Neutrinos, Spinodal Instability and the Dark Energy Problem

D. Cormier and R.H.

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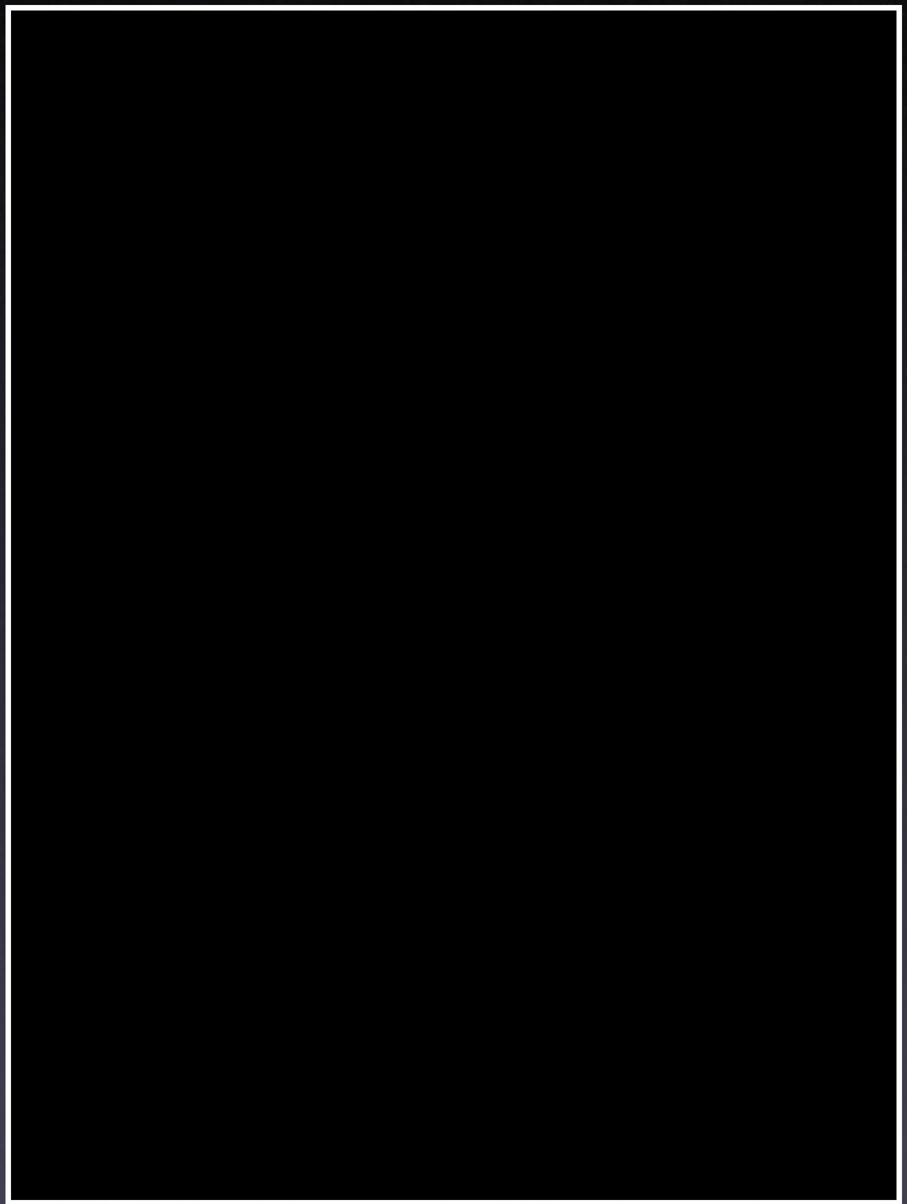
The dark energy problem



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  - SNIa Hubble diagram
  - WMAP estimates for Ω_{tot}
  - Cluster estimates for Ω_{matter}

