Lemaître-Tolman models: their interpretation and status

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Backreaction: where do we stand ?

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Outline

(1) Interpretation of spherical symmetry in Universe models

(2) Shape of the mass density profiles of LT models reproducing the ΛCDM $D_L(z)$

(3) Analysis of some test proposals and ruling out claims

(4) Conclusion

Spherical symmetry around the observer: an approximation (1)

It is a mere mathematical simplification used to perform more easily the calculations and to account roughly for the quasi-isotropy at very large scale.

The energy density is smoothed out over angles around us, i.e. only the radial inhomogeneities are taken into account. Comparing to FLRW models with 3 Killing vectors, these have only 2 Killing vectors, i.e. one symmetry less.

The real observer is not physically located at the center of any spherically symmetric universe.

It is a first step leading to more achieved models: Swiss-cheeses, meatballs, and so on.

The most often used: the LT models (2)

Metric in comoving coordinates and synchronous time gauge:

$$ds^2 = dt^2 - (R')^2/(1 + 2E(r))\, dr^2 - R^2(t,r)(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)$$

A first integral of the Einstein equations = a dynamical equation for R:

$$\dot{R}^2 = 2E + \frac{2M}{R} + \frac{\Lambda}{3}R^2$$

Mass density: $\kappa\rho = 2M'/(R^2R')$

For $\Lambda = 0$, the dynamical equation exhibits parametric solutions:

- $E < 0$:

  $$R(t,r) = M/(-2E)\,(1 - \cos\eta), \quad \eta - \sin\eta = (-2E)^{3/2}/M\,(t - t_B(r))$$

- $E = 0$: $R(t,r) = \left[9/2M(t - t_B(r))^2\right]^{1/3}$

- $E > 0$:

  $$R(t,r) = M/2E\,(\cosh\eta - 1) \quad \sinh\eta - \eta = (2E)^{3/2}/M\,(t - t_B(r))$$

Gauge choice: $M = M_0r^3$

Mass density profile of E = 0 LT models reproducing the ΛCDM $D_L(z)$ (2)

$$\kappa\rho = 4/(t - t_B)(3t - 3t_B - 2rt'_B)$$

$$\kappa\rho'/4 = \left[(t - t_B)(8t'_B + 2rt''_B) - 2rt'^2_B\right] / \left[(t - t_B)^2(3t - 3t_B - 2rt'_B)^2\right]$$

Near $r = 0$, $t_B(r) = \tau_1 r + \tau_2 r^2 + \mathcal{O}(r^3)$. **Hence,**

$$\kappa\rho(t = t_0, r \to 0) \to 4/3t_0^2$$

$$\kappa\rho'(t = t_0, r \to 0) \to 32\tau_1/9t_0^3.$$

⇒ at the center, the density profile is finite and exhibits a cusp. The sign of τ_1 says if the observer is in a void or an overdensity.

Noting that in Iguchi, Nakamura, Nakao 2002 PTP 108, 809 (INN), $dt_B/dz < 0$ **at the center, it is easy to show that the sign of τ_1 is < 0 (MNC 2011 arXiv: 1108.1373).**

⇒ the density profile at the center is a hump, not a void. This applies, e.g, to the example in MNC 2000 A& A 353, 63.

Mass density profile of $t_B = 0$ LT models reproducing the ΛCDM $D_L(z)$ (2)

Consider separately each case, $E < 0$ and $E > 0$, and apply the following method:

1. **Calculate $\kappa\rho$.**
2. **Since near $r = 0$, $E(r) = \mathcal{O}(r^2)$, ρ exhibits there a finite value**
3. **Calculate $\kappa\rho'$.**
4. **Noting from (INN) that, near the center, $d\rho/dz > 0$, complete a calculation analogous to that of the $E = 0$ case to conclude about the sign of ρ' near the origin.**

Result in both cases: the sign of ρ' is $> 0 \Rightarrow$ the observer is in a (local) void.

Conclusion of this analysis (2)

- **A LT model is fully specified by 2 functions of r ⇒ a random combination such as $E = 0$ or $t_B = 0$ and the ΛCDM $D_L(z)$ may well produce an LT model with unrealistic features.**

- **For pure decaying modes, $E = 0$, the density profile at the observer is a hump; for pure growing modes, $t_B = 0$, it is a void.**

- **Intermediate models implying both growing and decaying modes can also be used to solve the dark energy problem. What about the void or the hump in these models?**

- **MNC, Bolejko, Krasiński 2010 A&A 518, A21 and Kolb, Lamb arXiv:0911.3852 have computed the arbitrary functions of LT models reproducing ΛCDM features without a priori assumptions on their form. The shape of the current density profile is not a void, but a hump, not smooth at the observer. Note this property cannot be used as an argument to dismiss the models, since one can find it elsewhere in nature, e.g., on the surface of the Earth.**

The redshift drift (3)

Definition: temporal variation of the redshift due to the Universe expansion. **In FLRW models,** the drift is negative for a decelerating expansion and positive for an accelerating expansion Sandage 1962, ApJ 136,319.

