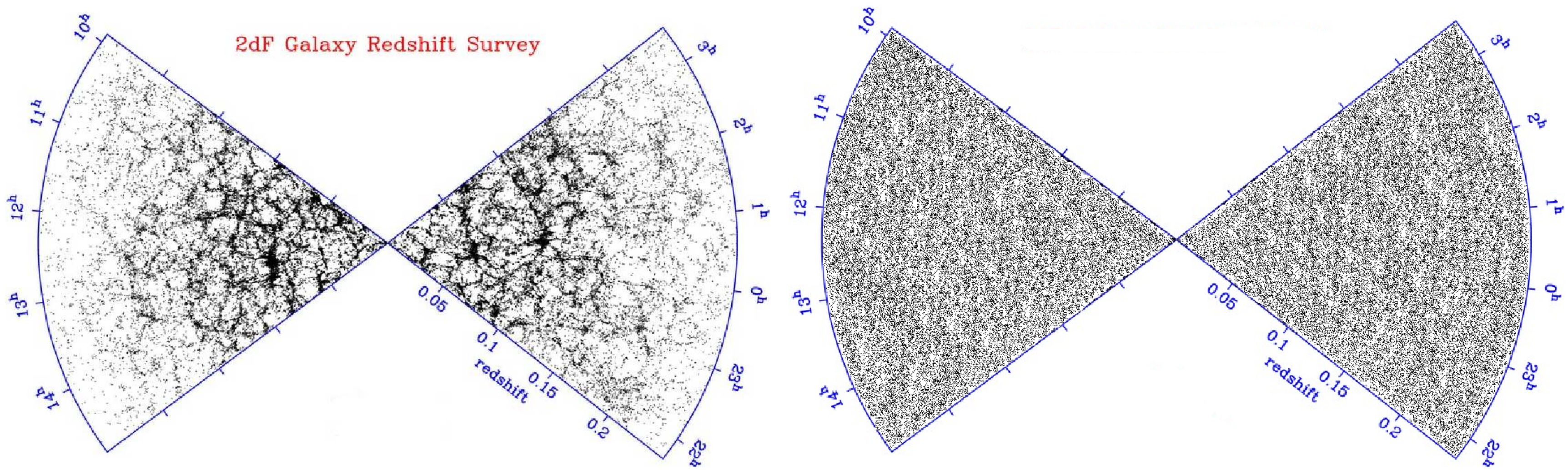
Light propagation in the inhomogeneous universe

Krzysztof Bolejko
University of Oxford

Paris, 22/11/2011

Backreaction



$$\mathrm{G}_{\alpha\beta}(\langle g_{\alpha\beta}\rangle) \neq \langle \mathrm{G}_{\alpha\beta}(g_{\alpha\beta})\rangle$$

- evolution
- light propagation

Angular distance

$$D_A^2 = \frac{\delta S}{\delta \Omega}$$

$$\frac{\mathrm{d}\delta S}{\mathrm{d}s} = 2\theta \delta S$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}s} + \theta^2 + |\sigma|^2 = -\frac{1}{2} R_{\alpha\beta} k^\alpha k^\beta$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}s} + 2\theta\sigma = C_{\alpha\beta\mu\nu} \epsilon^{*\alpha} k^\beta \epsilon^{*\mu} k^\nu,$$

$$\theta = \tfrac{1}{2} k^\alpha{}_{;\alpha} \qquad \sigma = \tfrac{1}{2} k_{(\alpha;\beta)} k^{(\alpha;\beta)} - \tfrac{1}{4} (k^\alpha{}_{;\alpha})^2$$

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R. K. Sachs *Proc. Roy. Soc. London* A **264** 309 (1961)

Angular distance

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- matter fluctuations: δ
- shear: σ
- evolution: $s(z)$

Ricci focusing

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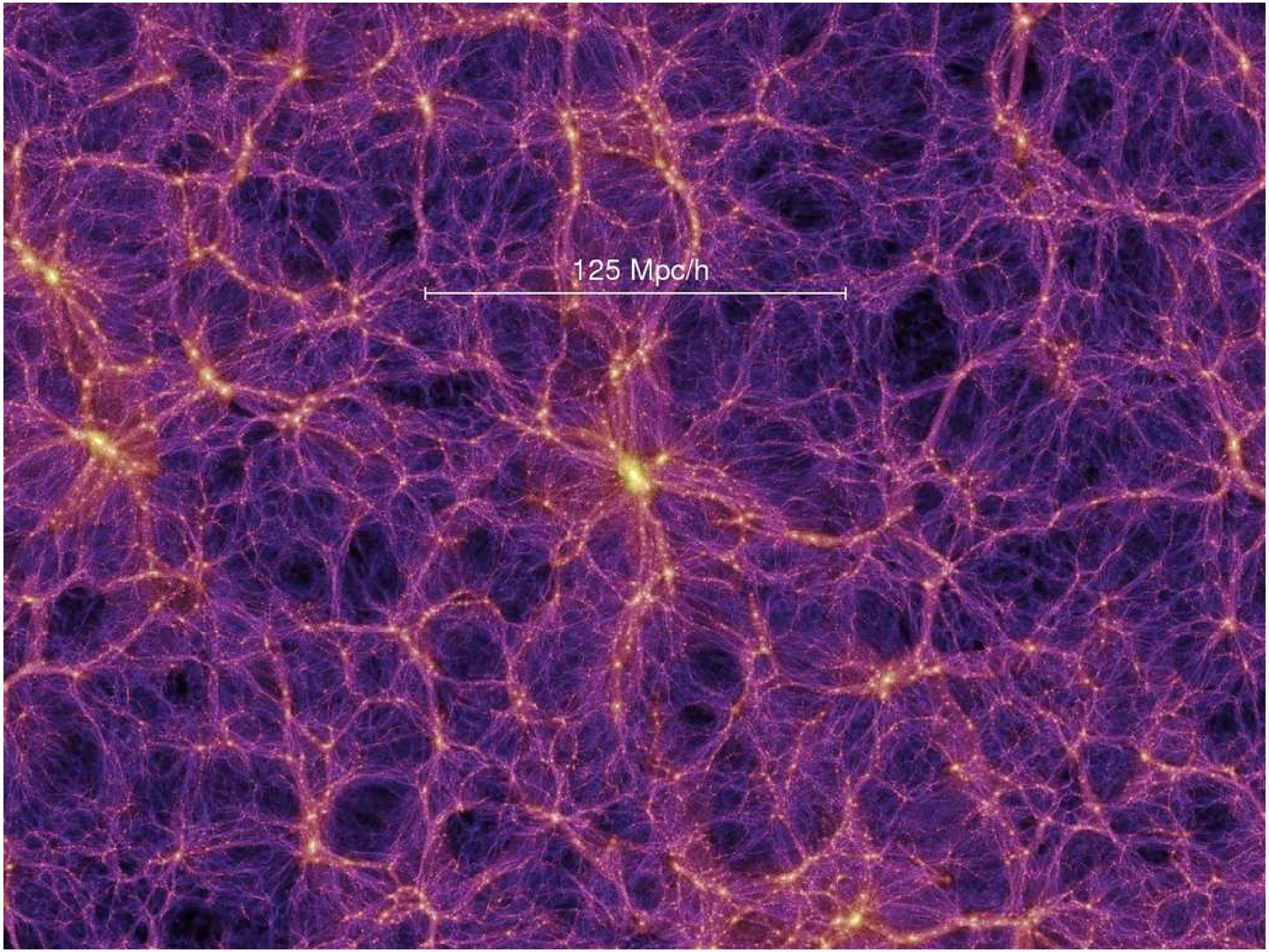
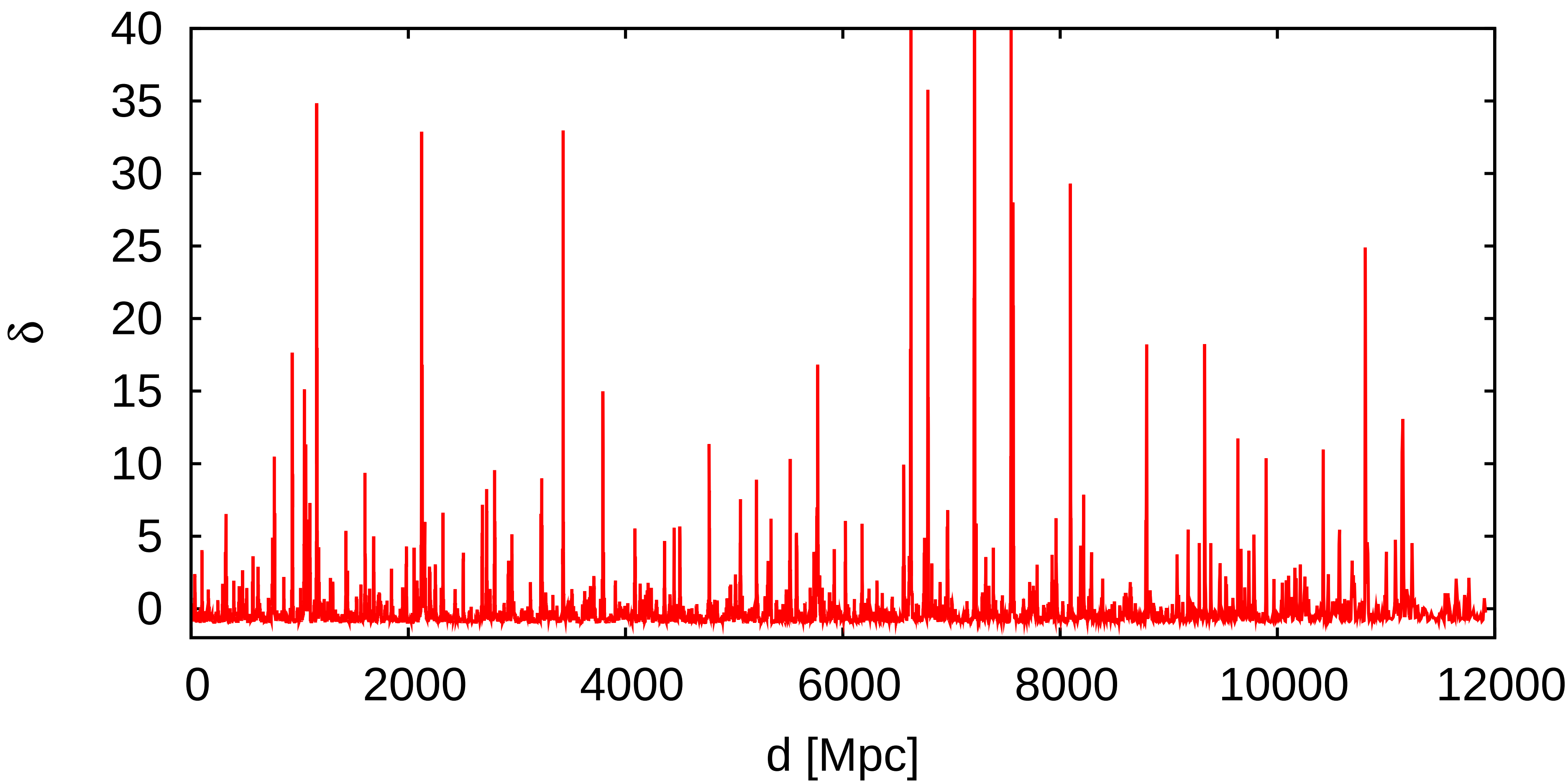
$$\frac{\mathrm{d}s}{\mathrm{d}z} = -\frac{1}{(1+z)^2 H(z)}$$