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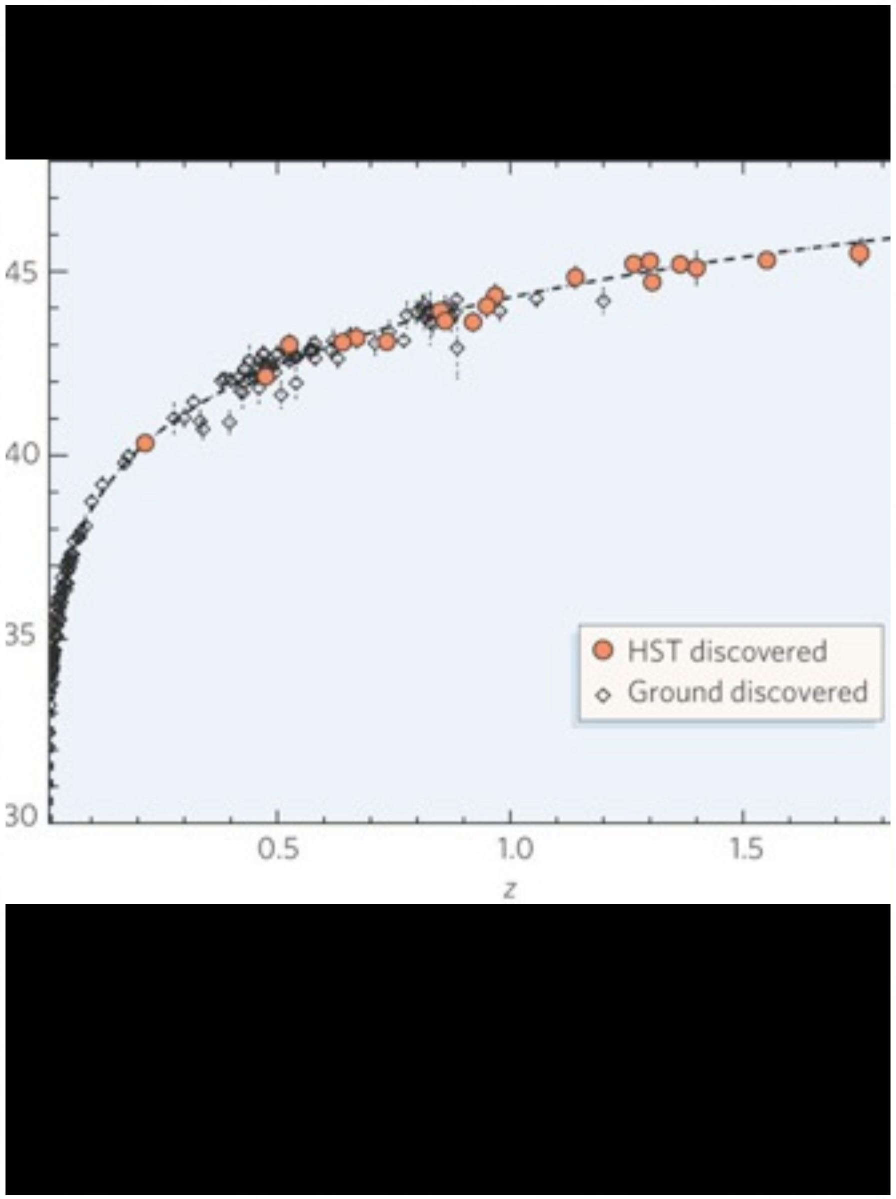

