Mass Determination of Groups of Galaxies: Effects of the Cosmological Constant

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Abstract

The spherical infall model first developed by Lemaître and Tolman was modified in order to include the effects of a dark energy term. The resulting velocity-distance relation was evaluated numerically. This equation, when fitted to actual data, permits the simultaneous evaluation of the central mass and of the Hubble parameter. Application of this relation to the Local Group, when the dark energy is modeled by a cosmological constant, yields a total mass for the M31-Milky Way pair of $(2.5 \pm 0.7) \times 10^{12} M_\odot$, a Hubble parameter $H_0 = 74 \pm 4\, kms^{-1}Mpc^{-1}$ and a 1-D velocity dispersion for the flow of about 39 kms^{-1}. A similar analysis for the Virgo cluster yields a mass of $(1.10 \pm 0.12) \times 10^{15} M_\odot$ and $H_0 = 65 \pm 9\, kms^{-1}Mpc^{-1}$. Several other groups of galaxies were also studied and the results obtained lead to an estimation of the Hubble parameter, namely $h = 0.67 \pm 0.03$.

The spherical infall model:

The evolution of a self-gravitating zero-pressure fluid with spherical symmetry was first considered by Lemaître (1933) and Tolman (1934). The Lemaître-Tolman model describes quite well the dynamics of an extended halo around a bound central core, asymptotically approaching a homogeneous Friedmann background. In this situation, three main distinct regions can be distinguished: (1) the central core, in which the shell crossing has already occurred, leading to energy exchanges which transform radial into transverse motion; (2) the zero-velocity surface, boundary which separates infalling and expanding bound shells and (3) the "marginally" bound surface (zero total energy), segregating bound and unbound shells (see Fig. 1).

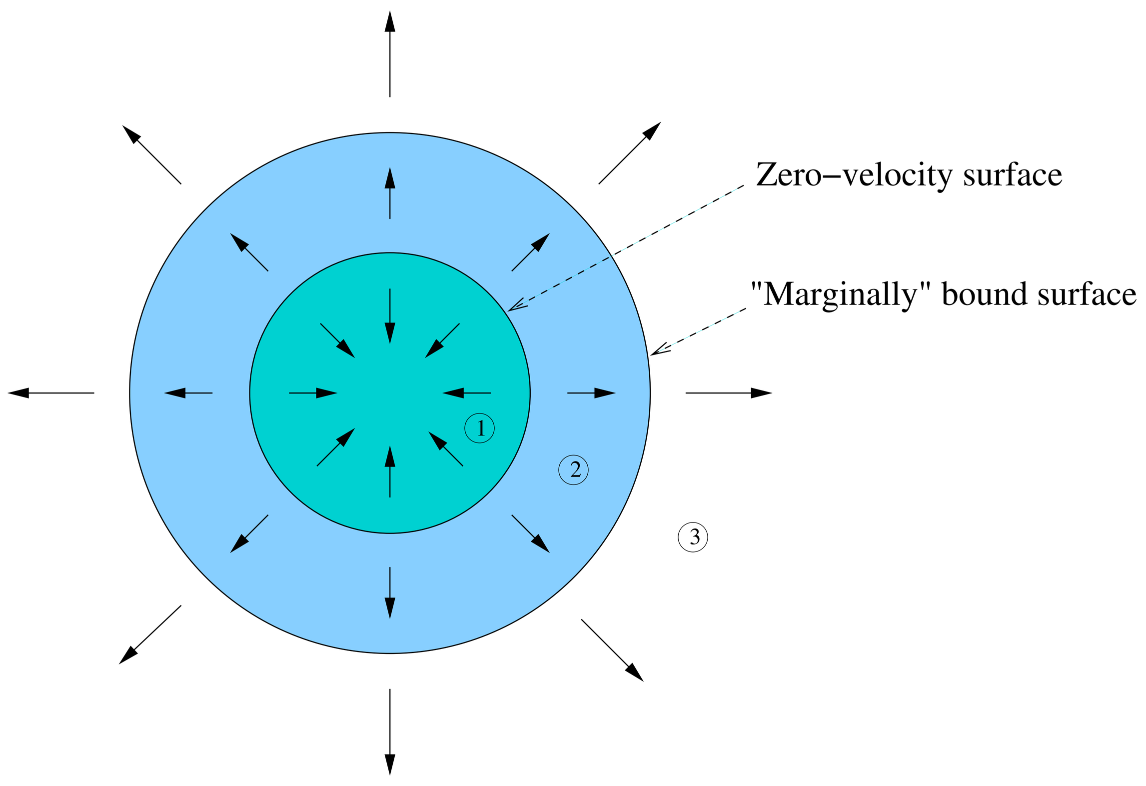

Figure 1: Diagram of the spherical infall model

Mass determination of group of galaxies:

Lynden-Bell (1981) and Sandage (1986) proposed an alternative method to the virial relation in order to estimate the mass of the Local Group, which can be extended to other systems dominated either by one or a pair of galaxies. Their analysis is essentially based on the spherical infall model. If the motion of bound satellites is supposed to be radial, the resulting parametric equations describe a cycloid. Initially, the radius of a given shell embedding a total mass M expands, attains a maximum value and then collapses. At maximum, when the turnaround radius R_0 is reached, the radial velocity with respect to the center of mass is zero. For a given group, if the velocity field close to the main body, probed by satellites, allows the determination of R_0, then the mass can be calculated straightforwardly from the relation

$$M = \frac{\pi^2 R_0^3}{8GT_0^2} \qquad (1)$$

where T_0 is the age of the universe and G is the gravitational constant.

The velocity-distance relation:

If displacements of galaxies, here associated to the outer halo shells, develop mainly at low redhsifts when the formation of the mass concentration around the core is nearly complete, then the equation of motion for a spherical shell of mass m, moving radially in the gravitational field created by a mass M inside a shell of radius R, including the dark energy term is

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2} - \frac{(1+3w)}{2}\Omega_v H_0^2 R\left(\frac{a_0}{a}\right)^{3(1+w)} \qquad (2)$$

where $M = 4\pi\int_0^R r^2\rho_m dr$ and a is the scale parameter (the present value is taken as a_0 = 1). The latter satisfies the Hubble equation

$$\left(\frac{dlg\, a}{dt}\right)^2 = H_0\left[\Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_v\left(\frac{a_0}{a}\right)^{3(1+w)}\right] \qquad (3)$$

Here, the common assumption that the dark energy can be modeled as being a fluid with an equation of state $P = w\epsilon$ was adopted and in the two equations above, the dependence of the dark energy on the scale parameter was obtained by solving the energy conservation for such a component. Eq. (2) is intended to describe the motion of shells in the halo, excluding the central region where shell crossing effects have probably already occurred.

Defining the dimensionless variables $y = R/R_0$, $\tau = tH_0$ and $x = a/a_0$, eqs. (2) and (3) can be rewritten as

$$\frac{d^2y}{d\tau^2} = -\frac{1}{2}\left[\frac{A}{y^2} + Byx^{-3(1+w)}\right] \qquad (4)$$

and

$$\frac{dx}{d\tau} = \sqrt{\frac{\Omega_m}{x} + \frac{\Omega_v}{x^{(1+3w)}}} \qquad (5)$$

where we have introduced the parameters $A = 2GM/(H_0^2R_0^3)$ and $B = (1+3w)\Omega_v$. These equations were solved numerically by adopting the following procedure. For a given redshift, the initial value of the scale parameter is derived as well as the corresponding instant of time from the Hubble equation. If initially, at high redshifts (here taken around $z \sim 100$), the dark energy term is negligible, then using a Taylor expansion of the standard Lemaître-Tolman solution, when the angle parameter $\theta << 1$, the initial values of y and its derivative $dy/d\tau$ can be estimated. For a given value of w, the parameter A is varied until the condition defining the zero-velocity surface, e.g., $dy/d\tau$ = 0 at y = 1 is satisfied. For the particular case $w = -1$, representing a cosmological constant, $A = 3.658$. Therefore, the mass inside the zero-velocity radius R_0 is

$$\boxed{M = 1.827\frac{H_0^2R_0^3}{G} = 4.1\times10^{12}h^2R_0^3\ M_\odot} \qquad (6)$$

Comparing with eq. (1), we notice that the inclusion of the dark energy term represents, for a given R_0, an *increase* of about 38% on the mass derived by such a procedure.