http://gavo.mpa-garching.mpg.de/Millennium/

Millennium



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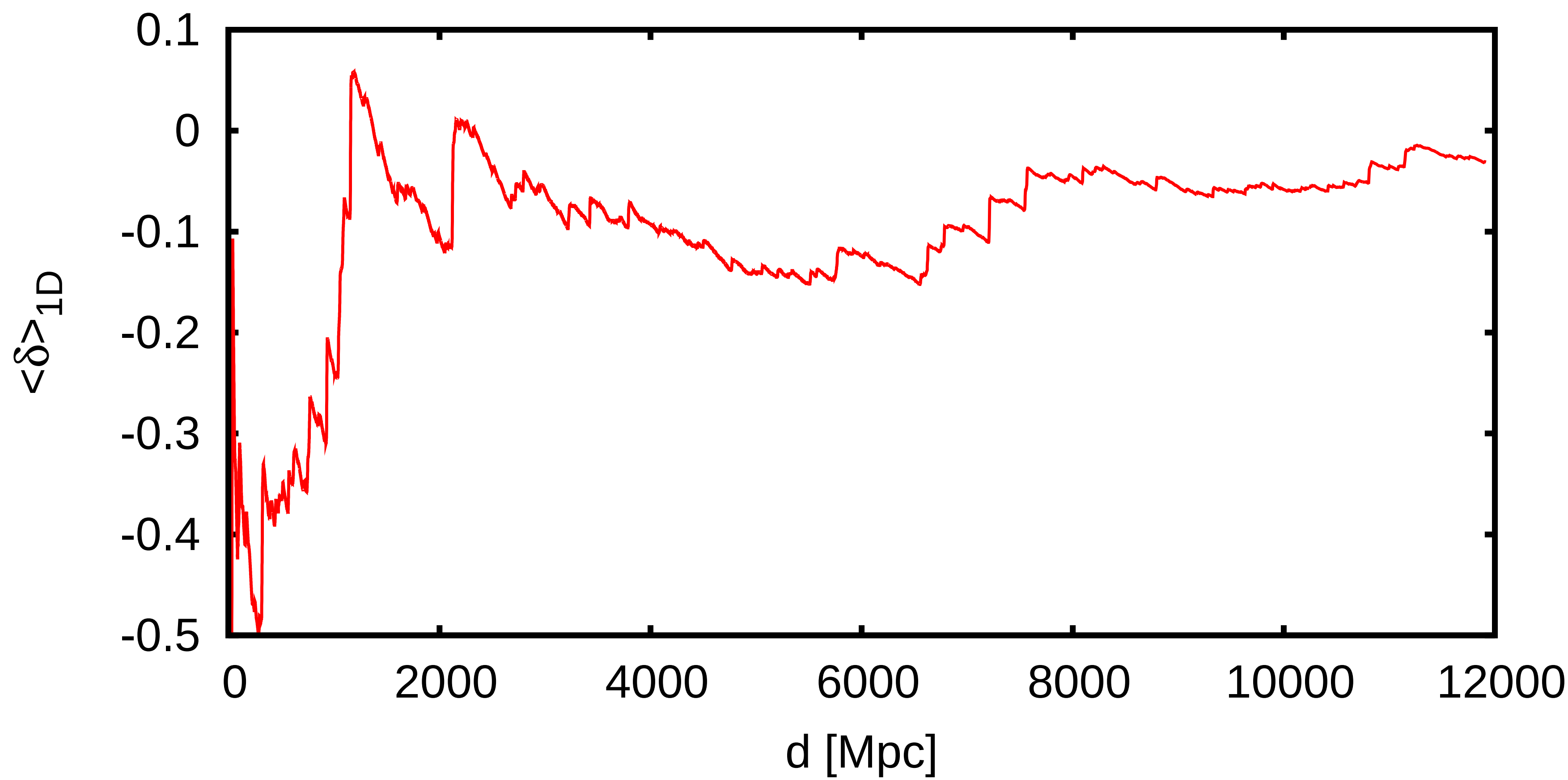
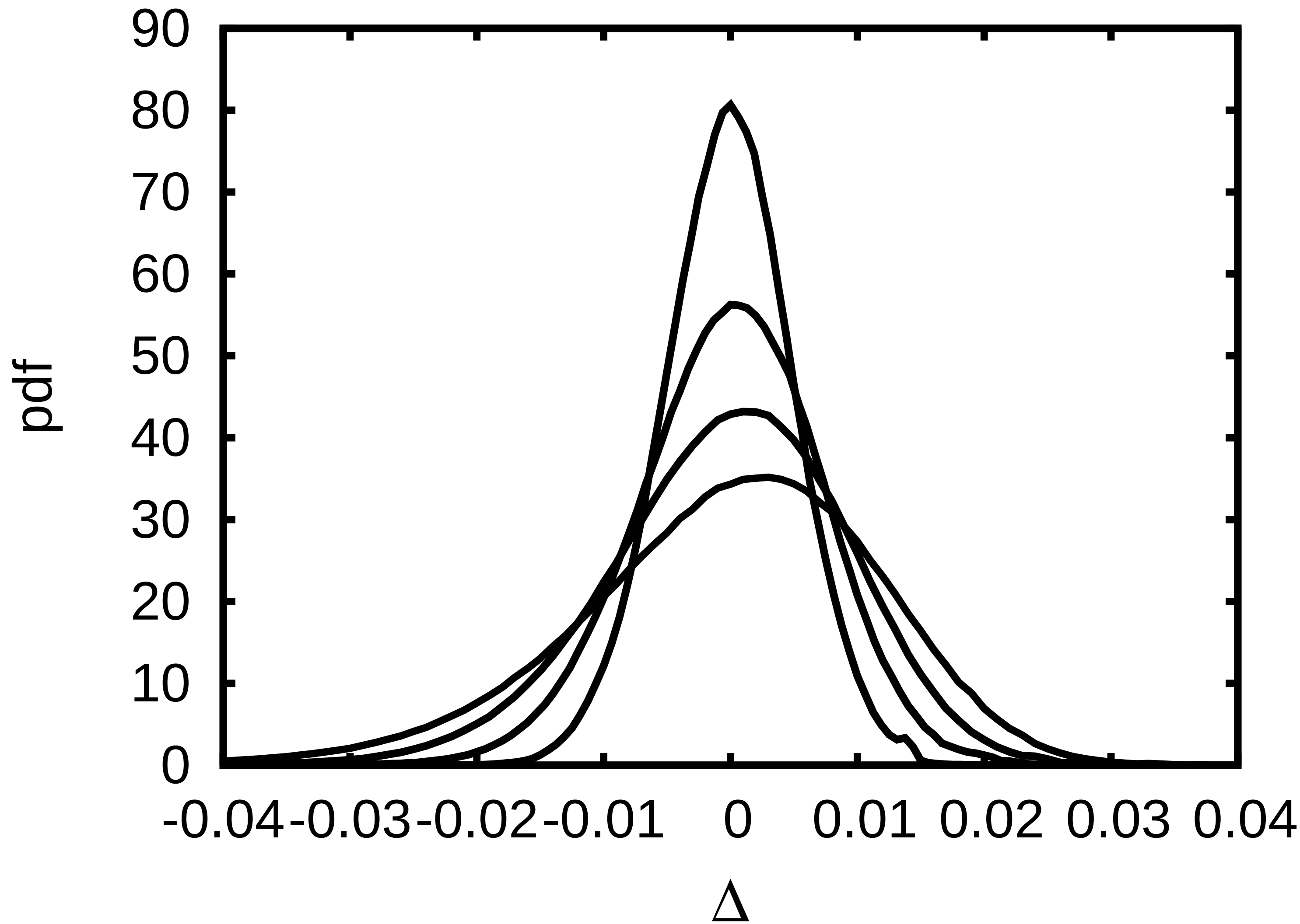
$$\rho = \rho_0 + \delta\rho \qquad \langle \delta\rho \rangle = 0$$

Millennium



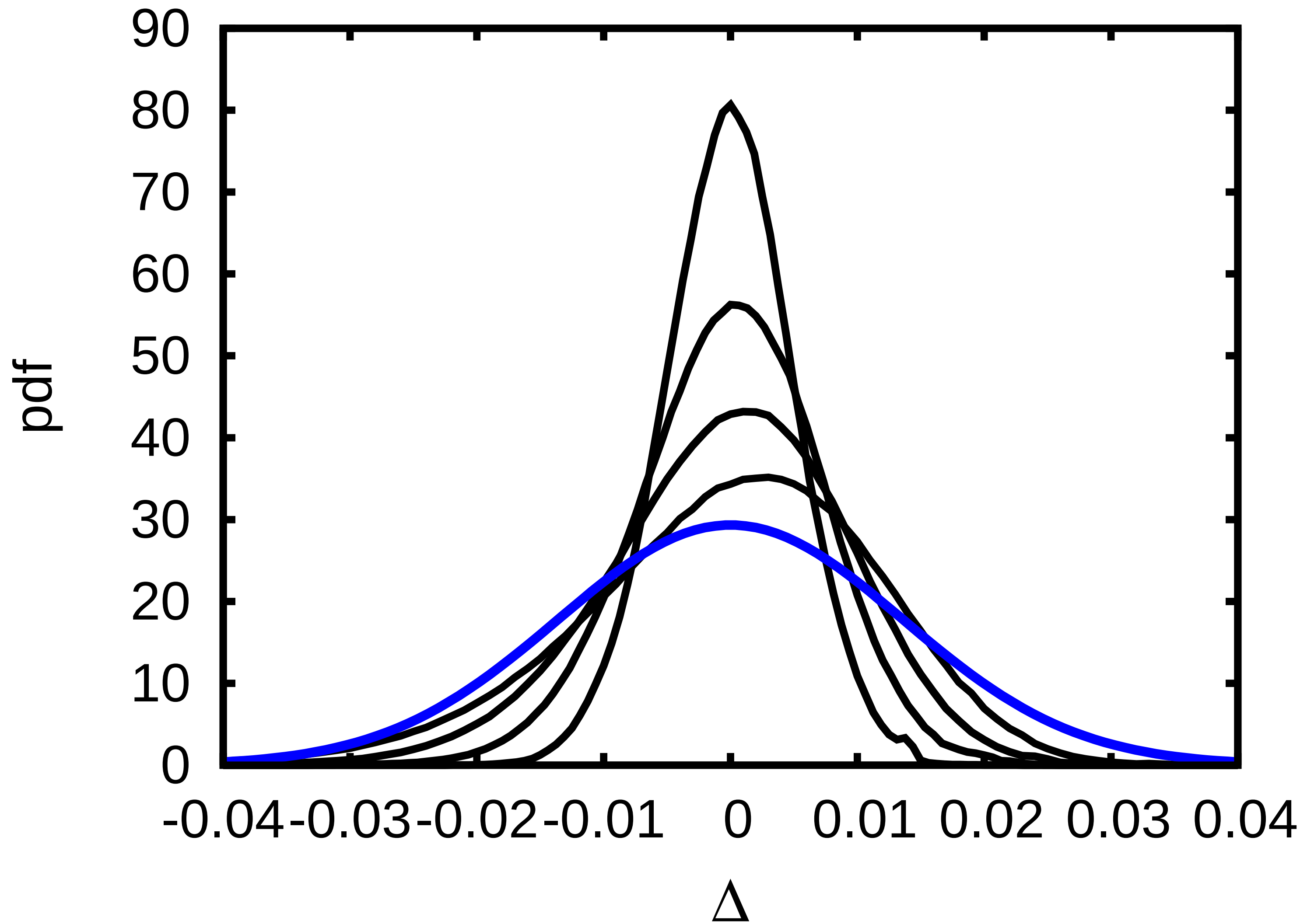
$$\langle\delta\rangle_{1D} = \frac{1}{L}\int_0^L \mathrm{d}r\,\delta$$

Δ at z=1.6



$$D_A = \bar{D}_A(1 + \Delta)$$

Δ at z=1.6



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- matter fluctuations: $\boldsymbol{\delta}$
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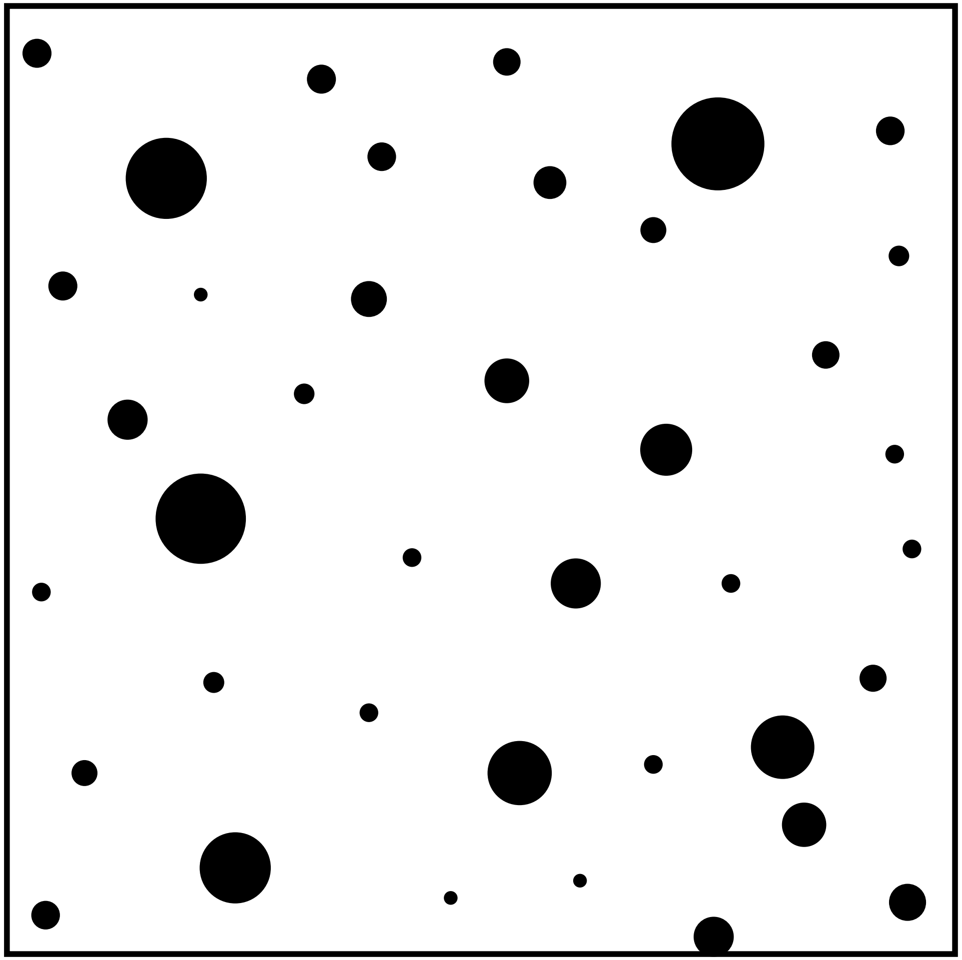

$$\mathcal{C} = \sum_i \mathcal{C}_i = \sum_i^N \frac{1}{2} \left(\frac{b_i}{R_i} \right)^2 (\rho - \bar{\rho}) \to - \sum_i 3 b_i \frac{m_i}{r_i^5}$$

