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CONSTRUCTING THE UNIVERSE WITH CLUSTERS OF GALAXIES

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Clusters of galaxies provide one of the most powerful tools in studying the universe. Clusters highlight the large-scale structure of the universe; they trace the evolution of structure with time; they constrain the amount and distribution of dark and baryonic matter in the universe; they reveal important clues to the formation and evolution of galaxies; and they provide critical implications for cosmology. In fact, some of the most powerful constraints placed so far on cosmological parameters, such as the mass density of the universe and the amplitude of mass fluctuations, have been obtained form clusters of galaxies. These yield, as discussed at this conference, $\Omega_m \simeq 0.2 - 0.3$ and $\sigma_8 \simeq 1$. The exciting and productive IAP2000 conference covered all the above topics. Here I summarize some of the topics discussed, with emphasis on "where we are" and "what's next" in the field of "Constructing the Universe with Clusters of Galaxies."

Introduction

Rich clusters of galaxies are the most massive virialized systems known. Even though they contain only a small fraction of all galaxies, rich clusters serve as powerful tools in the study of galaxy formation and evolution, the large scale structure of the universe, the amount and distribution of dark and baryonic matter, and the implications for cosmology. Some of the questions that can be addressed with clusters of galaxies include: How did galaxies and larger structure form and evolve? What is the amount, composition, and distribution of matter in clusters and larger structures? How does the cluster mass density relate to the mass density of the universe? What is the large-scale structure of the universe—superclusters, filaments, and voids, as well as the power spectrum and correlation function of clusters? And, what constraints can the data place on cosmology?

All these topics have been discussed at the conference. New results—from observations, simulations and modeling—have been presented. I summarize some of these results within a more global picture of "where we are" and "what's next?" More details of the different topics are presented by the relevant papers included within this proceedings volume.

I address topics within the following areas:

1. Clusters and Cosmology
2. New Cluster Surveys to $z \sim 1$
3. Evolution of Cluster Abundance
4. The S-Z Effect in Clusters
5. Scaling Relations and Profiles in Clusters
6. Large Scale Structure Traced by Clusters
7. Galaxy Evolution in Clusters
8. Non-Thermal Emission in Clusters
9. What's Next?

Each area includes contributions from multi-wavelength observations (e.g., optical, X-rays, S-Z effect, radio, lensing observations) as well as from theory and simulations. The multi-wavelength effort, combined with state-of-the-art simulations, have proven to be extremely successful in providing an improved understanding of clusters and their utilization in cosmology.

It is impressive to see the success that our field has achieved over the last several decades. The conference began with a beautiful historical perspective on science with clusters of galaxies; this review illustrates the important developments in the field over the last century.[16] At the beginning of the new millennium, when cluster properties are fairly well understood, when we can detect and study clusters at high redshifts ($z \gtrsim 1$) and use them to trace the structure, evolution, and dark matter in the universe, as well as simulate them in cosmological simulations, we are impressed with the progress made. The expectations for the future are just as promising; new cluster surveys are currently underway—in the optical, X-rays, S-Z, radio, lensing, and more. These will provide important information on the detailed properties and evolution of clusters and, hence, the universe. Similarly, large-scale high-resolution cosmological simulations are rapidly improving; these will continue to provide the basic comparisons needed for interpreting the observational data and testing popular cosmological models. We are fortunate to live in this exciting time and work in this exciting field.

For previous general reviews on clusters of galaxies see Zwicky (1957), Abell (1958), Bahcall (1977, 1988, 1999), Oort (1983), Dressler (1984), Rood (1988), Peebles (1993), and Biviano (2000, this volume).

1 Clusters and Cosmology

Clusters of galaxies place the most stringent constraints on the mass density of the universe. Several independent methods utilizing clusters all yield the same consistent result: a cosmological density parameter of $\Omega_m \simeq 0.2 - 0.3$. These methods include:

- The mass and mass-to-light ratio of clusters and superclusters
- The baryon fraction in clusters
- The evolution of cluster abundance with redshift.

I briefly summarize these results below.

