Implications of genuine gauge invariance & inflationary vacua

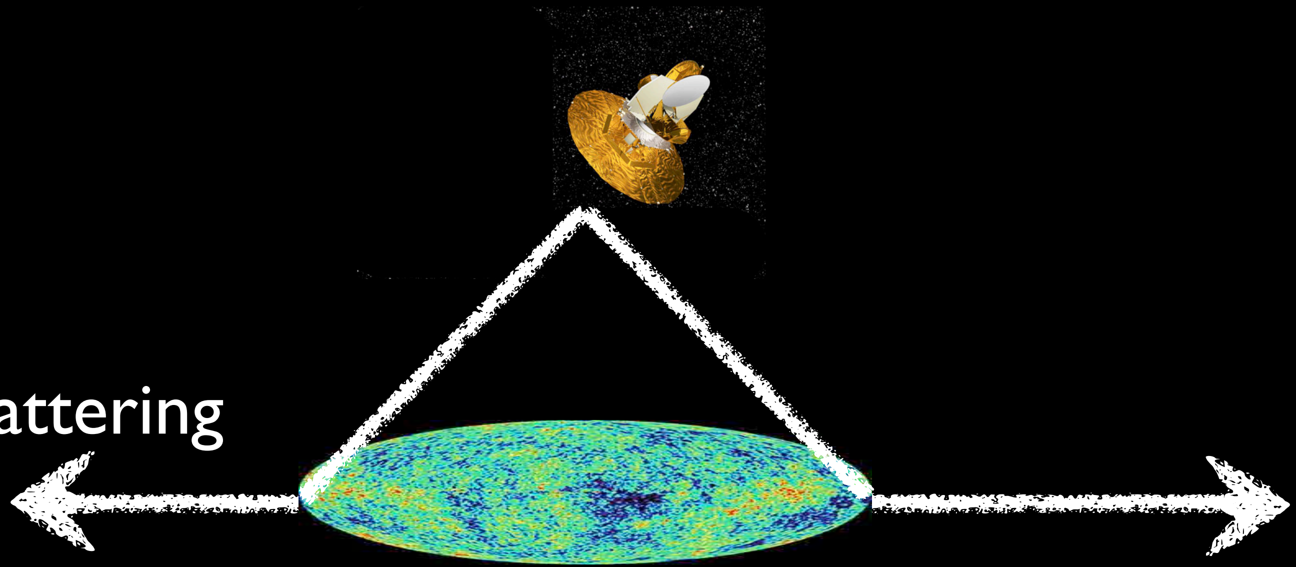
Yuko Urakawa (University of Barcelona)

Y.U. & T. Tanaka 1007.0468[hep-th], 1009.2947[hep-th], 1103.1251[astro-ph]

Y.U. 1105.1078[hep-th]

Y.U. & T. Tanaka 111.****[hep-th]*

Initial conditions of the universe



at Last scattering

- ✔ Gaussian distribution
- ✔ Scale invariant
- ✔ Adiabatic

$$\Delta^2(k) = \Delta^2(k_0)\left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$\Delta^2(k_0) = (2.445 \pm 0.096) \times 10^{-9}$$

$$n_s = 0.960 \pm 0.013$$

Initial conditions, models of early universe should yield

$\longrightarrow$ INFLATION!!

Cosmological fluctuations we observe

⇕

Gauge-invariant perturbations

$\left\{ \begin{array}{l} \text{at universe wt infinite vol.} \\ \text{at universe wt finite vol.} \end{array} \right.$

Gauge invariance

= Invariance under gauge-transformations

- Types of gauge transformations

$$x^\mu \rightarrow x^\mu + \delta x^\mu$$

- Whole universe with infinite (3dim-)vol.

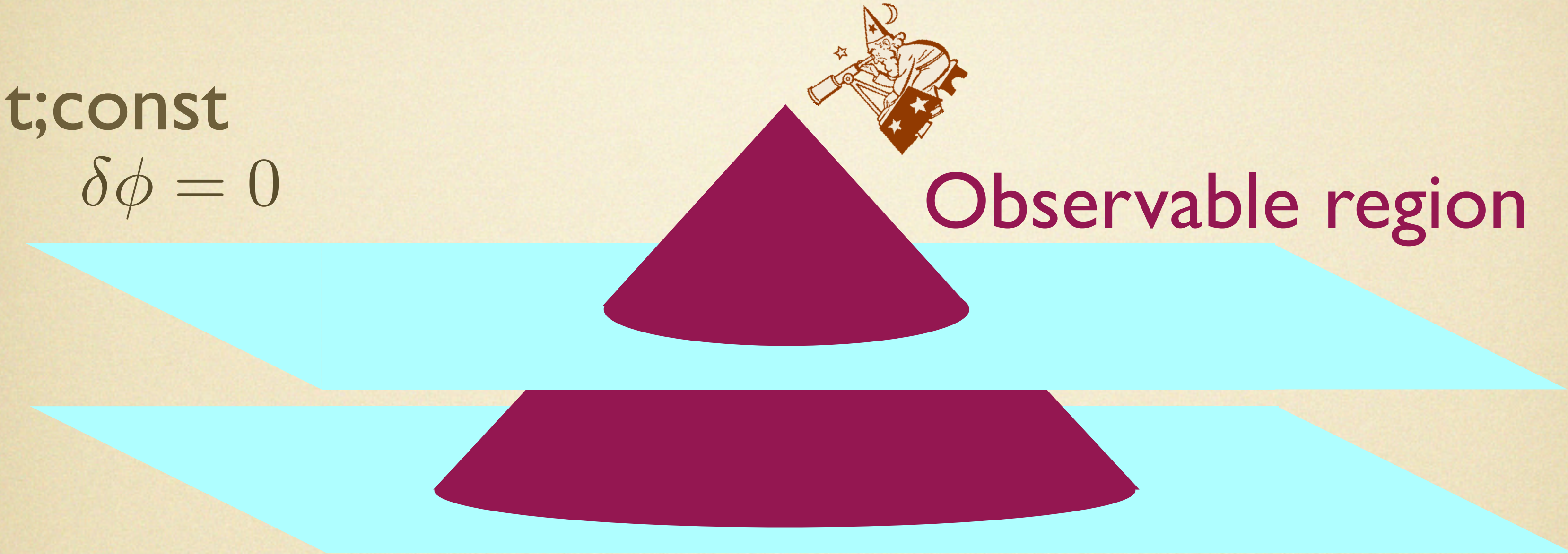
$|\delta x^\mu| \ll 1$ at whole universe

- A portion of universe with finite (3dim-)vol.

