# A Class of Nonmetric Couplings to Gravity

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A class of generically nonmetric couplings between nongravitational fields and the gravitational field is introduced. It is shown that these couplings violate the weak equivalence principle at a level (*a priori*) smaller than the current Eötvös-type experimental limits.

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Metric theories of gravity, i.e., theories embodying the Einstein equivalence principle, have deserved a privileged status among alternative theories for nearly eighty years. The prime example of a metric theory is of course Einstein's general relativity, but other famous examples include the 1914 Nordstrøm-Einstein-Fokker conformally flat theory [1,2], the Jordan-Brans-Dicke scalar-tensor theory [3,4], and the Rosen bimetric theory [5] (see Ni [6] and Will [7] for reviews on these and other theories). In a metric theory, the nongravitational fields of all types are coupled, in a universal way, to a single curved spacetime metric $g_{\mu\nu}$ associated with gravity ($g_{\mu\nu}$ is called the physical metric), in the local Lorentz frames of which the standard special-relativistic laws of physics are valid [8,9]. This coupling to gravity, which is referred to as the metric coupling, consists merely in the minimal replacement, in the Lagrangian density of special relativity (written in generally covariant form), of the Minkowski metric $\eta_{\mu\nu}$ by the physical metric $g_{\mu\nu}$. Metric theories of gravity differ from each other in their gravitational field equations, which include different types of fields, besides the physical metric, associated with gravity [8,9]. General relativity is essentially the unique theory possessing only the metric as a gravitational field.

The primary motivation underlying the postulates of metric theories of gravity is an empirical one. Indeed, the Einstein equivalence principle on which these theories are based has received numerous and precise experimental confirmations. Notably, the experiments performed by Roll, Krotkov, and Dicke [10] and by Braginsky and Panov [11] (following pioneering work by Eötvös, Pekar, and Fekete [12]), have verified that the equivalence principle is obeyed for typical metals to two parts in $10^{11}$ (Dicke experiment) and to one part in $10^{12}$ (Braginsky experiment). *A priori* the latter experiments test only that aspect of the equivalence principle known as the weak equivalence principle or principle of universality of free fall but, according to a conjecture first proposed by Schiff [13], they also provide evidence for the full Einstein equivalence principle, and thus for the metric-coupling postulates.

However, we show in this Letter that the empirical motivation for considering only metric theories of gravity may not be fully justified at present. We introduce a large class of generically *nonmetric* couplings to gravity, which thus do not in general satisfy the Einstein equivalence principle nor the weak equivalence principle. These couplings are obtained by an appropriate insertion of the gravitational field—a second-rank tensorial field—into the Lagrangian density of special relativity. The usual metric coupling to gravity belongs to this class, but appears as a simple, although notable, member of it. Our interest in the couplings of this class lies in the *a priori* very small violation of the (weak) equivalence principle that they predict. Indeed, we compute, in the restricted case of an electromagnetically bound matter system and using as the dominant gravitational potential that of the Sun at the level of the Earth, that the equivalence-principle violation is numerically equal, for gold and aluminum, to $1.2\times10^{-14}$ times a constant $b_2$ depending on the coupling in question. (We have $b_2=0$ in the case of the metric coupling.) Using the galactic potential as the dominant potential, we get a violation of the order of $10^{-12}b_2$. Thus, all couplings in this class that have a not too large value of $b_2$ seem *a priori* viable with respect to current Eötvös-type experiments, although only marginally viable if we use the galactic potential and make the comparison with the Braginsky experiment. All these couplings can potentially yield currently viable (in this respect) nonmetric theories of gravity. Note that this small level of weak-equivalence-principle violation should certainly be detected by future experiments testing the principle in space, like the STEP experiment [14]. (This experiment is now under assessment phase as a joint ESA-NASA mission.)

