Particle transport in Weibel-type and OTSI-type turbulence

G. Pelletier (IPAG) & M. Lemoine (IAP)
I. Plotnikov (IPAG)

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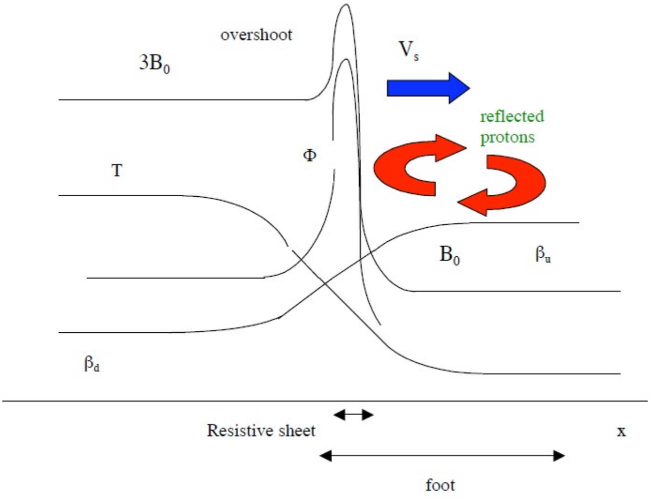
Relativistic shock

In the shock front rest frame.

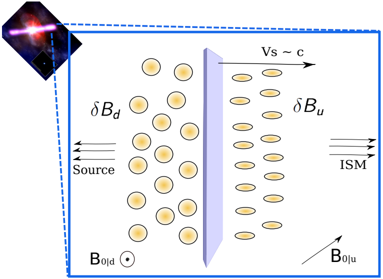


G.Pelletier et al. 2009

Instabilities and self-consistent fields at relativistic shocks

Length scale of the precursor in lab frame $l_{F|u} = m_p c^2/(eB_{0|u}\Gamma_S) \ll R_{L,0}$
$\rightarrow$ small-scale plasma instabilities.

Magnetic fields : generated upstream, transmitted downstream.



Families of relevant instabilities :

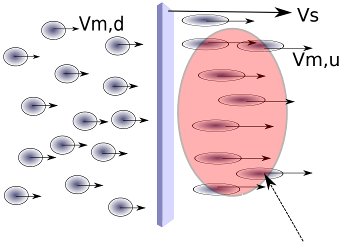
- Weibel-Filamentation : $\vec{k} \perp \vec{v}_b || \vec{E}$
- OTSI : $\vec{k} || \vec{E}$, $\vec{v}_b$ oblique

Generate intense and small-scale electomagnetic fields.

Transport in OTSI

OTSI instability : upstream frame $\rightarrow$ wave frame

Moving wave packets in upstream frame.



OTSI (Lemoine&Pelletier 2011) :

- $\Gamma_{OTSI} = (\xi_{cr} m_e / m_i)^{1/3} \omega_{pe}$.
- Frequency : $\omega \simeq \omega_{pe}$.
- Oblique : $k_\perp \sim k_{||}$ and $E_\perp \sim E_{||}$.
- Caracteristic field energy :

$$\boxed{\epsilon_0 = q\Delta\Phi = \xi_B^{1/2} m_p c^2.}$$

Where $\xi_B = B^2/(4\pi\Gamma_s^2 n m_p c^2)$

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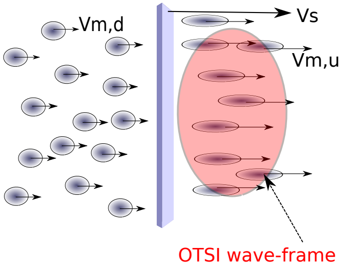

- In the wave frame : static E' and B' fields of the same strength
- And $\vec{k}_w = \vec{k}_{\perp,u} + \vec{k}_{||,u}/\gamma_w$

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Equations of motion in the wave-frame

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = qE'_x(1+\beta_w\beta_z)$$
$$\frac{\mathrm{d}p_y}{\mathrm{d}t} = qE'_y(1+\beta_w\beta_z)$$
$$\frac{\mathrm{d}p_z}{\mathrm{d}t} = qE'_z - q\beta_w(\beta_x E'_x + \beta_y E'_y)$$

Transport in OTSI : $k_z = 0$, invariants

$k_z = 0$ implies $E'_l = 0$.

Then 2 Invariants :

- Electrostatic field :
  $H = \epsilon(p) + q\Phi(x, y)$
- Generlized momentum :
  $\pi_z = p_z + q\Phi(x, y)/c$

Φ : electric potential.

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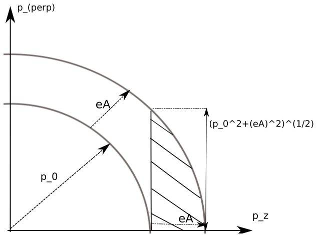
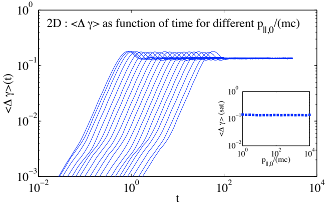
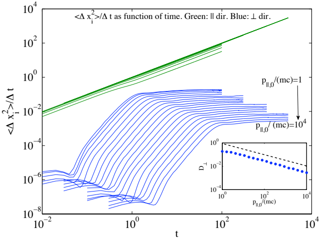
Momentum space constrained by π_z invariant.



