Cosmological backreaction via effective equations: the quantum counterpart

Giovanni Marozzi

College de France, Paris, France

Backreaction: where do we stand?
IAP, Paris
22 November 2011

Outline

- Motivation
- Cosmological backreaction via effective equations: a manifestly covariant and gauge invariant formalism
- Quantum backreaction via effective equations: from the free-falling to the isotropic observers
- Open Problems
- Conclusions

Based on:
M. Gasperini, G.M., G. Veneziano, JCAP 03 (2009) 011.
M. Gasperini, G.M., G. Veneziano, JCAP 02 (2010) 009.
G.M., JCAP 01 (2011) 012.
F. Finelli, G.M., G. P. Vacca, G. Venturi, Phys. Rev. Lett. 106, 121304 (2011).
G.M., G. P. Vacca, 1108.1363 [gr-qc].

Outline

- Motivation
- Cosmological backreaction via effective equations: a manifestly covariant and gauge invariant formalism
- Quantum backreaction via effective equations: from the free-falling to the isotropic observers
- Open Problems
- Conclusions

The Problem

The large scale properties of our Universe are usually described in the context of a homogeneous and isotropic FLRW space-time.

However:

The real Universe is not exactly homogeneous and isotropic

- neither in its present state (classic inhomogeneities)
- nor in its primordial state (quantum fluctuations)

Conventional Assumption and its Issues

Inhomogeneities are small on large scales, and the homogeneous Einstein equations provide a sufficiently good description of the averaged cosmological geometry.
But.....

- Inhomogeneities not always small!

  $$\Downarrow$$

  Inflation can amplify quantum fluctuations up to be comparable with the background ($\langle \delta\phi^2 \rangle \sim \langle \phi^2 \rangle$), making their effects non negligible (inflationary backreaction (Mukhanov, Abramo, Brandenberger (1997)).

- Inhomogeneities are there! We have to take in consideration their impact for a consistent comparison of the theory with the observational data.

Fitting Problem

Inhomogeneities could affect in a non-trivial way the cosmological evolution.

How to determine the true dynamical evolution of the averaged cosmological geometry?
Answer not obvious!

Differential operators and averaging procedure do not commute

$\Downarrow$ (Ellis (1984))

Averaged Einstein equations $\neq$ Einstein equations for the averaged geometry.

The dynamic of the averaged geometry is affected by the so-called "backreaction terms".

Gauge Issue

One needs a well defined averaging procedure for smoothing-out the perturbed (non-homogeneous) geometric parameters.

The computation of these averages is affected in principle by a well-known ambiguity due to the possible choice of different "gauges".

Standard averaging formalism (Buchert (2000))

$$\langle S \rangle_D = \frac{\int_D d^3x \sqrt{|\gamma|}\, S}{\int_D d^3x \sqrt{|\gamma|}}$$

Different gauge $\Rightarrow$ different results!

What does it mean to choose a gauge for this standard formalism? See later!!

Buchert Equations, 1

Considering the synchronous gauge one can define, with respect to an observer at rest in the cosmic medium, the following effective equations for a dust universe (Buchert (2000))

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}\rho_{eff}$$

$$-\frac{\ddot{a}_D}{a_D} = \frac{4\pi G}{3}\left(\rho_{eff} + 3p_{eff}\right)$$

with

$$V_D = \int_D d^3x \sqrt{|\gamma|} \quad , \quad a_D = \left(V_D / V_{D_0}\right)^{1/3}$$

Buchert Equations, 2

$$\rho_{eff} = \langle \rho \rangle_D - \frac{1}{16\pi G} \left(\langle Q \rangle_D + \langle \mathcal{R} \rangle_{\mathcal{D}} \right)$$

$$p_{eff} = -\frac{1}{16\pi G} \left(\langle Q \rangle_D - \frac{1}{3} \langle \mathcal{R} \rangle_{\mathcal{D}} \right)$$

and

$$\langle Q \rangle_D = \frac{2}{3} \left(\langle \Theta^2 \rangle_D - \langle \Theta \rangle_D^2 \right) - 2\langle \sigma^2 \rangle_D$$

the kinematical backreaction term.
Θ is the volume expansion, σ^2 the shear scalar, and $\mathcal{R}$ the spatial Ricci scalar.