1.1 Mass and Mass-to-Light Ratio

The mass and mass-to-light ratio of clusters of galaxies have been used for decades to illustrate the existence of dark matter in the universe; at the same time, the results suggest that if the clusters M/L ratio is representative, then applying this $<M/L_B>$ to the observed luminosity density of the universe yields a total mass density of only ~20%-30% of the critical density.

Observations of cluster velocity disperations, gas temperature and gravitational lensing, all yield cluster masses that are consistent with each other (typically within radii of $\sim$1 Mpc), indicating that cluster masses can be reliably determined within the observed scatter of $\sim \pm 30\%$. The observations also show that the mass profile in clusters follows the luminosity profile, i.e., $\rho_m(r) \sim \rho_L(r)$; they both exhibit a profile slope that is slightly steeper than isothermal at large radii ($r \sim 1$ Mpc). The typical mass-to-light ratio of rich clusters is observed to be $<M/L_B>=300 \pm 60h$, with scatter that ranges from $M/L_B \simeq 150h$ to $\sim 600h$ for individual clusters.[10 14 21 36 38 48 80]

Bahcall, Lubin, and Dorman (1995) showed that the M/L function (i.e., M/L as a function of scale, from galaxies to clusters to superclusters) increases linearly with scale on small scales (up to $\sim$200 kpc), but flattens on larger scales. This suggests that on large scales ($\gtrsim 1$ Mpc) mass is approximately proportional to light. Therefore, "voids" or other large-scale regions are not dominated by significant amounts of additional dark matter not accounted for by the galaxies and their large halos (as represented by the M/L_B ratios of clusters). The mass density of the universe can then be estimated from the observed value of M/L_B on large scales ($\gtrsim 1h^{-1}$ Mpc) and the observed luminosity density of the universe, L_B=2$\pm$0.4 $\times$ $10^8 h^3$ Mpc^{-3}; these yield Ω_m=0.2$\pm$0.1.

Recently, the first determination of the mass and mass-to-light ratio of a large supercluster was carried out using weak gravitational lensing observations.[46] The results yield an M/L value for the entire supercluster that is the same as the M/L ratio of the cluster members—i.e., the supercluster does not contain additional dark matter beyond that expected from the much smaller clusters as traced by light in the supercluster. This important result confirms the flattening of the M/L function on large scales.[10]

Comparisons with cosmological simulations (Figure 1) show that cold-dark-matter models yield the same M/L function as observed; both model and observations show flattening on large scales ($\gtrsim$1 Mpc).[14] The simulations, however, reveal that high overdensity regions such as clusters and superclusters of galaxies overestimate the global M/L_B value of the universe—i.e. these regions typically have lower L_B luminosity and thus higher M/L_B ratios than average. This is caused by the age effect: high density regions form early, and their L_B has decreased by the present time relative to the mean. Fitting the entire observed M/L function, from galaxies to groups, clusters, and superclusters, for the proper overdensities, we find

$$\Omega_m = 0.16 \pm 0.05 \quad (1\sigma) \tag{1}$$

(Bahcall et al. 2000) (see Figure 1). Higher values of Ω_m overestimate the observed M/L_B ratio on all scales—from galaxies to superclusters.

1.2 Baryon Fraction in Clusters

X-ray and Sunyaev-Zeldovich observations of clusters reveal a relatively high baryon fraction in clusters:

$$\begin{aligned} \Omega_b/\Omega_m &\gtrsim 0.07h^{-1.5} + 0.03 \quad \text{(X-rays)} \\ \Omega_b/\Omega_m &\gtrsim 0.077h^{-1} + 0.03 \quad \text{(S-Z).} \end{aligned} \tag{2}$$

The first term on the right represents the observed gas fraction in clusters and the second term reflects the approximate stellar contribution to the baryon fraction from the mostly early type galaxies in the clusters.[22 32 39] Combined with the baryon density allowed by nucleosynthesis,[75] Ω_b= 0.02 h^{-2}, the observed baryon fraction yields a powerful constraint on Ω_m:

$$\Omega_m \lesssim 0.3 \pm 0.05. \tag{3}$$

This constraint is consistent with the independent measure of Ω_m from the M/L function discussed above.