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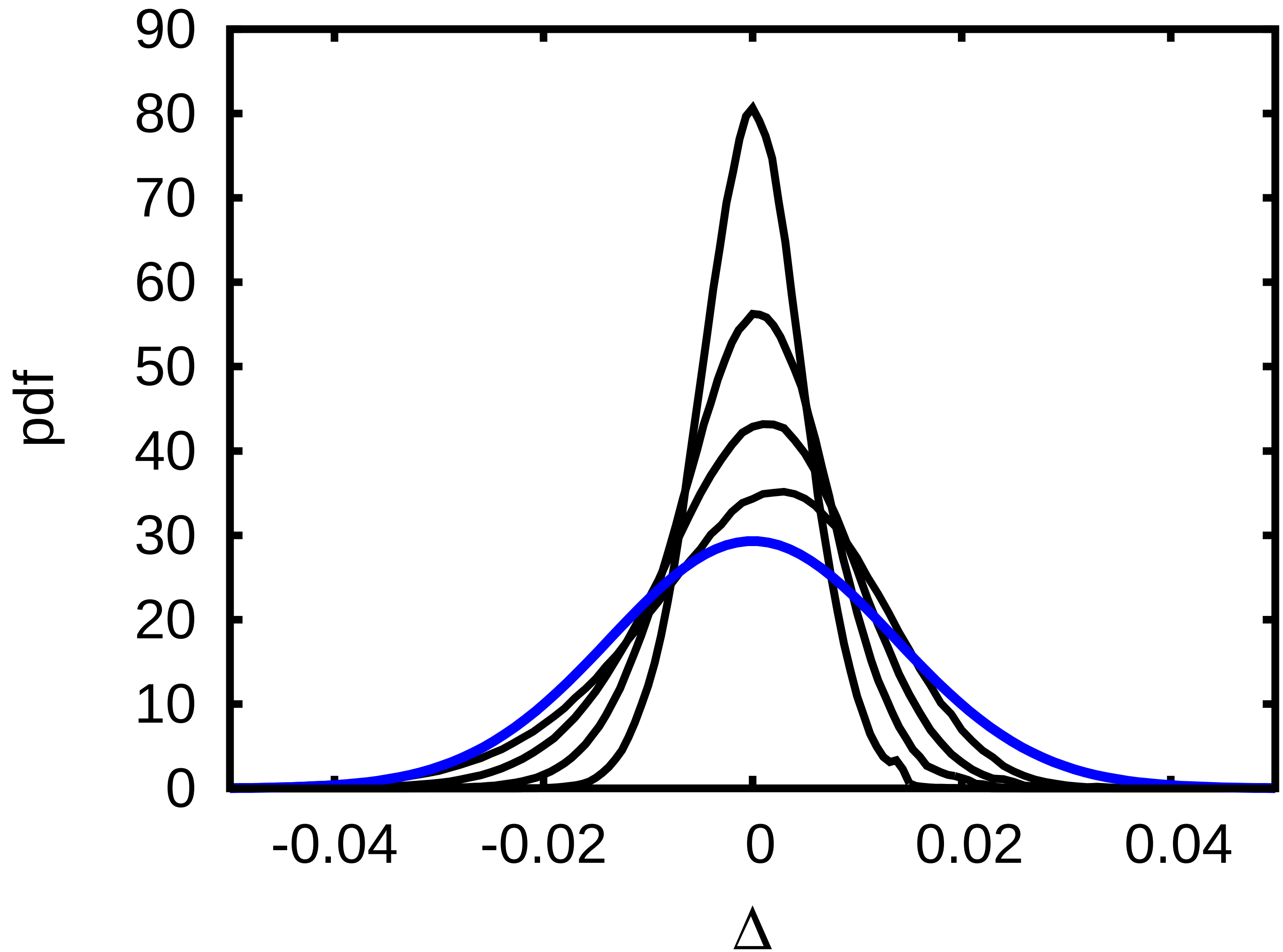

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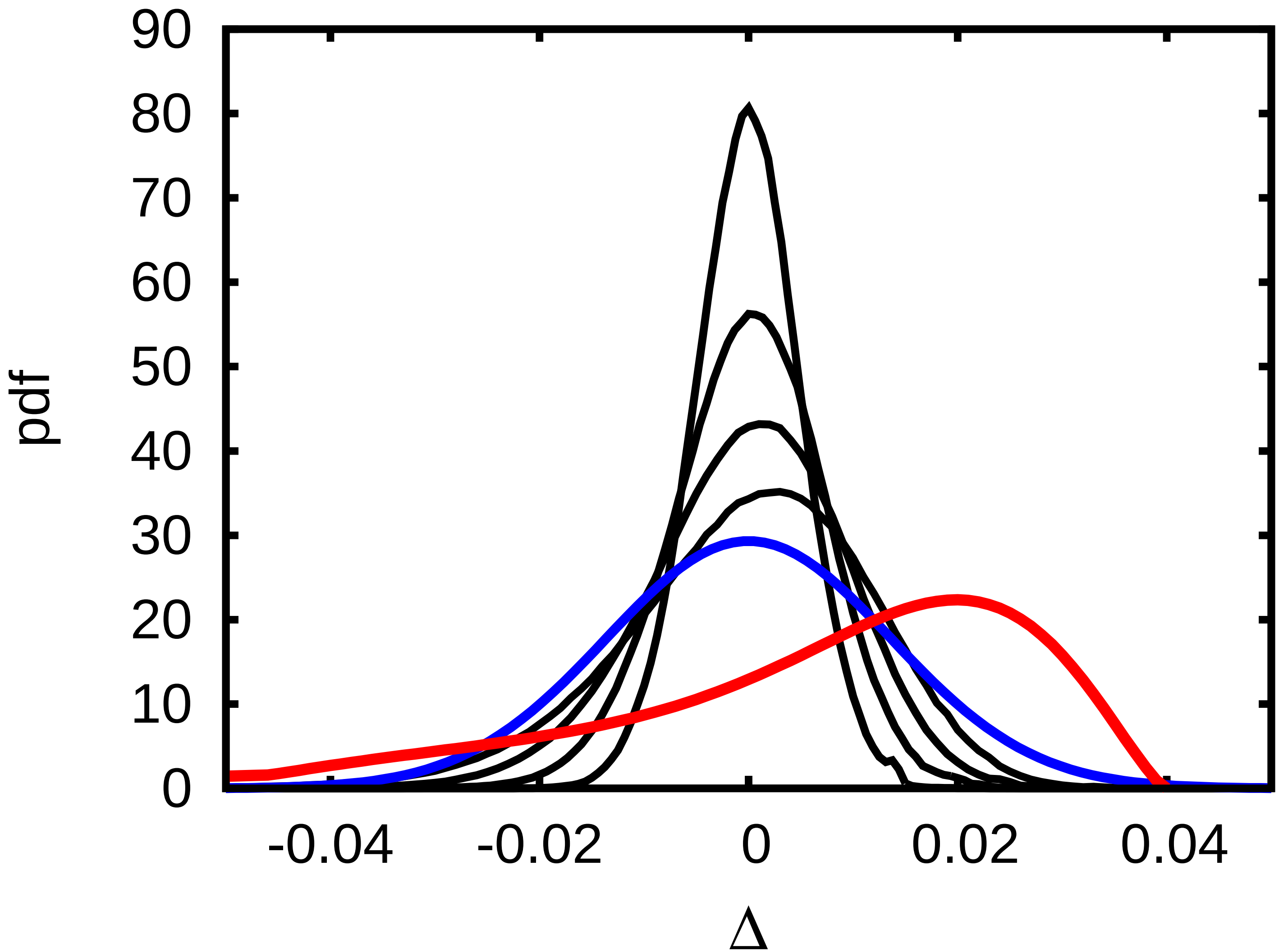

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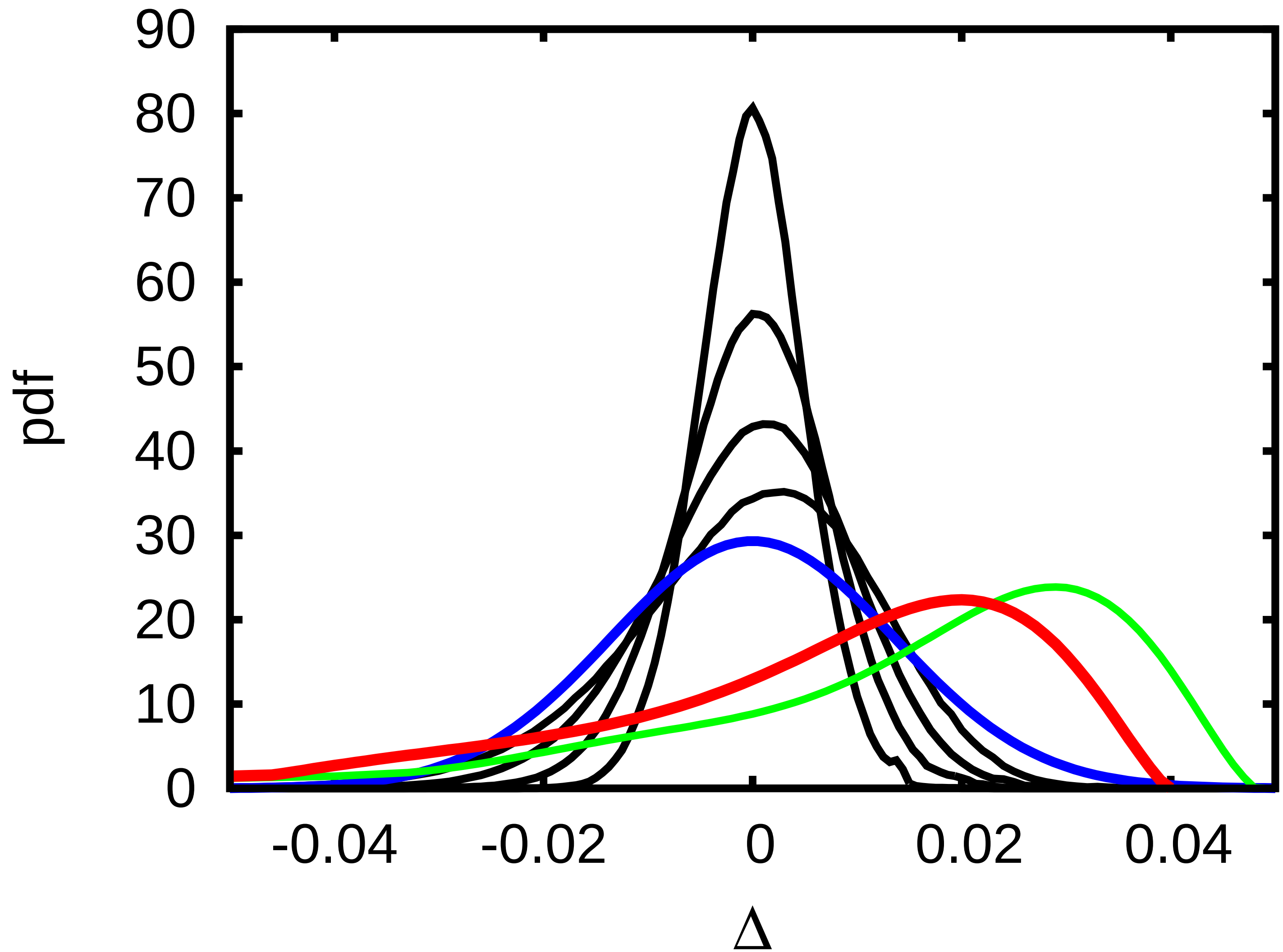