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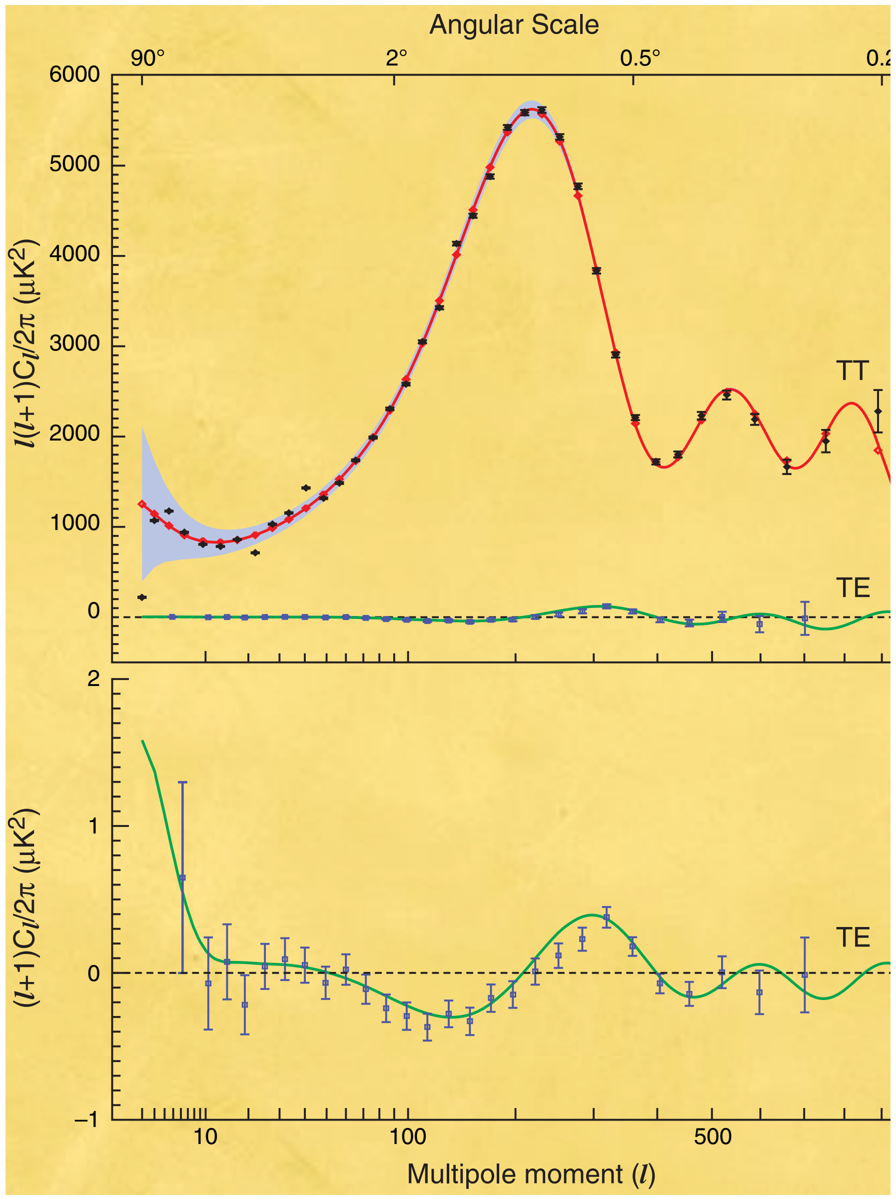