1.3 Evolution of Cluster Abundance

The evolution of cluster abundance provides another independent measure of the mass density of the universe, as well as the amplitude of mass fluctuations, σ_8. We discuss this topic in more detail in section 3. The results presented at this conference have been obtained from several independent surveys and methods including the evolution of the X-ray luminosity function, temperature function, and mass function; they yield consistent results that indicate only mild evolution of cluster abundance to $z \sim 1$, yielding

$$\Omega_m \simeq 0.25 \quad , \quad \sigma_8 \simeq 1. \tag{4}$$

2 New Cluster Samples to $z \sim 1$

Several new and upgraded cluster surveys in the optical and X-rays, with clusters in the redshift range from $z \sim 0$ to ~ 1, have been presented at the meeting . Important results have been obtained from some of the samples as discussed in the relevant sections of this paper. The surveys include the ENACS survey,[2] the NEP X-ray survey,[37 57] the REFLEX X-ray survey,[17 69] the RDCS X-ray survey,[18 66] the low surface brightness optical survey,[83] the EIS survey,[51] the DPOSS survey,[35] the WARPS X-ray survey,[45] the upcoming SDSS survey,[30] and a distant cluster sample.[25 71] These surveys and their results are discussed in this volume by the references indicated. Thousands of clusters are included in these new surveys, from $z \sim 0$ to $z \gtrsim 1$. Many clusters and candidate clusters are reported at high redshifts (z>0.5). For example, five high X-ray luminosity clusters ($L_x \gtrsim 1.5 \times 10^{45}$ erg/s) at $z = 0.7-1$ have been reported by the WARPS survey[45] over 73 deg^2. Temperatures and lensing masses will soon be obtained for these clusters. Assuming that these are indeed massive clusters at high redshift, as suggested by their high X-ray luminosity, this finding will greatly constrain the cosmological parameters Ω_m (suggesting a low value) and σ_8 (see section 3).

3 Evolution of Cluster Abundance

The evolution of cluster abundance with redshift provides a unique constraint on the mass density Ω_m and the amplitude of mass fluctuations on $8h^{-1}$ Mpc scale, σ_8. The present-day cluster abundance yields an important relation: $\sigma_8\Omega^{0.5} \simeq 0.5$[8 28 62 77 78]; this constraint, however, is degenerate in Ω_m and σ_8. The evolution of cluster abundance with redshift breaks this degeneracy and allows the determination of each of the two parameters independently.[11 12 21 28 29 41 60] An Ω_m=1 Gaussian model, with σ_8=0.5 (which is a strongly biased model, since σ_8(gal)=1), evolves very rapidly from $z \sim 1$ to $z \sim 0$; only very few massive clusters, at most, are expected to exist at $z \gtrsim 0.5$ in this model. On the other hand, a $\sigma_8 \simeq 1$ (where mass follows light) and $\Omega_m \simeq 0.25$ model yields only mild evolution in the cluster abundance to $z \sim 1$. Thus, if massive clusters are found at $z \simeq 0.5-1$, the mass density of the universe has to be subcritical, and $\sigma_8 \sim 1$.[11 12 21 26 40 41] The number of massive clusters observed at $z \simeq 0.5-0.8$ is found to be considerably higher—by orders of magnitude—than expected for an Ω_m=1 universe; the data yields $\Omega_m=0.25\pm^{0.15}_{0.1}$ and $\sigma_8=1\pm0.2$ (Figure 2).

New results presented at this conference, based on recent optical surveys, yield similar conclusions[2 83]; only a mild evolution of cluster abundance is seen, consistent with a low-density universe.

The evolution of the X-ray luminosity function to $z \sim 1$ shows similar results. Several new survey results have been reported at the conference.[18 37 45 57] All the surveys indicate no significant evolution of the X-ray luminosity function to $z \sim 1$ for clusters with $L_x \lesssim 2\times10^{44}$ erg/s, and only mild evolution (factor $\lesssim 3$) for higher X-ray luminosity clusters (to $z \sim 1$). (See above references for details.) The mild evolution of the luminosity function yields, again, $\Omega_m=0.2-0.3$ and $\sigma_8 \simeq 1$.