$|\delta x^\mu| \ll 1$ at the portion (→ local universe)

No restrictions on outside the local universe

Gauge inv.(=fixing) in local universe



- Choices of spatial coordinates

gauge cond. on spatial metric $\quad h^i{}_i = h^i{}_{j,i} = 0$

$$\delta_g h_{ij} = \xi_{i,j} + \xi_{j,i} \qquad \Delta\xi_i = \cdots$$

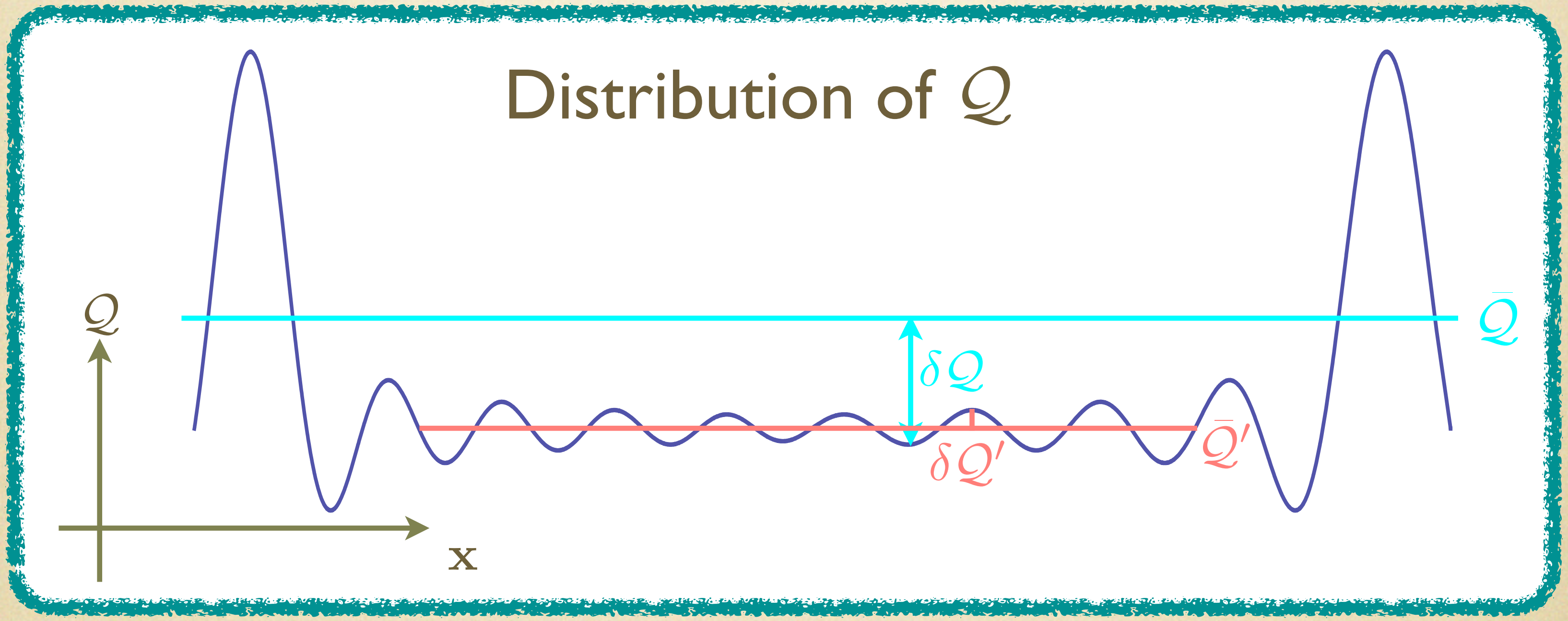
DOFs in boundary cond. ⟷ Residual gauges

* in whole universe   Boundary cond. is trivial

Residual gauge modes

Def. fluctuation $\delta\mathcal{Q} := \mathcal{Q} - \bar{\mathcal{Q}}$ $\mathcal{Q} = \zeta,\, \delta\gamma_{ij}$

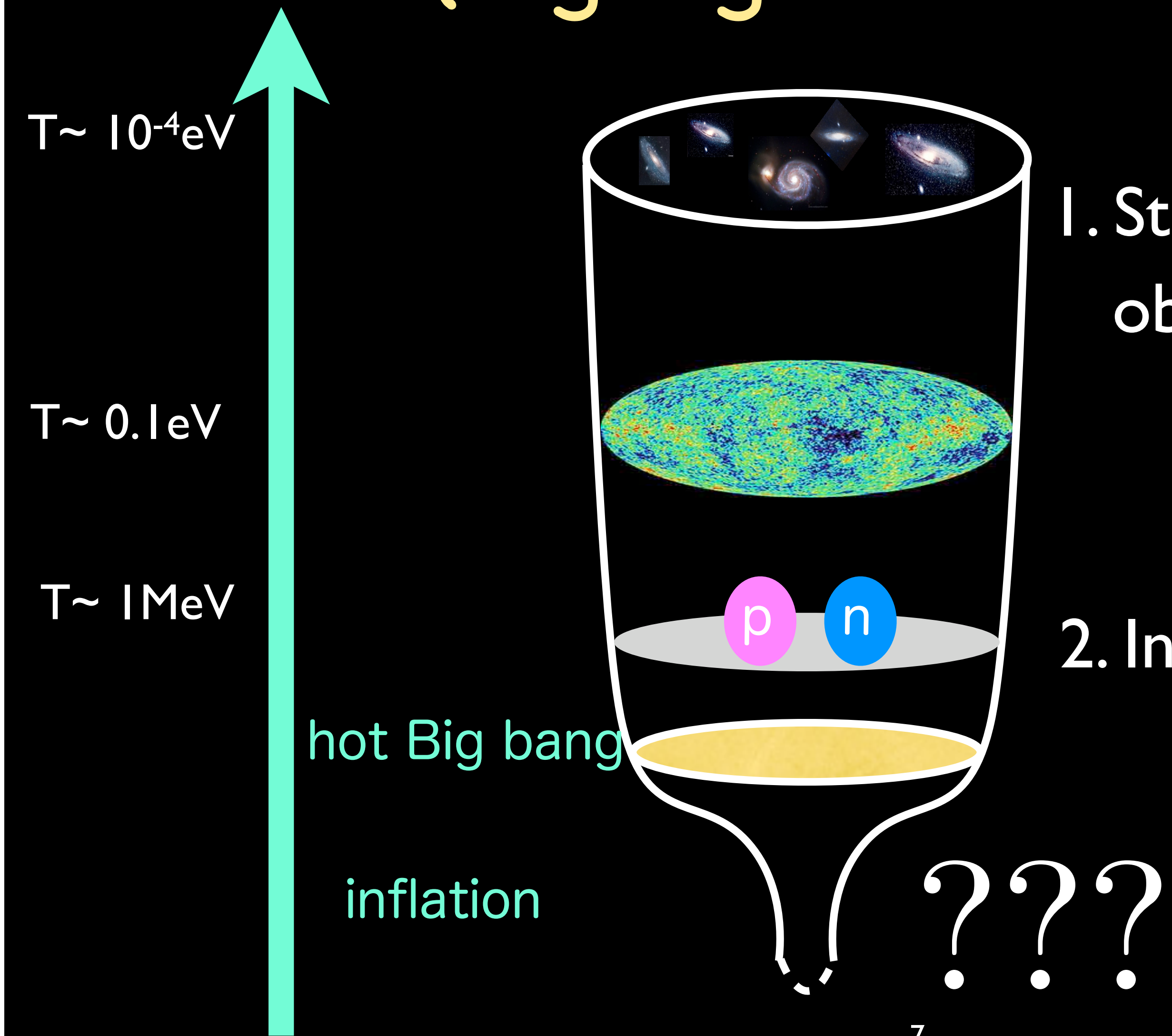
Averaged value $\bar{\mathcal{Q}} := \int d^3\mathbf{x}\, \mathcal{Q} \,/\, \int d^3\mathbf{x}$



Gauge transformation : $\bar{\mathcal{Q}} \rightarrow \bar{\mathcal{Q}}'$ Change averages!