Transport in OTSI : $k_z = 0$, simulations

Beam-like configuration : $\vec{p}_0 = p_{||,0}\vec{e}_z$
Simulations : $p_{||,0} \in [mc, 10^4 mc]$.





Diffusion in $\perp$ direction.

$\Delta\gamma$: $H = \epsilon(p_{||,0}) + q\bar{\Phi}$.

Time-scales

- Non-linear ballistic : $t_{nl} = \sqrt{\frac{2p_{||,0}l_c}{qE_t c}}$.
- Linear coherence time : $t_c = l_c/c$ (not-seen).

2 types of particles :

- Thermal particles : $\epsilon \ll \epsilon_0$. Considerable enegy gain.
- Beam particles : $\epsilon \gg \epsilon_0$. Negligible influence by field.

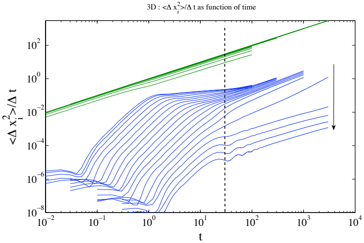
Transport in OTSI : $k_z \neq 0$, short term

$k_z = k_\perp/\gamma_w$. $E'_l = E'_t/\gamma_w$.
Simulations : $p_{||,0} \in [mc, 10^4 mc]$.

New time-scales in z direction :
$t_{c,||} = \gamma_w l_c/c$ and $t_z = \gamma_w \frac{l_c}{c}\left(\frac{p_{||,0}c}{qE_z\gamma_w l_c}\right)^2$

For $t > t_{nl}$ 2D-like behaviour dissapears. Change in transport regime when $t_{nl} > t_{c,||} = \gamma_w l_c/c$. Transverse diffusion dissapears.

Green :$\langle \Delta x_{||}^2 \rangle/\Delta t$, Blue : $\langle \Delta x_\perp^2 \rangle/\Delta t$

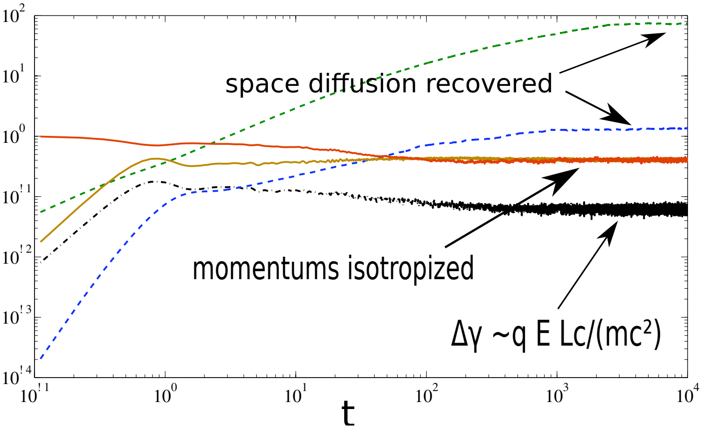


More complicated picture. Long term behaviour $t \gg t_z$?

Transport in OTSI : $k_z \neq 0$, long term behaviour

Time-scale in z direction : $t_z = \xi \frac{l_c}{c} \left(\frac{p_{||,0} c}{q E_z \gamma_w l_c} \right)^2$, with $\xi = l_{\parallel}/l_{\perp}$.

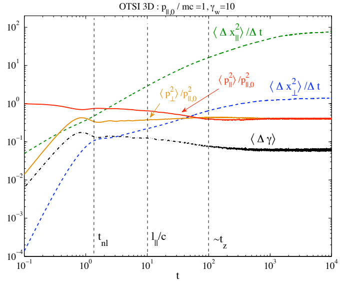
Particle with : $p_{||,0} = mc$.



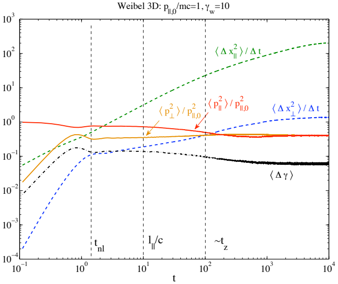
Weibel type vs. OTSI type turbulence

Same set of equtions in 2D limit, because ...
In 3D similar behavior. The only difference is the dispersion of Weibel modes in z direction.

OTSI :



Weibel :



Conclusions

1. OTSI upstream : energy transfer from rms energy of waves to particles. Electron preheating up to $T_e = \xi_b m_p c^2$.
2. Weibel upstream ($k_z = 0$) : similar behaviour as OTSI in $k_z = 0$ limit.
3. Tridimensionnalisation time is too long to be seen in PIC simulations.
4. Issues at relativistic shocks (e.g. Guy Pelletier)