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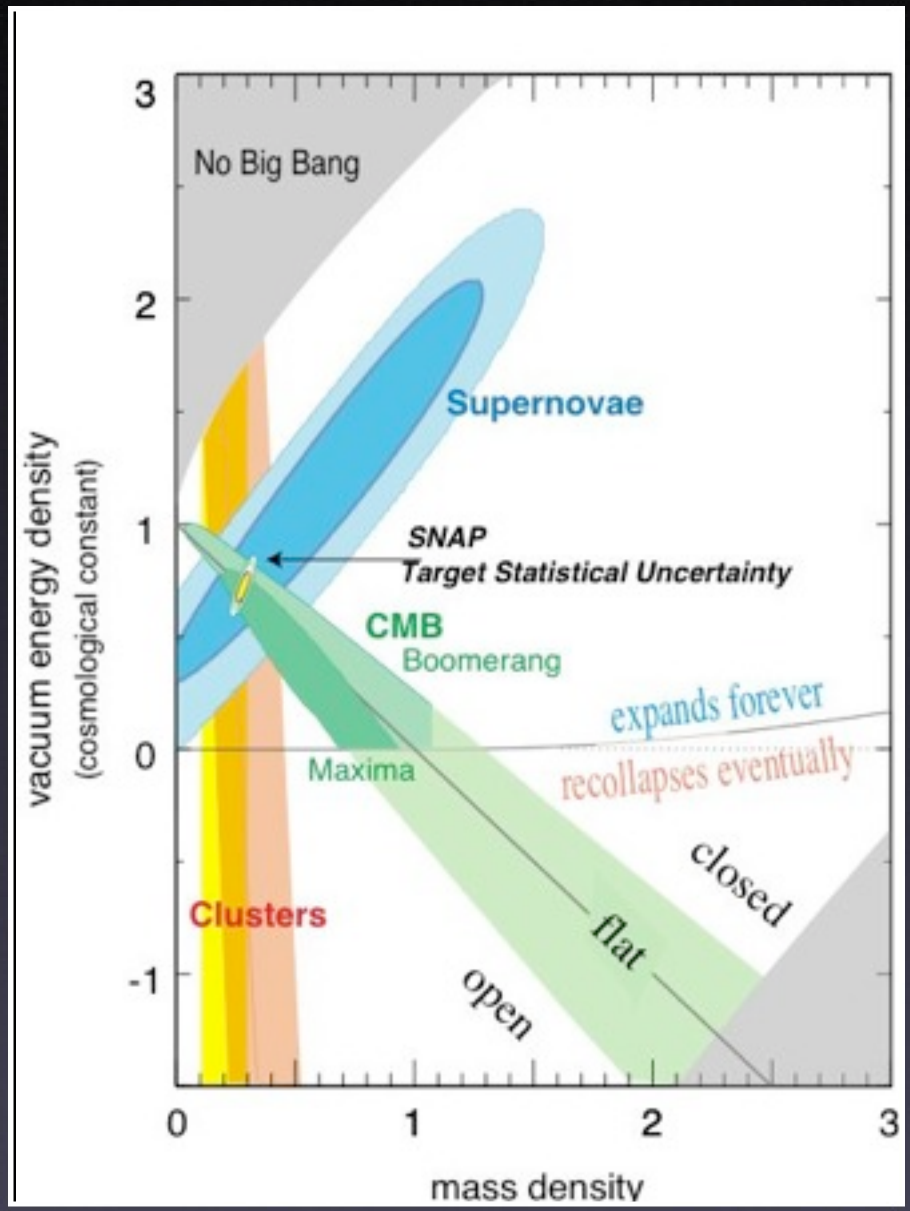

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The Magic Numbers

To have DE be
due to a scalar field rolling
in a potential we need:

$$m_\phi \sim 10^{-33}\ \mathrm{eV}$$

$$V \propto \mu^4,\ \ \mu \sim \text{few} \times 10^{-3}\ \mathrm{eV}$$

These are
interesting numbers:

$$m_\phi \sim H_0, \quad \mu \sim m_\nu$$

These are also
UNNATURAL numbers!

- In general, the potential energy gets renormalized by an amount Λ^4

- Furthermore, unless the potential is constant, the mass will also get renormalized by a term $\propto \Lambda^2$

- EXTREME fine tuning is required to make this work. Also have to deal with the Nancy Kerrigan problem: Why me, Why now?

PNGB's, Naturalness and all that

One way to deal with this is to find a symmetry to protect the scalar field parameters

Shift symmetry: $\phi \to \phi + c$

SUSY $\Rightarrow$ Energy scale of SUSY breaking is too large!

Pure shift symmetry prevents ANY potential for the scalar field. This is too much symmetry.

Pseudo-Nambu Goldstone Bosons:
Shift symmetry + explicit symmetry breaking terms

Examples of PNGB's : pions in QCD (chiral symmetry+ quark masses).
Axions (chiral PQ symmetry and chiral anomaly)

t’Hooft Naturalness:

Potential terms are due only to the explicit breaking.

Parameters are NATURALLY small if when it is set to 0, the symmetry of the Lagrangian increases.

Natural parameters are MULTIPLICATIVELY renormalized!