In addition, five high luminosity ($L_x \gtrsim 1.5\times10^{45}$ erg/s) clusters have been detected at $z = 0.7-1$ over 73 deg^2 of the WRAPS survey.[45] If confirmed as massive clusters, as expected based on their high L_x, the existence of such massive clusters at these early times will strongly support a low-density universe of $\Omega_m \sim 0.2$.

With the continued successful detection of high redshift clusters and follow-up observations of their properties, especially their mass, it is expected that tighter constraints will soon be placed on both Ω_m and σ_8 (assuming Gaussian fluctuations).

4 The S-Z Effect in Clusters

The Sunyaev-Zeldovich decrement in clusters has been imaged for a sample of clusters by Carlstrom and collaborators.[22 39 42 64] Using the S-Z data, the gas fraction in clusters has been determined, $\Omega_{\rm gas}/\Omega_m=0.077h^{-1}$, and used to constrain the mass density of the universe (section 2); the results yield $\Omega_m \lesssim 0.3$, consistent with the other independent methods discussed above.

Combining the S-Z and X-ray observations of clusters, the cluster distances can be obtained and hence a measure of the Hubble constant can be derived,[64]

$$H_o = 63 \pm 5 \pm 18 \quad km \quad s^{-1} \ \ \mathrm{Mpc}^{-1}. \tag{5}$$

The possibility of measuring the deceleration parameter, q_o, using a larger and more accurate sample of S-Z clusters at higher redshifts, may also become real in the near future!

Upcoming surveys of S-Z clusters, currently being planned, will provide important implications for cosmology and for the evolution of structure in the universe. S-Z clusters can be uniquely and accurately detected to high redshifts, thus enabling the evolutionary studies discussed in section 3 to be carried out with complete and accurate samples of high redshift clusters.[15 23] (See references listed above, this volume.)

5 Scaling Relations and Profiles in Clusters

The steep $L_x \sim T^3$ relation observed for clusters of galaxies, which flattens for smaller groups, is not reproduced in most simulations; the latter typically yield $L_x \sim T^2$. The simulated relation, however, steepens somewhat when cooling flows are present in the clusters.[17 30 32 43 44 52 63 72 74 82]

High resolution simulations of cold-dark-matter models reveal cluster density profiles that have steep central densities, following approximately a $\rho(r) \sim r^{-1.5}$ central profile.[34 56 58 72] Some observations suggest a flat core in clusters, but others, like A370, suggest a steep central profile. Systematic observations on these small scales of cluster cores ($\lesssim$ 100 kpc) are not yet conclusive at this time.

The temperature profile in poor clusters studied recently with XMM shows cooling in the central regions, as well as an increase in metalicity in the cluster cores.[4 44 73 81]

Comparison of cluster properties with various high resolution simulations show, in general, an excellent agreement with observations. However, some significant discrepancies, and thus warnings, are apparent; these include the L_x–T relation and the central density cusps in clusters discussed above.[30 47 49 54 56 67 72 79 81]

6 Large-Scale Structure Traced by Clusters

The cluster correlation function has been determined from the new optical survey of clusters at $z \simeq 0.5$ by Zaritsky *et al.*[83] The cluster correlation length increases from $r_\circ \simeq 20h^{-1}$ Mpc to $\sim 30h^{-1}$ Mpc for the richest clusters. The results are consistent with those obtained for the Abell and other clusters by Bahcall and Soneira (1983), Bahcall and West (1992), Croft *et al.* (1996), and support the richness-dependent cluster correlation function.[6 7 9]

Similarly, the large REFLEX X-ray cluster sample at $z \sim 0$ reveals a cluster correlation length that ranges from $\sim 20h^{-1}$ to $\sim 30h^{-1}$ Mpc, and an X-ray cluster power spectrum that exhibits a shape parameter of $\Gamma = 0.195 \pm 0.05$ and slope $n = -1.9$ on cluster scales.[17 69] The amplitude of the power spectrum increases with increasing L_x, as does the correlation amplitude—consistent with the richness-dependent clustering of clusters.[7 9] The bias parameter between the cluster power spectrum and that of galaxies appears to be constant on large scales (see above references). No sharp excess features are seen in the REFLEX cluster power spectrum on large scales.

