ARES 2 Biarritz school
L2 : Atmospheric structure & radiative transfer

I. Thermal profiles, thermal balance, and atmospheric structure
(P. Drossart, IAP)
II – Radiative transfer equations (Q. Changeat, UCL)

I. Photometry and thermal profiles of a planet

1. Radiative balance : bond albedo, effective temperature, greenhouse effects
2. Atmospheric structure : troposphere, adiabatic gradient, stratosphere, etc.
3. Homopauses and eddy diffusion coefficients
4. Escape phenomena

Introduction to Thermal Profiles for Exoplanets

- The presence of an atmosphere on an exoplanet has deep consequences on the temperature at the surface, compared to the thermal equilibrium of atmosphereless bodies
- In particular the question of the stability of liquid water, of paramount importance for the habitability of exoplanets, is strongly dependent on the atmospheric structure.
- The observation of exoplanets with spectroscopic instruments gives constraints on thermal structure as well as on atmospheric composition in gases, clouds and aerosols : it is therefore necessary to understand the full complexity of the atmospheric thermal models to modelize exoplanetary spectra

1. Radiative balance

A planet is a thermodynamical engine, with the energy source from the central star and the work producing the atmospheric circulation

Fundamental question : how is the energy received from the central star redistributed, converted and then radiated to outer space ?

1. Radiative balance

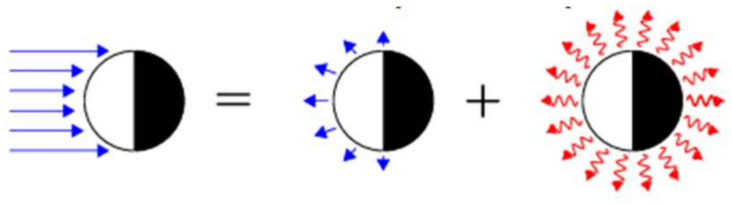
Definition of equilibrium temperature : basic calculation

Temperature of black body emitting the same energy flux as received from the star

Parameters:

- Planetary radius
- 1. Energy intercepted from the star (blue)
- 2. Energy reflected by the planet (blue)
- 3. Energy radiated by the planet (red)

Definition of Bond albedo : A_B = E2 / E1



E1 E2 E3

Basic calculation of the equilibrium temperature

With
R = planetary radius
F = stellar flux
σ = Stefan constant
A = Bond albedo
d = distance to the star
T_* = stellar temperature
T_{eq} = equilibrium temperature of the planet

$$\pi R^2 (1 - A) F = 4\pi R^2 \sigma T_{eq}^4$$

$$T_{eq} = \left[\frac{(1-A)F}{4\sigma}\right]^{1/4} = \sqrt{\frac{R_*}{d}} \left(\frac{1-A}{4}\right)^{1/4} T_*$$

Remarks :

1. Factor 4 = sphericity of the planet (don't forget ! see www.realclimate.org and the order of the knights of the flat Earth)
2. T_{eq} is independent on the planetary radius (at this level of approximation)
3. The calculations assumes a uniform temperature : if the planet is slowly rotating, this is valid only if the redistribution of heat from day to night hemisphere is efficient

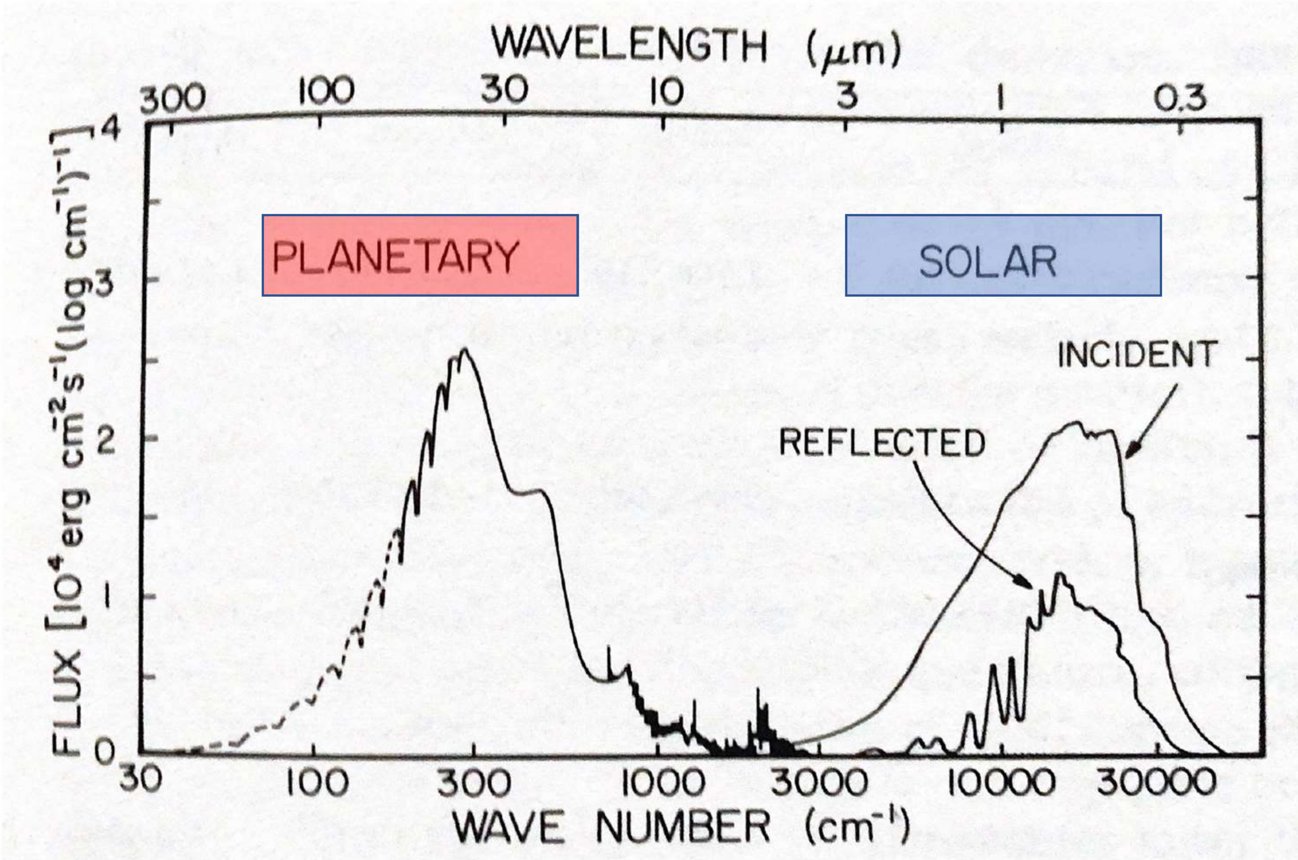
Radiative balance on a planet

Spectral variations of the three components:

- Solar incident flux
- Reflected flux
- Planetary emitted flux

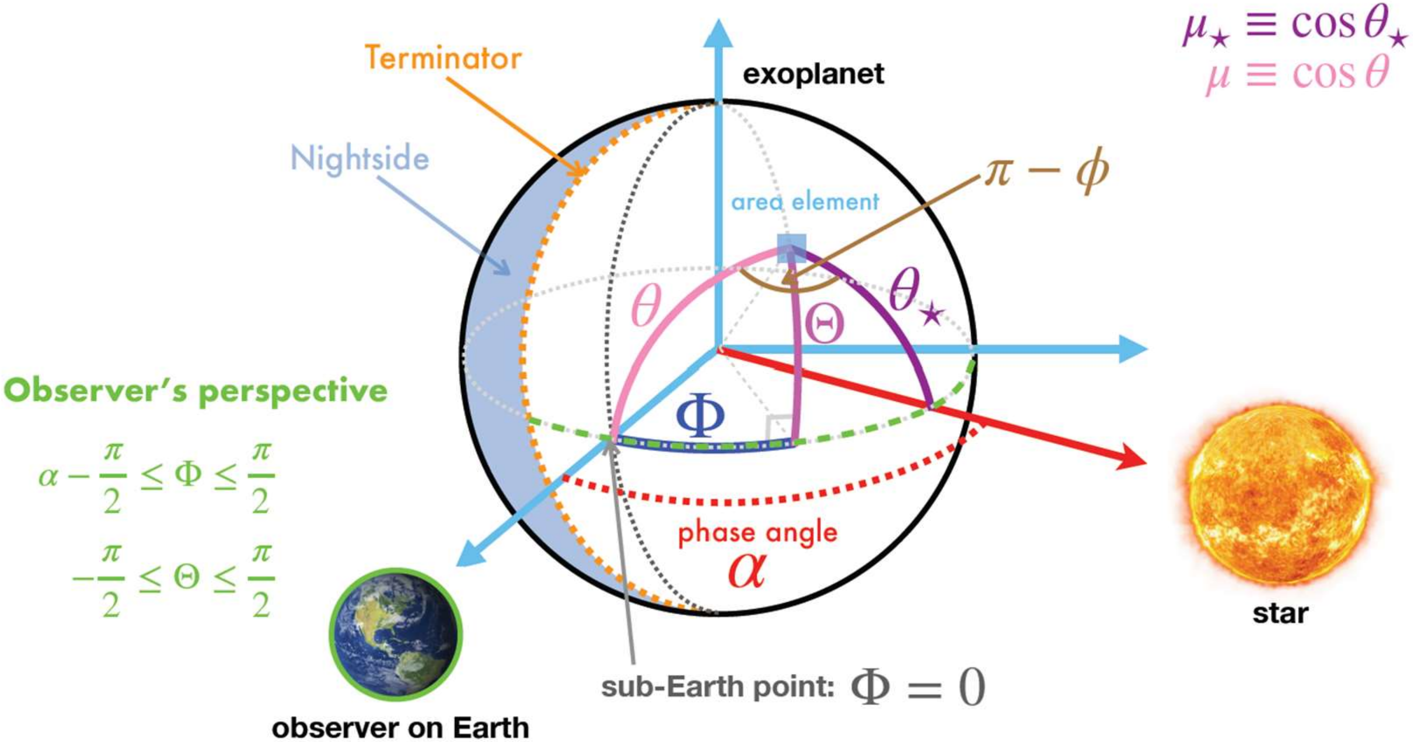
Integrations :