Homogeneous modes of ζ & $\delta\gamma_{ij}$ are pure gauges

Let's discuss genuine gauge inv. !!
(= gauge inv. in local universe)



1. Study of observable fluctuations

2. Initial state of universe

Aspects of genuine gauge invariance

1. Genuine gauge-invariant perturbations & IR divergence problem

Y.U.&T.Tanaka (09, 10[1], 10[2])

2. Primordial non-Gaussianity

T.Tanaka&Y.U. (11), Y.U. (11)

3. Genuine gauge-inv. and initial states

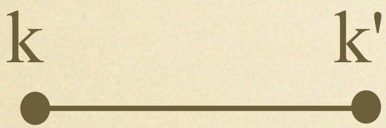
T. Tanaka&Y.U. (in preparation)

Infrared(IR) divergence

- Two point function $\langle \zeta_k \zeta_{k'} \rangle$ $\mathcal{L}_{\rm int} \propto \zeta^4$

$\mathcal{L}_{\rm int} \propto \zeta^4$ ζ : mass-less field

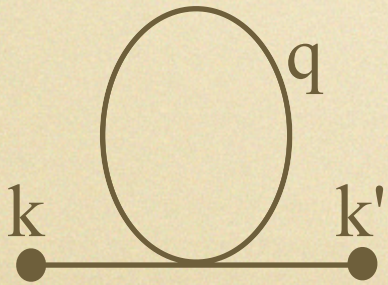
■ Leading order



$\langle \zeta_k \, \zeta_{k'} \rangle = |\zeta_k|^2 \propto k^{-3}$

Scale-invariant

■ Next to leading order



Momentum (Loop)integral

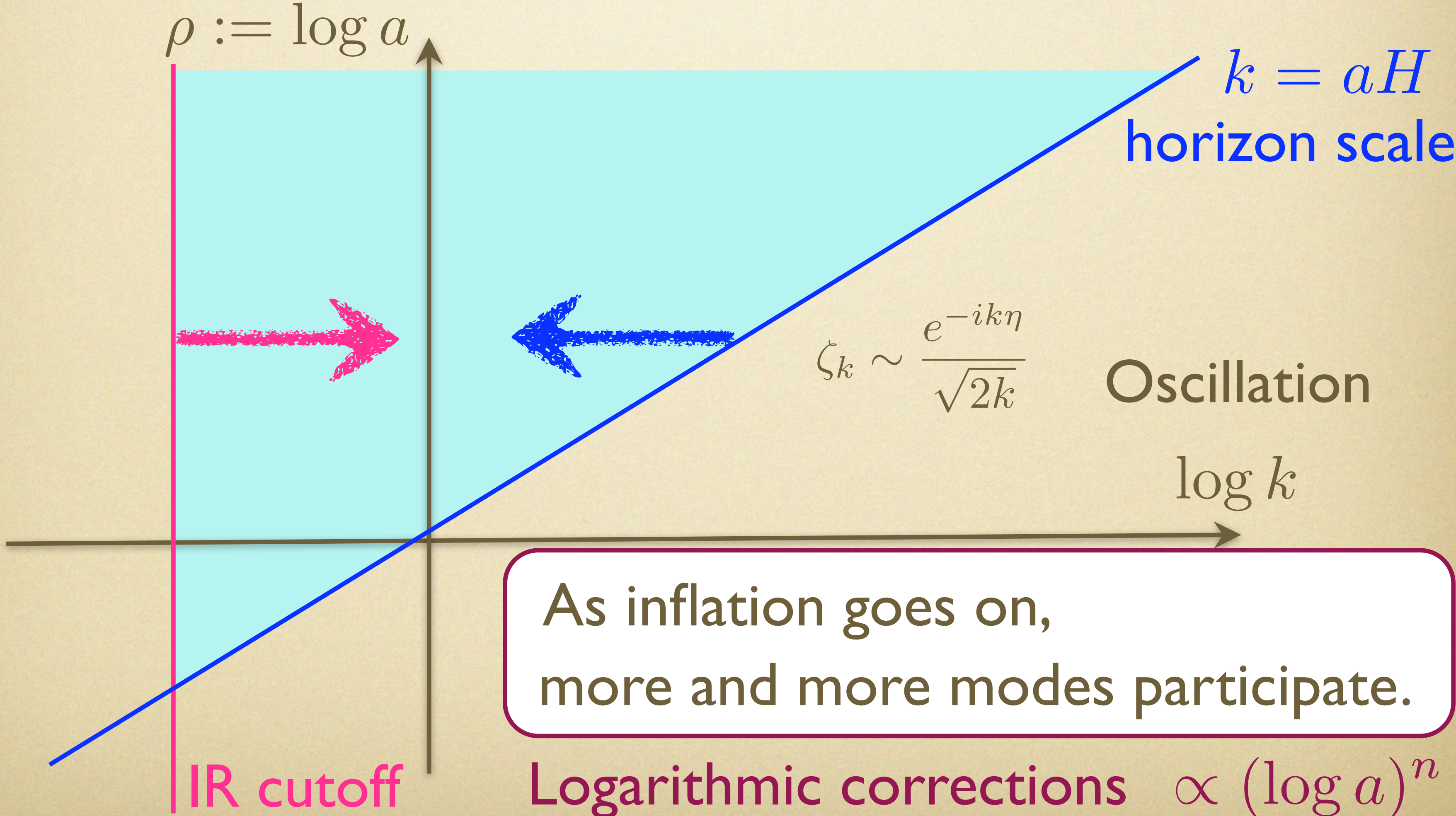
$$\int \mathrm{d}^3 q \, |\zeta_q|^2 = \int \mathrm{d}^3 q / q^3$$

Logarithmic divergence

Introduction of IR cutoff

$$\langle \zeta\zeta\cdots\rangle = \prod_\alpha \int \mathrm{d}(\log a_\alpha)\mathrm{d}^3\mathbf{k}_\alpha \cdots$$

Which modes participate in loop corrections?

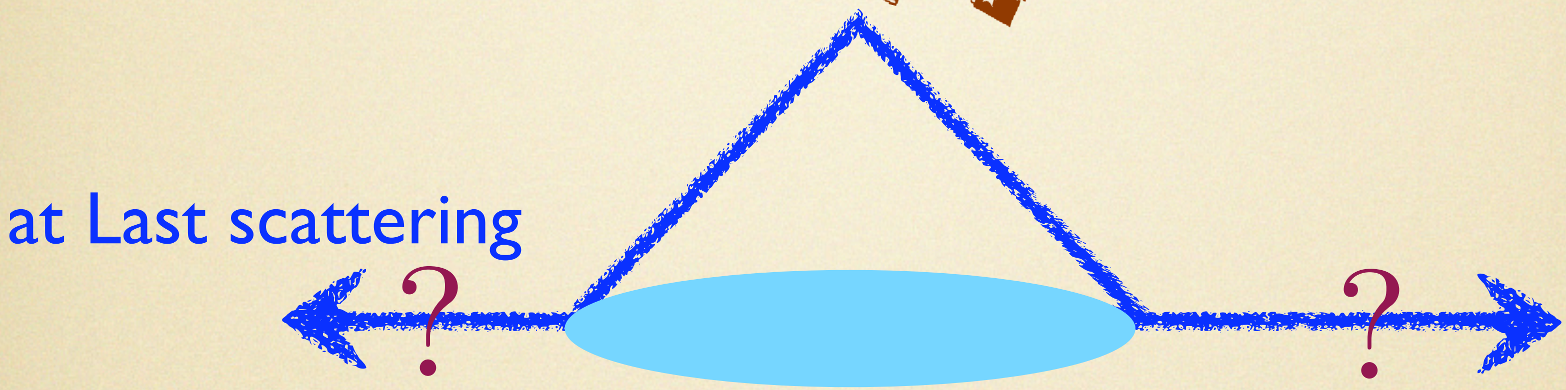


As inflation goes on,
more and more modes participate.