We can understand the small magnitude of the equivalence-principle violation as follows. First of all, the nonmetric couplings we consider have the property of being metric at first order in the field (in a sense made precise below). For such couplings, the passive gravitational to inertial mass ratio of a body in an exterior gravitational field differs from unity by a correction which is dominantly linear in the gravitational potential $U/c^2$. Second, for a body made of charged particles and electromagnetic fields with negligible weak and strong interactions, the correction in the mass ratio is made, at dominant order, of two contributions, one involving the specific electrostatic energy of the body, and one involving its specific magnetostatic energy [15,16]. In our computation, both contributions have $b_2$ in front of them so that they are zero in the metric-coupling case. However, the first electrostatic contribution has also, in addition to $b_2$, a factor in front of it which is proportional to $\gamma-1$, where $\gamma$ is the usual Eddington-Robertson [17] post-Newtonian parameter. Note that the use of the parameter $\gamma$ makes sense here because our couplings are metric to first order in the field. Thus, if the couplings are to yield the correct observational deflection of electromagnetic waves by the Sun and the correct Shapiro time delay, $\gamma-1$ will be very small (at present $|\gamma-1|\lesssim 2\times 10^{-3}$) and the electrostatic contribution in the gravitational to inertial mass ratio will be strongly suppressed with respect to the magnetostatic contribution. Specific magnetostatic energies are smaller than specific electrostatic energies by a factor of about $10^{-4}$ for typical metals (Haugan and Will [16]) and thus one finally has, if $\gamma=1$, the above quoted violation (if $\gamma$ is equal to $\pm 2\times 10^{-3}$ the violation is 10 times larger).

Let us denote by [18]

$$\mathcal{L}^0_{\rm NG}=\mathcal{L}^0_{\rm NG}[\eta_{\mu\nu},q_A] \qquad (1)$$

the special-relativistic Lagrangian density describing the dynamics of the nongravitational fields in the absence of gravity. It is a functional of the flat spacetime metric $\eta_{\mu\nu}(x)$, of the nongravitational fields $q_A(x)$ (labeled by the integer $A$), and of the spacetime derivatives of $\eta_{\mu\nu}$ and $q_A$. The Lagrangian density (1) is a scalar density with respect to arbitrary coordinate transformations. By inserting into (1) the physical metric $g_{\mu\nu}$ in place of the flat metric $\eta_{\mu\nu}$, and by posing $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ (in the arbitrary coordinate system we use), we get the Lagrangian density of the metric coupling to gravity in the form

$$\mathcal{L}^{(\rm metric)}_{\rm NG}=\mathcal{L}^0_{\rm NG}[\eta_{\mu\nu}+h_{\mu\nu},q_A]\,. \qquad (2)$$

Hereafter we regard the gravitational field as being described by the symmetric second-rank tensor field $h_{\mu\nu}$. Let us expand the right-hand side of Eq. (2) in perturbation series about $h_{\mu\nu}=0$; i.e., let us consider the formal *weak-field* expansion of the metric coupling. After discarding a total divergence, we get

$$\mathcal{L}^{(\rm metric)}_{\rm NG}=\sum_{n=0}^{+\infty}\frac{1}{n!}\left(h_{\mu\nu}\frac{\delta}{\delta\eta_{\mu\nu}}\right)^n\mathcal{L}^0_{\rm NG}[\eta_{\mu\nu},q_A]\,, \qquad (3)$$

where $\delta/\delta\eta_{\mu\nu}$ is the variational derivative operator that is defined by

$$\frac{\delta\mathcal{L}}{\delta\eta_{\mu\nu}}=\frac{\partial\mathcal{L}}{\partial\eta_{\mu\nu}}-\partial_\lambda\left(\frac{\partial\mathcal{L}}{\partial\partial_\lambda\eta_{\mu\nu}}\right)+\partial_\lambda\partial_\sigma\left(\frac{\partial\mathcal{L}}{\partial\partial_\lambda\partial_\sigma\eta_{\mu\nu}}\right)-\cdots, \qquad (4)$$

which acts in Eq. (3) with fixed $h_{\mu\nu}$ and $q_A$.

