The UV cut-off and Trans-Planckian Physics

Marcos Brum, Sergio Jorás
Physics Insitute - UFRJ
Backreaction: Where do we stand?

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Sumrio

Inflation and Generation of Perturbations

The biggest virtue of cosmic inflation is to provide a causal mechanism for the generation of perturbations that give rise to anisotropies on the CMB.

Scale factor - acceleration

$$a(\tau) \propto \tau^{p/(1-p)} \,,\, p > 1$$

Inflaton fluctuations

$$\phi(\mathbf{x}, \tau) = \phi_0(\tau) + \delta\phi(\mathbf{x}, \tau)$$

Metric fluctuations

$$ds^2 = a^2(\tau)[-(1+2\varphi)d\tau^2 - 2B_{,i}\, d\tau dx^i + ((1-2\psi)\delta_{ij} - 2E_{,ij})dx^i dx^j]$$

Gauge-invariant variable

$$\upsilon = a\left(\delta\phi + \frac{\phi'_{(0)}}{\mathcal{H}}\psi\right)$$

Fourier modes' EoM

$$\upsilon''_{\mathbf{k}} + \left(k^2 - \frac{(2p^2-p)}{(p-1)^2}\frac{1}{\tau^2}\right)\upsilon_{\mathbf{k}} = 0$$

The curvature perturbation

$$\zeta = -\frac{\mathcal{H}}{\phi'_{(0)}}\frac{1}{a}\upsilon$$

The Power spectrum

$$\mathcal{P}_\zeta = \left(\frac{\mathcal{H}}{\phi'_{(0)}}\right)^2 \frac{k^3}{2\pi^2}\left|\frac{\upsilon_{\mathbf{k}}}{a}\right|^2$$

And the spectral behavior

$$n_s = 1 + \frac{d\ln\mathcal{P}_\zeta}{d\ln k}$$

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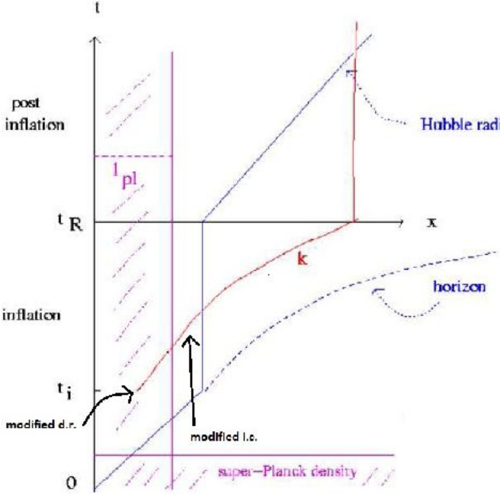
Inflation and Trans-Planckian Physics

The huge stretching of length scales during inflation opens the possibility that the presently observable scales have been of the order of the Planck lenght at the beginning of (or before) it.



The Approach to TPP

During the Plank Era, Quantum Physics certainly was not as it is today. We expect some modification on the dispersion relation.

After the Planck Era, Quantum Physics will become as we know it. Thus, the EoM will come back to

$$v''_{\mathbf{k}} + \left(k^2 - \frac{(2p^2 - p)}{(p-1)^2}\frac{1}{\tau^2}\right) v_{\mathbf{k}} = 0$$

But the initial conditions will not be the ordinarily taken ones, assumed without consideration of the Planck Era. At the beginning of inflation, as $k \gg |\tau|^{-1}$, the field modes' solution is

$$v_{\mathbf{k}}(\tau) = c_{\mathbf{k}} e^{-ik\tau} + d_{\mathbf{k}} e^{ik\tau}$$

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Cut $\times$ Depopulation

The matter of cutting high energy modes has been extensively discussed in the literature. Most of it is concerned with the UV catastrophe, that is, if we naively cut modes above the Planck Era, we find