Logarithmic corrections $\propto (\log a)^n$

Origin of IR divergence

Y.U.&T.Tanaka (09, 10)



Fix gauge conditions in the local region

→ Fix average values in local universe

IR divergences no longer appear

For single field, genuine gauge-invariant variables

⟶ NO IR divergence!!

Gauge-inv. operator

Gauge invariance regarding $x^i \to \tilde{x}^i = x^i + \delta x^i$

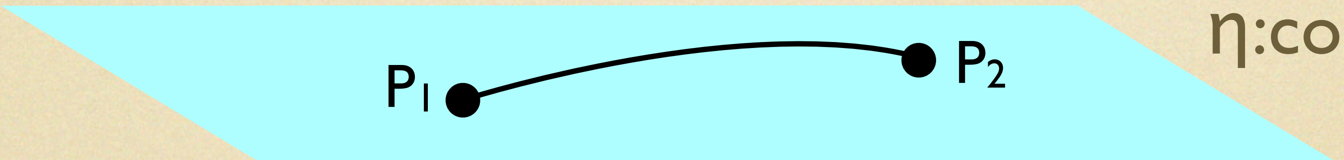
● Geodesic normal coordinate   Y.U.&T.Tanaka (10)

Scalar quantity, labeled by the gauge-invariant argument

→ Gauge-invariant

■ 3D scalar curvature ${}^sR$ Byrnes et al. (10)

Two-point function on t:const surface   Giddings&Sloth (10)



η:const

$\delta\phi = 0$

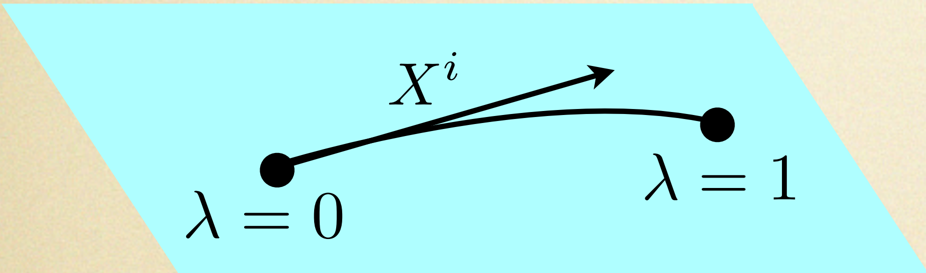
$\langle {}^sR(P_1) {}^sR(P_2) \rangle \longrightarrow \langle {}^sR {}^sR \rangle (l)$

l: Geodesic distance between P_1 and P_2

Geodesic normal coordinate

$$\begin{cases} x^i & \text{: Global coordinates} \\ X^i & \text{: Geodesic normal coordinates} \end{cases}$$

- 3D geodesics



η:const

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}\lambda^2} + {}^s\Gamma^i{}_{jk} \frac{\mathrm{d}x^j}{\mathrm{d}\lambda}\frac{\mathrm{d}x^k}{\mathrm{d}\lambda} = 0$$

$$X^i = \left.\frac{\mathrm{d}x^i}{\mathrm{d}\lambda}\right|_{\lambda=0}$$

spatial metric $\quad \mathrm{d}l^2 = e^{2\zeta}\left[\delta_{ij} + \delta\gamma_{ij}\right]\mathrm{d}x^i \mathrm{d}x^j$

large scale limit →

$$x^i(X) \simeq e^{-\zeta}\left[e^{-\delta\gamma/2}\right]^i{}_j X^j$$

$$\delta x^i := x^i - X^i \simeq \left(e^{-\zeta}\left[e^{-\delta\gamma/2}\right]^i{}_j - \delta^i_j\right) X^j$$

Properties of ${}^{g}R$

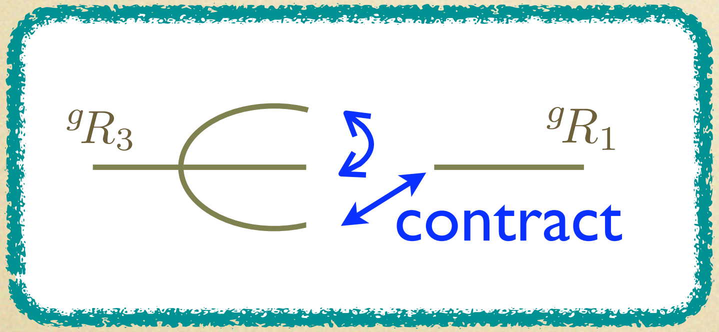
$$ {}^{g}R(\eta,\, X^i) := {}^{s}R(\eta,\, x^i = X^i + \delta x^i(X)) $$
$$ = {}^{s}R(\eta,\, X^i) + \delta x^i \partial_i {}^{s}R(\eta,\, X^i) + \cdots $$

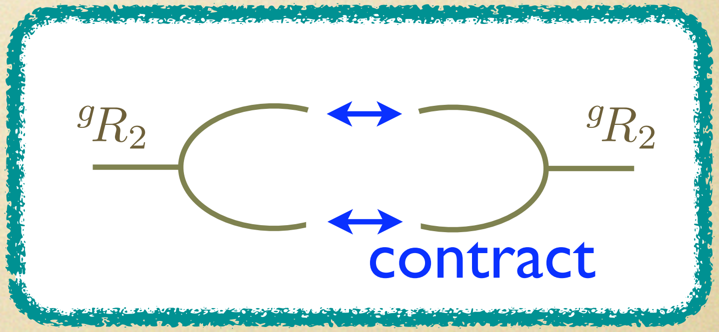
Cancel gauge-dependence

- Perturbation ${}^{g}R = {}^{g}R_1 + {}^{g}R_2 + \cdots$

$$ \langle\, {}^{g}R\, {}^{g}R \,\rangle = \langle\, {}^{g}R_1\, {}^{g}R_1 \,\rangle $$ tree

$$ + \langle\, {}^{g}R_1\, {}^{g}R_3 \,\rangle + \langle\, {}^{g}R_2\, {}^{g}R_2 \,\rangle + \langle\, {}^{g}R_3\, {}^{g}R_1 \,\rangle $$ 1-loop





$$ {}^{g}R_3 \ni \zeta^2 \partial^2 \zeta \longrightarrow \langle \zeta^2 \rangle = \infty \qquad {}^{g}R_2 \ni \zeta \partial^2 \zeta \longrightarrow \langle \zeta^2 \rangle = \infty $$

Gauge-inv. initial state

Y.U.&T.Tanaka(10)