The class of couplings to gravity we consider in this Letter is the class of couplings which admit a formal weak-field expansion analogous to the expansion (3), namely,

$$\mathcal{L}_{\rm NG}=\sum_{n=0}^{+\infty}a_n\left(h_{\mu\nu}\frac{\delta}{\delta\eta_{\mu\nu}}\right)^n\mathcal{L}^0_{\rm NG}[\eta_{\mu\nu},q_A]\,, \qquad (5)$$

but in which the $a_n$'s are *arbitrary* coefficients for $n\geq 2$. For $n=0$ and 1 we impose

$$a_0=a_1=1\,. \qquad (6)$$

The first constraint of (6) is imposed in order that $\mathcal{L}_{\rm NG}$ reduces to $\mathcal{L}^0_{\rm NG}$ in the absence of gravity, and the second one ensures that the couplings have the correct Newtonian limit when the 00 component of $h_{\mu\nu}$ satisfies in this limit $h_{00}=2U/c^2$, where $U$ is the Newtonian potential (this can always be assumed, after a possible redefinition of the field variable). It follows from these two constraints (6) that the general coupling (5) can be put into metric form at first order in the field in the sense

$$\mathcal{L}_{\rm NG}=\mathcal{L}^0_{\rm NG}[\eta_{\mu\nu}+h_{\mu\nu},q_A]+O(h^2)\,. \qquad (7)$$

Thus, $\eta_{\mu\nu}+h_{\mu\nu}$ appears as an approximate physical metric.

It is interesting to note [19] that a nonperturbative representation can be given to the general coupling (5). Indeed, we introduce a function, or distribution, $f(s)$ of the real variable $s$, and satisfy for any $n$,

$$\int_{-\infty}^{+\infty}ds\frac{s^n}{n!}f(s)=a_n\,. \qquad (8)$$

The function $f(s)$ can be written as the formal series $f(s)=\sum(-)^n a_n(d/ds)^n\delta(s)$, where $\delta$ is the Dirac distribution. The general coupling (5) then reads

$$\mathcal{L}_{\rm NG}=\int_{-\infty}^{+\infty}ds\,f(s)\,\mathcal{L}^0_{\rm NG}[\eta_{\mu\nu}+sh_{\mu\nu},q_A] \qquad (9)$$

and the two constraints (6) become

$$\int_{-\infty}^{+\infty}ds\,f(s)=\int_{-\infty}^{+\infty}ds\,sf(s)=1\,. \qquad (10)$$

We can now adopt Eqs. (9),(10) as the defining equations of our class of couplings, where $f(s)$ is any function, or distribution, which is sufficiently well behaved so as to give to the definition a well-defined mathematical meaning. The form (9) of the coupling will be valid in a regime of strong gravitational fields.

The usual metric coupling (2) belongs to the class of couplings (5),(6) or (9),(10) and is characterized by $a_n=1/n!$ or $f(s)=\delta(s-1)$. Obviously, the other couplings of the class are nonmetric but we can easily find some approximate metric couplings by using adequate functions of $s$ which are picked around the value $s=1$. In this way we can define a subfamily of "Gaussian" nonmetric couplings by choosing, for any $\sigma$, the function $f(s)=(2\pi\sigma^2)^{-1/2}\exp[-(s-1)^2/2\sigma^2]$. These couplings tend to the metric coupling when $\sigma\to 0$. However, more fundamentally nonmetric couplings can also easily be obtained. For instance, a linear coupling (linear in the gravitational field) follows from the choice $f(s)=\delta(s)-d\delta(s)/ds$ (hence $a_n=0$ for $n\geq 2$). In this case one has

$$\mathcal{L}^{(\rm linear)}_{\rm NG}=\mathcal{L}^0_{\rm NG}+h_{\mu\nu}\frac{\delta\mathcal{L}^0_{\rm NG}}{\delta\eta_{\mu\nu}}\,. \qquad (11)$$

which can be rewritten as

$$\mathcal{L}_{\rm NG}^{(\rm linear)}=\mathcal{L}_{\rm NG}^{0}+\frac{\sqrt{-\eta}}{2}h_{\mu\nu}T^{\mu\nu}, \tag{12}$$