7 Galaxy Evolution in Clusters

The evolution of galaxies in clusters and the gradients in the density distribution of ellipticals, spirals, and SO galaxies, were discussed both observationally and theoretically, the latter using semi-analytic models (SAM).[31 47 49 53 70] The semi-analytic models succeed in producing a generally good agreement with the data for a wide range of observations. (See relevant papers, this volume).

The luminosity function of galaxies in clusters was discussed by Andreon[3] and by Ulmer[76], with reported faint-end slopes that range from $\alpha = -1$ to $\alpha = -2$.[3]

8 Non-Thermal Emission in Clusters

For discussions of non-thermal emission from clusters in the radio, hard X-rays, γ-rays, and EUV (thermal and non-thermal) see relevant papers in this volume. [4 19 24 33 50 68]

9 What's Next?

Some important open questions that are likely to be resolved or improved in the next few years include the following:

1. Determination of the mass and mass-to-light ratios of groups, clusters, superclusters, and large-scale structure using gravitational lensing (as well as optical and X-ray observations when relevant) and the study of these parameters as a function of scale, temperature, mass, and overdensity (see e.g., section 1 and figure 1.) These observations, especially weak lensing on large scale, should provide accurate determination of the mass density of the universe.

2. Baryon Census

   a. Determination of the baryon fraction in clusters and superclusters as a function of scale, overdensity, and redshift.

   b. Understanding the baryon census in groups and clusters: what is the amount of gas, stars, and other components?

   c. Where are the "missing" baryons? Are they mainly in warm gas of superclusters and filament? Observations in the X-ray, S-Z, optical, and lensing will all contribute to these topics.

3. Statistical and Internal Cluster Properties

   a. Determination of the mass function, temperature function, and X-ray luminosity function of clusters from $z \sim 0$ to $z>1$.

   b. Determination of the relations between cluster mass, temperature, overdensity, L_x and radius and their evolution to $z>1$.

   c. Determination of the mass density profiles in clusters, from the inner parts of the cores to the outer scales ($\gtrsim 2h^{-1}$ Mpc).

   d. Determination of arc lensing statistics by clusters; this will provide strong constraints on the abundance of high redshift clusters, with direct implications for cosmology.

4. Studies of non-thermal effects in clusters.

5. Large-scale structure traced by clusters (power spectrum, correlation function) from $z \sim 0$ to $z>1$.

6. Improved high-resolution large-scale simulations: Is the "canonical" model o.k.? Are there other models that match the data?

7. Surveys needed: wide and deep searches in optical, X-rays, S-Z, weak and strong lensing, radio/EUV/γ-rays.

8. "Will clusters be useful for cosmology?" The answer is undoubtedly YES. They already have been and will continue to be. The goal is to accurately "construct the universe with clusters of galaxies" and carry out "precision cosmology" over the next decade.

