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FROM MESSIER TO ABELL: 200 YEARS OF SCIENCE WITH GALAXY CLUSTERS

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1 Introduction

The history of the scientific investigation of galaxy clusters starts with the XVIII century, when Charles Messier and F. Wilhelm Herschel independently produced the first catalogues of nebulæ, and noticed remarkable concentrations of nebulæ on the sky. Many astronomers of the XIX and early XX century investigated the distribution of nebulæ in order to understand their relation to the local *"sidereal system"*, the Milky Way. The question they were trying to answer was whether or not the nebulæ are external to our own galaxy. The answer came at the beginning of the XX century, mainly through the works of V.M. Slipher and E. Hubble (see, e.g., Smith[424]).

The extragalactic nature of nebulæ being established, astronomers started to consider clusters of galaxies as physical systems. The issue of how clusters form attracted the attention of K. Lundmark[287] as early as in 1927. Six years later, F. Zwicky[512] first estimated the mass of a galaxy cluster, thus establishing the need for dark matter. The role of clusters as laboratories for studying the evolution of galaxies was also soon realized (notably with the collisional stripping theory of Spitzer & Baade[430]).

In the 50's the investigation of galaxy clusters started to cover all aspects, from the distribution and properties of galaxies in clusters, to the existence of sub- and super-clustering, from the origin and evolution of clusters, to their dynamical status, and the nature of dark matter (or *"positive energy"*, see e.g., Ambartsumian[29]). As a matter of fact, the topic expanded so much that in 1959 a new separate section specifically devoted to galaxy clusters – *Galaxienhaufen* – appeared in the *Astronomischer Jahresbericht*. Galaxy clusters had become one of the main research topics in extragalactic astrophysics.

In this historical review I have tried to cover all aspects of astrophysics research on galaxy clusters, spanning a temporal range of exactly 200 years, from 1784 to 1983. In 1784, Charles Messier[303] was the first to write about a cluster of galaxies, Virgo, in his *Catalogue des nébuleuses et des amas d'étoiles que l'on découvre parmi les étoiles fixes, sur l'horizon de Paris.* In 1983, on

October 7^{th}, George O. Abell, the eponymous of nearby rich clusters of galaxies, prematurely died at the age of 56. A practical reason for stopping this review with 1983, is that the exponential increase of publications makes it increasingly difficult for the historian to keep pace with the new scientific results.

This review is divided into four main topics:

1. THE DISTRIBUTION OF CLUSTERS, including:
    - the discovery of clusters
    - cluster catalogues
    - the large scale structure (superclusters)
    - distribution functions of cluster properties
2. THE CLUSTER COMPONENTS, including:
    - the properties and distribution of cluster galaxies
    - the properties and distribution of intracluster (IC hereafter) hot gas
    - cluster radio-sources
3. THE CLUSTER STRUCTURE, including:
    - the dynamical status of clusters (stability and subclustering)
    - cluster masses
    - cluster luminosities (the luminosity function)
    - the nature of the *missing mass*
4. THE EVOLUTION OF CLUSTERS, including:
    - the evolution of clustering
    - the evolution of galaxies in clusters
    - the evolution of the IC gas
    - cooling flows and the evolution of cD galaxies

I consider here both theoretical and observational aspects. However, I rarely mention technical aspects, such as the development of new telescopes and instruments, which were certainly very relevant to our understanding of galaxy clusters. In this respect, this review traces the history of the scientific thought, rather than the history of science.

For the sake of homogeneity, all quantities that are H_0-dependent, have been re-scaled to the same value the Hubble constant, $H_0 = 75$ km s^{-1} Mpc^{-1}.

2 The distribution of clusters

2.1 Early days

The first written reference to a cluster of galaxies is probably that of the French astronomer Charles Messier[303] in 1784. In his *Catalogue des nébuleuses et des amas d'étoiles que l'on découvre parmi les étoiles fixes, sur l'horizon de Paris*, he listed 103 nebulæ, 30 of which we now identify as galaxies[a]. Messier already noticed the exceptional concentration of nebulæ in the

Figure 1: Portraits of C. Messier (left) and F. Wilhelm Herschel.

Virgo constellation. However, Messier's interest in nebulæ was very marginal. He seeked to define the positions of nebulæ in order not to misidentify them with new comets[b].