Outline

- Motivation
- Cosmological backreaction via effective equations: a manifestly covariant and gauge invariant formalism
- Quantum backreaction via effective equations: from the free-falling to the isotropic observers
- Open Problems
- Conclusions

Gauge freedom in a FLRW universe, 1

Let us consider a cosmological background sourced by a scalar field ϕ and described by the simple four-dimensional action

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

with spatially flat FLRW background geometry

$$ds^2 = -dt^2 + a(t)^2\, \delta_{ij}\, dx^i dx^j$$

Gauge freedom in a FLRW universe, 2

The background fields $\{\phi, g_{\mu\nu}\}$ can be expanded in non-homogeneous perturbations as follows:

$$\phi(t,\vec{x}) = \phi^{(0)}(t) + \delta\phi(t,\vec{x}),$$
$$g_{00} = -1 - 2\alpha, \qquad g_{i0} = -\frac{a}{2}\left(\beta_{,i} + B_i\right),$$
$$g_{ij} = a^2\left[\delta_{ij}\left(1-2\psi\right) + D_{ij}E + \frac{1}{2}\left(\chi_{i,j} + \chi_{j,i} + h_{ij}\right)\right],$$

where $D_{ij} = \partial_i\partial_j - \delta_{ij}(\nabla^2/3)$.
One obtains 11 degrees of freedom which are in part redundant.
To obtain a set of equations (Einstein equations + equation of motion of ϕ) well defined, order by order, we have to set to zero two scalar perturbations among $\delta\phi$, α, β, ψ and E, and one vector perturbation between B_i and χ_i.

Gauge freedom in a FLRW universe, 3

The choice of such variables is called a choice of gauge.

For the scalar sector we may have:

$\psi = 0$, $E = 0$ Uniform Curvature Gauge
$\beta = 0$, $E = 0$ Longitudinal Gauge
$\alpha = 0$, $\beta = 0$ Synchronous Gauge
$\delta\phi = 0$, β or ψ or $E = 0$ Uniform Field Gauge
etc.

Gauge trasformation vs Gauge Invariant variables, 1

To connect different gauge we need an infinitesimal coordinates transformation.
This can be parametrized by a first-order, $\epsilon^\mu_{(1)}$, and a second-order, $\epsilon^\mu_{(2)}$, vector generator, and is given by (Bruni, Matarrese, Mollerach, Sonego (1997)):

$$x^\mu \to \tilde{x}^\mu = x^\mu + \epsilon^\mu_{(1)} + \frac{1}{2}\left(\epsilon^\mu_{(1),\nu}\epsilon^\nu_{(1)} + \epsilon^\mu_{(2)}\right).$$

Under the associated GT old and new fields are evaluated at the same space-time point x and a tensor T changes as

$$T^{(1)} \to \tilde{T}^{(1)} = T^{(1)} - \mathcal{L}_{\epsilon_{(1)}} T^{(0)},$$

$$T^{(2)} \to \tilde{T}^{(2)} = T^{(2)} - \mathcal{L}_{\epsilon_{(1)}} T^{(1)} + \frac{1}{2}\left(\mathcal{L}^2_{\epsilon_{(1)}} T^{(0)} - \mathcal{L}_{\epsilon_{(2)}} T^{(0)}\right)$$

Gauge trasformation vs Gauge Invariant variables, 2

Request: Physics results should not depend on the gauge chosen to describe these.

Answer: Gauge Invariant (GI) formalism (Bardeen (1980), for a review see: Mukhanov, Feldman, Brandenberger(1992)).

Physically meaningful variable $\leftrightarrow$ GI variable.