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- matter fluctuations: $\boldsymbol{\delta(z)}$
- shear: $\boldsymbol{\sigma(z)}$
- evolution: $s(z)$

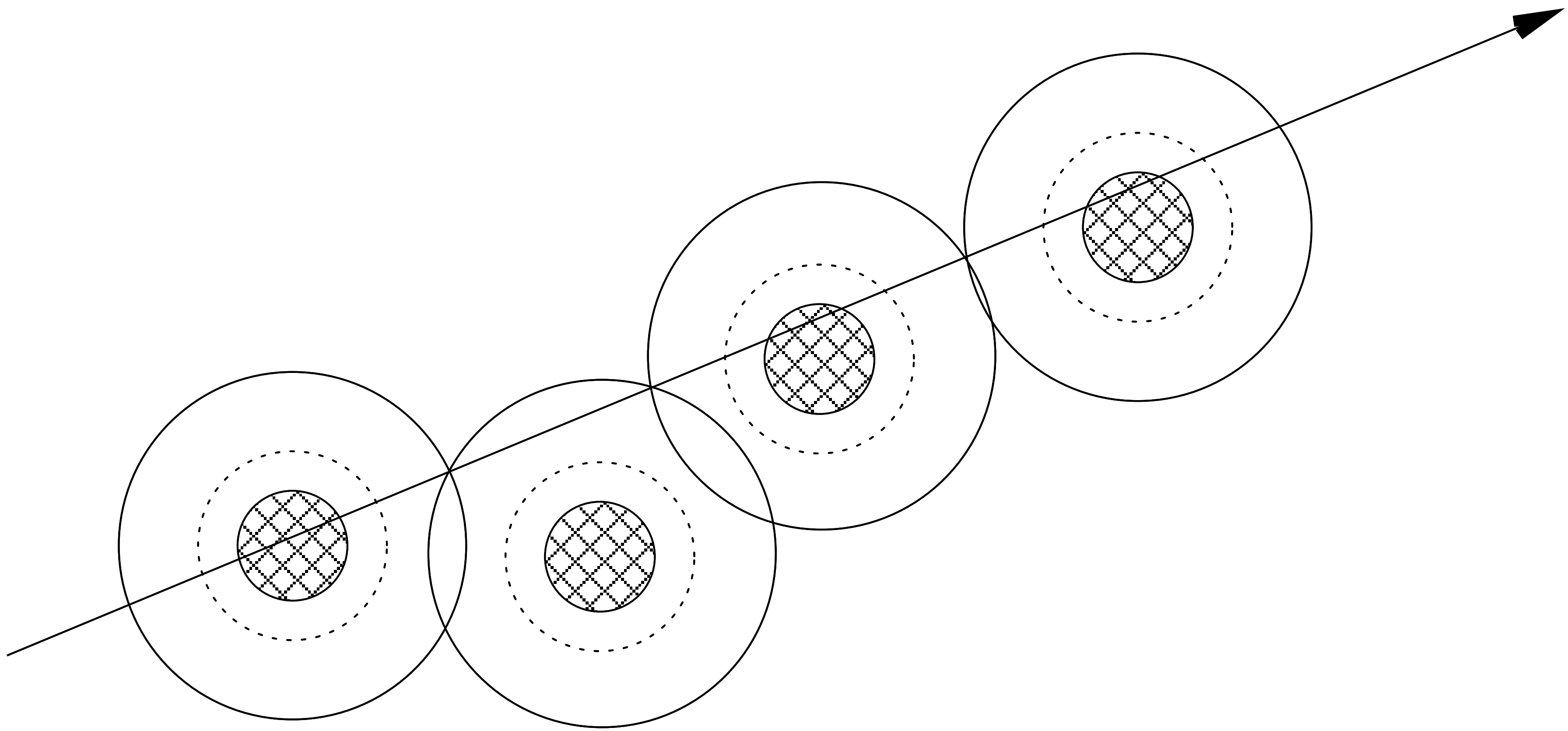
Lemaître–Tolman model

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \frac{R,_r^2(r,t)}{1+2E(r)}\,\mathrm{d}r^2 - R^2(t,r)\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right),$$

FLRW limit

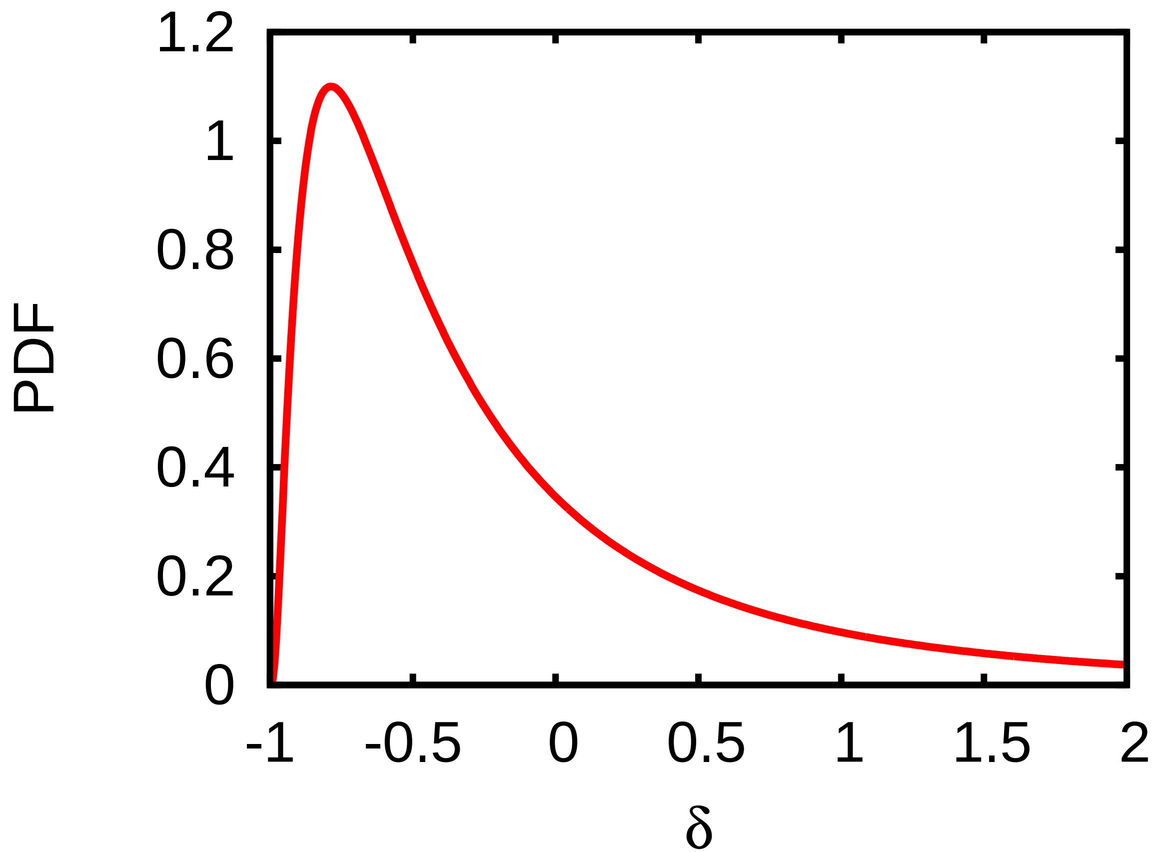
$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \frac{a^2(t)}{1-kr^2}\,\mathrm{d}r^2 - a^2(t)r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right).$$

LT Swiss Cheese model



Log-normal PDF

$$P(\delta) = \frac{1}{\sqrt{2\pi\sigma_{nl}^2}} \exp\left[-\frac{(\ln(1+\delta)+\sigma_{nl}^2/2)^2}{2\sigma_{nl}^2}\right]\frac{1}{1+\delta},$$



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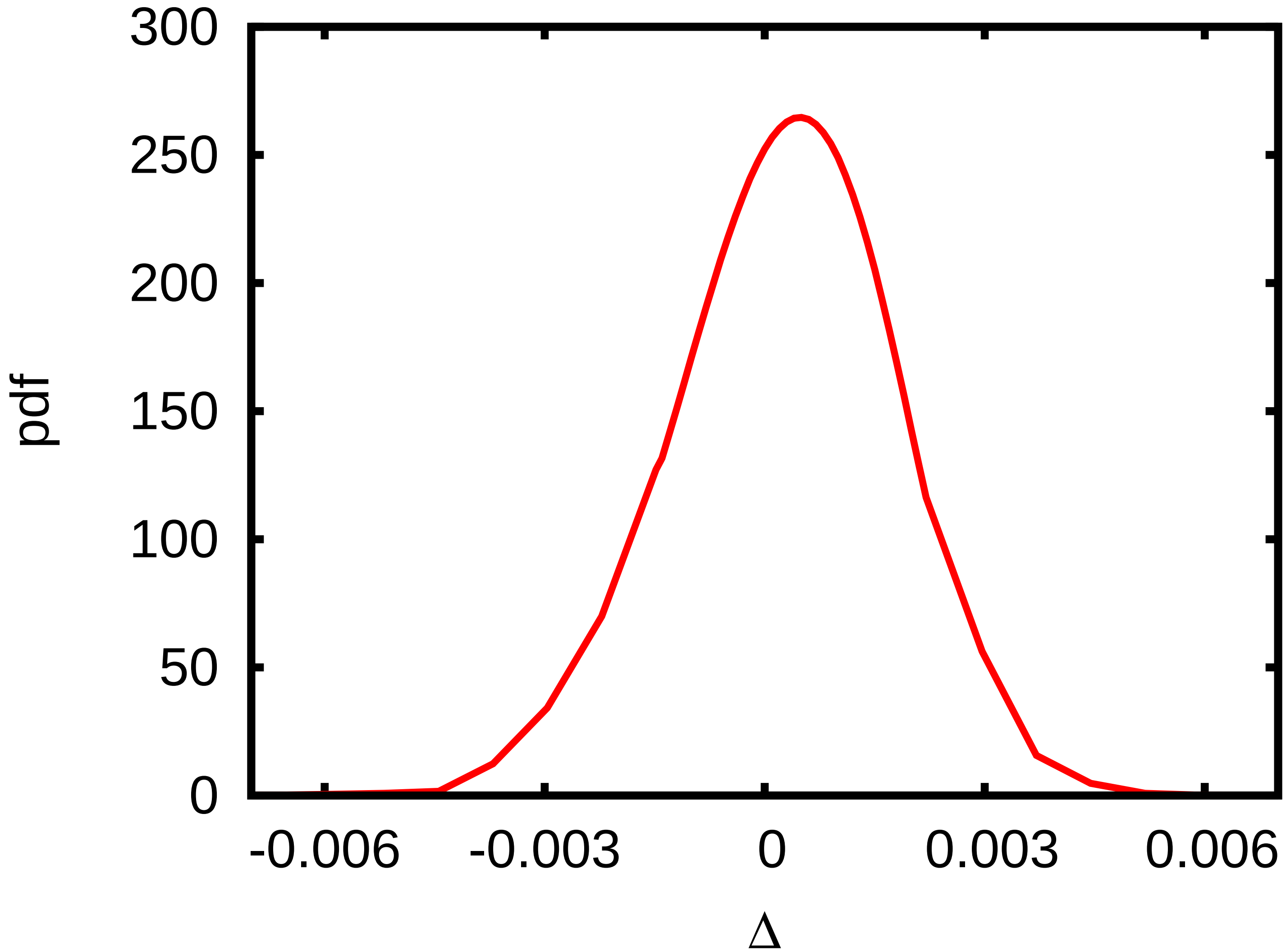