- Angular integration
- spherical integrations needed to obtain the total flux



The example of Jupiter (Hanel et al, 1982)

Geometry of a spherical planet illumination



From Heng et al arxiv 2021

More definitions for spherical photometry

- Geometric albedo p : amount of light reflected by a planetary body divided by the amount of light of a perfect lambertian disk of the same cross section area

$$p = \frac{I(0)}{I_{lam}}$$

- Phase function : directional dependence of light scattered in all directions by a planet
- Phase integral : q = 2 * integral of phase function
- Bond albedo or spherical albedo: fraction of total incident power scattered into space

$$A = p\,q$$

How to measure a Bond albedo ?

1) Measure the geometric albedo (for some phase) for every wavelength
2) Measure the phase integral, by repeating the observation for phase angle α = 0 to π

<u>Example of Venus :</u>

Phase functions => access to full disk viewing geometry

$\Rightarrow$Limited to inner planets !

or complete the phase curve with modeling…

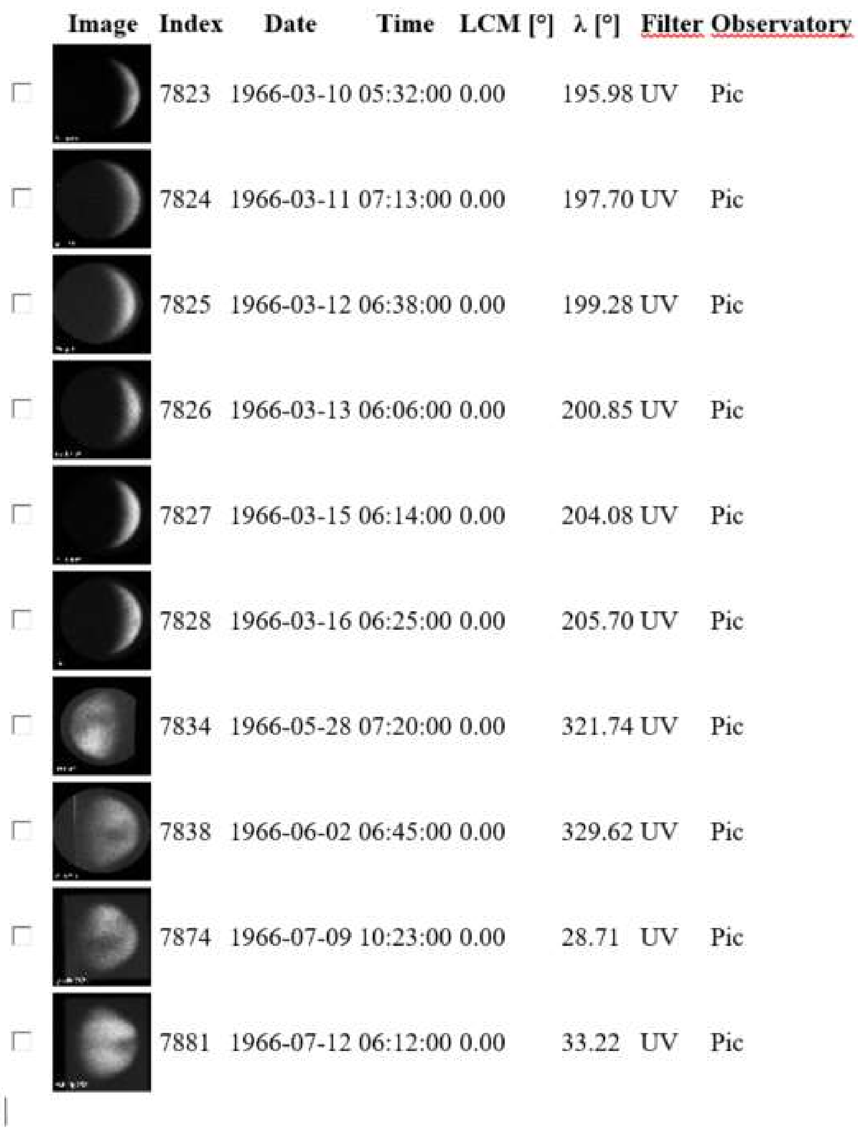


Image	Index	Date	Time	LCM [°]	λ [°]	Filter	Observatory
	7823	1966-03-10	05:32:00	0.00	195.98	UV	Pic
	7824	1966-03-11	07:13:00	0.00	197.70	UV	Pic
	7825	1966-03-12	06:38:00	0.00	199.28	UV	Pic
	7826	1966-03-13	06:06:00	0.00	200.85	UV	Pic
	7827	1966-03-15	06:14:00	0.00	204.08	UV	Pic
	7828	1966-03-16	06:25:00	0.00	205.70	UV	Pic
	7834	1966-05-28	07:20:00	0.00	321.74	UV	Pic
	7838	1966-06-02	06:45:00	0.00	329.62	UV	Pic
	7874	1966-07-09	10:23:00	0.00	28.71	UV	Pic
	7881	1966-07-12	06:12:00	0.00	33.22	UV	Pic

Modeling of spherical (or Bond) albedo

- Lambert reflection
- Rayleigh diffusion
- Scattering by cloud particles :

Parameters for a homogeneous cloud : single scattering albedo (ϖ) , phase function P(α)

models:

- isotropic scattering (P=1) for semi-infinite atmosphere: cf Chandrasekhar => fully analytical models available
- More general models : P calculated for particules (Mie for spherical particles ; Henyey Greenstein for empirical models of aspherical particles, etc.)

Modeling the reflection component through scattering models

From Heng et al arxiv 2021

Dyudina et al, 2016

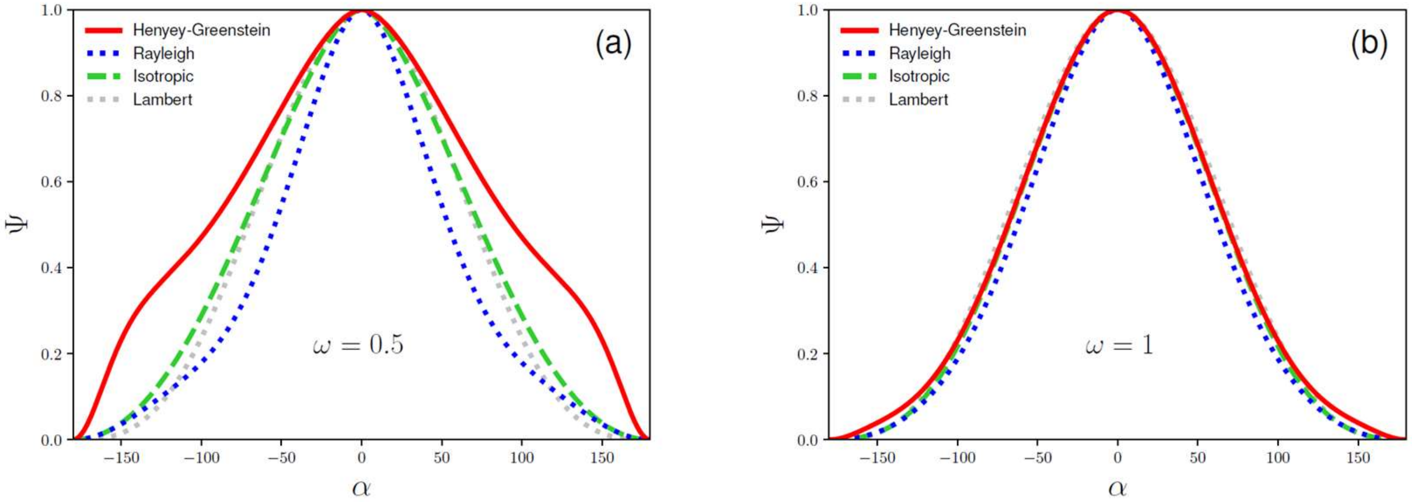
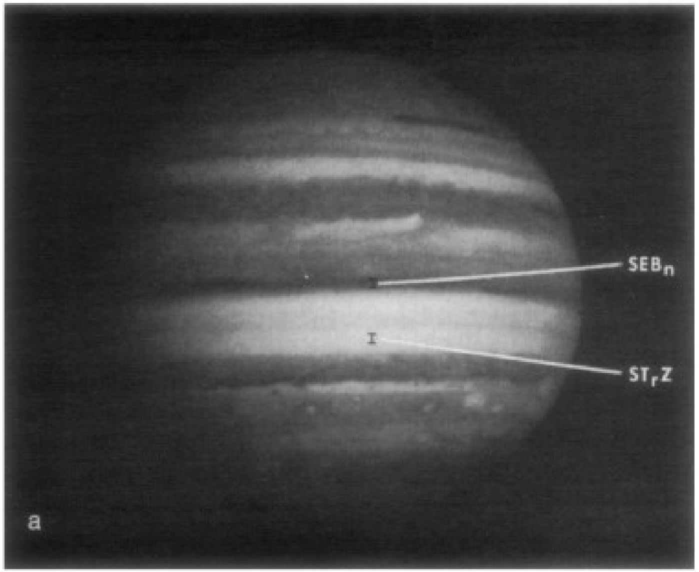