Small natural parameters remain small to ALL orders in PT

The Plan

- Dark Energy should come from PNGB's
- Try to feed in neutrino masses into PNGB potential
- Make scale of spontaneous breaking large enough so that $m_\phi \sim \dfrac{m_\nu^2}{f_{\rm spont}}$ is small enough.
- Study dynamics.

The Model

Consider an effective theory of N neutrinos coupled to a scalar via a Yukawa coupling of the form

$$-\mathcal{L}_{\text{Yuk}} = \sum_{j=0}^{N-1} \left(m_0 + \varepsilon \exp i \left(\frac{\Phi}{f} + \frac{2\pi j}{N} \right) \right) \overline{\nu}_{jL} \nu_{jR} + \text{h.c.}$$

$m_0 = 0$ ⇒ Continuous chiral $U(1)$ symmetry

$m_0 \neq 0$ ⇒ Only discrete Z_N remains

$$\nu_j \to \nu_{j+1}, \nu_{\mathrm{N}-1} \to \nu_0, \Phi \to \Phi + \frac{2\pi \mathrm{f}}{\mathrm{N}}$$

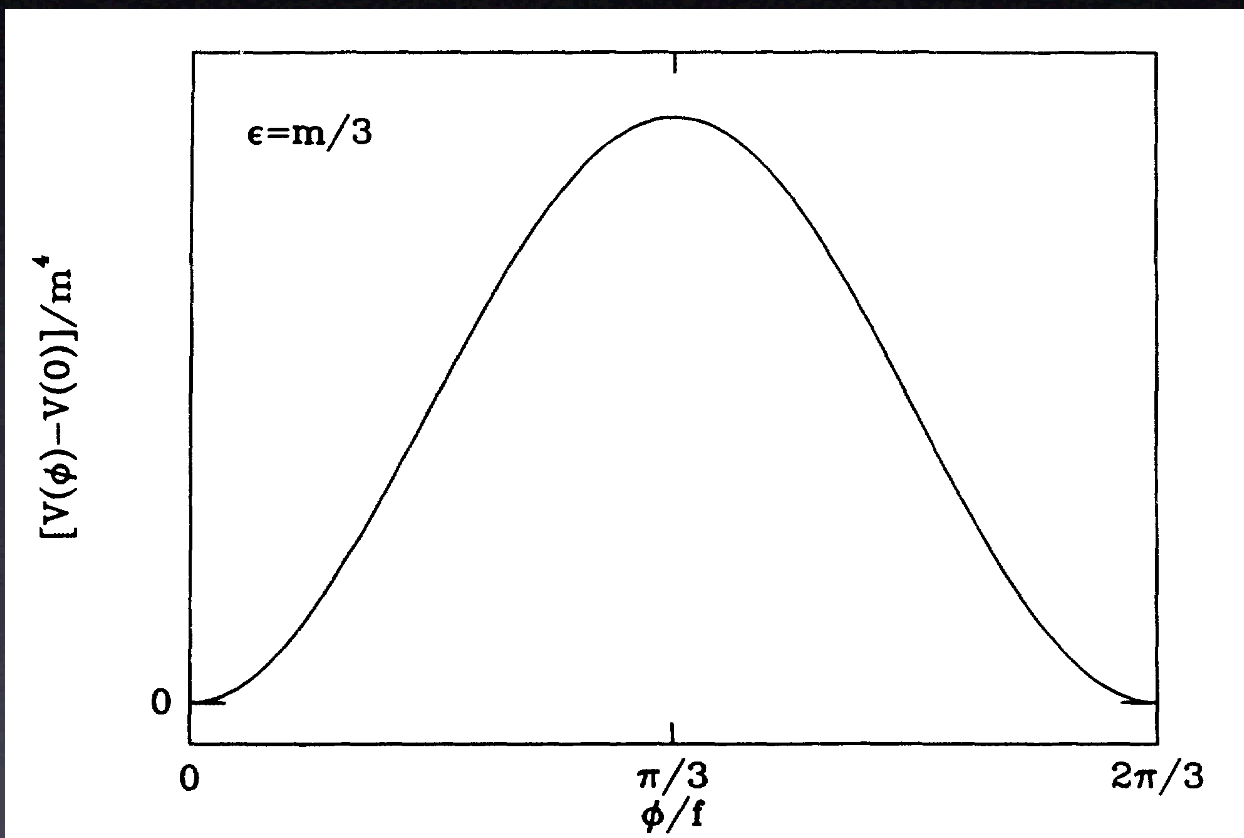
- When $m_0 = 0$ there can be NO potential for Φ due to the chiral Z_N symmetry.
- When $m_0 = 0$, neutrino masses vanish!
- Thus, mass scale associated with the potential induced for Φ, when $m_0 \neq 0$ is a neutrino mass scale!
- The smallness of this scale is protected against radiative corrections since symmetry is increased when $m_0 \to 0$