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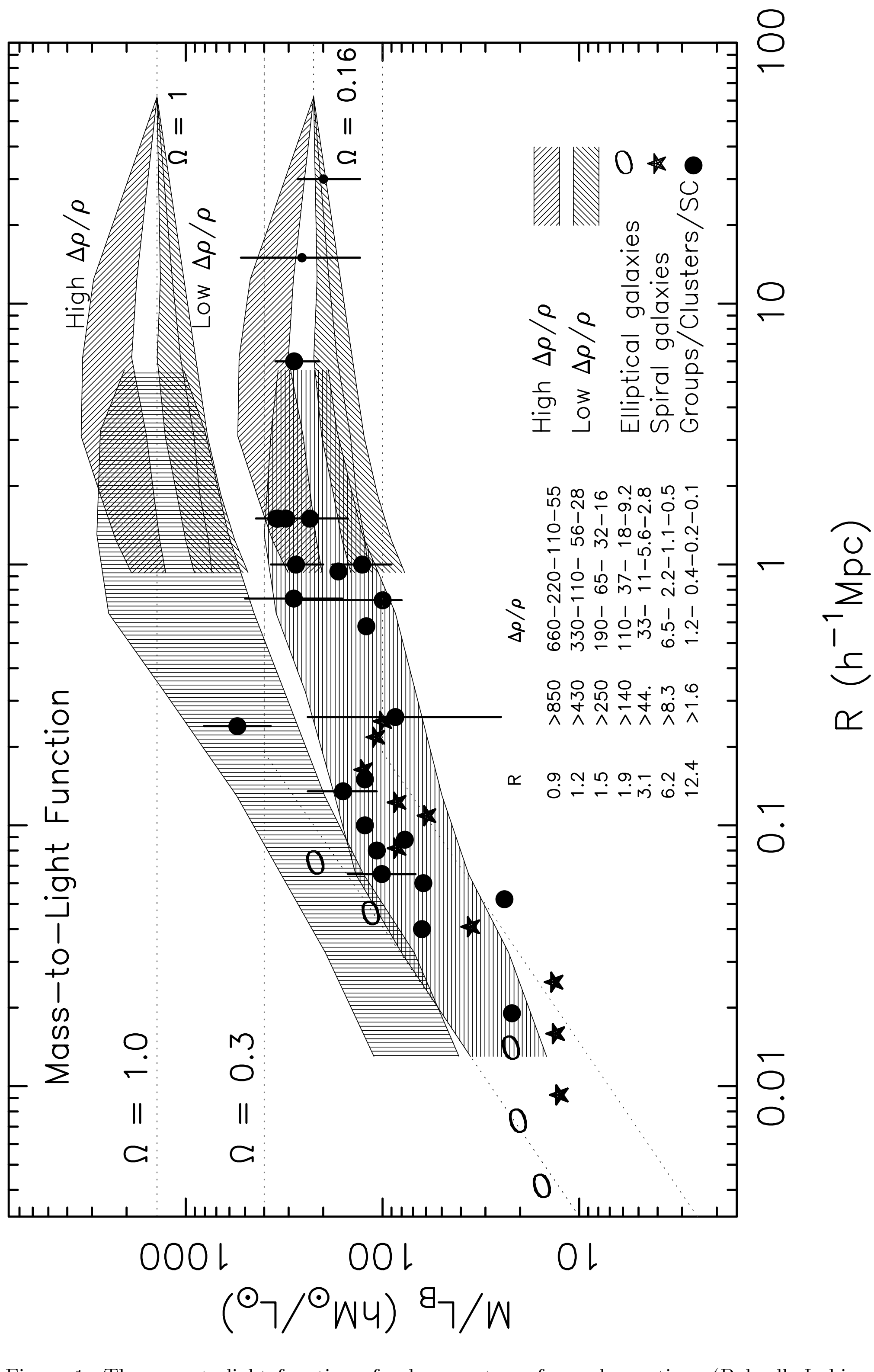

Figure 1: The mass-to-light function of galaxy systems from observations (Bahcall, Lubin, and Dorman 1995) and simulations (Bahcall et al. 2000). The observations are presented by the data points for medians of galaxies, groups, clusters, and a supercluster. The simulation results (for cold-dark-matter models) are presented by the shaded bands for $\Omega_m = 1$ and 0.16 (our best fit value). On scales >1Mpc, the simulation results for both high- and low- density regions are presented (where these correspond roughly to the overdensities of rich-clusters and groups, respectively). See Bahcall et al. 2000 for more details.