Once the parameter A is known, the velocity-distance relation, v = v(R), for different shells at a *given time* is obtained by varying their energy. Shells with negative energy will expand, halt and fall back toward the center, while shells with positive energy expand forever, according to the aforementioned characterization of regions (2) and (3). At a given time, there is a critical energy E_c which defines the zero-velocity radius. Shells having $E < E_c$ have already crossed the turnaround point and are collapsing. Consequently, they have *negative* velocities. Shells with $E > E_c$ are still expanding and thus have *positive* velocities.

For the case $w = -1$, the resulting numerical values are quite well fitted by the relation

$$\boxed{v(R) = -\frac{0.875H_0}{R^n}\left(\frac{GM}{H_0^2}\right)^{(n+1)/3} + 1.274H_0R} \qquad (7)$$

with $n = 0.865$.

Applications:

In this section, the derived velocity-distance relation is applied to the Local Group, the Virgo cluster, and groups relating to M81, Sculptor and IC342/Maffei. The necessary observationnal data can be found in Karachentsev et al. (2002, 2003, 2005). For each group, the velocity and the distance for galaxies with respect to the mass center of the system and the best fit solution to $v = v(R)$ relation for eq. (7) are represented in Fig. 2.

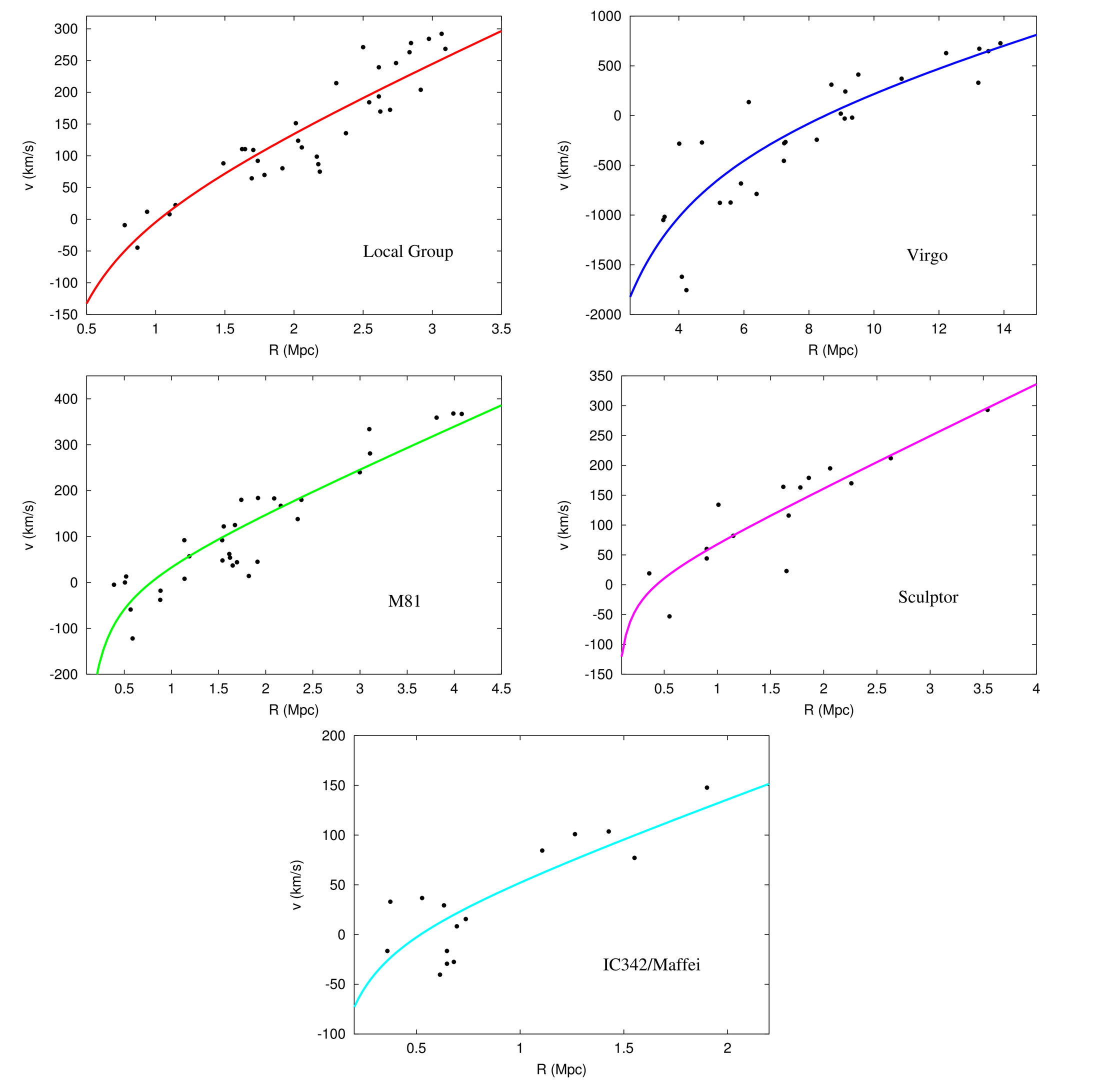

Figure 2: Velocity and distance data with respect to the mass center of the system for galaxies belonging to a studied group.