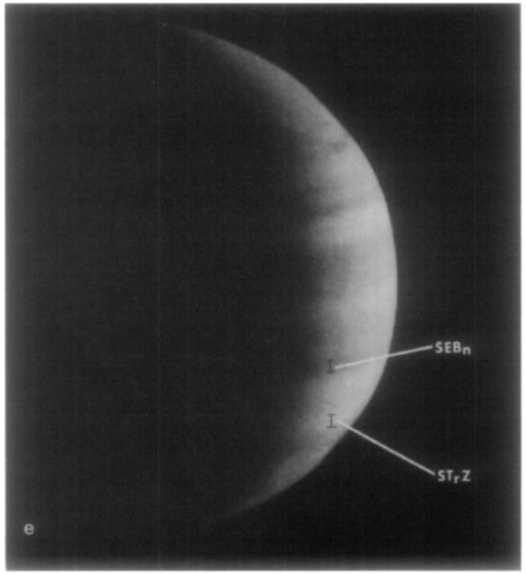
Figure 3: Integral phase function for the isotropic, Lambertian, Rayleigh and Henyey-Greenstein reflection laws for (a) $\omega = 0.5$ and (b) $\omega = 1$. As the strength of scattering increases, the integral phase function becomes approximately Lambertian. For illustration, the scattering asymmetry factor of the Henyey-Greenstein reflection law is chosen to be $g = 0.508$ such that the phase integral (single scattering only) has a value of 3.5, which matches the ensemble averaged value of a population of hot Jupiters [25].

Measurement of phase integral for Jupiter: Pioneer observations (1976)

TOMASKO, WEST, AND CASTILLO



PHOTOMETRY OF JUPITER



DYUDINA ET AL.

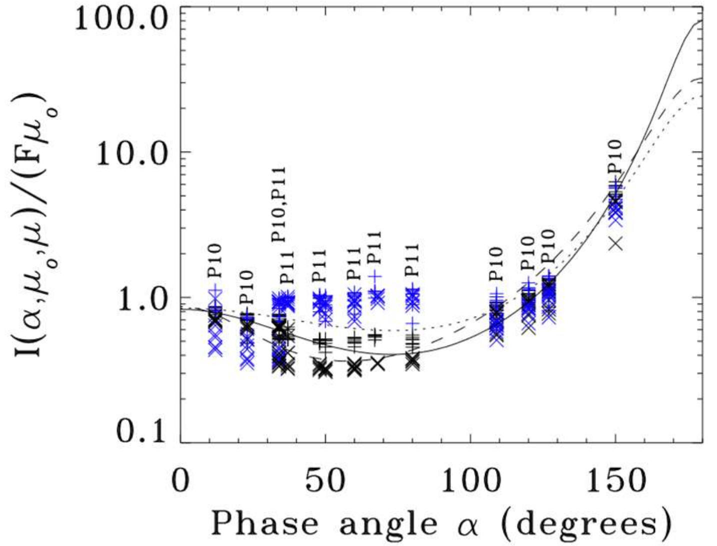


Figure 1. Fits of Henyey–Greenstein functions to the *Pioneer* 10 data, labeled *P*10 (Tomasko et al. 1978), and *Pioneer* 11 data, labeled *P*11 (Smith & Tomasko 1984). The data represent belts (× symbols) and zones (+ symbols) on Jupiter observed with the red (0.595–0.720 μm, black symbols) and blue (0.390–0.500 μm, blue symbols) filters. The solid, dashed, and dotted lines demonstrate the range of possible fits to the red filter points (black symbols).

Difficulty : obtain a global phase function for the disk by averaging belt/zones contrasts