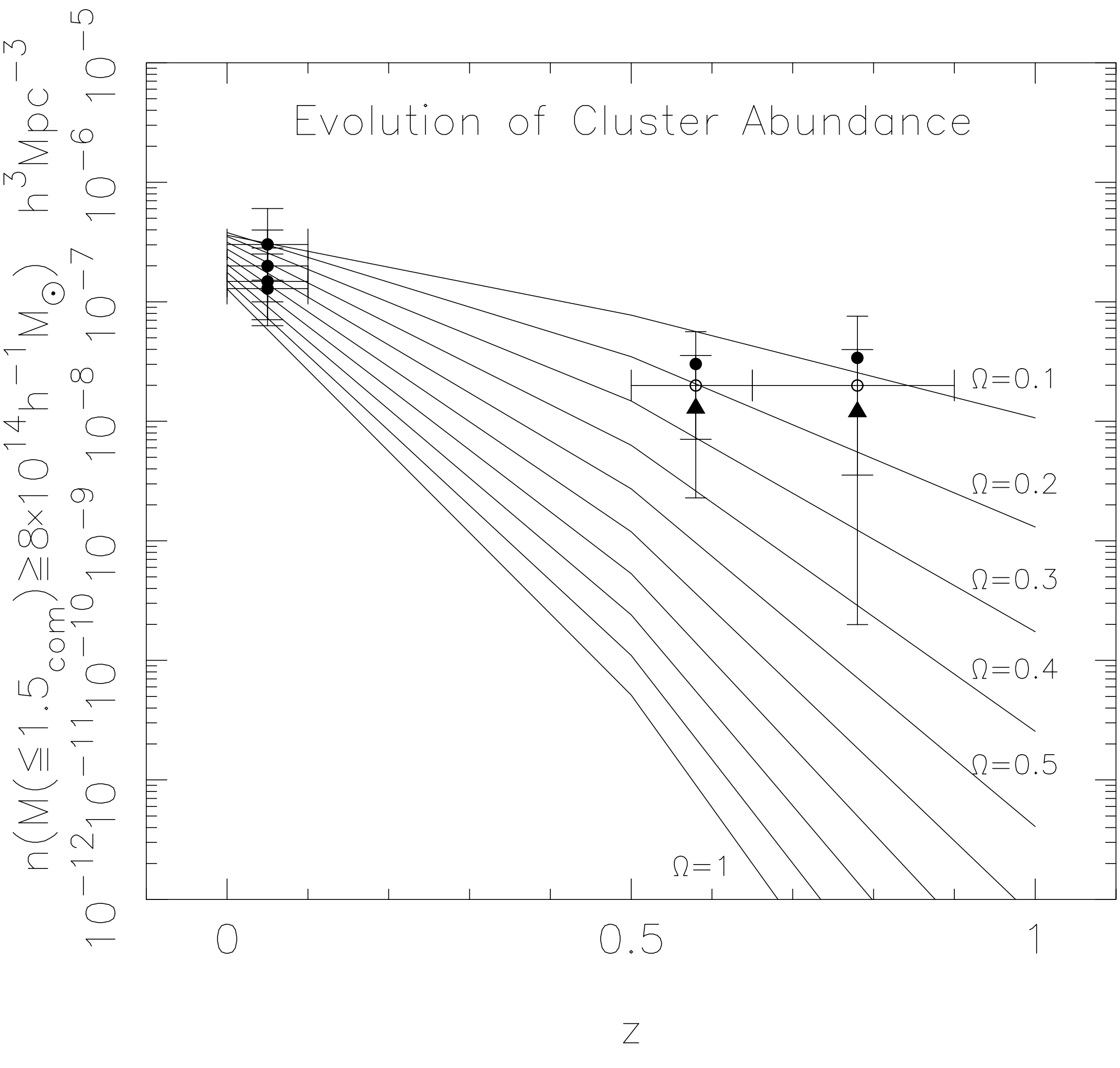

Figure 2: Evolution of the cluster abundance with redshift for massive clusters (with mass $>8\times10^{14}h^{-1}$ Mo within a comoving radius of $1.5h-1$ Mpc). (From Bahcall and Fan 1998.) The data points represent the observational data (see text), and the curves represent the expected cluster abundance for different Ω_m values (based on the Press-Schechter method). Similar results are obtained from direct simulations (Bode et al., in preparation).