Effect proposed as a test of ΛCDM models where the drift is positive at $z \lesssim 2.5$ and of typical void LT models by Uzan, Clarkson, Ellis 2008 PRL 100, 191303 and Quartin, Amendola 2010 PRD 81, 043522. Claimed to be negative in void models and measurable by the CODEX experiment within a decade or so.

But remember: **LT models are not exact representations of our Universe** $\Rightarrow$ they cannot be put to the test at the level of some $10^{-10} - 10^{-9}$ order of magnitude, i.e, by the redshift drift effect measurable in a foreseeable future.

Moreover: some large hump LT models can exhibit a positive redshift drift for some z range (Yoo, Kai, Nakao 2011 PRD 83, 043527).

The kinematic SZ effect and the CMB spectral distortion (3)

Other proposals for trying to go beyond the observer's light cone to put tight constraints on the models. The kSZ effect (García-Bellido, Haugbølle 2008 JCAP09(2008)016 +) and the spectral distortion of the CMB (Caldwell, Stebins 2008 PRL 100, 191302 +).

The kSZ effect: if we are located very near to the center of a void, we can observe distant off-center sources where corresponding observers should see a large dipole in the CMB spectrum. Such a dipole would manifest to us through a kSZ effect not observed.

The CMB spectral distortion: same philosophy. A Compton scatterer at a given redshift observes an anisotropic CMB which will be reflected back at us in the form of spectral distortions, i.e. deviations from a black body spectrum not observed.

Drawback of the reasonings: since we are not physically at the center of a spherically symmetric universe, the observer in the distant source or the Compton scatterer is neither physically off-center.

Cosmic parallax (3)

The idea: put bounds on the departure of a possible off-center observer from the symmetry center of LT models, arguing that such observers see an anisotropic space. If the expansion is anisotropic, the angular separation between two distant sources varies in time, thereby inducing a cosmic parallax effect supposed to be measurable by future space missions such as GAIA or SIM.

Loophole: LT models are only relevant with a central observer, since they must be considered as smoothing out all the anisotropies of our Universe. The cosmic parallax effect can provide an interesting measurement of the general anisotropy of the Universe but can say nothing about LT models.

CMB spectrum and the value of H_0 (3)

For LT models matched to an EdS background, the most stringent constraint on H_0 comes from the CMB power spectrum which is the result of (1) the imprint of the primordial perturbations onto the LSS (determined by the EdS cosmological parameters), (2) the geometry of the Universe between this surface and ourselves (influenced by the LT model). The claim that LT models exhibit a too low H_0 compared to that measured by Riess et al. 2011 ApJ 730, 119 is based on the simplified scheme, EdS background + $t_B = 0$, put forward by Moss, Zibin, Scott 2010 PRD 83, 103515.

Improvements:

- Make our region of the Universe younger (Clifton, Ferreira, Zuntz 2009 JCAP07, 029; Bull, Clifton, Ferreira arXiv: 1108.2222).
- Increase the size of the void (García-Bellido, Haugbølle 2008 JCAP04, 003).
- Include a nonzero overall curvature and a variation of the density profile of the void (Biswas, Notari, Valkenburg 2010 JCAP11, 030).
- Take into account the dynamical effect of radiation (Clarkson, Regis 2011 JCAP02, 013).
- Non-scale invariant primordial power spectrum (Nadathur, Sarkar 2011 PRD 83, 063506).

The first 3 reproduce only the first peak, but the last 2 reproduce the full spectrum.

Conclusion: claims that all LT models are ruled out by H_0 are highly premature.

Conclusions (4)

- **Use of spherically symmetric models with a central observer = a mathematical simplification, not to be taken at face value.**

- **It is therefore misleading to put such models to the test with methods which are mainly designed to test their simplifying assumptions: spherical symmetry and/or central spatial location of the observer.**

- **The mass density profiles of LT models: 1) case E=0 ⇔ a central hump. 2) case $t_B = 0$ ⇔ a central void. 3) Mixed case ⇔ a central void or hump.**

- **The redshift drift: too many years of duration to yield conclusions in a foreseeable future.**

- **The kSZ, the CMB spectral distortion and the cosmic parallax effects: irrelevant to put to the test models whose spherical symmetry is a mere approximation smoothing out the anisotropies of the Universe.**

- **The value of H_0: a good test. However, contrary to some claims, there are LT models (even voids) compatible with the highest measured values.**

- **The future: develop more sophisticated models and finally use numerical relativity to be able to fit all the cosmological data.**

WAS THE NOBEL COMMITTEE TOO MUCH IN A RUSH WHEN IT SUPPORTED THE ACCELERATED EXPANSION OF THE UNIVERSE?