F. Wilhelm Herschel had a quite different approach to the investigation of nebulæ. German born, he escaped from Hanover and reached England during the War of the Seven Years. A musician, he became interested in astronomy after reading a popular book. After the first successful discoveries[c] with his self-made telescopes, the king of England granted him the money to build the largest telescope of his times, a 1.47 m aperture, 12.2 m focal length refractor. Herschel was interested in what we would now call the Large Scale Structure of the Universe. In 1785 he published *On the Construction of the Heavens*[214], where he suggested that the *"sidereal system we inhabit"* is a nebula, common in appearance to many others, which therefore must be external to our own. Most relevant here is W. Herschel's description of the Coma cluster of galaxies:

> *"that remarkable collection of many hundreds if nebulæ which are to be seen in what I have called the nebulous stratum of Coma Berenices"*

In the same paper, W. Herschel mentioned her sister's discovery of the second small companion of M 31, NGC 205. With M 32, these three galaxies make a triplet similar to that composed by the Milky Way and the two Magellanic clouds. The other giant galaxy in the Local Group, M 33 was listed in Messier's catalogue. So, 7 members of the Local Group of galaxies were already known at that time. Their distances being unknown, it was only in 1936 that E. Hubble[233] pointed out that these galaxies (and a few more) belong to the same system, which he named *"The Local Group"* (see, e.g., van den Bergh[480]).

In the course of his life, W. Herschel[215] classified some 2500 nebulæ and recognized several other nearby clusters and groups of galaxies, such as Leo, Ursa Major, Hydra, NGC4169, etc. His work was continued by his son, John F.W. Herschel. J. Herschel surveyed the southern sky from Cape of Good Hope, and catalogued over 6000 nebulæ that in 1864 he collected in his *General Catalogue of Nebulæ and Clusters of Stars*. During the first part of the XIX century, J. Herschel noted that the northern hemisphere has an excess of nebulæ with respect

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[a] Of the 30 extragalactic objects in Messier's catalogue, only 13 are listed in the Virgo Cluster Catalogue of Binggeli et al.[63].

[b] Charles Messier was nicknamed *"le furet des comètes"* by Louis XV.

[c] W. Herschel became very famous after his discovery of Uranus in 1781.

to the southern hemisphere, and he recognized several concentrations of nebulæ (in Pisces and Fornax, in particular). He already hinted at the existence of the Local Supercluster, with the Virgo concentration *"being regarded as the main body of this system"*, and our own Galaxy *"placed somewhat beyond the borders of its densest portion, yet involved among its outlying members"* (see, e.g., Flin[164]).

In J. Herschel's times, d'Arrest[119] and Proctor[364] published new positions and finding charts of nebulæ in the Coma and Virgo clusters, Stephan[433] discovered the famous galaxy quintet, and Dreyer[148] published his *New General Catalogue*. Complemented by the *Index Catalogues*, the *NGC* listed roughly 13000 nebulæ in 1908.

At the beginning of the new century, the extensive photographic work of Max Wolf[500,501,502] led to a detailed description of the Coma and Perseus clusters. In 1918 Curtis[117] added more nebulæ to Wolf's list, reaching a total of 300 nebulæ in the Coma cluster.

In the early years of the XX century, intensive photographic observations of nebulæ were done mostly with the aim of establishing whether they were external to our own galaxy or not. The *Great Debate* on the nature of nebulæ between Shapley and Curtis, took place on April, 26th 1920, with no clear winner. Not only were astronomers trying to determine the distribution of nebulæ with respect to the galactic plane, they were also trying to count them! Curtis[118]' estimate of 722,000 nebulæ in 1918, was revised to 60 millions by Hubble[233] in 1936.

In 1904 Easton[150] noted an asymmetry in the distribution of the nebulæ with respect to the galactic plane, with an excess of nebulæ in the northern hemisphere. Nineteen years later, this asymmetry was re-discovered by Reynolds[371,372] who noted that

> *"many of the spirals 10' diameter and upwards lie along 100°, and form part of a well-marked band of nebulæ passing over the north galactic pole, which comes out conspicuously if the spirals ranging down to 2' diameter are plotted together."*

A clear reference to the Local Supercluster! In the same years, C. Wirtz, using Dreyer's catalogues and Curtis' surveys, called the attention to several conspicuous well-defined centers of clustering (see, e.g., Abell[15]).

In the early twenties, Edwin Hubble discovered cepheids in M31, and definitely established the extragalactic nature of nebulæ. A few years later he published his work[232] on the velocity-distance relation for extragalactic nebulæ. Extending this relation to higher redshifts became the main driver for Hubble & Humason's great observational work on extragalactic nebulæ[234]. In 1934 and 1936 Milton Humason[236,237] measured velocities of 39,200 km/s and 42,000 km/s for galaxies in the Boötis and Ursa Major II clusters, making them the most distant clusters known at that time.