The previous fit solutions yield to estimation of the mass of each group of galaxies as well as the Hubble parameter. The results obtained are summarized in the following table.

Group of galaxies	mass ($M_\odot$)	h
Local Group	$(2.5 \pm 0.7) \times 10^{12}$	0.74 ± 0.04
Virgo	$(1.10 \pm 0.12) \times 10^{15}$	0.65 ± 0.09
M81	$(9.7 \pm 3.4) \times 10^{11}$	0.69 ± 0.05
Sculptor	$(1.5 \pm 1.3) \times 10^{11}$	0.67 ± 0.06
IC342/Maffei	$(2.0 \pm 1.2) \times 10^{11}$	0.58 ± 0.10

Now, if we take into account all the derived values of the Hubble parameter, we find an interesting result which is in good agreement with previous studies:

$$\boxed{h = 0.67 \pm 0.03} \qquad (8)$$

The enigma of the cold Hubble flow:

The velocity-distance relation gives also an indication of the dispersion of the peculiar velocities over the Hubble flow. Indeed, the local velocity dispersion is known to be quite small (Giraud 1986; Schlegel et al. 1994), a fact referred usually as the "coldness" of the local flow. Thus, an investigation of the dynamics of the Local Group and its environment by using numerical simulations permits to test different Cold Dark Matter models.

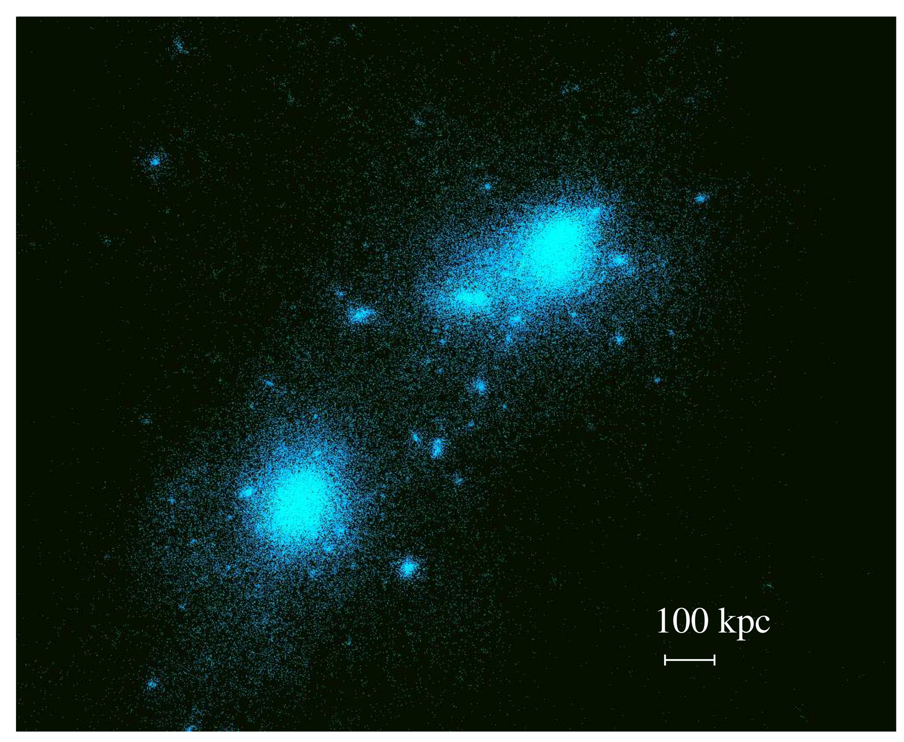

Figure 4: Simulation of the Local Group with the technique of "re-simulation". This example of pair of dark matter halos have physical characteristics similar to the Milky Way-M31 pair.

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