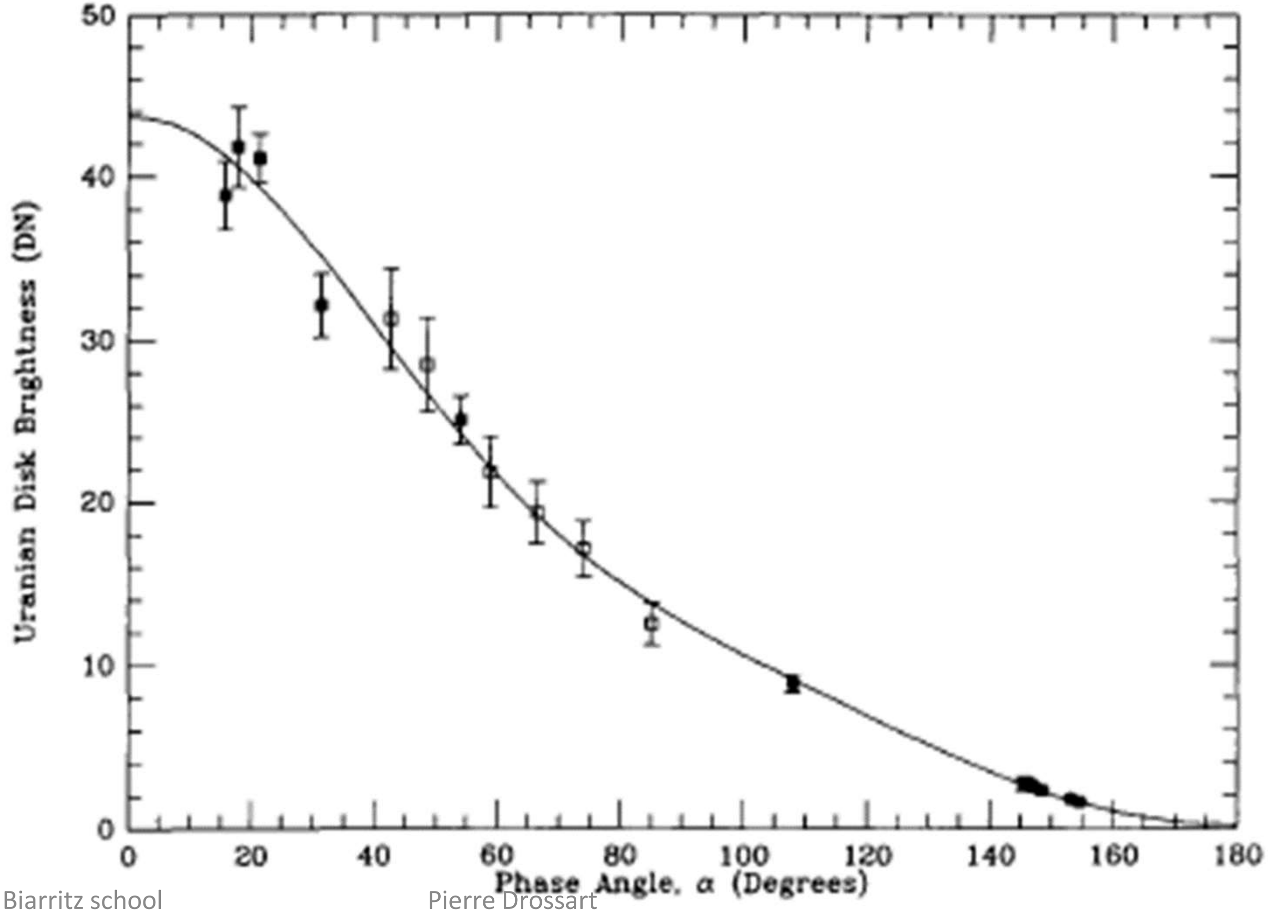
Tomasko et al, 1978

Dyudina et al, 2016

Phase curve for Uranus

Retrieval of the phase curve of Uranus from Voyager / IRIS observations

Pearl et al, 1990

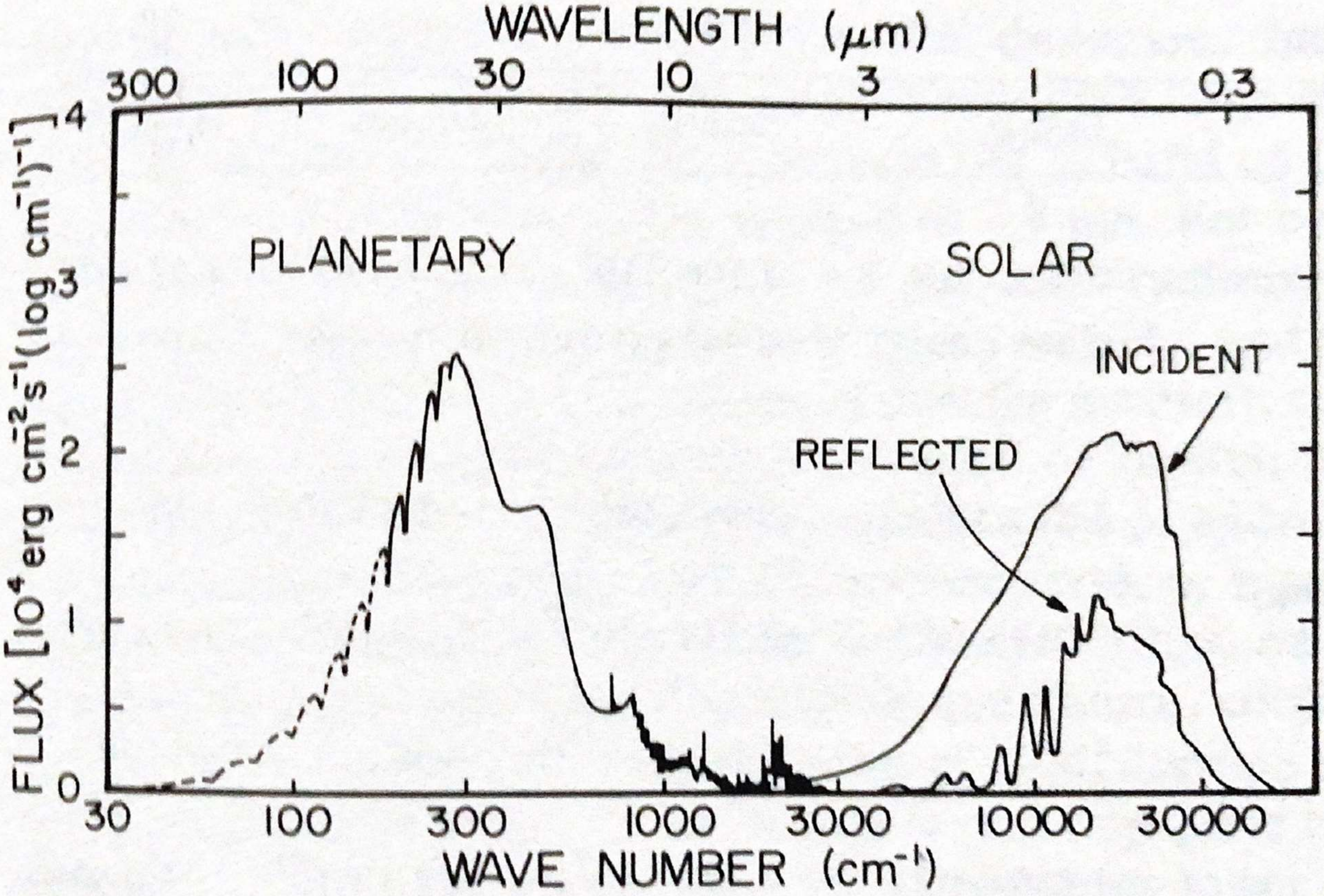


Jupiter

TABLE 3. Summary of Interferometer Measurement

Quantity	Numerical Value and Uncertainty	Probable Error, %
Average integrated radiance of five disk spectra between 175 and 2300 cm^{-1} and standard deviation	$3.482 \pm 0.006 \times 10^{-4}$ W cm^{-2} sr^{-1}	±0.16
Same spectra between 230 and 2300 cm^{-1}	$2.819 \pm 0.006 \times 10^{-4}$ W cm^{-2} sr^{-1}	±0.21
Possible error due to shape of Jupiter image, 5 times standard deviation	$2.819 \pm 0.030 \times 10^{-4}$ W cm^{-2} sr^{-1}	±1.06
Model calculation and error in extrapolation to low wave numers, 2% of radiance up to 230 cm^{-1}	$1.507 \pm 0.030 \times 10^{-4}$ W cm^{-2} sr^{-1}	±2.0
Total radiance (0–2300 cm^{-1})	$4.326 \pm 0.043 \times 10^{-4}$ W cm^{-2} sr^{-1}	±1.0
Possible error due to temperature sensor in instrument		±0.2
Thermal flux (0–2300 cm^{-1})	$1.359 \pm 0.014 \times 10^{-3}$ W cm^{-2}	±1.0
Equivalent blackbody temperature	124.4 ± 0.3 K	

From Hanel et al, 1981


WAVELENGTH (μm)
300
100
30
10
3
1
0.3
FLUX [10^4 erg $cm^{-2}s^{-1}$(log cm^{-1})$^{-1}$]
0
1
2
3
4
PLANETARY
SOLAR
INCIDENT
REFLECTED
30
100
300
1000
3000
10000
30000
WAVE NUMBER (cm^{-1})

Jupiter

TABLE 4. Summary of Energy Balance Quantities

Quantity	Numerical Value and Uncertainty	Probable Error, %
Bond albedo	0.343 ± 0.032	± 9.3
Solar constant at the earth	0.1374 ± 0.0007	± 0.5
Solar constant at Jupiter's mean distance, 5.203 AU	$5.076 \pm 0.025 \times 10^{-3}$ W cm^{-2}	± 0.5
Reflected solar energy	$1.741 \pm 0.163 \times 10^{-3}$ W cm^{-2}	± 9.4
Absorbed solar energy	$3.335 \pm 0.165 \times 10^{-3}$ W cm^{-2}	± 4.9
Total solar energy absorbed by Jovian disk (cross section = 1.5035×10^{20} cm^2)	$5.014 \pm 0.248 \times 10^{17}$ W	± 4.9
Thermal emission	$1.359 \pm 0.014 \times 10^{-3}$ W cm^{-2}	± 1.0
Total thermal energy emitted by Jovian ellipsoid (surface area = 6.1551×10^{20} cm^2)	$8.365 \pm 0.084 \times 10^{17}$ W	± 1.0
Total internal heat source	$3.351 \pm 0.262 \times 10^{17}$ W	± 7.8
Internal heat flux	$5.444 \pm 0.425 \times 10^{-4}$ W cm^{-2}	± 7.8
Energy balance, total emitted/absorbed solar energy	1.668 ± 0.085	± 5.1

From Hanel et al, 1981

Application to exoplanets : Kepler 7b

Measurement of a phase integral for Kepler 7b (Heng et al, 2021)

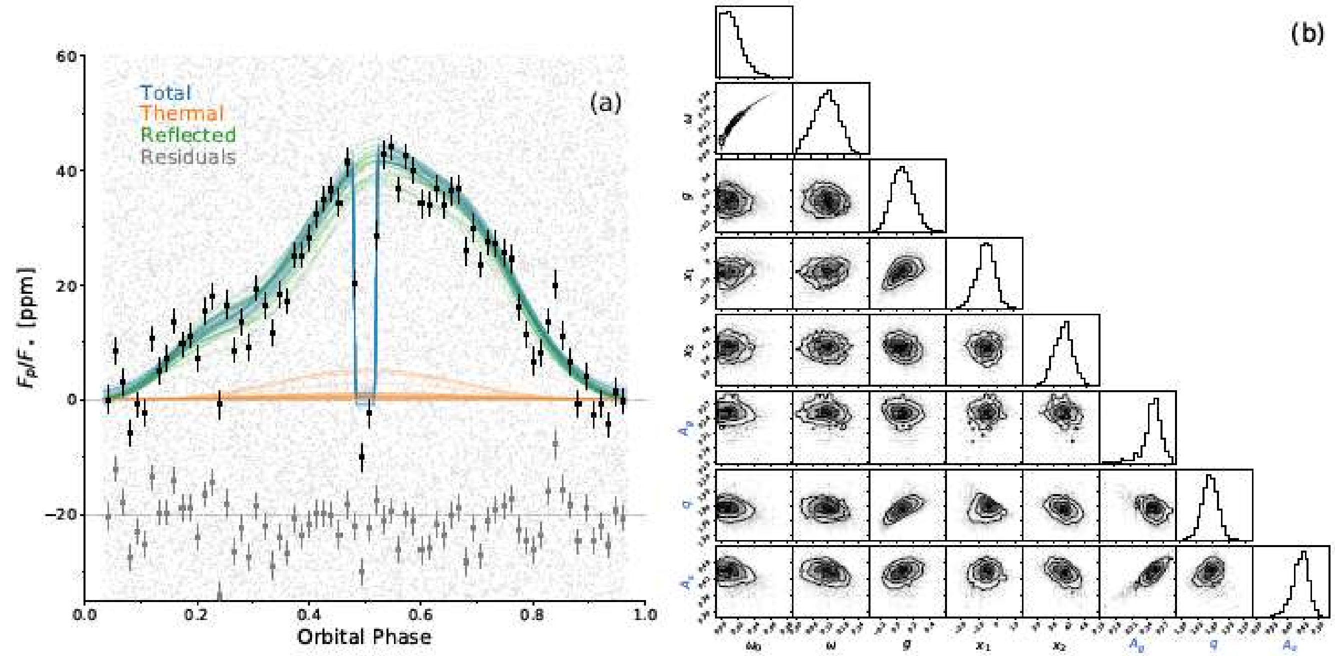


Figure 4: (a) We simultaneously fit (blue curve) for the reflected light (green curve), thermal emission (orange curve), secondary eclipse and a stellar/Gaussian process model for the Kepler phase curve of the hot Jupiter Kepler-7b, assuming an atmosphere with inhomogeneous cloud/haze cover. The observations are shown phase-folded and binned only for clarity (black squares). The fit is performed in the time domain for all 60,000 long-cadence fluxes without binning or phase folding. (b) Posterior distributions of physical parameters (black labels): the baseline single-scattering albedo ω_0, the total single-scattering albedo ω, the scattering asymmetry factor g, the local longitudes x_1 and x_2 between which the single-scattering albedo is ω_0. Also shown are the posterior distributions for the derived quantities (blue labels), which can be computed from the physical parameters: the geometric albedo A_g, the phase integral q, and the spherical albedo A_s in the Kepler bandpass.