■ Necessary cond. for gauge invariance

$$\langle {}^g\!R\, {}^g\!R \rangle \simeq \langle \zeta^2 \rangle \mathcal{F}_{\rm div}[\zeta] \ + \text{(Regular terms)}$$

Choose initial conditions

$$\int d(\log k)\partial_{\log k}(\ldots)$$ Total derivatives

Fourier mode of $\zeta_{\rm liner}$

ρ:e-folding

$$(1+\varepsilon)\,\partial_\rho \zeta_k - x^i \partial_i \zeta_k + \varepsilon \zeta_k + \ldots = -\left(\partial_{\log k} + 3/2\right)\zeta_k$$

with assumption; "usual" UV behavior

→ Determined to adiabatic vacuum wt scale invariance

Aspects of genuine gauge invariance

1. Genuine gauge-invariant perturbations
   & IR divergence problem

Y.U.&T.Tanaka (09, 10[1], 10[2])

2. Primordial non-Gaussianity
   - → 2. 1 Single-field models — T.Tanaka&Y.U. (11), Y.U. (11)
   - 2. 2 Multi-field models

3. Genuine gauge-inv. and initial states

T. Tanaka&Y.U. (in preparation)

Consistency relation

Maldacena (02), Creminell & Zaldarriaga (04)

$$ {}^{g}\zeta_{\mathbf{k}}(\rho) = \int \frac{\mathrm{d}^3\mathbf{x}}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}}\, {}^{g}\zeta(\rho,\, \mathbf{x}) $$

$$ \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \underset{k_1 \to 0}{\to} -(2\pi)^3 \delta^{(3)}\left(\Sigma \mathbf{k}_i\right) (n_s - 1) P_{k_1} P_{k_2} $$

In general single field models of inflation,

bi-spectrum in $k \to 0$ is related with power-spectrum.

- Revisit of consistency relation
  - Gauge-invariant operator $\quad$ cf. ${}^{g}R_2 \simeq e^{-2\,{}^{g}\zeta_2} \partial^2\, {}^{g}\zeta_2$

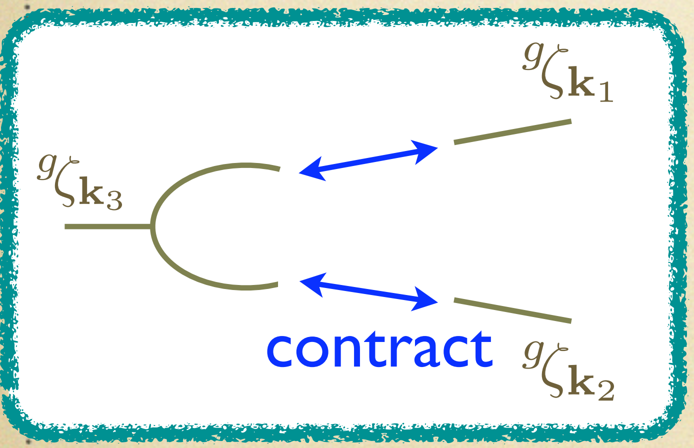
    $$ {}^{g}\zeta(\eta, X^i) = \zeta(\eta, x^i(X^i)) $$

  - Gauge-invariant (vacuum) state

Non-Gaussianity

$$ {}^{g}F(\mathbf{k}_1,\,\mathbf{k}_2,\,\mathbf{k}_3) = \langle\, {}^{g}\zeta_{(\mathbf{k}_1}\, {}^{g}\zeta_{\mathbf{k}_2}\, {}^{g}\zeta_{\mathbf{k}_3)} \rangle $$

$$ {}^{g}\zeta_{\mathbf{k}}(\rho) = \int \frac{\mathrm{d}^3\mathbf{X}}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{X}}\, {}^{g}\zeta(\rho,\,\mathbf{X}) $$



Squeezed limit $k_1 \ll k_2\,, k_3$

T. Tanaka&Y.U.(11)

$$ {}^{g}F(\mathbf{k}_1,\,\mathbf{k}_2,\,\mathbf{k}_3) $$

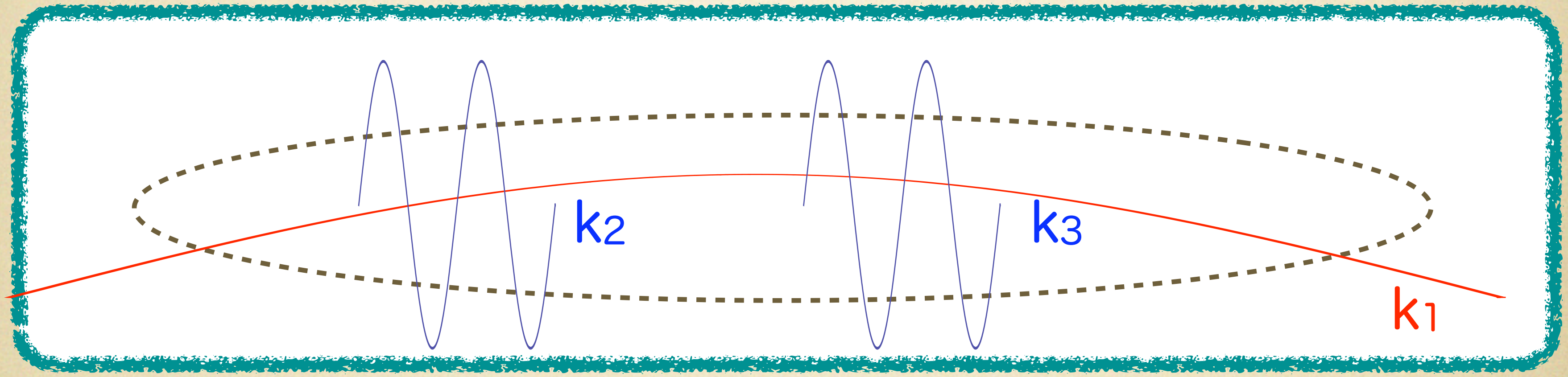
$$ \simeq P(k_1)P(k_2) \times \mathcal{O}\left(\frac{k_1}{k_2}, \frac{k_1}{aH}, \cdots\right) $$

Dominance of gauge artifact

T.Tanaka & Y.U. (11)

- Local non-Gaussianity

$$\langle {}^g\zeta_{\mathbf{k}_1} {}^g\zeta_{\mathbf{k}_2} {}^g\zeta_{\mathbf{k}_3} \rangle \qquad k_1/k_2, k_1/k_3 \ll 1$$



k_1 modifies the background for k_2 & k_3

↑

Gauged away by using unbounded gauge trans.