A GI variable F is defined as a function of our perturbations which takes always the same value independently of the gauge chosen

$$F(\delta\phi, \alpha, \beta, ...) \to F(\delta\tilde{\phi}, \tilde{\alpha}, \tilde{\beta}, ...) = F(\delta\phi, \alpha, \beta, ...)$$

Gauge trasformation vs Gauge Invariant variables, 3

Scalar Power Spectrum

The scalar power spectrum associated with a model of inflation is defined using the GI curvature perturbation ξ. Such perturbation is given, to first order, by

$$\xi^{(1)} = \frac{H}{\dot{\phi}} Q^{(1)} \quad \textit{with} \quad Q^{(1)} = \delta\phi^{(1)} + \frac{\dot{\phi}}{H}\left(\psi^{(1)} + \frac{1}{6}\nabla^2 E^{(1)}\right)$$

where $Q^{(1)}$ is the first order GI Mukhanov variable (Mukhanov (1988)). So one obtains

$$P_\zeta(k) = \frac{k^3}{2\pi^2}\left(\frac{H}{\dot{\phi}}\right)^2 |Q_k|^2$$

Covariant averaging prescription, 1

Depending on the context in which the backreaction is considered, there are two types of averaging procedure: spatial average of classical variables, and (vacuum) expectation values of quantized fields.

In both cases, one has to face the problem of the gauge dependence of the results.

Spatial average of classical scalar variable $\equiv$ average with respect to the observers seat on a hypersurface defined by another scalar $A(x)$ through the condition $A(x) = A_0$.

In this way

- the scalar $A(x)$ (with time-like gradient) determines the temporal boundary and the observers which perform the measure.
- another function $B(x)$ (with space-like gradient) determines the spatial boundary by the coordinate condition $B(x) < r_0$.

Covariant averaging prescription, 2

The averaging prescription is so defined as (Gasperini, GM, Veneziano (2009,2010)):

$$\langle S \rangle_{A_0,r_0} = \frac{F(S,\Omega)}{F(1,\Omega)} = \frac{\int d^4x \sqrt{-g}\, S\, u^\mu \nabla_\mu \theta(A(x) - A_0)\theta(r_0 - B(x))}{\int d^4x \sqrt{-g}\, u^\mu \nabla_\mu \theta(A(x) - A_0)\theta(r_0 - B(x))}$$

and, in the bar coordinates where A is homogeneous, one obtains

$$\langle S \rangle_{A_0,r_0} = \frac{\int_{\Sigma_{A_0}} d^3x \sqrt{|\overline{\gamma}(t_0,\vec{x})|}\ \overline{S}(t_0,\vec{x})\, \theta(r_0 - B(h(t_0,\vec{x}),\vec{x})}{\int_{\Sigma_{A_0}} d^3x \sqrt{|\overline{\gamma}(t_0,\vec{x})|}\, \theta(r_0 - B(h(t_0,\vec{x}),\vec{x}))}$$

where we have called t_0 the time $\bar{t}$ when $A^{(0)}(\bar{t})$ takes the constant values A_0. The averaging prescription will be strictly gauge invariant only if also $B(x)$ is a scalar.

On the other hand, considering the limit of an infinite spatial volume the possible "residual" gauge dependence goes to zero and we obtain a gauge invariant average (Gasperini, G.M., Veneziano (2009), G.M. (2011)).

Covariant averaging prescription, 3

Going back to the standard averaging prescription!

The Buchert averaging formalism

$$\langle S \rangle_D = \frac{\int_D d^3x \sqrt{|\gamma|}\, S}{\int_D d^3x \sqrt{|\gamma|}}$$

can be seen as a particular case of

$$\langle S \rangle_{A_0, r_0} = \frac{\int_{\Sigma_{A_0}} d^3x \sqrt{|\overline{\gamma}(t_0, \vec{x})|}\;\; \overline{S}(t_0, \vec{x})\, \theta(r_0 - B(h(t_0, \vec{x}), \vec{x}))}{\int_{\Sigma_{A_0}} d^3x \sqrt{|\overline{\gamma}(t_0, \vec{x})|}\, \theta(r_0 - B(h(t_0, \vec{x}), \vec{x}))},$$

where D is defined by $\theta(r_0 - B(h(t_0, \vec{x}), \vec{x}))$, and we take a scalar $A(x)$ which is homogeneous in the particular gauge chosen to make the averages.