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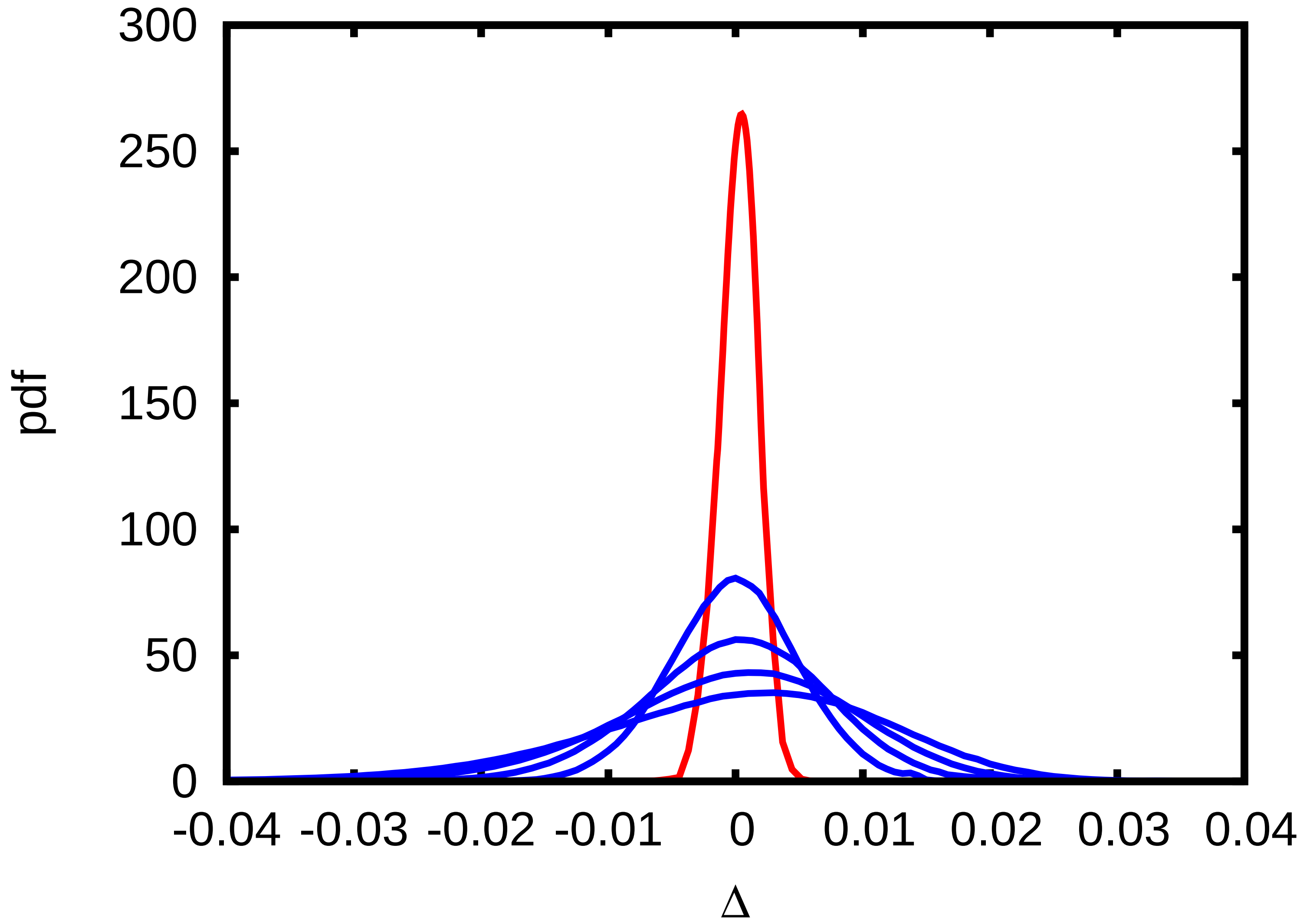

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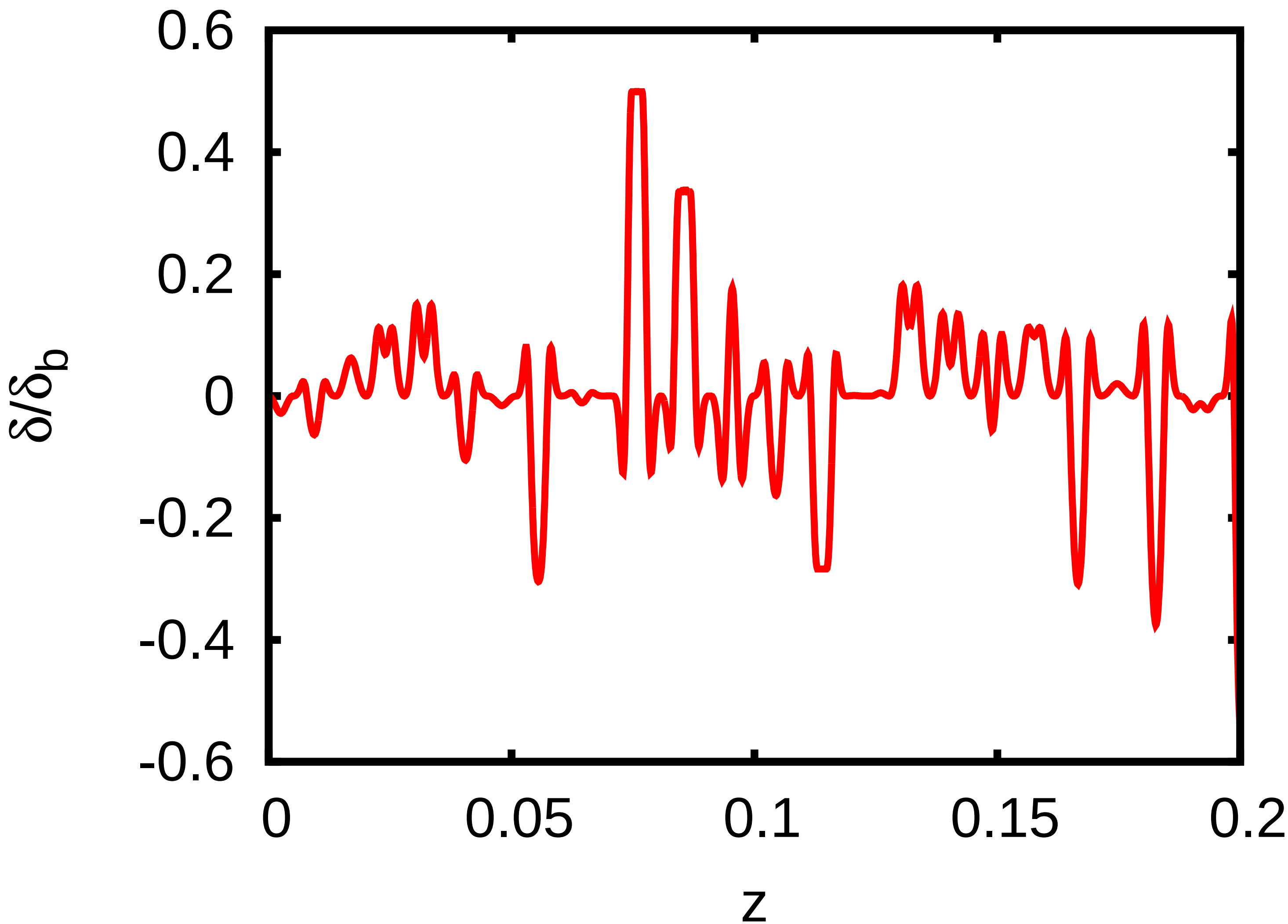
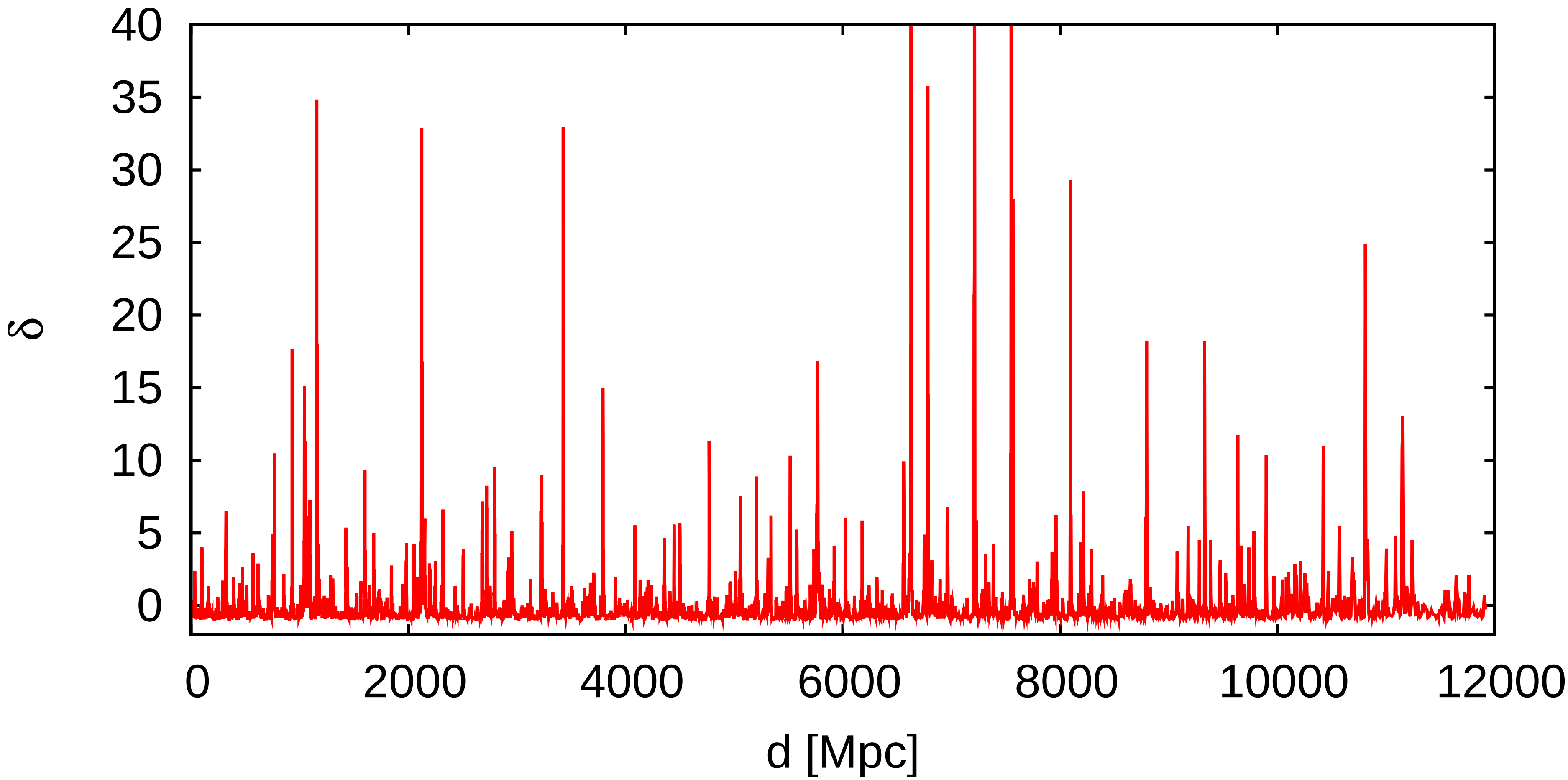
Density along a random l.o.s.