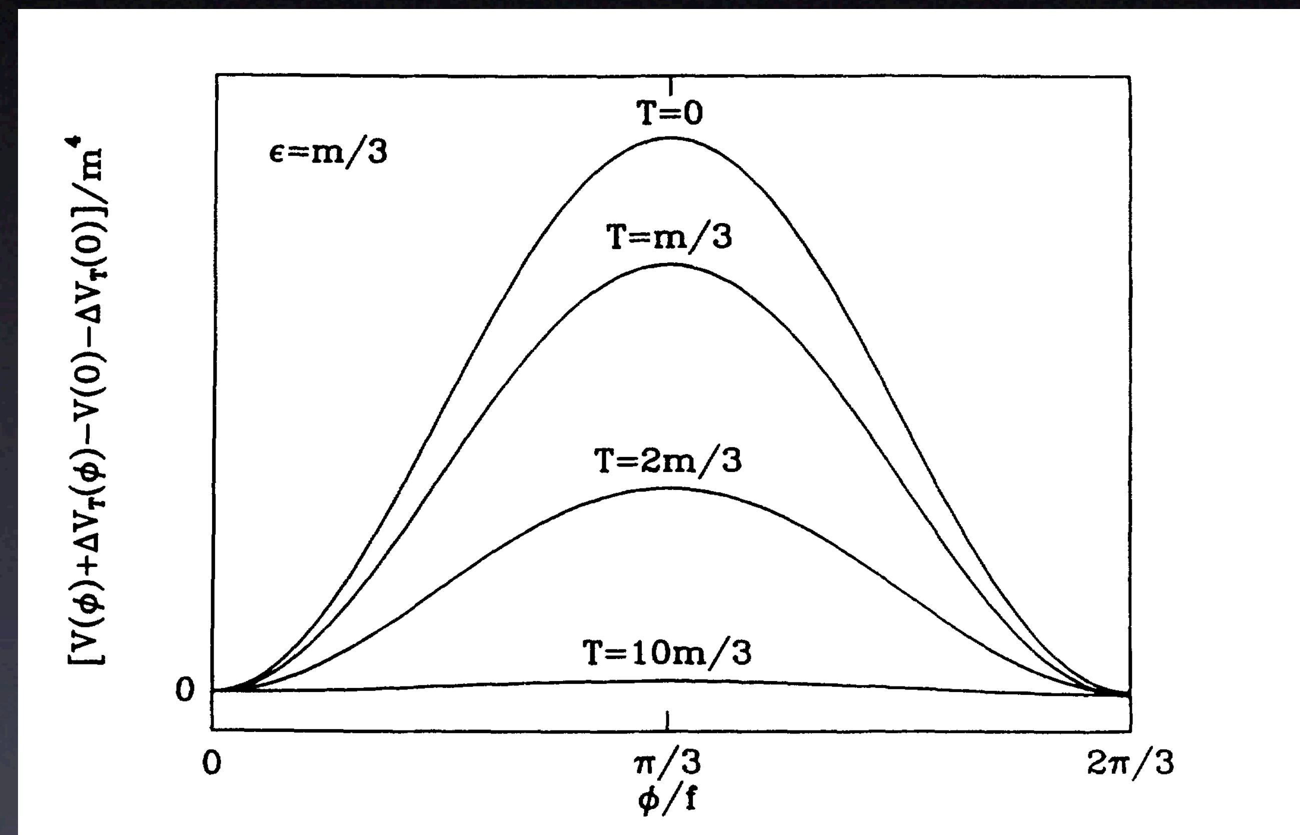
- Since Φ is an angular variable, the effective potential is of the form $V(\Phi) = m_\nu^4(1 + \cos\frac{N\Phi}{f})$.