To fix the gauge in the standard formalism corresponds to fix the observers with respect to which we perform the average.

Effective Equation for the Cosmological Backreaction: a Covariant and Gauge Invariant approach, 1

We are now in the position to give a covariant and gauge invariant generalization of the effective equations for the cosmological backreaction. Considering the covariant averaging prescription defined and deriving with respect to A_0, we can obtain the gauge invariant generalization of the Buchert-Ehlers commutation rule (Buchert, Ehlers (1997)) as

$$\frac{\partial\langle S\rangle_{A_0}}{\partial A_0} = \left\langle \frac{\partial_\mu A \partial^\mu S}{\partial_\mu A \partial^\mu A} \right\rangle_{A_0} + \left\langle \frac{S\,\Theta}{(-\partial_\mu A \partial^\mu A)^{1/2}} \right\rangle_{A_0} - \langle S\rangle_{A_0} \left\langle \frac{\Theta}{(-\partial_\mu A \partial^\mu A)^{1/2}} \right\rangle_{A_0} .$$

Starting from this, and using a scalar form of the ADM Hamiltonian constraint and Raychaudhuri's equation, we obtain the following generalization for the effective equations (see Gasperini, G.M., Veneziano (2010), for details)

$$\left(\frac{1}{a_{eff}}\frac{\partial\, a_{eff}}{\partial A_0}\right)^2 = \frac{8\pi G}{3}\left\langle \frac{\varepsilon}{Z_A}\right\rangle_{A_0} - \frac{1}{6}\left\langle \frac{\mathcal{R}_s}{Z_A}\right\rangle_{A_0} - \frac{1}{9}\left[\left\langle \frac{\Theta^2}{Z_A}\right\rangle_{A_0} - \left\langle \frac{\Theta}{Z_A^{1/2}}\right\rangle_{A_0}^2\right] + \frac{1}{3}\left\langle \frac{\sigma^2}{Z_A}\right\rangle_{A_0}$$

$$= \frac{1}{9}\left\langle \frac{\Theta}{Z_A^{1/2}}\right\rangle_{A_0}^2$$

Effective Equation for the Cosmological Backreaction: a Covariant and Gauge Invariant approach, 2

and

$$-\frac{1}{a_{eff}}\frac{\partial^2 a_{eff}}{\partial A_0^2} = \frac{4\pi G}{3}\left\langle\frac{\varepsilon+3\pi}{Z_A}\right\rangle_{A_0} - \frac{1}{3}\left\langle\frac{\nabla^\nu(n^\mu\nabla_\mu n_\nu)}{Z_A}\right\rangle_{A_0} + \frac{1}{6}\left\langle\frac{n_\mu\partial^\mu Z_A}{Z_A^2}\Theta\right\rangle_{A_0}$$
$$-\frac{2}{9}\left[\left\langle\frac{\Theta^2}{Z_A}\right\rangle_{A_0} - \left\langle\frac{\Theta}{Z_A^{1/2}}\right\rangle_{A_0}^2\right] + \frac{2}{3}\left\langle\frac{\sigma^2}{Z_A}\right\rangle_{A_0}$$

with $Z_A = -\partial^\mu A \partial_\mu A$ and where $\mathcal{R}_s$ is a generalization of the intrinsic scalar curvature, $\Theta = \nabla_\mu n^\mu$ the expansion scalar and σ^2 the shear scalar with respect to the observer. We then define

$$\varepsilon = \rho - (\rho+p)\left(1-(u^\mu n_\mu)^2\right),$$
$$\pi = p - \frac{1}{3}(\rho+p)\left(1-(u^\mu n_\mu)^2\right)$$

with u_μ the 4-velocity comoving with the perfect fluid and ρ and p are, respectively, the (scalar) energy density and pressure in the fluid's rest frame.