where we have introduced the stress-energy tensor $T^{\mu\nu}=(2/\sqrt{-\eta})\delta\mathcal{L}_{\rm NG}^{0}/\delta\eta_{\mu\nu}$ of the nongravitational fields in special relativity. The linear coupling has been adopted in several nonmetric theories, including the Fierz-Pauli theory [20] and the Capella-Naida theory [21]. (The coupling used in the Belinfante-Swihart theory [22,23] does not belong to our class of couplings.) Another example of a fundamentally nonmetric coupling, which is more interesting (in the author's opinion) than the linear coupling, can be obtained by assuming $a_n=1$ for any $n$, or by using the truncated exponential function $f(s)=Y(s)e^{-s}$ where $Y(s)$ is the usual Heaviside step function. Then the Lagrangian density, $\mathcal{L}_{\rm NG}^{(\rm exp)}$ say, satisfies the equation

$$\mathcal{L}_{\rm NG}^{(\rm exp)}=\mathcal{L}_{\rm NG}^{0}+h_{\mu\nu}\frac{\delta\mathcal{L}_{\rm NG}^{(\rm exp)}}{\delta\eta_{\mu\nu}} \tag{13}$$

or, equivalently,

$$\mathcal{L}_{\rm NG}^{(\rm exp)}=\mathcal{L}_{\rm NG}^{0}+\frac{\sqrt{-\eta}}{2}h_{\mu\nu}\Theta^{\mu\nu}=\int_0^{+\infty}ds\,e^{-s}\mathcal{L}_{\rm NG}^{0}[\eta_{\mu\nu}+sh_{\mu\nu},q_A], \tag{14}$$

where the second equality holds modulo a total divergence, and where $\Theta^{\mu\nu}=(2/\sqrt{-\eta})\delta\mathcal{L}_{\rm NG}^{(\rm exp)}/\delta\eta_{\mu\nu}$ is the (physical) stress-energy distribution of the matter fields in the presence of the gravitational field itself. Evidently, the coupling term in Eq. (14) represents a nonlinear interaction of the gravitational field with this physical distribution. The metric coupling, the Gaussian coupling, the linear coupling, and the "exponential" coupling have, respectively, $b_2=0$, $b_2=\sigma^2$, $b_2=-1$, and $b_2=+1$ in the notation of Eq. (19) below.

We now outline the computation of the dominant equivalence-principle violation as predicted by the couplings defined by Eqs. (5),(6) or by Eqs. (9),(10). We consider a matter system made of charged particles and electromagnetic fields described in special relativity by the Lagrangian density

$$\mathcal{L}_{\rm NG}^{0}=-c(-\eta_{\mu\nu}J_*^{\mu}J_*^{\nu})^{1/2}+(1/c)j_*^{\mu}A_\mu-(\sqrt{-\eta}/16\pi)\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}, \tag{15}$$

where $J_*^{\mu}$ and $j_*^{\mu}$ are the conserved mass and charged currents of the particles and where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. We insert this expression into Eq. (5) and perform the differentiations with respect to the metric holding $J_*^{\mu}$, $j_*^{\mu}$, and $A_\mu$ fixed. We assume that the gravitational field $h_{\mu\nu}$ takes a static and isotropic form of the type

$$h_{00}=\frac{2U}{c^2},\quad h_{0i}=0,\quad h_{ij}=\frac{2\gamma U}{c^2}\delta_{ij}, \tag{16}$$

where $\gamma$ is the usual post-Newtonian parameter, and that the coordinate system is Minkowskian for the metric: $\eta_{\mu\nu}={\rm diag}(-1,1,1,1)$. Finally we neglect all terms which are at least of formal order $O(c^{-6})$. The result of the computation is then the standard $TH\epsilon\mu$ expression [15]

$$\mathcal{L}_{\rm NG}=-\rho_*c^2\left(T-H\frac{\mathbf{v}^2}{c^2}\right)^{1/2}+\frac{1}{c}j_*^{\mu}A_\mu+\frac{1}{8\pi}\left\{\epsilon\mathbf{E}^2-\frac{\mathbf{B}^2}{\mu}\right\}+O\left(\frac{1}{c^6}\right), \tag{17}$$