2. Thermal profiles : Atmospheric layers

Thermosphere

- Collisionless region

Mesosphere

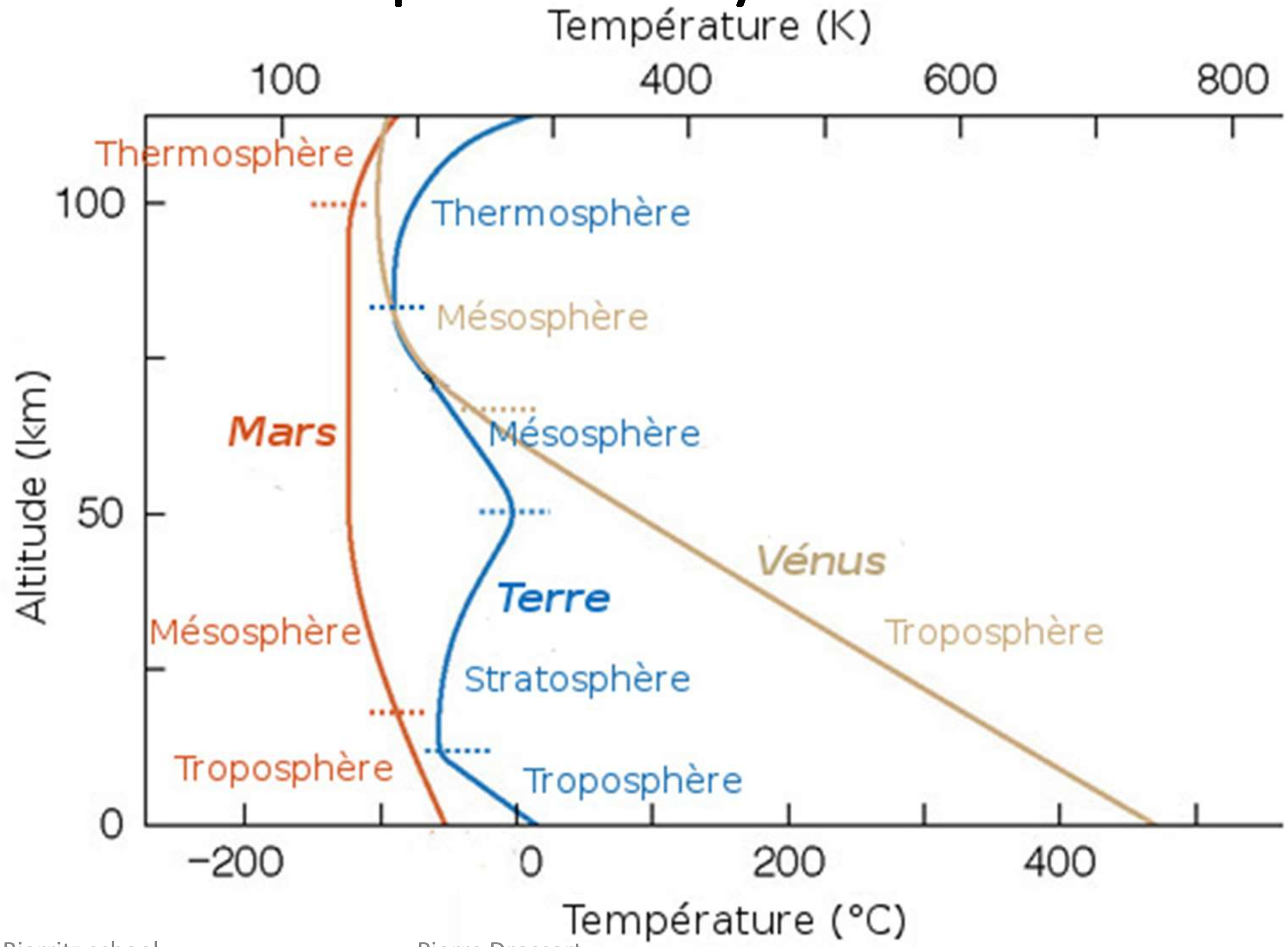
- Radiative layer

Stratosphere

- Between mesosphere and troposphere when temperature inversion is present

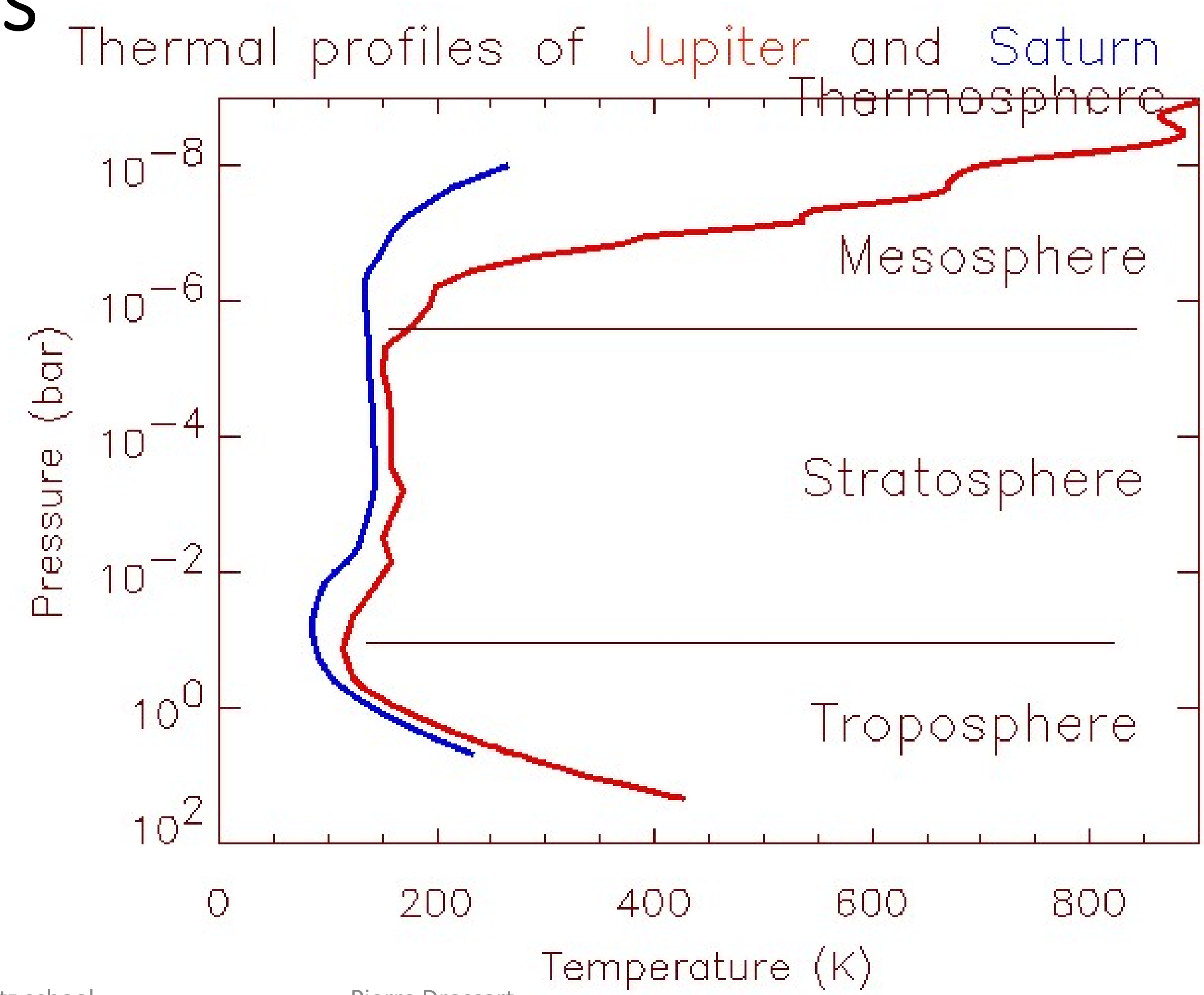
Troposphere

- Convective layer



Giant planets profiles

Jupiter and Saturn atmospheres



Energy transfer in atmospheres

Three modes of heat transfer:

- Electromagnetic radiation : long range – radiative transfer equation
- Conduction : thermal conduction is direct tranfer of thermal motions : local equations of conduction (or thermal diffusion in gases)
- Convection : in fluids only, corresponds to dynamical transport through fluid motions

Example : cooking of a nooddle can…

Atmospheric scale heights

It is of current experience that atmospheric density decreases with height. The scale parameter of this decrease is called « atmospheric scale heigth » - as models will show, the decrease is usually exponential, and the scale height is therefore simply defined as the altitude difference corresponding to a decrease by a factor e=2.718

Examples :

Planet	Venus	Earth	Mars	Jupiter	Io	Saturn	Titan	Uranus	Neptune
Scale height (in km)	10	8.4	11	25	7.9	48	21	27	22

Hydrostatic equilibrium

$$-\frac{GM_r\rho}{r^2}-\frac{dP}{dr}=\rho\frac{d^2r}{dt^2}$$

If we now assume the gas is static, the acceleration must be zero. This gives us the equation of *hydrostatic equilibrium* (HSE).

$$\boxed{\frac{dP}{dr}=-\frac{GM_r\rho}{r^2}}$$

- It is the pressure *gradient* that supports the star against gravity
- The derivative is always negative. Pressure must get stronger toward the centre

Hydrostatic atmosphere : plane parallel calculations

$dP = - \rho g dz$

With z=altitude ; g = acceleration of gravity (constant if $z << r_p$)

For an ideal gas : $P = \rho R T$ (with R = ideal gas constant)

$\Rightarrow$Definition of the Scale height : $dp/p = - dz/H$

With ***$H=R T / M g$*** (with M the molar mass of the atmosphere)

For an isothermal atmosphere $\rho = \rho_0 e^{-z/H}$ and $P = P_0 e^{-z/H}$

Hypothesis :

- variations of z << r (plane parallel atmosphere)
- Isothermal atmosphere for exponential decrease of P, ρ

Remarks $H=RT/Mg \Rightarrow m g H = k T$ (with m the molecular mass and k the Boltzmann constant)

Interpretation of H : a molecule traveling vertically on a distance H sees a difference in potential gravitational energy equivalent to its thermal energy

Effective temperature

The effective temperature is a measure of the power of thermal energy of a planet

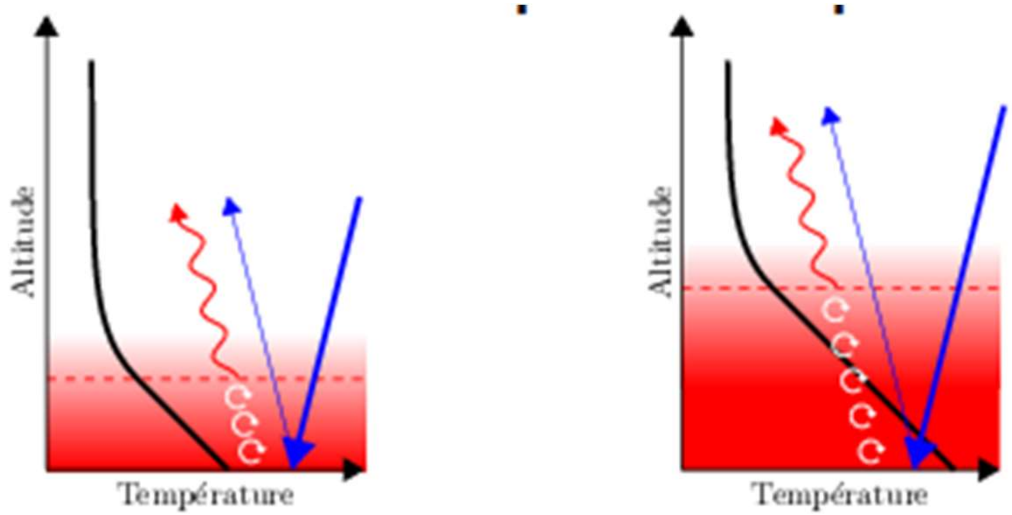
Teff=black body temperature emitting the same total power as the planet per m^2

When Tsurf > Teff : greenhouse effect

Planetary temperature in Kelvin	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Titan	Uranus	Neptune
Equilibrium	434	231	254	210	110	81	82	58	46
Effective	434	231	254	210	124	95	82	59	59
Mean surface	434	735	288	215	N/A	N/A	94	N/A	N/A

Greenhouse effect

Cause : Differential absorption by the atmosphere in visible/UV light (star illumination) and infrared emission (planetary emission)

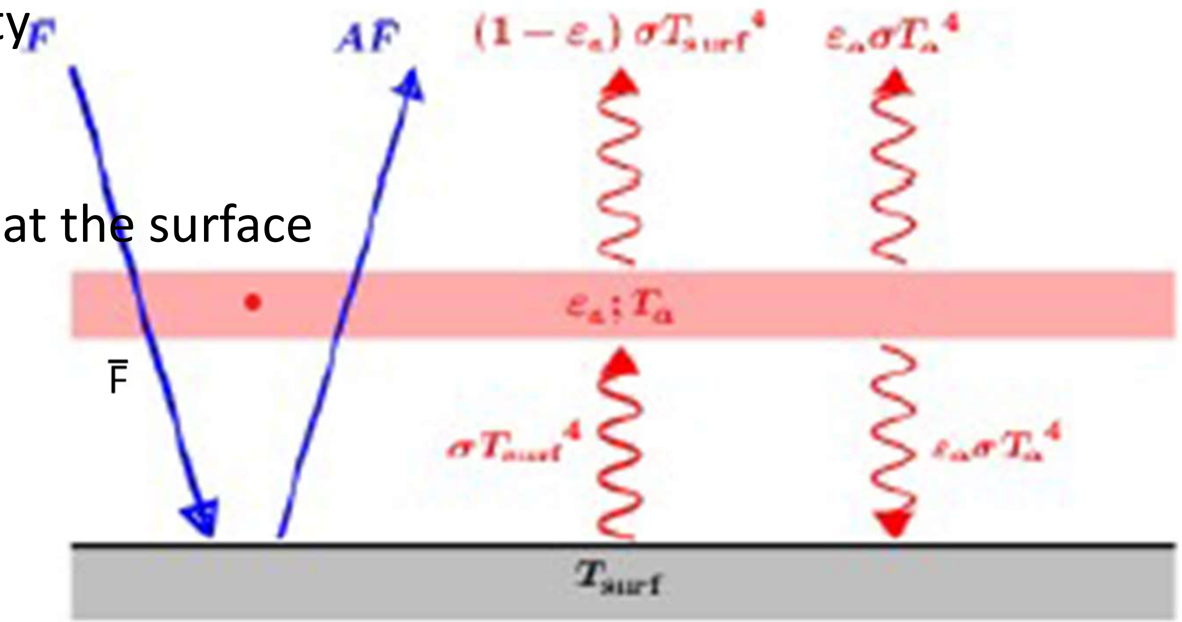


Mechanism : since Teff = Teq, the blackbody temperature of the surface must be higher

Greenhouse effect : one layer atmosphere

Calculation of the greenhouse effect,

Kirchhoff law : absorbance = emissivity



If F = stellar flux and Φ the mean flux at the surface of the planet then

$\pi R^2 F = 4 \pi R^2 \bar{F} \Rightarrow \bar{F} = F/4$

Energy balance equation :

1. $\bar{F} + \varepsilon_a \sigma T_a^4 = \bar{F} A + \sigma T_{surf}^4$
2. $\sigma T_{surf}^4 = (1 - \varepsilon_a) \sigma T_{surf}^4 + 2 \varepsilon_a \sigma T_a^4$

Solution with $T_{eq}^4 = (1-A) \bar{F}/4 \sigma$: $T_{surf}^4 = 2/(2 - \varepsilon_a) T_{eq}^4$ and $T_a^4 = 1/(2- \varepsilon_a) T_{eq}$

2 cases

1. $\varepsilon_a = 0 \Rightarrow T_{surf} = T_{eq}$ as was retrieved in the zero layer model – note that $T_a < T_{eq}$
2. $\varepsilon_a = 1 \Rightarrow T_a = T_{eq}$ and $T_{surf} > T_{eq}$ greenhouse factor = $2^{1/4}$

Internal sources

For Jupiter, Saturn and Neptune , it is observed that $T_{eff} > T_{eq}$

This denotes the presence of an internal source of energy

- Residual of gravitational contraction
- Demixing of He in the metallic hydrogen interior (chemical potential differentiation)

Adiabatic gradient

If convection is dominant : the atmospheric gradient (K/km) is called the adiabatic gradient.