Outline

- Motivation
- Cosmological backreaction via effective equations: a manifestly covariant and gauge invariant formalism
- Quantum backreaction via effective equations: from the free-falling to the isotropic observers
- Open Problems
- Conclusions

Gauge invariant averaging for the quantum BR, 1

In the limit of an infinite spatial volume the covariant average prescription defined is strictly gauge invariant.
In this limit the step-like boundary disappears, and we obtain:

$$\langle S \rangle_{A_0} = \frac{\int_{\Sigma_{A_0}} d^3x \sqrt{|\overline{\gamma}(t_0, x)|} \; \overline{S}(t_0, x)}{\int_{\Sigma_{A_0}} d^3x \sqrt{|\overline{\gamma}(t_0, x)|}} .$$

This results can be generalized to the quantum case.
Expectation values of quantum operators can be extensively interpreted (and re-written) as spatial integrals weighted by the integration volume V, according to the general prescription

$$\langle \dots \rangle \quad \rightarrow \quad V^{-1} \int_V d^3x \, (\dots) \; ,$$

where V extends to all three-dimensional space.

Gauge invariant averaging for the quantum BR, 2

In this way the above gauge invariant prescription becomes

$$\langle S \rangle_{A_0} = \frac{\langle \sqrt{|\overline{\gamma}(t_0, x)|}\ \overline{S}(t_0, x) \rangle}{\langle \sqrt{|\overline{\gamma}(t_0, x)|} \rangle}$$

where it is important to note that the two entries of this ratio are not separately gauge invariant, but the ratio itself, equivalent to the above prescription, is indeed invariant.

Following the results above one can consider an effective scale factor $a_{eff} = \langle \sqrt{|\bar{\gamma}|}\, \rangle^{1/3}$ and obtain a quantum gauge invariant version of the effective cosmological equations for the averaged geometry.

Gauge invariant BR in chaotic $m^2\phi^2$ inflation, 1

Taking advantage of these results we want evaluate the backreaction of quantum fluctuations during a chaotic model of inflation. Such analysis will be GI but dependent on the different observers intrinsically used in the GI construction.

Considering the long wavelength (LW) limit, one obtains a simple expression for the first effective equation for the cosmological backreaction

$$\left(\frac{1}{a_{eff}}\frac{\partial\, a_{eff}}{\partial A_0}\right)^2 = H^2\left[1 + \frac{2}{H}\langle\bar{\psi}\dot{\bar{\psi}}\rangle - \frac{2}{H}\langle\dot{\bar{\psi}}^{(2)}\rangle\right]$$

This is the starting point of our analysis.

Gauge invariant BR in chaotic $m^2\phi^2$ inflation, 2

Let us define our observers through a scalar field A homogeneous in a particular gauge.

We have a correspondence between a class of gauges and a class of observers with their physical properties.

In the long wavelength limit such physical properties are characterized by the time gauge condition on the vector generator $\epsilon^0_{(1)}$ and $\epsilon^0_{(2)}$ to go from a general gauge to the class of gauges chosen.

We can devide the observers in 3 different classes:

(a) the ones which correspond to gauges with $\psi = 0$, with trivially zero backreaction.

(b) the ones which correspond to gauges with $\alpha = 0$ (or $\varphi = 0$ (UFG)), which are geodesic, or free falling, observers.

(c) the ones which correspond to the gauges with $\beta = 0$ and $E = 0$ (longitudinal gauge), which have zero scalar and tensor shear. These are called isotropic observers and see an inhomogeneous and isotropic space.

Geodesic Observers, 1

The dynamic of a free falling observer is determined by the equation $t_\mu = n^\nu \nabla_\nu n_\mu = 0$ for its velocity n_μ.

The scalar field $A(x)$ associated with this observer is, for example, the one homogeneous in the SG (see G.M. (2011) for details).