- For $N > 2$, finite T corrections change this to $c(T)m_\nu^4 \cos\frac{N\Phi}{f}$, where $c(T)$ vanishes above a critical temperature of order m_ν .

- Note: Finite T calculation makes sense since neutrinos will keep their thermal distribution past their decoupling.

Z_3 Effective Potential





- At high T (but still below the SSB scale f), the potential becomes flat with a value m_ν^4.

- Note that the potential has spinodal regions where $V''(\Phi) < 0$. This drives the growth of unstable modes that can become non-perturbatively large.

- In order to study time evolution of the system, need to go beyond PT to deal with these modes.

Want to follow the evolution of the expectation value of in the presence of expansion AND including the backreaction from the growing fluctuations

Write:

$$\Phi(\vec{x},t) = \phi(t) + \psi(\vec{x},t)$$

and calculate

$$\langle \psi\,(\vec{x},t) \rangle = 0$$

Hartree approx:

$$\cos\frac{N\psi}{f} \longmapsto \left(1 - \frac{N^2\,(\psi^2 - \langle\psi^2\rangle)}{2f^2}\right)\exp - \frac{N^2\,\langle\psi^2\rangle}{2f^2}, .$$

$$\sin\frac{N\psi}{f} \longmapsto \frac{N\psi}{f}\exp - \frac{N^2\,\langle\psi^2\rangle}{2f^2}$$

EOM's: $$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{Nm_\nu^4}{f}\exp - \frac{N\langle\psi^2\rangle}{2f^2}\sin\frac{N\phi}{f} = 0$$

$$\ddot{f}_k + 3\frac{\dot{a}}{a}\dot{f}_k + (\frac{k^2}{a^2} - \frac{N^2 m_\nu^4}{f^2}\exp - \frac{N\langle\psi^2\rangle}{2f^2}\cos\frac{N\phi}{f})f_k = 0$$

