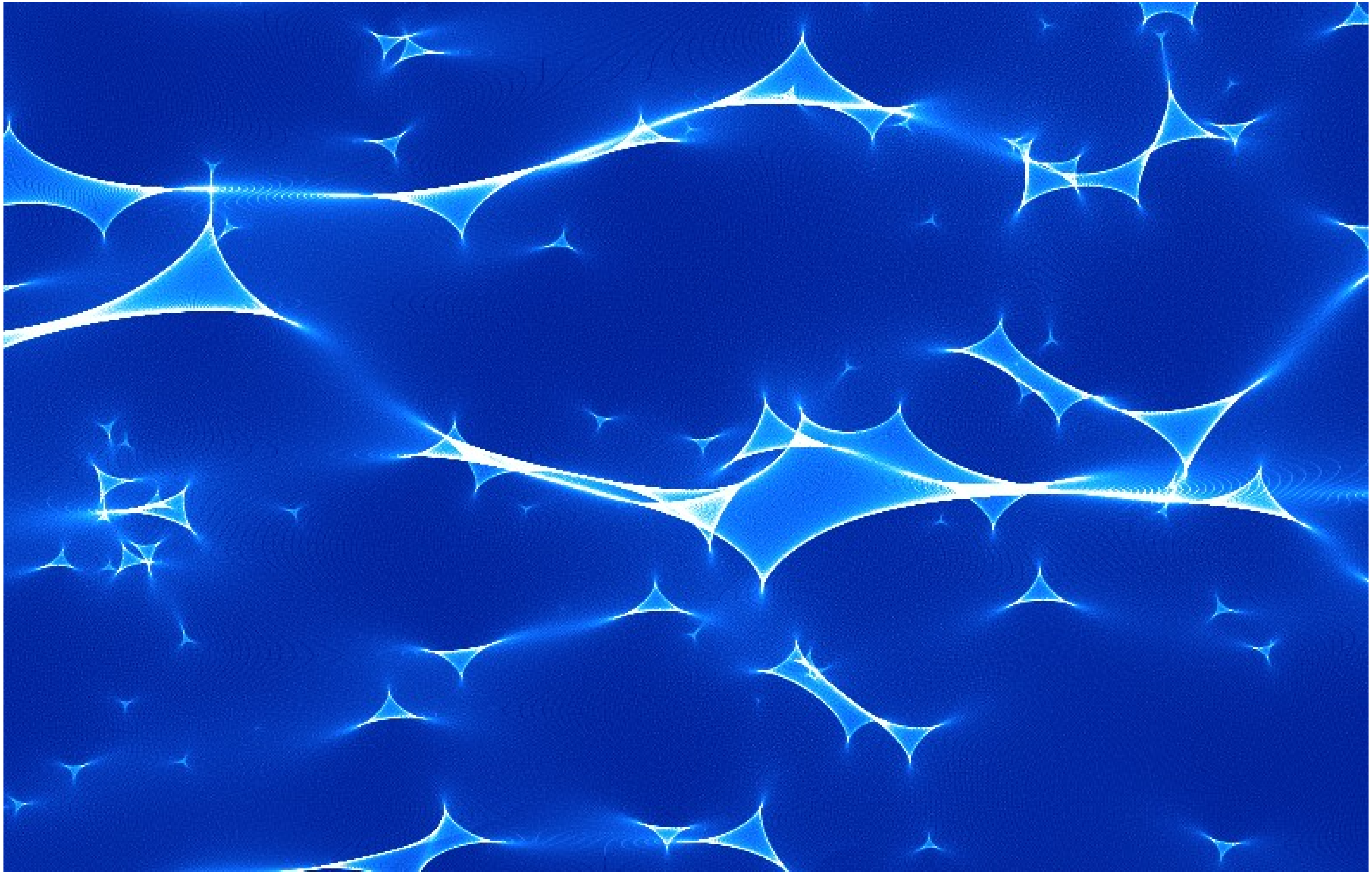
$$\phi=\sqrt{(1-\eta)x^2+(1+\eta)y^2}+\Sigma_i \mu_i \log(|\vec{r}-\vec{r}_i|)$$