Millennium



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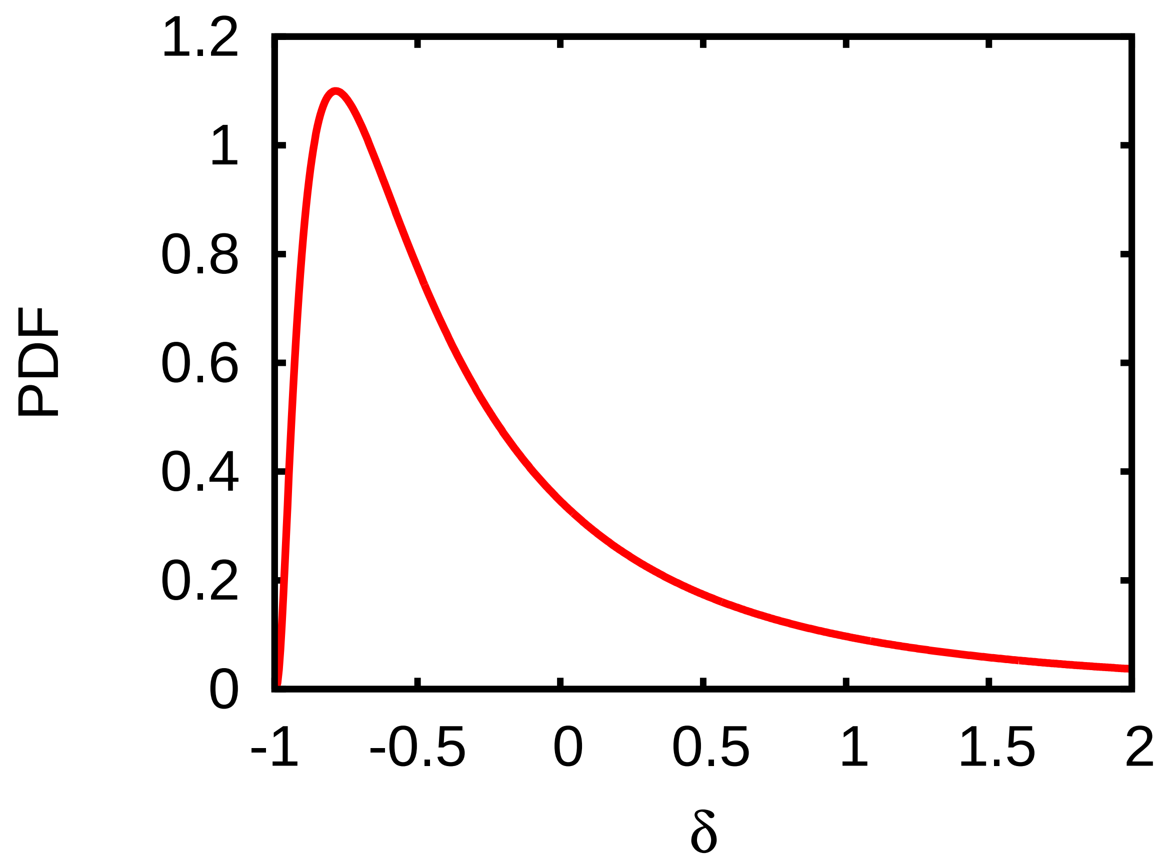

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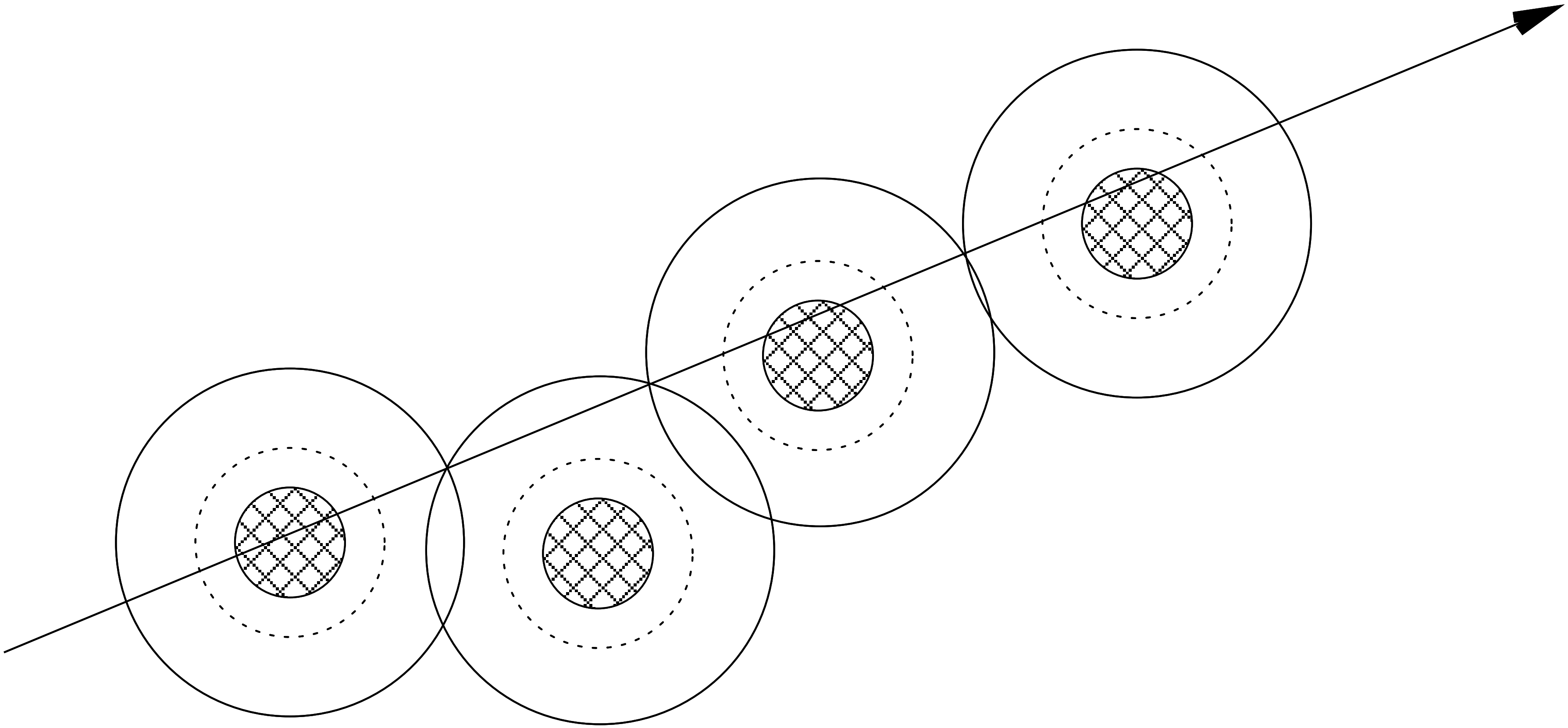

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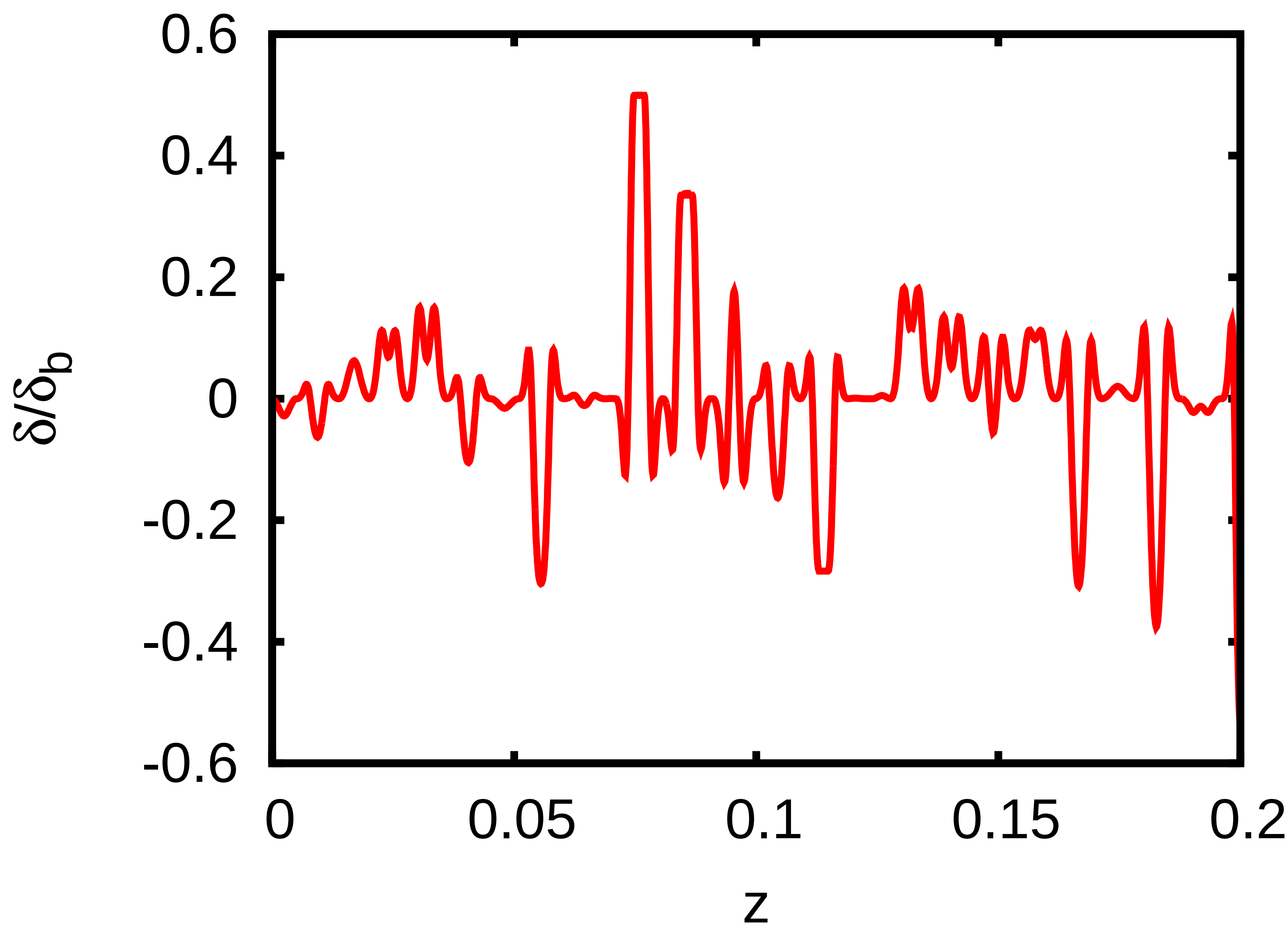

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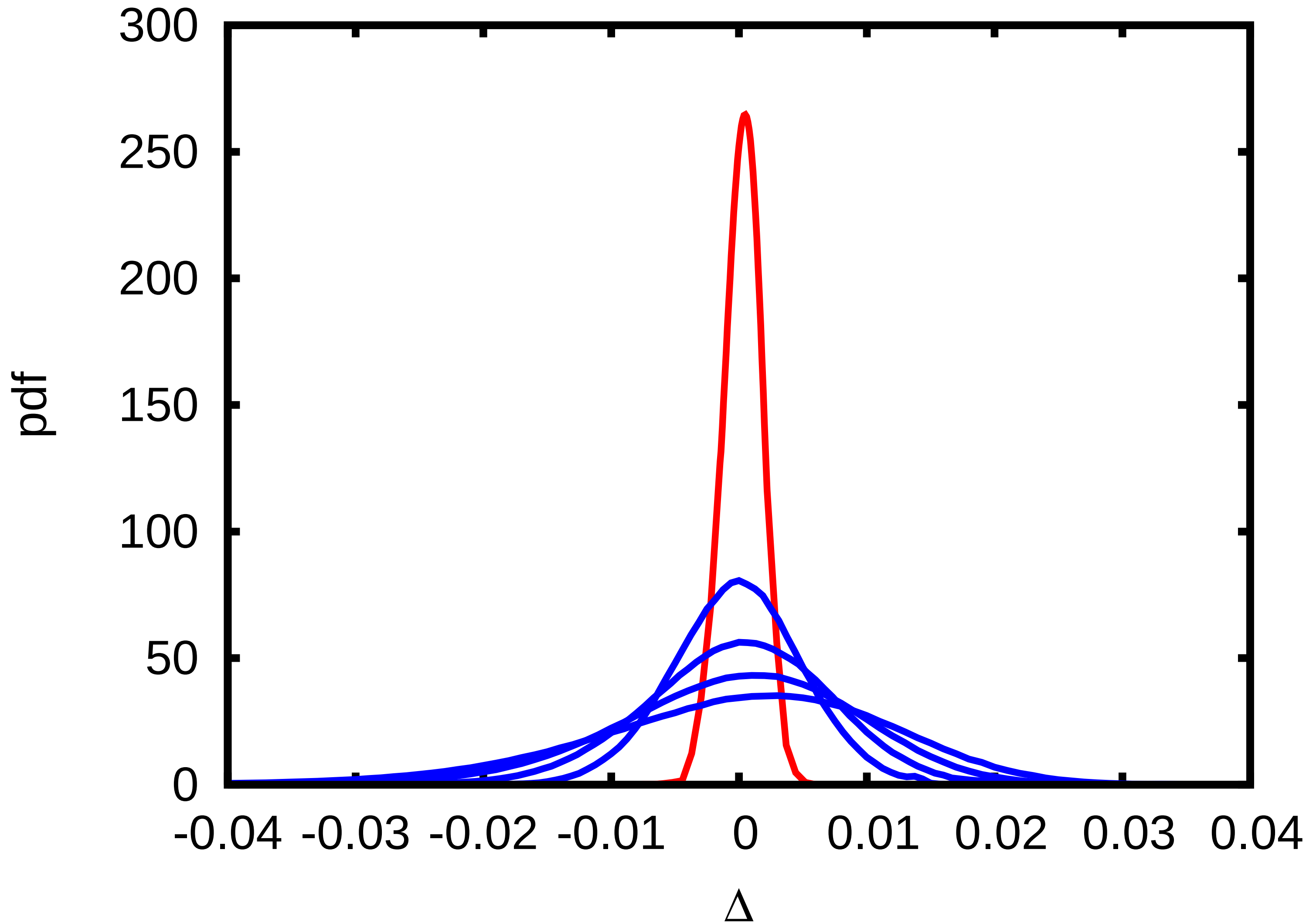

