Einstein-Cartan Theory and Averaging

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$\Rightarrow$ Such a manifold is called a **Riemann-Cartan spacetime** U_4.

- In a Riemann-Cartan spacetime the difference between the affine connection and the Levi-Civita connection defines the **contortion tensor**
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$\Rightarrow$

$$(L_4, g) \xrightarrow{Q=0} U_4 \xrightarrow{T=0} V_4 \xrightarrow{R=0} R_4$$

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- In terms of Weitzenböck connection

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Averaging Process and Averaged Connection

$$\langle T^{\mu}{}_{\nu}\rangle(x) = \frac{1}{V_\Sigma}\int_\Sigma P^{\mu}{}_{\mu'}(x,x')P_{\nu}{}^{\nu'}(x,x')\,T^{\mu'}{}_{\nu'}(x')e(x')d^4x'$$

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$\Rightarrow$ The averaged geometry is a Riemann-Cartan geometry

Poincaré Gauge Theory

Hehl, von der Heyde, Kerlick, and Nester:

- Lagrangian invariant under Poincaré gauge transformations

$$\mathcal{L} = \mathcal{L}(\Psi, \partial\Psi, g, \partial g, T)$$

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Hehl, von der Heyde, Kerlick, and Nester:

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- They are related according to
$$\mu^{\lambda\mu\nu} = -\tau^{\lambda\mu\nu} + \tau^{\mu\nu\lambda} - \tau^{\nu\lambda\mu}$$
and
$$\Sigma^{\mu\nu} = \sigma^{\mu\nu} - (\nabla_\lambda + 2T^\rho{}_{\lambda\rho})\mu^{\mu\nu\lambda}$$

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$$\begin{aligned}&\mathring{R}^{\mu\nu} - \tfrac{1}{2}\mathring{R}g^{\mu\nu}\\ &= 8\pi G\sigma^{\mu\nu} + 8\pi G(-2\tau^{\mu\rho}{}_{\lambda}\tau^{\nu\lambda}{}_{\rho} + 2\tau^{\mu\rho}{}_{\rho}\tau^{\nu\lambda}{}_{\lambda} - 2\tau^{\mu\rho\lambda}\tau^{\nu}{}_{\rho\lambda}\\ &+\tau^{\rho\lambda\mu}\tau_{\rho\lambda}{}^{\nu} + \tfrac{1}{2}g^{\mu\nu}(2\tau_{\sigma}{}^{\rho}{}_{\lambda}\tau^{\sigma\lambda}{}_{\rho} - 2\tau_{\sigma}{}^{\rho}{}_{\rho}\tau^{\sigma\lambda}{}_{\lambda} + \tau^{\sigma\rho\lambda}\tau_{\sigma\rho\lambda}))\end{aligned}$$

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- Nonminimal coupling of the Riemannian connection to the matter field Ψ

$$\bar{\Gamma}^{\lambda}{}_{\mu\nu} = \mathring{\Gamma}^{\lambda}{}_{\mu\nu} + 8\pi G \left(\tau_{\mu\nu}{}^{\lambda} - \tau_{\nu}{}^{\lambda}{}_{\mu} + \tau^{\lambda}{}_{\mu\nu} + \delta^{\lambda}_{\mu}\tau_{\nu\sigma}{}^{\sigma} - g_{\mu\nu}\tau^{\lambda\sigma}{}_{\sigma}\right)$$

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$\Rightarrow$ Apply to cosmological perturbation theory

Cosmological Perturbation Theory

- Linear scalar perturbations about a FLRW background

$$ds^2 = -(1+2\phi)dt^2 + a^2(t)(1-2\phi)\delta_{ij}dx^i dx^j$$

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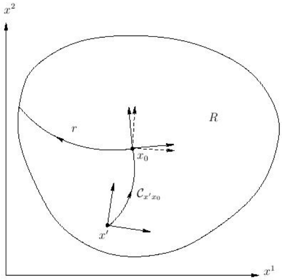
$$\nabla^2\phi(t_0) = 4\pi G a^2 \bar{\rho}(t_0)\delta(t_0)$$
- G^0_i component: momentum constraint equation
$$\dot{a}(\partial_i\phi) + a(\partial_i\dot{\phi}) = -4\pi G a\bar{\rho}v_i$$
- Trace of the G^i_j component: evolution equation
$$\ddot{\phi} + 4H\dot{\phi} + 3H^2\phi + 2\frac{\ddot{a}}{a}\phi - 2H^2\phi = 0$$

How to Choose the Tetrad Field



Parallel transport along geodesics $\mathcal{C}_{x_0 x'}$ realized by Wegner-Wilson line operator

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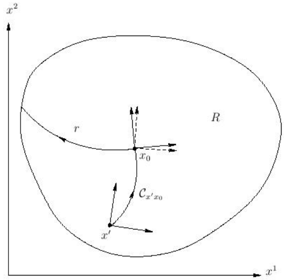

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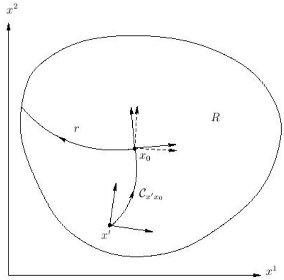

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$$V_j{}^i(\tau, 0, ; \mathcal{C}_{0\tau}) = (1 + \phi(\tau) - \phi(0))\,\delta_j{}^i - \delta^{ik}\epsilon_{kjl}\frac{dv^l}{d\tau}(\tau)$$

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- In a cosmological setting, what is the magnitude of the additional term in the field equation and does it behave like dark energy?