$$\vec{r}_S=\vec{r}-\vec{\nabla}\phi$$

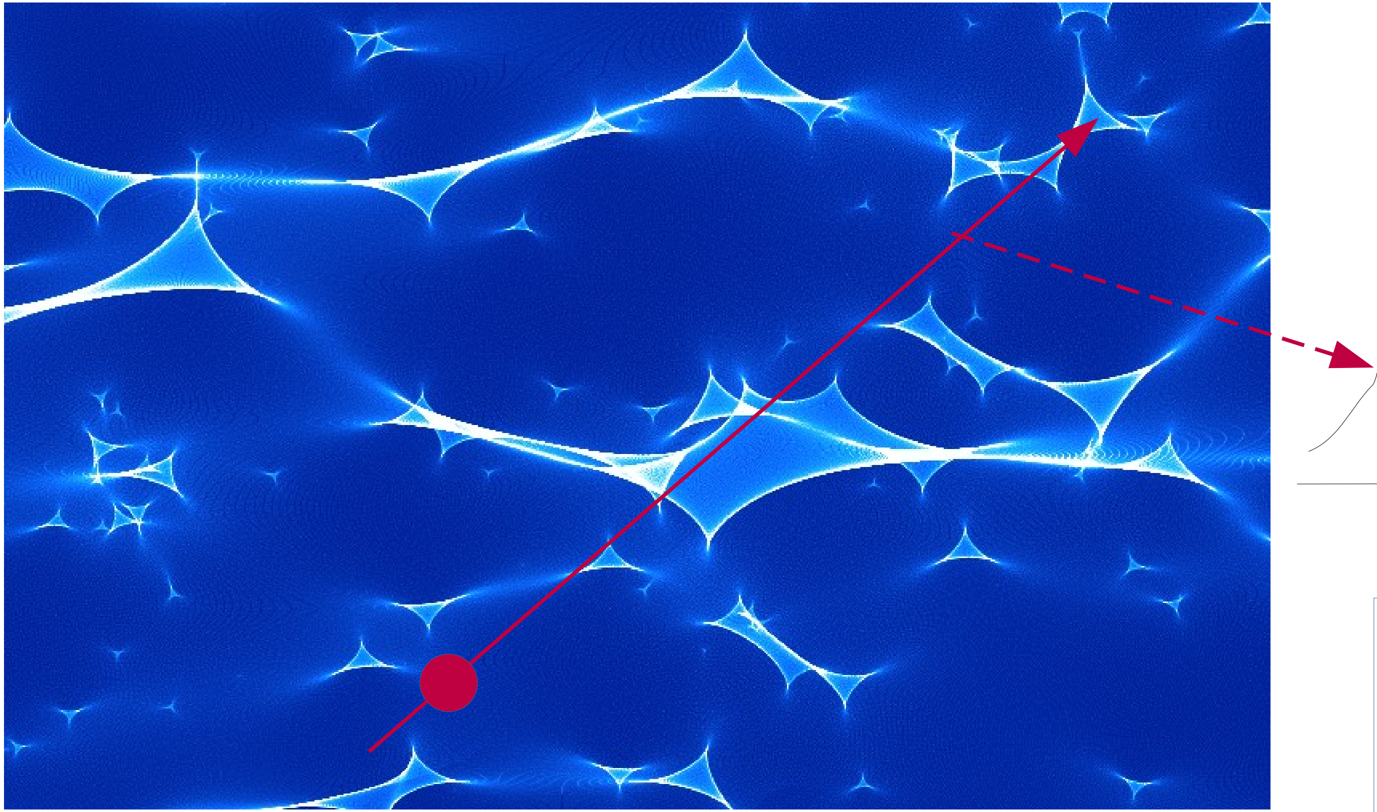
→ Ray-tracing and amplification maps / caustics reconstruction

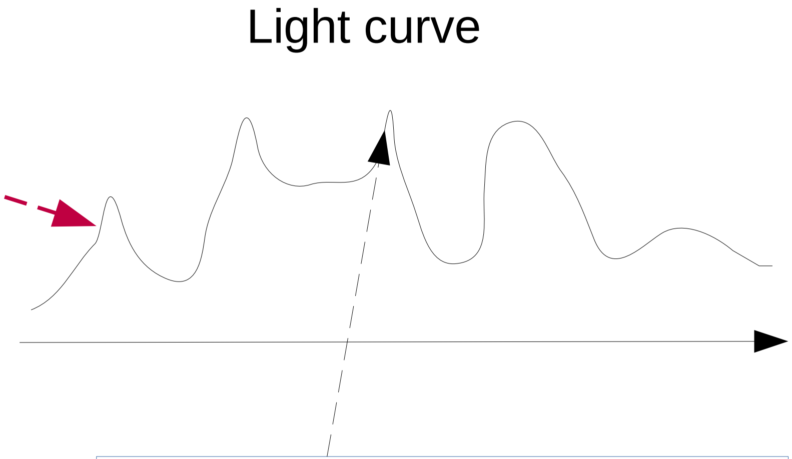
$\mu_i=\frac{m_i}{M_0}$ ratio of the mass of the star m_i to the mass of the galaxy M_0

Practical result



In practice we observe a trajectory of the quasar in this map





The larger the source
The stronger is the attenuation
Of the peaks

The structure of the source (quasar) as infered from caustic crossing
(Finite source size effect)

Shalyapin *etal.* (2002)

Best model

standard accretion disk around a supermassive black hole

90% of the light is emitted by a region with size less than : $1.2\ \ 10^{-2}\,pc$