$$\langle\psi^2\rangle = \int\frac{d^3k}{(2\pi)^3}|f_k|^2$$

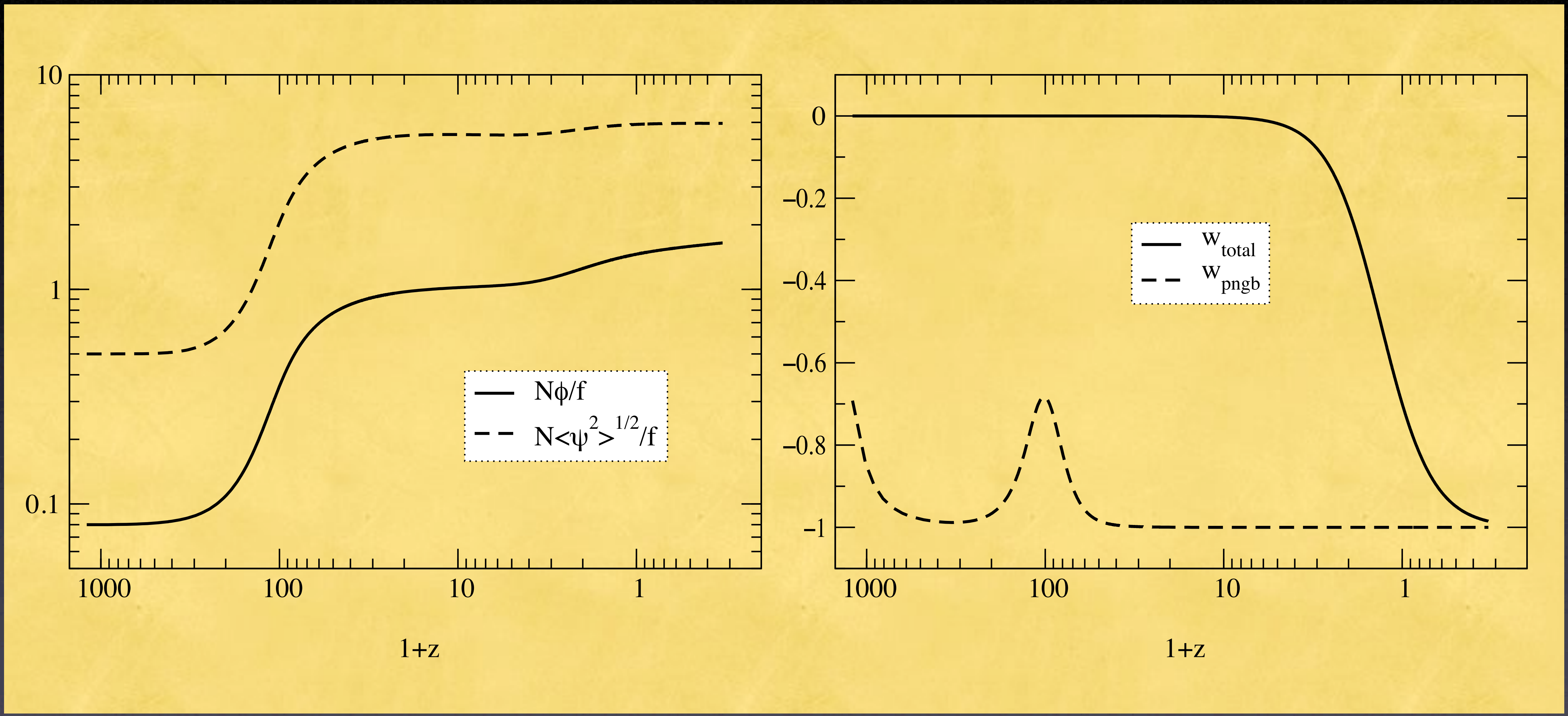
$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3M_p^2}[\rho_m(t) + \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}\langle\dot{\psi}^2\rangle$$
$$+ \frac{1}{2a^2}\langle(\vec{\nabla}\psi)^2\rangle + M^4(1 + \cos(N\phi/f)\exp - \frac{N^2\langle\psi^2\rangle}{2f^2})]$$

The Scenario

- Since potential is flat at high T, expect that we should be able to find the zero mode above the spinodal line with $P \sim \frac{1}{2}$. If inflation occurs, we can take the zero mode to have the same value throughout the observable Universe.

- Need to compare two time scales: How long does it take the zero mode to hit the spinodal line NEGLECTING fluctuations ($t_{\rm spin}$), vs. how long does it take the fluctuations to sample the minima of the potential ($t_{\rm fluct}$)

- Need $t_{\text{spin}} \gg t_{\text{fluct}} \Rightarrow \phi^2(t_i) \ll \langle\psi^2(t_i)\rangle$.
- If we have inflation, $\frac{\langle\psi^2\rangle(t_i)}{f^2} \simeq \frac{H_{DS}^2}{4\pi^2 f^2}(N_{\text{e-folds}} - 60).$ For reasonable values of the parameters the conditions becomes $\phi(t_i)/f \ll 0.5$



10
1
0.1
1000
100
10
1
1+z
Nφ/f
N<ψ2>1/2/f
0
−0.2
−0.4
−0.6
−0.8
−1
wtotal
wpngb

Conclusions

- Dark energy is pretty strange so it may require pretty weird ideas!
- Before getting really carried away, though, check limits of what field theory and known particles can do.
- What we show: Back reaction + naturalness can be strong enough to flatten potential naturally!
- However, REALLY need to deal with cosmological constant problem!