In general the condition for a scalar field $A(x)$ to be associated with free falling observers at first order is given by (zero order condition is trivially satisfied for any scalar)

$$\frac{d}{dt}\left(\frac{A^{(1)}}{\dot{A}^{(0)}}\right) - \alpha = 0\,.$$

Geodesic Observers, 2

The coordinate transformations needed to go from a general gauge to the SG one are characterized by

$$\epsilon^0_{(1)} = \int^t dt'\alpha \ , \ \ \epsilon^0_{(2)} = -\alpha \int^t dt'\alpha + \int^t dt' \left(2\alpha^{(2)} - \alpha^2\right)$$

so for the free falling observer we have

$$\bar{\psi} = \psi + H \int^t dt'\alpha \, ,$$

$$\begin{aligned} \bar{\psi}^{(2)} \ &= \ \psi^{(2)} - H\alpha \int^t dt'\alpha - \frac{1}{2}\left(\dot{H} + 2H^2\right)\left[\int^t dt'\alpha\right]^2 \\ &\quad - \left(2H\psi + \dot{\psi}\right)\int^t dt'\alpha + \frac{H}{2}\int^t dt'\left(2\alpha^{(2)} - \alpha^2\right) \end{aligned}$$

Geodesic Observers, 3

The backreaction results are gauge independent, so we can choose the gauge in which performs the calculations at our convenience. We consider the UCG (Finelli, G.M., Vacca, Venturi (2004)) and we find that

$$\left(\frac{1}{a_{eff}}\frac{\partial\, a_{eff}}{\partial A_0}\right)^2 = H^2\left[1 + \mathcal{O}\left(\epsilon^2\right)\frac{\langle\delta\phi^2\rangle}{M_{pl}^2}\right],$$

No leading backreaction in the slow-roll parameter ϵ on the effective Hubble factor induced by scalar fluctuations.

No leading backreaction in the slow-roll parameter ϵ on the effective equation of state induced by scalar fluctuations. (see later)

UFG Observers

The scalar associated to these observers is given by

$$A(x) = A^{(0)} + \frac{\dot{A}^{(0)}}{\dot{\phi}}\delta\phi + \frac{\dot{A}^{(0)}}{\dot{\phi}}\delta\phi^{(2)} + \frac{\dot{A}^{(0)}}{2\dot{\phi}^2}\left(\frac{\ddot{A}^{(0)}}{\dot{A}^{(0)}} - \frac{\ddot{\phi}}{\dot{\phi}}\right)\delta\phi^2 \,.$$

The condition to have geodesic observers to first order becomes

$$\frac{d}{dt}\left(\frac{\delta\phi}{\dot{\phi}}\right) - \alpha = 0 \,.$$

Such a condition is trivially satisfied in the LW limit and we have similar result to second order.

The UFG observers are physically equivalent to the free falling ones and experience the same backreaction

Isotropic Observers, 1

Let us consider the observers define by the scalar homogeneous in the longitudinal gauge.

This is defined to first order by

$$A(x) = A^{(0)} + \dot{A}^{(0)}\left[\frac{a}{2}\beta + \frac{a^2}{2}\dot{E}\right]$$

and is not fee-falling.

The shear scalar (neglecting tensor perturbations) is given by

$$(\sigma^2)^{(0)} = 0 \quad , \quad (\sigma^2)^{(1)} = 0$$

$$(\sigma^2)^{(2)} = \frac{1}{2a^4\dot{A}^{(0)\,2}}\left[A^{(1)}_{,ij}A^{(1),ij} - \frac{1}{3}(\nabla^2 A^{(1)})^2\right] + \frac{1}{8a^2}\left[\beta_{,ij}\beta^{,ij} - \frac{1}{3}(\nabla^2\beta)^2\right]$$
$$-\frac{1}{2a^3\dot{A}^{(0)}}\left[A^{(1)}_{,ij}\beta^{,ij} - \frac{1}{3}(\nabla^2 A^{(1)})(\nabla^2\beta)\right] - \frac{1}{4a^2\dot{A}^{(0)}}A^{(1)}_{,ij}\dot{\hat{h}}^{ij} + \frac{1}{8a}\beta_{,ij}\dot{\hat{h}}^{ij} + \frac{1}{32}\dot{\hat{h}}_{ij}\dot{\hat{h}}^{ij}$$

where $\hat{h}_{ij} = 2D_{ij}E$.