where $\rho_*=J_*^0/c$, $v^i=J_*^i/\rho_*$, $E_i=F_{i0}$, $B_i=\frac{1}{2}\epsilon_{ijk}F_{jk}$, and where the functions $T$, $H$, $\epsilon$, and $\mu$ are given by

$$T=1-\frac{2U}{c^2}-b_2\frac{U^2}{c^4}+O\left(\frac{1}{c^6}\right), \tag{18a}$$

$$H=1+\frac{2\gamma U}{c^2}+(2\gamma+1)b_2\frac{U^2}{c^4}+O\left(\frac{1}{c^6}\right), \tag{18b}$$

$$\epsilon=1+(\gamma+1)\frac{U}{c^2}-\frac{1}{2}(\gamma+1)(\gamma-3)(b_2+1)\frac{U^2}{c^4}+O\left(\frac{1}{c^6}\right), \tag{18c}$$

$$\mu^{-1}=1-(\gamma+1)\frac{U}{c^2}+\frac{1}{2}(\gamma+1)(3\gamma-1)(b_2+1)\frac{U^2}{c^4}+O\left(\frac{1}{c^6}\right). \tag{18d}$$

The constant $b_2$ in these expressions is given by

$$b_2=2a_2-1, \tag{19}$$

where $a_2$ is the coefficient of the quadratic nonlinearity in the Lagrangian, or the second-order moment of the function $f$ [see Eqs. (5) and (8)]. Note that our class of couplings naturally yields the $TH\epsilon\mu$ expression without coupling to the pseudoscalar field suggested by Ni [24] (see Coley [25] for another nonmetric formalism which does not fit into our work). We can now compute, following Refs. [15], [21], and [7], the passive gravitational mass to inertial mass ratio of a (neutral) test body in the external field (16). The result is

$$\frac{m_P}{m_I}=1+2b_2(\gamma+1)\left\{(\gamma-1)\left(\frac{E_e}{m_I c^2}\right)+(3\gamma+1)\left(\frac{E_m}{m_I c^2}\right)\right\}\left(\frac{U}{c^2}\right). \quad (20)$$

We have retained here only the dominant contributions, and $E_e$ and $E_m$ are the electrostatic and magnetostatic self-energies of the body [we use the conventions of Eq. (2.118) of Ref. [7]]. Inserting into this expression $|\gamma-1|\leq 2\times10^{-3}$ as required by solar system observations, the values of specific electrostatic and magnetostatic energies given in Haugan and Will [16], and the Sun potential $U_\odot/c^2=9.8\times10^{-9}$, we arrive at

$$\left|\left\{\left(\frac{m_P}{m_I}\right)_{\rm Au}-\left(\frac{m_P}{m_I}\right)_{\rm Al}\right\}_e\right|\leq 10^{-13}|b_2|, \quad (21)$$

$$\left\{\left(\frac{m_P}{m_I}\right)_{\rm Au}-\left(\frac{m_P}{m_I}\right)_{\rm Al}\right\}_m=+1.2\times10^{-14}b_2, \quad (22)$$

for, respectively, the electrostatic and magnetostatic contributions to the difference of mass ratios of gold and aluminum. As we see, the couplings of the class (5),(6) having a not too large value of $b_2$ are (*a priori*) viable with respect to the Eötvös-Dicke-Braginsky experiments. Note, however, that the combination $1-TH^{-1}\epsilon\mu$, which measures the violation of local Lorentz invariance [7], is by Eqs. (18) equal to $3.8\times10^{-16}\,b_2$ for the Sun potential and $4\times10^{-12}b_2$ for the galactic potential. On the other hand, the various Hughes-Drever-type experiments [26] set a limit ranging from $10^{-22}$ to $10^{-26}$ on the product of $1-TH^{-1}\epsilon\mu$ with $(V/c)^2$, where $V$ is (typically) the velocity of the laboratory. Using $V/c\geq10^{-4}$, we see that the most precise of these experiments conflict with the predictions, unless $b_2$ is very small.

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