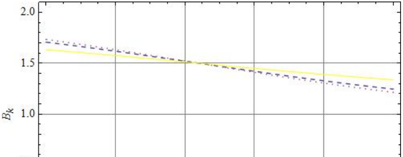
$$\rho_{vac} \gtrsim 10^{120}\rho_c$$

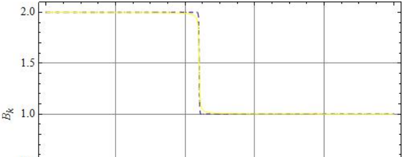
We will concern ourselves here with possible effects of attenuating the contribution of high energy modes to the inflationary spectrum.

$$v_{\mathbf{k}}(\tau) = \frac{1}{2}\sqrt{\pi|\tau|}\left[A_k H_\nu^{(1)}(k/aH) + B_k H_\nu^{(2)}(k/aH)\right]$$

We will choose the parameters B_k and A_k such that $|B_k|^2 - |A_k|^2 = 1$ and that the usual vacuum structure is recovered, if the attenuation is removed.

$$B_k = m + \frac{1}{2}\left\{\begin{array}{c}\tanh(x)\\ \mathrm{erf}(x)\\ \arctan(x)\end{array}\right. , \qquad\qquad x \equiv \frac{\alpha}{M_{Pl}}\left(\Lambda - \frac{k}{a(\tau_i)}\right) .$$





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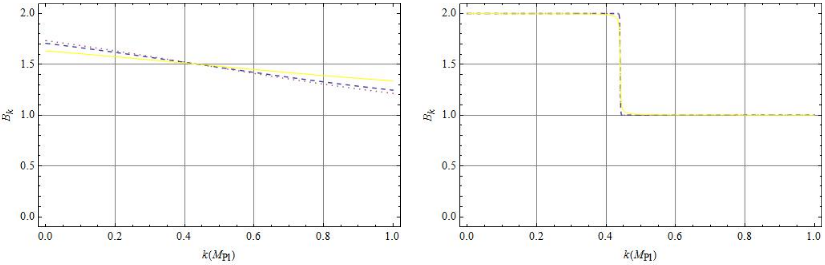

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Without regard to TPP, the field is chosen to be, initially, at the Bunch-Davies vacuum state, which is expressed by the choice of parameters $B_k = 1,\ A_k = 0$.

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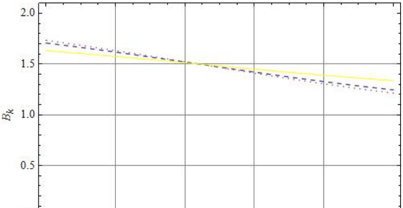

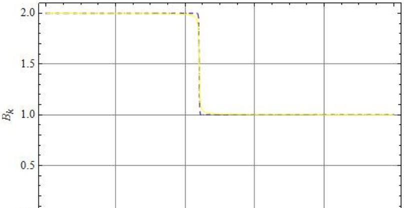

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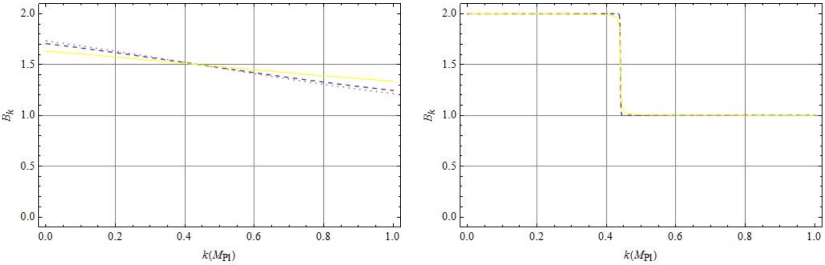


Inflationary, Trans-Planckian spectrum

The spectrum of perturbations, for general parameters B_k and A_k is

$$\mathcal{P}_{Tp} = |A_k - B_k|^2 \mathcal{P}$$

For a convenient choice of parameters,

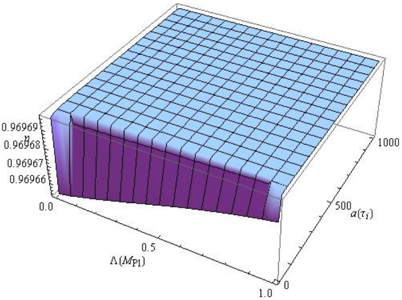


Figure: $\alpha = 1$, $\theta = \pi$

For inflation beginning **AT** the Planck time, $\Delta = \frac{n_s^{obs} - n_s}{n_s^{obs}}$,