Isotropic Observers, 2

Considering the LG scalar in the evaluation of the shear scalar we obtain a identically zero value.

In the same way also the shear tensor $\sigma_{\mu\nu}$ turns out to be zero when evaluated with respect to the LG observers.

As a consequence $\rightarrow \Theta_{\mu\nu} = \frac{1}{3} h_{\mu\nu} \Theta$

The expansion is seen as isotropic from all the observers associated with the longitudinal gauge!

We call these the Isotropic Observers.

Isotropic Observers, 3

Using the UCG as starting point the coordinate transformations needed to go to the SG are characterized by

$$\epsilon^0_{(1)} = \frac{a}{2}\beta + \frac{a^2}{2}\dot{E} = \frac{a}{2}\beta \quad , \quad \epsilon_{(1)} = \frac{E}{2} = 0$$

$$\epsilon_{(2)} = \frac{3}{8}\frac{1}{\nabla^2}\left(\frac{\partial^i\partial^j}{\nabla^2} - \frac{1}{3}\delta^{ij}\right)\partial_i\beta\,\partial_j\beta\,,$$

$$\epsilon^0_{(2)} = a^2\dot{\epsilon}_{(2)} + a\beta^{(2)} - \frac{a}{2}\alpha\beta - a\frac{\partial^i}{\nabla^2}(\alpha\,\partial_i\beta)$$

so for the isotropic observer we have

$$\bar{\psi} = \frac{aH}{2}\beta,$$

$$\bar{\psi}^{(2)} = \frac{Ha}{2}\left[a\dot{\epsilon}_{(2)} + \beta^{(2)} - \frac{\partial^i}{\nabla^2}(\alpha\,\partial_i\beta)\right] - \frac{aH}{2}\alpha\beta - \frac{a^2}{8}\left(\dot{H} + H^2\right)\beta^2$$

Isotropic Observers, 4

and one obtains the following final result

$$\left(\frac{1}{a_{eff}}\frac{\partial\, a_{eff}}{\partial A_0}\right)^2 = H^2\left[1+\frac{3}{5}\frac{\dot{H}}{H^2}\frac{\langle\delta\phi^2\rangle}{M_{pl}^2}+\mathcal{O}\left(\frac{\dot{H}^2}{H^4}\right)\frac{\langle\delta\phi^2\rangle}{M_{pl}^2}\right].$$

Leading backreaction, in the slow-roll parameter, on the effective Hubble factor induced by scalar fluctuations!

The observers associated to the longitudinal gauge foliation, which are not free-falling, but see an inhomogeneous isotropic space, experiences a backreaction such that $H_{eff}^2 < H^2$.

A valuable information is also given by the effective equation of state which is defined with respect to such observers. Therefore we want now study the quantity $w_{eff} = p_{eff}/\rho_{eff}$.

General BR in a slow-roll inflationary model

Let us start with a more general result valid for any observer and slow-roll inflationary models.
We consider for the effective Hubble factor the following relation

$$\left(\frac{1}{a_{eff}}\frac{\partial\, a_{eff}}{\partial A_0}\right)^2 = H^2\left[1+\left(c\frac{\dot{H}}{H^2}+d\frac{\dot{H}^2}{H^4}+\mathcal{O}\left(\frac{\dot{H}^3}{H^6}\right)\right)\frac{\langle\delta\phi^2\rangle}{M_{pl}^2}\right],$$

where c and d are parameters which encodes the possible non zero backreaction at first and second order in the slow-roll approximation.
Then, from the consistency between the effective equations for the averaged geometry, one obtains