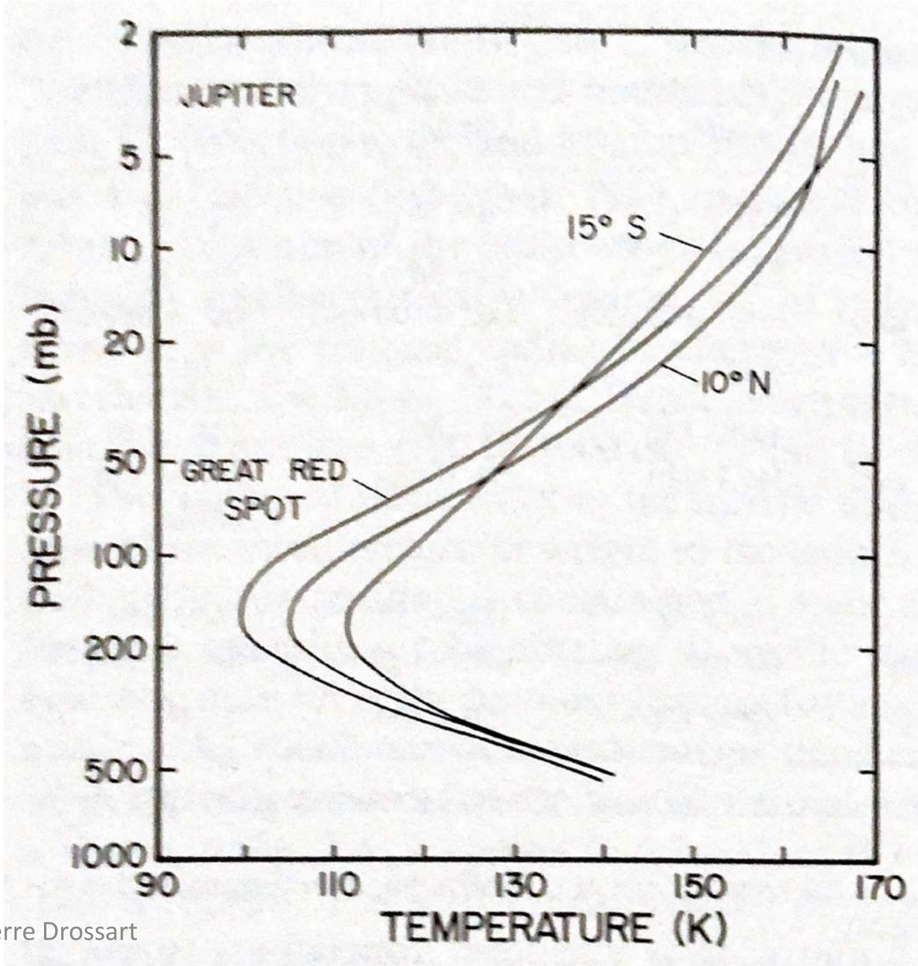
Convection implies conversion of gravitational potential energy and thermal energy to transport a gas parcel vertically => temperature decreases with height

Dry adiabat, humid adiabat

Definition of the troposphere : region where the energy transport is predominently by convection

Variations of atmospheric profiles for Jupiter

Large variations are observed in the upper atmosphere, but the deeper atmosphere becomes more uniform, below the meteorological layer defined as the region of solar energy absorption



Calculation of the Adiabatic gradient

Thermodynamics of the atmosphere
For an adiabatic (or isentropic) displacement :

dH= m C_p DT = TdS + VdP = VdP
Hydrostatic law : dP= -ρ g dz = - m/V g dz

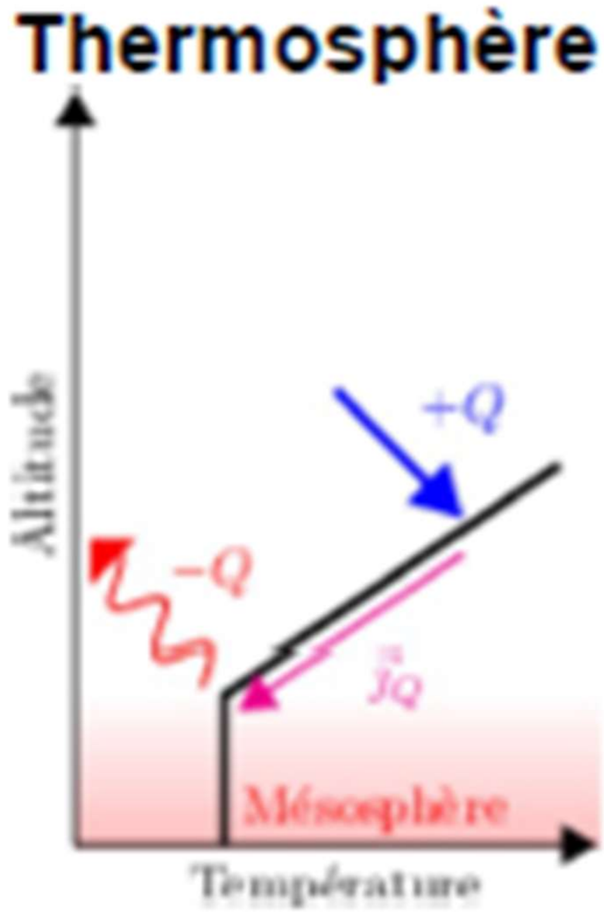
dT= - Γ dz with Γ = -g/C_p = adiabatic gradient

Interpretation : thermal energy is converted to potential gravitational energy when a parcel is moved isentropically

Stratospheres

• General condition for the formation of a stratosphere

If the source of heat +Q and the radiative well are at different altitudes, a conductive profile has to be generated with a heat current j_q forcing a positive thermal gradient



3. Upper atmospheres

Molecular composition is homogeneous in the troposphere, due to the turbulent mixing

This is not true in a regime of molecular diffusion

Equation of diffusion in an atmosphere

Mass conservation equation for the flux of a i^{th} consitutent in the atmosphere:

$d\varphi/dz = 0$

(in the absence of chemical/photochemical sources & loss, see O. Venot course)

Lower atmosphere : turbulent mixing => one scale height $H_a = RT/M_a g$

Molecular diffusion : one scale height per constituent $H_i = RT/M_i g$

General Equation of diffusion

$$\phi_i = n_i \left[-D_i \left(\frac{1}{n_i} \frac{dn_i}{dz} + \frac{1}{H_i} + \frac{1}{T} \frac{dT}{dz} \right) - K \left(\frac{1}{n_i} \frac{dn_i}{dz} + \frac{1}{H_a} + \frac{1}{T} \frac{dT}{dz} \right) \right]$$

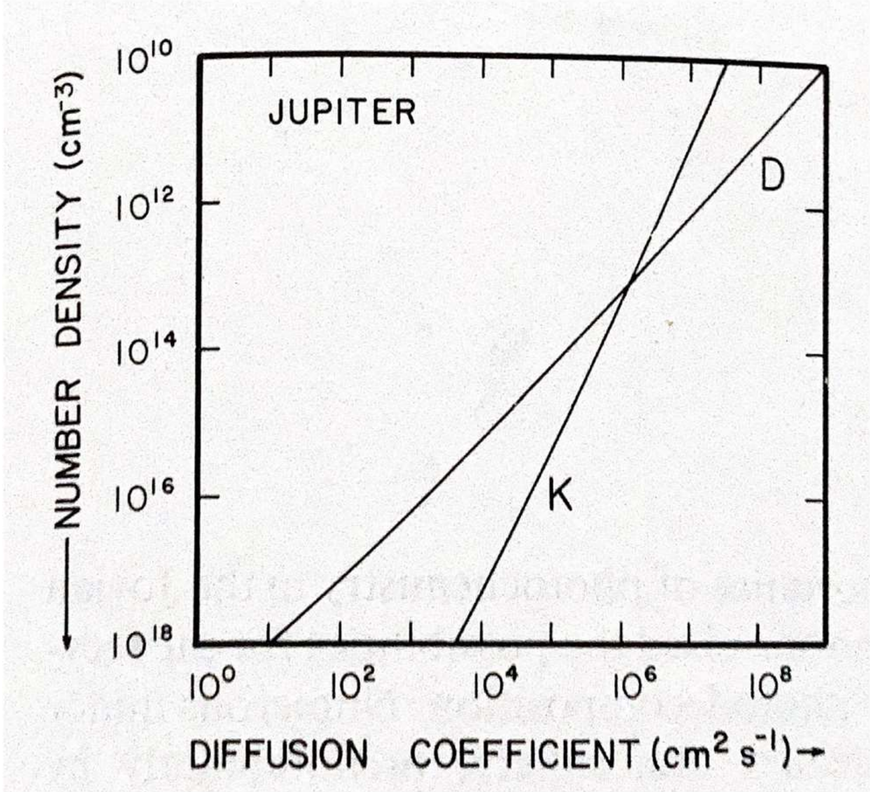
$$n_i(z) = n_i(z_0)(T_0/T) \exp\left(-\int_{z_0}^{z} dz/H_i \right)$$

Variation of number density in molecular regime $D_i >> K$

$$n_i(z) = n_i(z_0)(T_0/T) \exp\left(-\int_{z_0}^{z} dz/H_a \right)$$

Variation of number density in turbulent regime $D_i << K$

Molecular diffusion coefficient



From the kinetic gas theory we have :