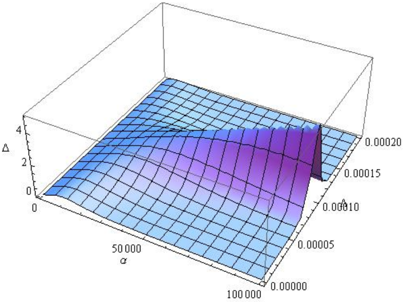


Figure: $\theta = \pi$, $a(\tau_i) = 1$

k is fixed at $k = 10^{-4} M_{Pl}$, the pivot scale measured by the WMAP team.

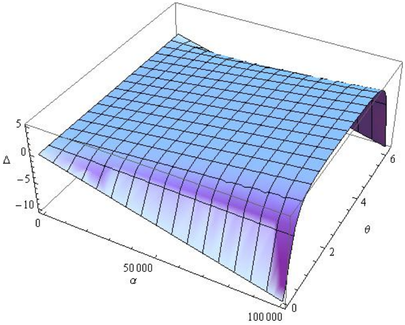


Figure: Examining the dependence on θ. The spectrum can be both strongly red or blue-shifted, depending on the parameters.

For which values of θ and α can the deviation be made null?

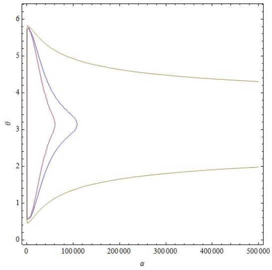


Figure: magenta = arc tangent, blue = hyperbolic tangent, pink = error function

For inflation starting slightly after the Planck time, there is no deviation from the observed spectrum!

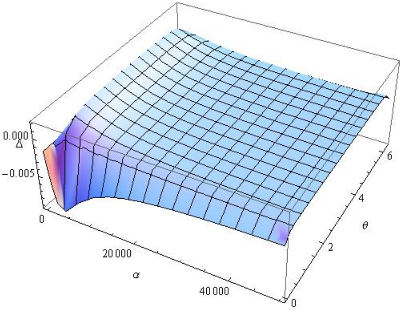
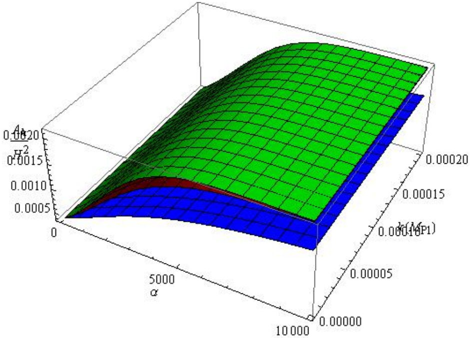


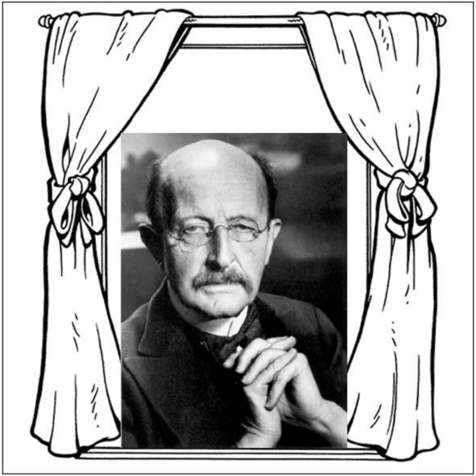
Figure: At $t = 1.03496 t_{Pl}$, a(t)=10, and $\Lambda = 3 \times 10^{-4} M_{Pl}$.

Besides, it is important to know how much matter is created by the modification of initial conditions. The energy density of the inflation is $\rho \sim V \simeq H^2$, so



Planck, where are you??

Are you searching for me?



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