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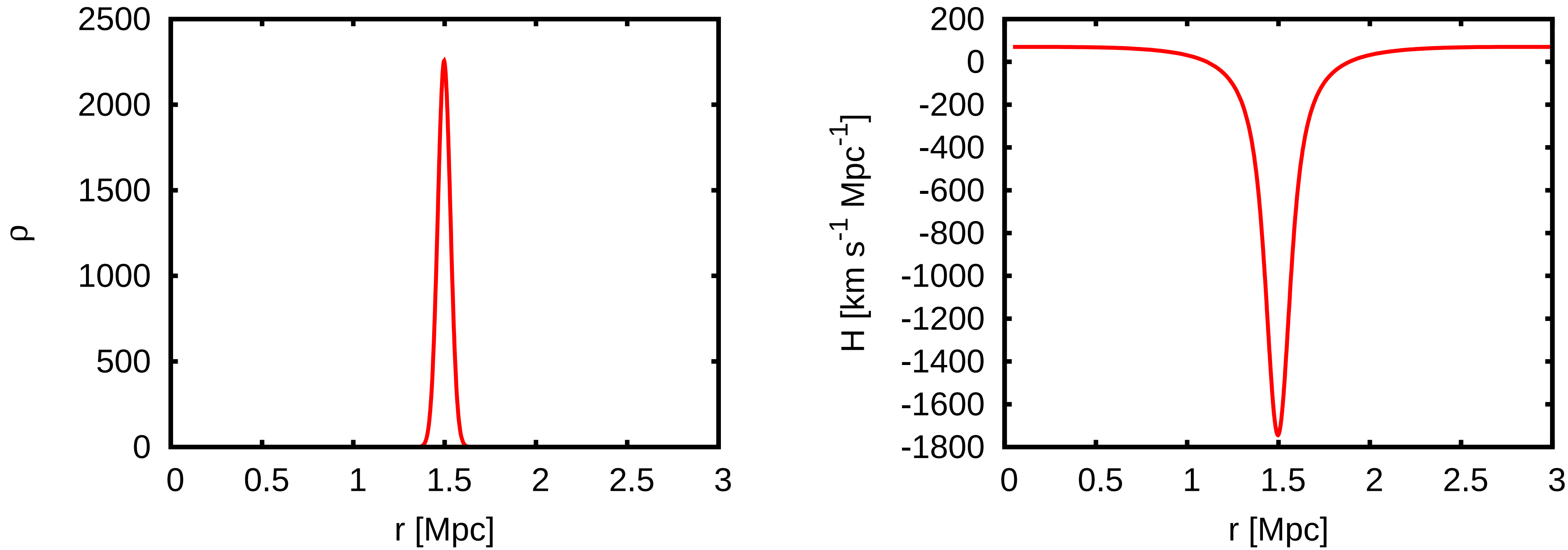
Extreme LT Swiss Cheese model



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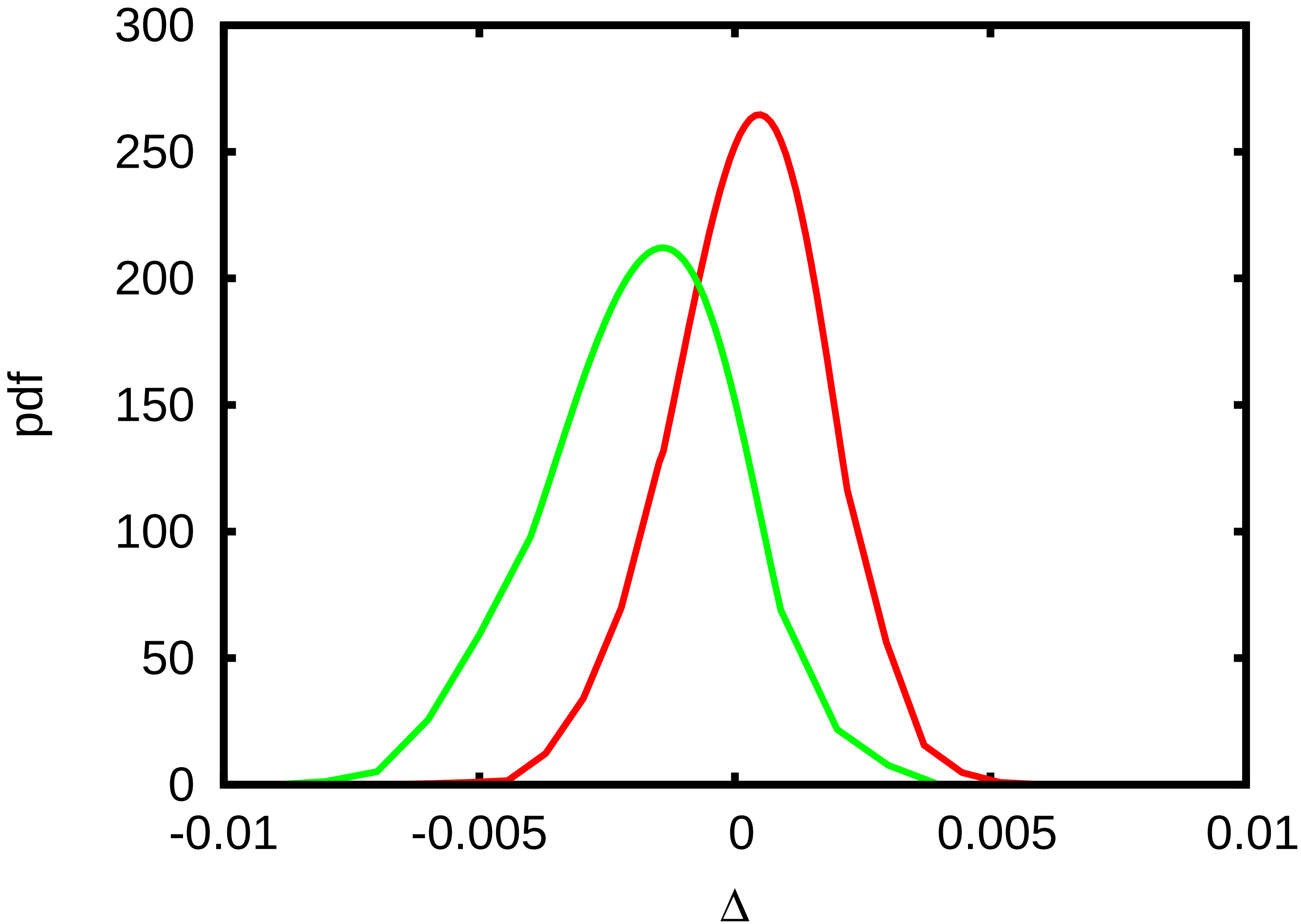

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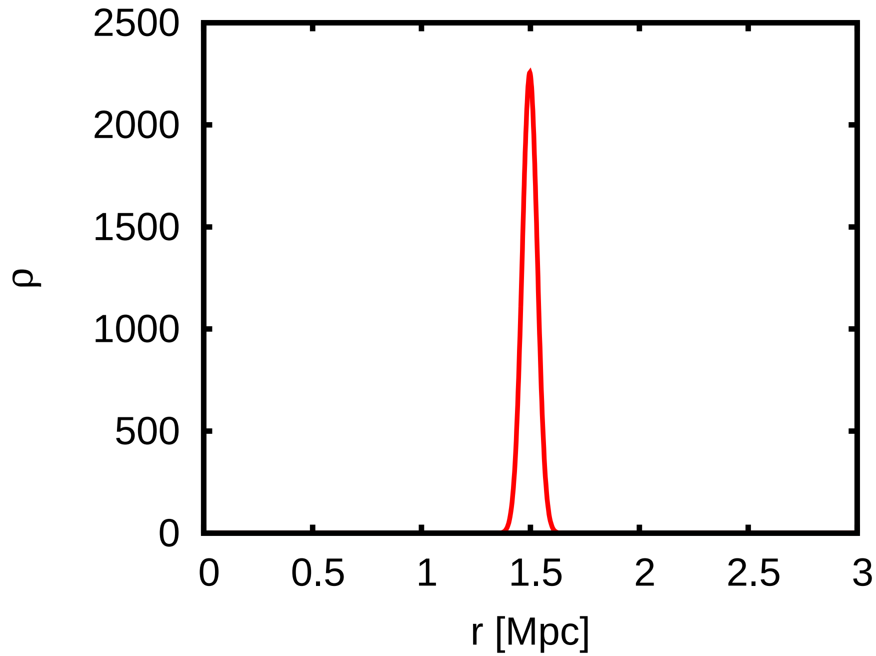