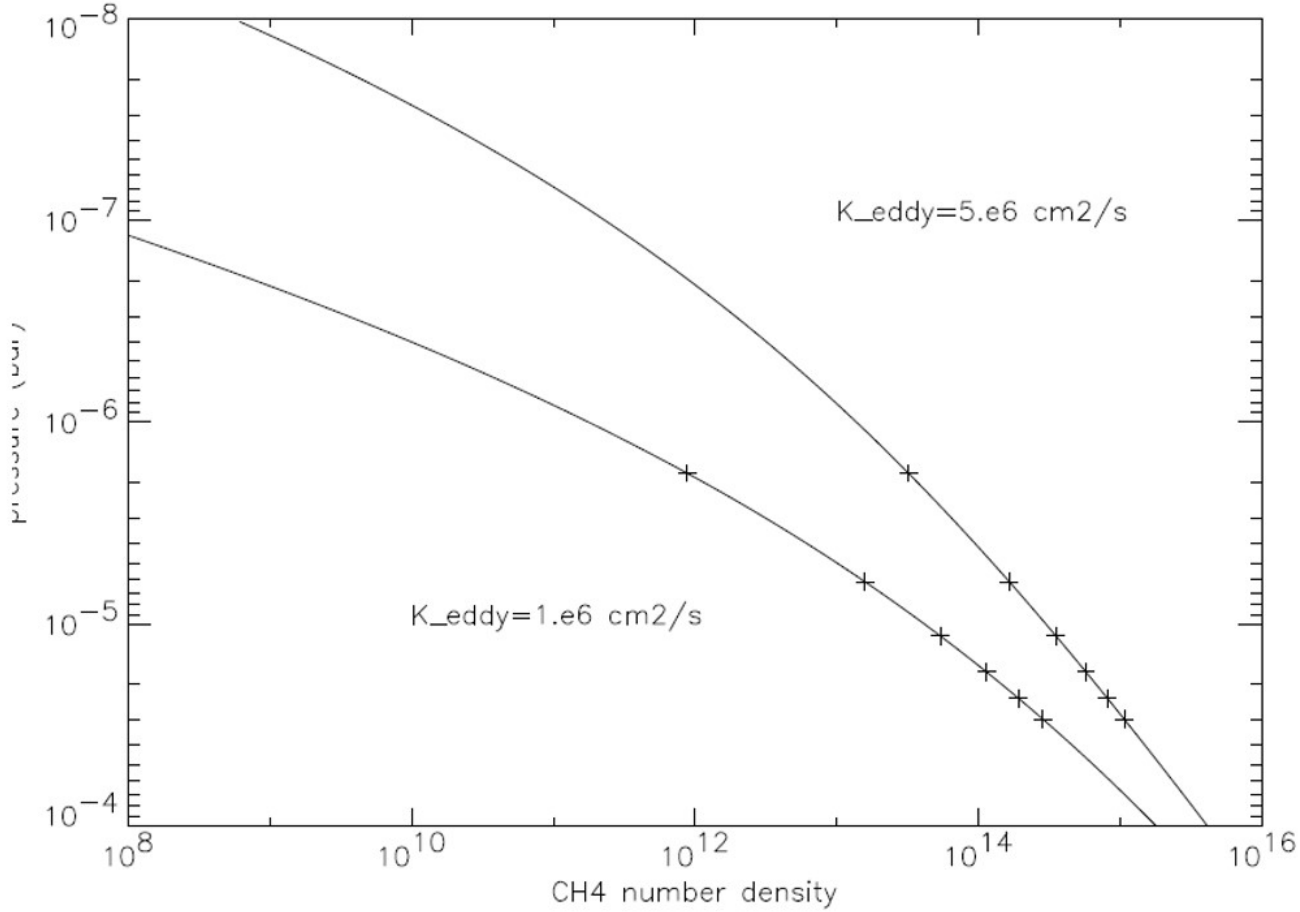
- $D_i = A\ T^s / n$
- $K \sim n^{-0.5}$

Thererefore the D coefficient will dominate at high altitude

$K = D_i$ defines the homopause

Jupiter CH_4 measurement

Contrary to Saturn, on Jupiter, the eddy diffusion coefficient is low enough to put the homopause of CH_4 below the main photodissociation level



4) Escape phenomena in planetary atmospheres

High interest for atmospheric escape to define the stability of planetary atmospheres and estimate the evolution of their composition.

Examples:

- stability of a H_2/He primary atmosphere for telluric planets after their formation
- Atmospheric escape of secondary atmospheres like on Venus (loss of H_2O proved by the D/H ratio) or Mars

=> Strong need for parametric modeling for long term evolution

Problem : even for Solar System planets, the escape phenomena are still not fully understood !

« 2010 crisis » : discrepancy between observations and models for Titan atmosphere

Physical parameters in escape models

Simplest physical model for a one component, spherically symmetric and 1D modeling, with energy deposition at a level R_0 below the simulation region: models show a dependance in only two parameters :

Jeans number :

$$\lambda(r) = \frac{{v_{esc}}^2}{U(r)^2} = \frac{G\,M\,m}{r\,k\,T(r)} = \frac{r}{H(r)} = \frac{\text{gravitational PE}}{\text{random thermal KE}}$$

With $v_{esc} = \sqrt{2GM/r}$

With $v_{th} = \sqrt{2kT/m}$

Knudsen number :

$$Kn(r) = \frac{l(r)}{H(r)} = \frac{\text{molecular mean free path}}{\text{atmospheric scale height}}$$

Small values of λ (comets) : hydrodynamical outflow / Large values of λ (giant or terrestrial planets) : Jeans escape

Kn << 1 corresponds to hydrodynamical escape; Kn >~1 : molecule-by-molecule escape (or Jeans escape)

Definition of exobase Kn ~ 1 : for Pluto at the exobase λ ~ 8.5 for CH_4 (moderately gravitationally bound atmosphere)

Pluto temperature : 68K / rp=1190km / Rexobase= 2900 km

Strobel, Pluto Atmospheric Escape, University of Arizona Press, Pluto book 2019

Kinetic theory of gases applied to escape

Boltzmann equation

$$\frac{\partial f_s}{\partial t} + \overleftrightarrow{v_s} \cdot \nabla f_s + \overleftrightarrow{g} \cdot \nabla_{v_s} f = \left(\frac{\delta f_s}{\delta t} \right)$$

Here $f_s = f_s(\overleftrightarrow{r}, \overleftrightarrow{v_s}, t)$ is the distribution function of species s,

$\overleftrightarrow{v_s}$ $\left(\frac{\delta f_s}{\delta t} \right)$, the Boltzmann collision integral.

Boltzmann equation is unfortunately difficult to apply, but can be handled with MC simulation

Strobel, Pluto Atmospheric Escape, University of Arizona Press, Pluto book 2019

Kinetic theory of gases applied to escape

Hydrodynamical escape valid for Kn < 0.2 and l <1 (derivation of Navier-Stokes and thermal conduction equations from Boltzmann equation)

Thermal heating equation is reduced to a Bernoulli equation

$$\frac{1}{2}v^2 + c_p T + \Phi_g \approx c_p T_0 + \Phi_{g0}$$

Strobel, Pluto Atmospheric Escape, University of Arizona Press, Pluto book 2019

Kinetic theory of gases applied to escape

Jeans escape takes place when $\lambda > 3$ at the exobase (located at Kn ~ 1)

f_{exo} = distribution function at the exobase (truncated Maxwellian) ; μ = cosine of v with radial direction

integration over the velocity volume. Escape flux is usually calculated at the exobase ($r=r_{exo}$)

$$\Phi(r \to \infty) = \iiint v \mu f_{exo} d^3 v = n_{exo}(r_0) \frac{n v_{th}}{4} (1 + \lambda_0) e^{-\lambda_0}$$

Strobel, Pluto Atmospheric Escape, University of Arizona Press, Pluto book 2019

Jeans escape : kinetic escape

Exact calculation of the distribution function possible, with some assumptions:

- No collision above exosphere
- Distribution function = truncated maxwellian
- Consequence : column density above the exobase ~ n0 x H
- Escape flux calculation

Intermediate escape model

- Direct Simulation with Monte-Carlo (DSMC) models have revisited the intermediate case
- With boundary condition at R_0 (exobase) : maxwellian distribution and no energy deposition above R_0
- For collisional flow, transition parameter λ~ 2 between molecule-by-molecule escape (Jeans) to organized outflow (hydrodynamical)

Strobel, 2019 (PSS, Pluto special issue)
On the theoretical front the two papers by Volkov et al. (2011a, 2011b) have clarified the general problem of thermal escape from planetary atmospheres and the transition, as the gravitational binding energy relative to thermal energy increases, from organized supersonic outflow to random evaporation of individual atoms/molecules at the exobase known as Jeans escape. This transition is narrow and very abrupt. They also found that escape rates were enhanced over traditional Jeans escape rates by approximately a factor of 2 in thermal escape regime for moderately gravitationally bound atmospheres such as Pluto's.

Strobel, *Pluto Atmospheric Escape*, Planetary Space Science, Pluto Special Issue 2019
Volkov et al. a*, Kinetic simulations of thermal escape from a single component atmosphere*. Phys. Fluids, 2011
Volkov et al. b,*Thermally-driven atmospheric escape: Transition from hydrodynamic to Jeans escape*. ApJ Lett., 2011

List of references

A list of publications for reference will be provided at the end of the school in a separate document.

A special mention for documents included in this presentation is from Emmanuel Marcq on-line course availablel on the site

sesp.esep.pro : thermal structure of atmospheres (in French)