Applicability

- Super-horizon approximation $k \ll aH$

- Smaller than the observable scale $1/L_{\rm obs} \ll k$

$${}^{g}R_2 \simeq e^{-2\,{}^{g}\zeta_2}\partial^2\,{}^{g}\zeta_2$$

Fourier transformation $\partial^2 \rightarrow k^2$

c.f. during inflation $1/L_{\rm obs} \ll k \ll aH$

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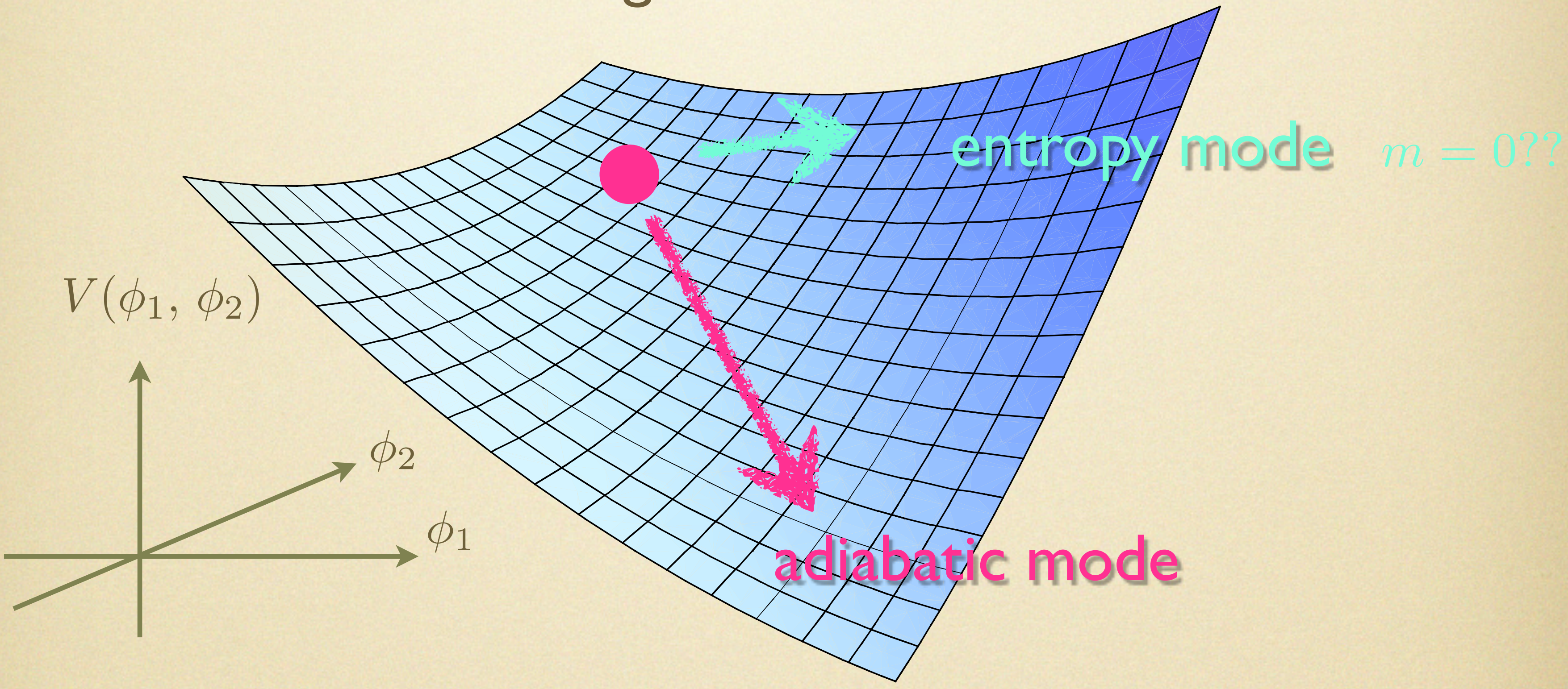
T. Tanaka&Y.U. (in preparation)

Multi-filed models

Inflation with multi light scalar fields



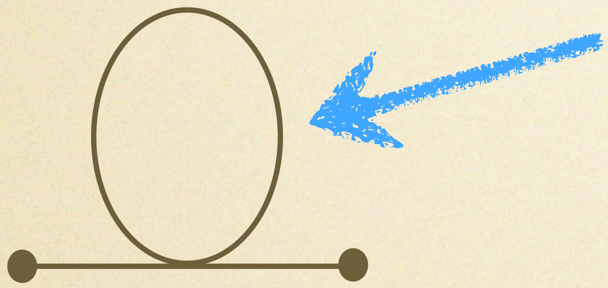
- Entropy mode

Intrinsically gauge independent

(If exist) IR div. for entropy mode $\neq$ Gauge artifact

Gauge-invariance conditions

- Potentially divergent terms in $\langle\, {}^g\!R\, {}^g\!R\, \rangle$ Y.U.(11)



$\langle \zeta^2 \rangle (,\ \langle \zeta \mathcal{S} \rangle),\ \langle \mathcal{S}^2 \rangle$

Regularized

- Gauge invariance condition

$$(1+\varepsilon)\,\partial_\rho \zeta_k - x^i \partial_i \zeta_k + \varepsilon \zeta_k + \ldots = -\left(\partial_{\log k} + 3/2\right) \zeta_k$$

Almost determined to adiabatic vacuum

Non-Gaussianity

Y.U. (11)

in the squeezed limit $k_1/k_2, k_1/k_3 \to 0$

$$\langle {}^g\zeta_{\mathbf{k}_1} {}^g\zeta_{\mathbf{k}_2} {}^g\zeta_{\mathbf{k}_3} \rangle \propto |\zeta_{k_2}|^2 \, \mathrm{Re}\left[\zeta_{k_1} \partial_\eta \zeta^*_{k_1}\right]$$

EOM for ζ $\quad \zeta'' + 2\frac{z'}{z}\zeta' - \partial^2\zeta \propto \theta' \mathcal{S} + \cdots$

θ : local rotation angle

¡Remark!

If trajectory is curved $\quad i.e. \ \theta' \neq 0$

$\Longrightarrow$ $\partial_\eta \zeta_{k_1} \not\to 0\,, \ \langle {}^g\zeta_{\mathbf{k}_1} {}^g\zeta_{\mathbf{k}_2} {}^g\zeta_{\mathbf{k}_3} \rangle \not\to 0$

Time variation in ζ → Non-vanishing contributions

Cannot be gauged away

Aspects of genuine gauge invariance

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→ 3. Genuine gauge-inv. and initial states

T. Tanaka&Y.U. (in preparation)

"Does genuine gauge-invariance(= IR stability) restrict initial states of universe?"

Yes!

Y.U.&T.Tanaka (10,11)

We need to take "scale-invariant" vacuum.

(Maybe) No!

IR stability can be ensured for arbitrary vacua.

Giddings&Sloth (10)

Chialva&Mazumdar (11)

.....

Revisit of IR stability

- Non-linear EOM (= Heisenberg eq.)

$$\mathcal{L}\zeta = \mathcal{S}_{\rm NL}[\zeta] := -2\psi\partial^2\psi + \cdots$$

$$\mathcal{L} := \partial_\eta^2 + 2(\log z)'\partial_\eta - \partial^2 \qquad \psi := \zeta_{\rm liner}$$

- Boundary conditions of $\mathcal{L}^{-1}$
- Spectrum of ψ

$$\langle\, {}^gR\, {}^gR\,\rangle \simeq \langle\psi^2\rangle \mathcal{F}_{\rm div}[\zeta] + \text{(Regular terms)}$$

Solution A $\quad \zeta_A = \mathcal{L}_A^{-1}\mathcal{S}_{\rm NL}$

$\mathcal{F}_{\rm div}[\zeta_A]$: total derivative for ψ : adiabatic vac.