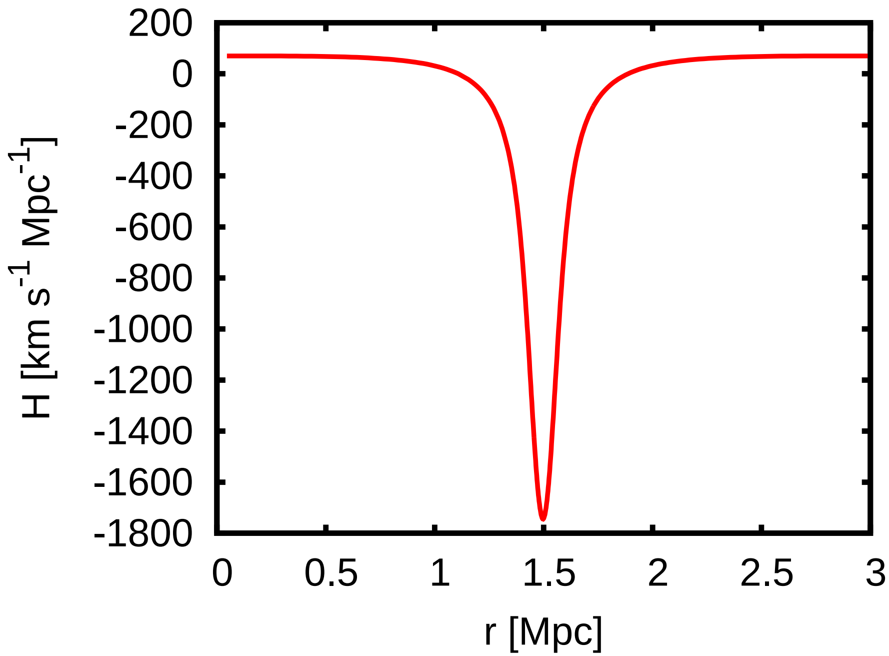

Millennium: δ_ρ, δ_H

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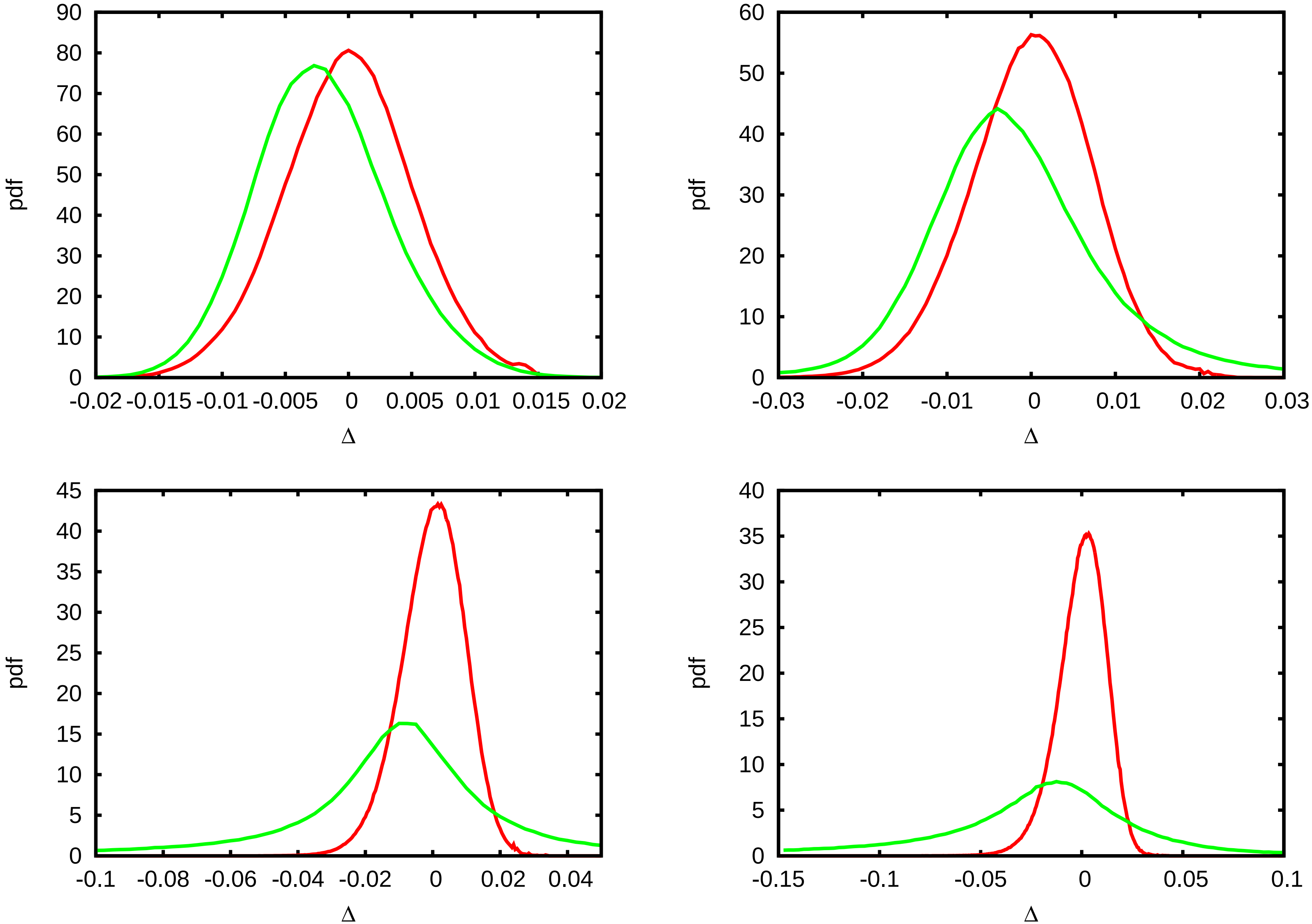
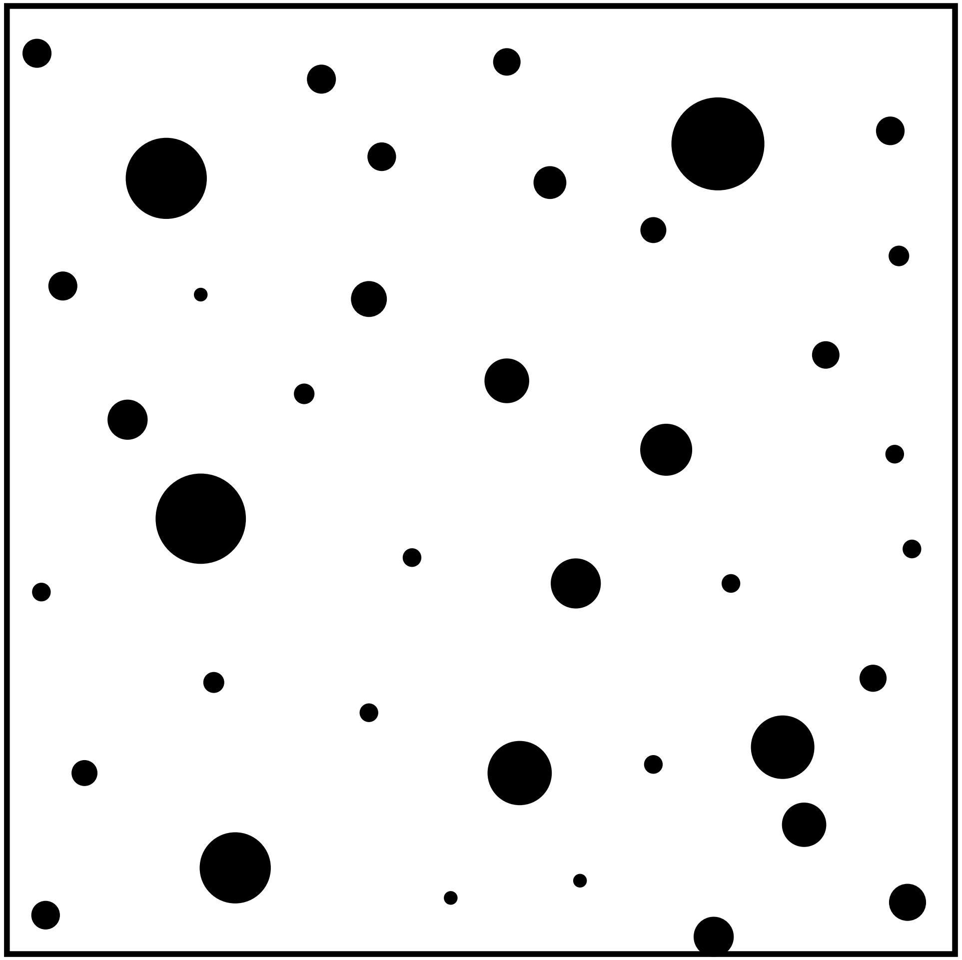
$$\rho = \rho_b(1+\delta)$$

$$H = H_b(1+\delta_H)$$

Millennium: δ_ρ, δ_H





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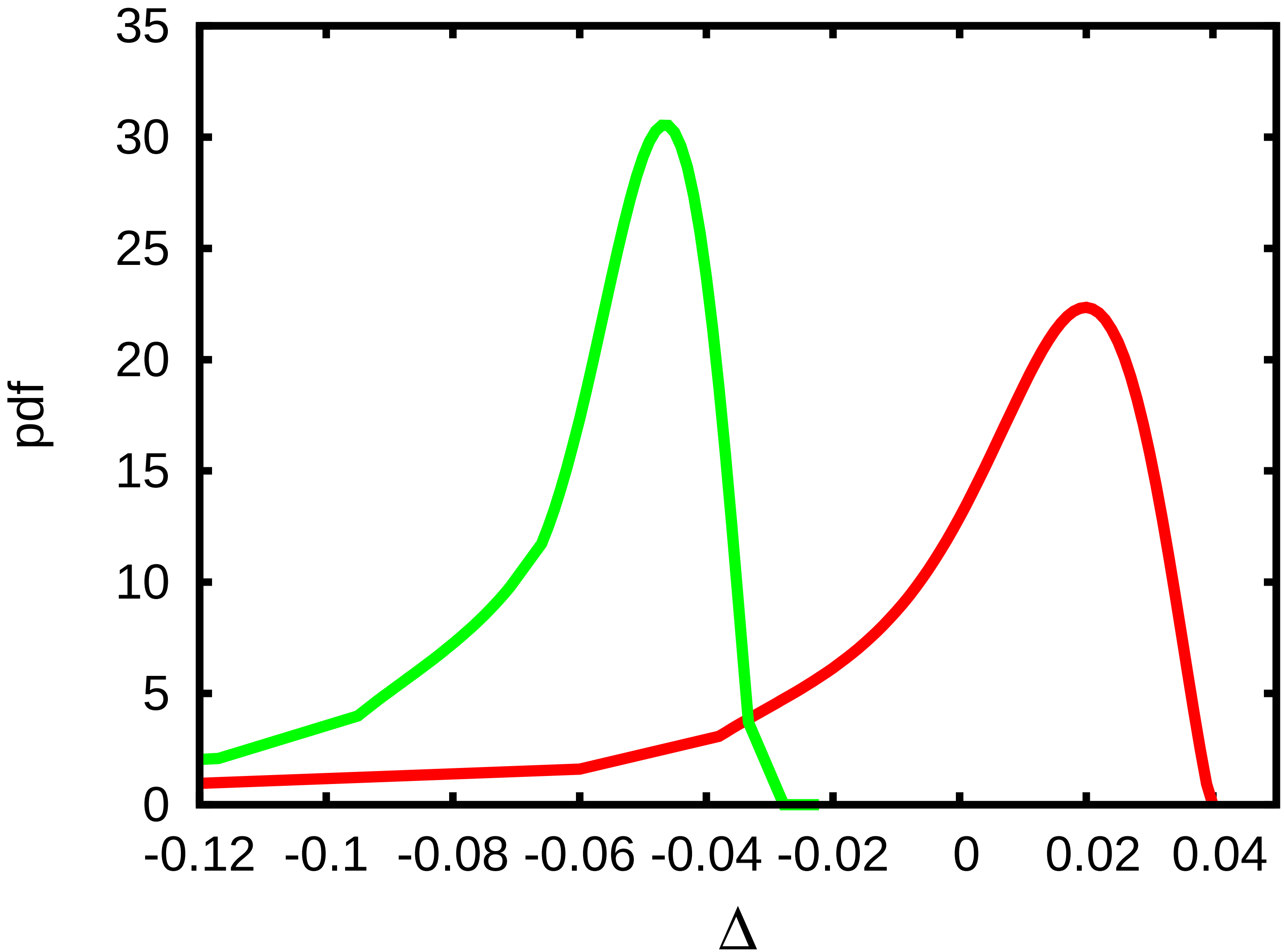

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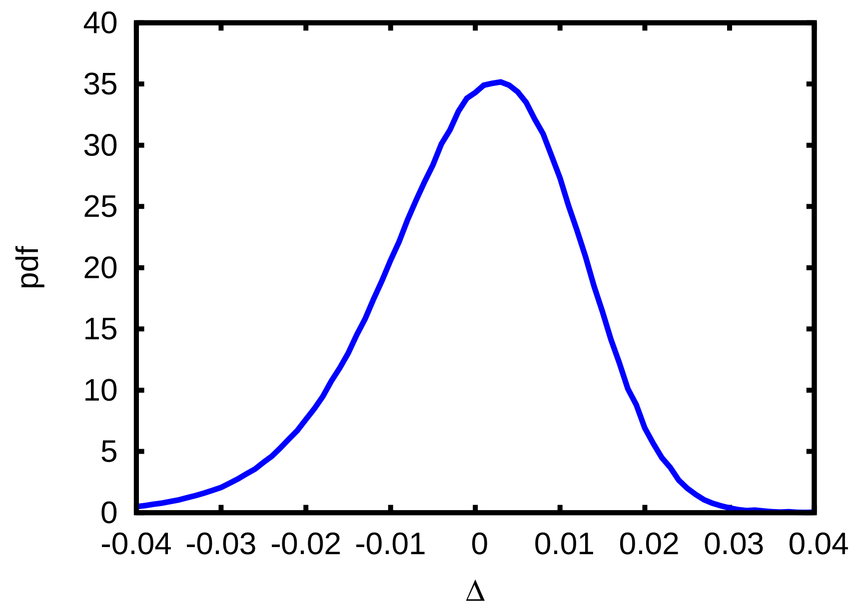

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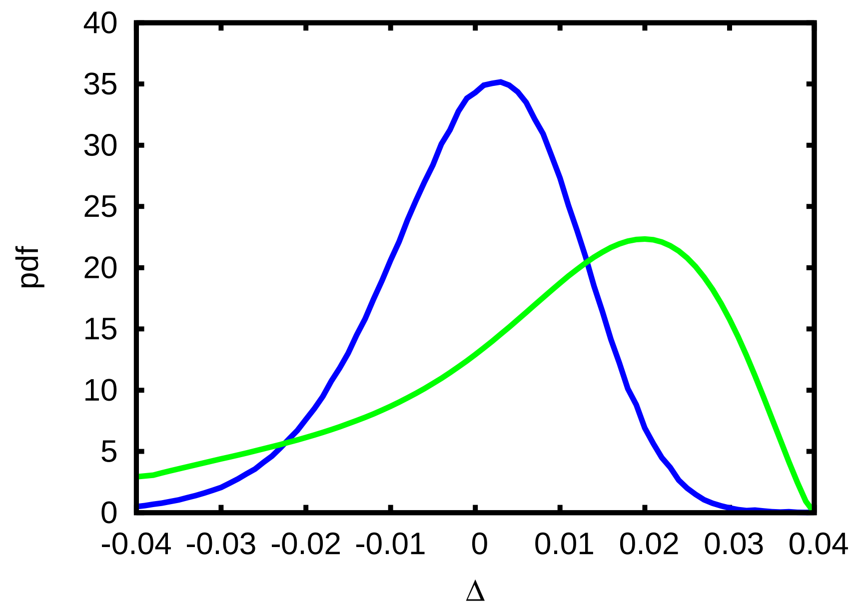

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