$$-\frac{1}{a_{eff}}\frac{\partial^2\, a_{eff}}{\partial A_0^2} = -\dot{H}-H^2-H^2\left[c\frac{\dot{H}}{H^2}+\left(d-\frac{c}{2}\right)\frac{\dot{H}^2}{H^4}+c\frac{\ddot{H}}{H^3}+\mathcal{O}\left(\frac{\dot{H}^3}{H^6}\right)\right]\frac{\langle\delta\phi^2\rangle}{M_{pl}^2}$$

and it is easy to see that the effective equation of state to the first non trivial order is given by

$$w_{eff} = \frac{p_{eff}}{\rho_{eff}} = -1-\frac{2}{3}\frac{\dot{H}}{H^2}+\left[(c-c^2)\frac{\dot{H}^2}{H^4}-\frac{2}{3}c\frac{\ddot{H}}{H^3}+\mathcal{O}\left(\frac{\dot{H}^3}{H^6}\right)\right]\frac{\langle\delta\phi^2\rangle}{M_{pl}^2},$$

BR on the effective equation of state

For the isotropic observers and a $m^2\phi^2$ chaotic model $c = 3/5$ and we obtain the following result ($\ddot{H}/H^3$ is, for this case, of third order in the slow-roll parameter ϵ):

$$w_{eff} = \frac{p_{eff}}{\rho_{eff}} = -1 + \frac{2}{3}\epsilon + \left[-\frac{24}{25}\epsilon^2 + \mathcal{O}\left(\epsilon^3\right)\right] \frac{\langle \delta\phi^2 \rangle}{M_{pl}^2} .$$

The correction to w_{eff} goes in the direction of a more de Sitter like equation of state.

Summarizing the result, these isotropic non free-falling observers see a slightly smaller expansion rate, more de Sitter like.

On the other hand, for the geodesic observers $c = 0$ and there is no leading BR on the effective equation of state.

Is Quantum BR important?

For a massive chaotic model in the LW limit and $H_i = H(t_i) \gg H$, one has

$$\frac{\langle \delta\phi^2 \rangle}{M_{pl}^2} \simeq -\frac{1}{24\pi^2}\frac{H_i^6}{M_{pl}^2 H^2 \dot{H}} \sim \frac{H_i^4}{H^2 M_{pl}^2} \ln a$$

If the coefficient of $\langle \delta\phi^2 \rangle$ is not zero (as for the isotropic observers), quantum backreaction appears with a secular term related to the infrared growth of inflaton fluctuations. On the other hand such a growth gives a negligible effect whenever $\frac{\langle \delta\phi^2 \rangle}{M_{pl}^2} \ll \epsilon^{-1}$.

In general non negligible effects could appear at the end of inflation ($H \sim m$) only for $H(t_i) \sim (m^2 M_{pl})^{1/3}$. Such values give a typical number of e-folds of the order of $\mathcal{O}(10^4)$, for $M_{pl} = 10^5 m$, and correspond to the case where non-linear corrections become really important (Finelli, G.M., Starobinsky, Vacca, Venturi (2009), Finelli, G.M., Vacca, Venturi (2006)).

Open Problems

- Quantum backreaction in multi fields inflationary models where a more ”physical” definition of the observer is possible.
- Quantum backreaction in a growing-curvature model. Impact of the backreaction on the possible graceful exit from the model.
- Quantum backreaction at/after the end of inflation (preheating/reheating, relativistic era, etc.).
- Possible application of the light-cone averaging formalism (see Nugier talk) to the early Universe.

Conclusions

- We have proposed a general-covariant and gauge invariant formulation of the so-called “cosmological backreation”
- We have applied our gauge-invariant observer-dependent approach to the evaluation of backreaction effects induced by long wavelength scalar fluctuations generated by an inflationary era in the early universe.
- Different observables, non local but gauge invariant, and the associated measurement can probe for some of them backreaction effects and for others no backreaction at all.
- Not Wrong! The observables are observers dependent!! Different observers $\Longleftrightarrow$ different features of the Universe dynamics.

THANKS FOR THE ATTENTION!