Solution B $\quad \zeta_B = \mathcal{L}_B^{-1}\mathcal{S}_{\rm NL}$

$\zeta_B = \mathcal{L}_B^{-1}\mathcal{S}_{\rm NL} \qquad \mathcal{F}_{\rm div}[\zeta_B] = 0$ for arbitrary ψ

Revisit of IR stability 2

Solution A

$$\zeta_A = \psi + \psi \partial_\rho \psi + \cdots$$

Y.U.&T.Tanaka (10[1], 10[2])

$$\zeta_A(\eta) \supset U(\eta)\psi(\eta_i)U^\dagger(\eta) = \psi(\eta) + \int d\eta'\, G_R(\eta, \eta') \cdots$$

- Consistent with CTP/ Sol. wt retarded Green fn.
- Initial condition can be given by $i\epsilon$ prescription

Solution B

special choice $\mathcal{L}_B^{-1}$

$$\zeta_B = \psi + \psi X^i \partial_i \psi + \cdots$$

$$\zeta_B(\eta) \not\supset U(\eta)\psi(\eta_i)U^\dagger(\eta) = \psi(\eta) + \int d\eta'\, G_R(\eta, \eta') \cdots$$

Subtleties
- Time evolution is unitary??
- Commutation relation is satisfied??
- Solution wt $X^i \partial_i$ is acceptable??

How robust?

Formal conditions on $\mathcal{L}^{-1}\mathcal{S}_{\rm NL}$ and ψ

T. Tanaka&Y.U. (in preparation)

if these conditions are satisfied,

- IR stability
- Suppression of local Non-Gaussianity

for general single field models of inflation

(Ex.)
- k-inflation

  Explicit conditions Y. Misonoh&Y.U. (in preparation)

- Break down of slow-roll conditions

* Similar story for graviton loops

Summary

- Observable fluctuations should be genuinely gauge invariant.

- Implications of genuine gauge invariance

  1. No IR divergence in single field models
  2. Consistency relation for bi-spectrum is dominated by gauge modes.
  3. Time variation of ζ can generate observable fluc.

- Genuine gauge-invariance almost determines

  initial states to the adiabatic vacuum

  ＊Still cannot exclude one particular exception


Merci Beaucoup!
...come & enjoy Barcelona

Supplement

As an example...

Single field inflation driven by ϕ

- Complete gauge fixing in whole universe

$$\delta\phi = 0 \qquad h_{ij} = a^2(\eta)e^{2\zeta}[e^{\delta\gamma}]_{ij}$$

Maldacena (2002)

$$\delta\gamma_{ii} = 0 \quad \partial_j\delta\gamma_{ij} = 0$$

- in local universe

(i) Time slicing fixed

(ii) Spatial coordinates not fixed

Residual gauge DOFs $x^i \to x^i + \delta x^i$

$$\delta x^i \simeq C^i{}_{i_1 \ldots i_n}(\eta) x^{i_1} \cdots x^{i_n}$$

Symmetric traceless

Un-bounded at spatial infinity

Residual gauge modes

Single field inflation

$$S_\phi = -\frac{1}{2}\int \sqrt{-g}\,[g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2V(\phi)]\mathrm{d}^4x$$

- General solutions of $\delta N,\ \check{N}_i = e^{-\rho}N_i$

From Hamiltonian&Momentum constraints at 1st order

$$\delta N_1(x) = \frac{1}{\rho'}\left(\zeta_1'(x) \; -\frac{1}{4}\partial^i G_{i,1}(x)\right) \qquad \partial^2 G_{i,1}(x) = 0$$

$$\check{N}_{i,1}(x) = \partial_i\left(\frac{\phi'^{\,2}}{2\rho'^{\,2}}\partial^{-2}\zeta_1'(x) - \frac{1}{\rho'}\zeta_1(x)\right)$$

$$-\frac{1}{4}\left(1+\frac{\phi'^{\,2}}{2\rho'^{\,2}}\right)\partial_i\partial^{-2}\partial^j G_{j,1}(x) + G_{i,1}(x)$$

DOFs in $\delta N \,\&\, N_i$ → Residual gauge DOFs

Residual gauge modes 2

$$\left(\delta N,\, \check{N}_i\right) \text{ for } G_i = 0 \longrightarrow \left(\delta \tilde{N},\, \tilde{\check{N}}_i\right) \text{ for } G_i \neq 0$$

- Gauge transformation: $(t,\, x^i) \to (t + \delta t,\, x^i + \delta x^i)$
  - Time coordinate Fixed by $\delta\phi = 0$
  - Spatial coordinates $\gamma^{ij}\delta\gamma_{ij} = 0$ $\partial^i \delta\gamma_{ij} = 0$

$^\exists$Residual gauge modes

$$\delta x_i = -\int \mathrm{d}\eta\, G_i + \frac{1}{4}\int \mathrm{d}\eta \partial_i \partial^{-2}\partial^j G_j + \frac{1}{4}\int \frac{\mathrm{d}\eta}{\rho'}\int \mathrm{d}\eta \partial_i \partial^j G_j + \cdots$$

(i) Scale transformation

$$x^i \to e^{f(\eta)} x^i \qquad \zeta(x) \to \tilde{\zeta}(x) = \zeta(x) - f(\eta) + \cdots$$

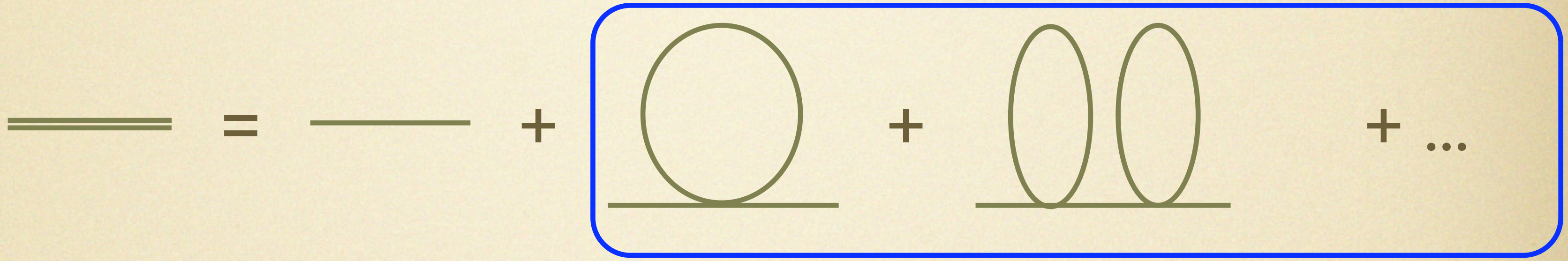
(ii) Shear transformation

$$x^i \to x^i + C^i{}_j(\eta) x^j \qquad C^i{}_i = 0,\, C_{ij} = C_{ji}$$

$$\delta\gamma_{ij}(x) \to \delta\tilde{\gamma}_{ij}(x) = \delta\gamma_{ij}(x) - 2C_{ij}(\eta) + \cdots$$

Other IR issues

- Re-summation can cure IR singularity? C. Burges et al. (09,10)



Generate effective mass

Singular behavior in mass-less limit

→ Break-down of perturbation theory?

- Effects of decoherence can cure IR singularity?

Y.U.&T.Tanaka (09)

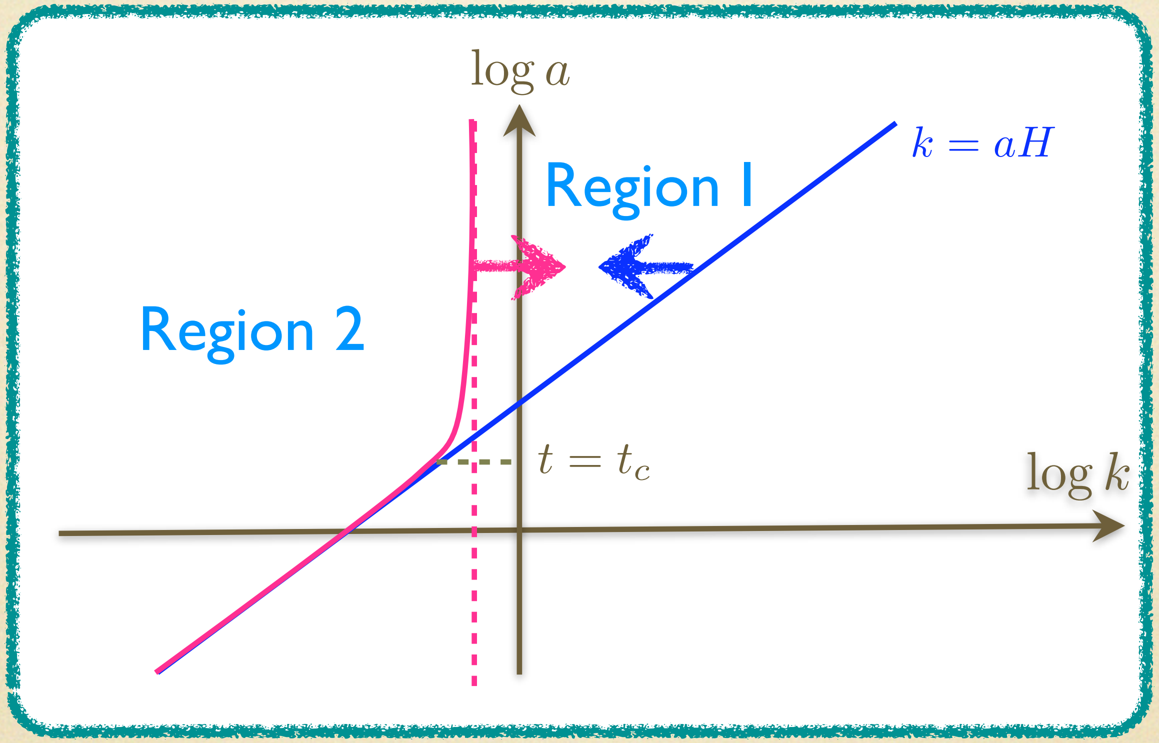
Momentum integral can be regularized. Time integral?

- Stability of de Sitter spacetime

Polyakov (07, 09), Marolf and Morrison (09,10)

Analytic continuation from Euclidean S^5

More about secular growth



ζ_n { Contributions from region I $\left[\frac{H(t)}{M_{\rm pl}\varepsilon^{1/2}}\right]^n$

Contributions from region II $\{a_i H_i L(t)\}\left[\frac{H_i}{M_{\rm pl}\varepsilon^{1/2}}\right]^n$

For $n > n_c$, suppression is not enough to eliminate the contributions from the distant past.

Revisit of consistency relation

Leading terms for $k_1 \ll k_2\,, k_3$

Y.U. & T. Tanaka (11)

$${}^gF(\mathbf{k}_1,\,\mathbf{k}_2,\,\mathbf{k}_3)$$

$$\simeq -(1+\lambda_2)|v_{k_1}|^2(2\pi)^{-3/2}\left\{\frac{1}{2}\mathrm{Re}\left[v_{k_2}v^*_{|k_1+k_3|}+v_{k_3}v^*_{|k_1+k_2|}\right]+|v_{k_2}|^2+|v_{k_3}|^2\right\}\mathbf{k}_1\cdot\partial_{\mathbf{K}}\delta^{(3)}(\mathbf{K})$$

$$+\frac{1}{2}(1+\lambda_2)|v_{k_1}|^2(2\pi)^{-3/2}\mathrm{Re}\left[v_{k_2}v^*_{|k_3+k_1|}-v_{|k_2+k_1|}v_{k_3}\right](\mathbf{k}_2-\mathbf{k}_3)\cdot\partial_K\delta^{(3)}(\mathbf{K})$$

$$-2|v_{k_1}|^2(2\pi)^{-3/2}\delta^{(3)}(\mathbf{K})\sum_{a=2}^{3}\mathrm{Re}\left[v_{k_a}\left\{\mathcal{L}^{-1}_{|k_a+k_1|}\mu_3 v^*_{k_a}-\mathcal{L}^{-1}_{k_a}\mu_3 v^*_{k_a}\right\}\right]$$

$$+\mathcal{O}(\varepsilon^3)$$

→ ${}^gF(\mathbf{k}_1,\,\mathbf{k}_2,\,\mathbf{k}_3)\;\rightarrow\;0$ in the limit $k_1\rightarrow 0$

Gauge-invariance conditions

up to 2nd order in perturbation

$$ {}^{g}R = -4e^{-2\rho}\partial^2\left[\psi + \zeta_2 - \psi X^i\partial_i\psi + \cdots\right] $$

$$ \mathcal{L}\zeta = \mathcal{S}_{\rm NL}[\zeta] := -2\psi\partial^2\psi + \cdots $$

$$ \mathcal{L} := \partial_\eta^2 + 2(\log z)'\partial_\eta - \partial^2 \qquad \psi := \zeta_{\rm liner} $$

pick up div. terms

$$ \zeta_2 \simeq -2\psi\mathcal{L}^{-1}\partial^2\psi $$

$$ {}^{g}R \simeq -4e^{-2\rho}\partial^2\left[\psi - \psi(2\mathcal{L}^{-1}\Delta + X^i\partial_i)\psi\right] $$

Solution A

$$ (2\mathcal{L}^{-1}\Delta + X^i\partial_i)\psi_k = (\partial_{\log k} + 3/2)\psi_k $$

Solution B

$$ (2\mathcal{L}^{-1}\Delta + X^i\partial_i)\psi = 0 $$

Graviton loops

up to 2nd order in perturbation

$$^{g}R \simeq -4e^{-2\rho}\partial^2\Big[\psi - \psi(2\mathcal{L}^{-1} + X^i\partial_i)\psi$$
$$-\frac{1}{2}\delta\gamma^{ij}\psi(2\mathcal{L}^{-1}\nabla_i\nabla_j + X_i\partial_j)\psi\Big]$$

polarization tensor of GW

Solution A

$$e^{ij}(\mathbf{k}')\psi(2\mathcal{L}^{-1}\nabla_i\nabla_j + X_i\partial_j)\psi_k = \mathcal{C}(\partial_{\log k} + 3/2)\psi_k$$

$$\mathcal{C}(\mathbf{k}',\,\mathbf{k}/k) = (k^i/k)e_{ij}(\mathbf{k}')(k^j/k)$$

Solution B

$$e^{ij}(\mathbf{k}')(2\mathcal{L}^{-1}\nabla_i\nabla_j + X_i\partial_j)\psi_k = 0$$