More galaxy systems were discovered in those years: Cancer, Hercules, Leo, and notably the *"Centaurus cloud"*, today's *Shapley concentration* (see, e.g., Bardelli et al.[52]). Shapley[414] correctly estimated it to be 14 times more distant than Virgo, and 10 times as rich in nebulæ. All these discoveries were serendipitous; as an example, the Perseus-Pisces stratum was noted by Tombaugh[461] as *"a by-product of the extensive trans-Neptunian planet search"* which eventually led to the discovery of Pluto. Knut Lundmark[287] plotted the sky distribution of 55 clusters of *"anagalactic nebulæ"* – see Fig. 2. Coordinates of these clusters were not listed, but it is likely that many of them were *groups* rather than *clusters*. Lundmark noted *"the most characteristic feature in the charts of the nebular distribution is the clustering tendency"*, a tendency confirmed in the Harvard survey[418]. While presenting results from this survey, Shapley[415] provided a list of 25 clusters and suggested the existence of *"metagalactic clouds"* (today's superclusters), such as those in Coma, Centaurus and Hercules[416]. E.F. Carpenter[96] described clusters as the extremes of a continuous non-uniform spatial distribution of galaxies, thus anticipating the works of Neyman & Scott[320] and Peebles[352].

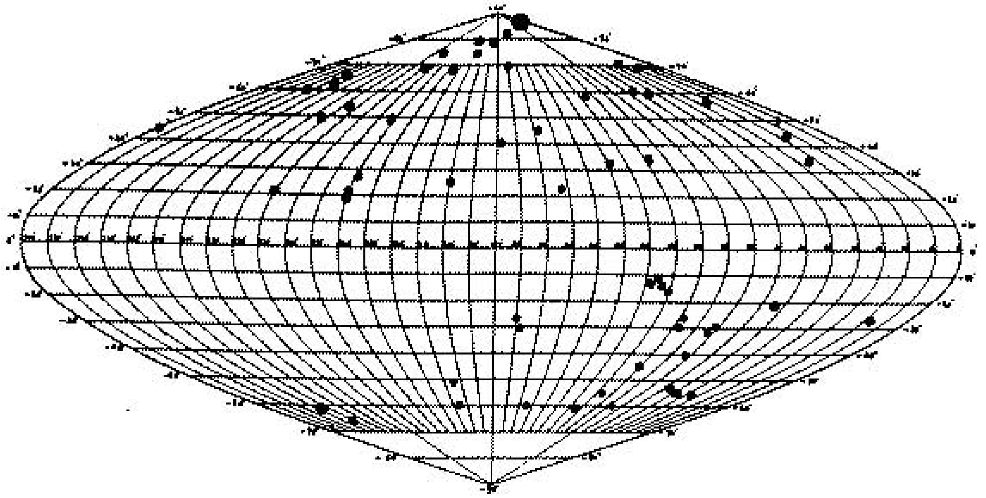

Figure 2: Galactic distribution of the clusters of anagalactic nebulæ. From Lundmark (1927).

In contrast to the growing dominant opinion, in 1936 Hubble[233] described the distribution of nebulæ as *"moderately uniform"* and noted that *"no organization on a scale larger than the great clusters"* was definitely known. However, he recognized our own Galaxy as a member of a galaxy system, which he named *"The Local Group"*. Zwicky[514] noted that the local group may well be part of the Virgo galaxy system, that Holmberg[224] described as a *"Metagalactic cloud"* of $\sim$ 100 Mpc size.

2.2 Surveys and catalogues

After the Second World War, the Lick and Palomar sky surveys and the spectroscopic observations of Humason, Mayall & Sandage[238] provided the essential data-base for the analysis of the distribution of galaxies. The 1956 paper of Humason et al.[238] collected the results of twenty years of spectroscopic observations, providing more than 800 redshifts of galaxies, of which 75 in Virgo, 23 in Coma, and a few dozens in several other clusters. They noted that there was *"increasing evidence"* for a general clustering phenomenon, and dismissed Hubble's view of a uniform galaxy distribution with a few sporadic isolated clusters.

The evidence for the *"Local Supergalaxy"* and for many other superclusters grew stronger mainly through the works of de Vaucouleurs[128,129] – see Fig. 3 – Shane & Wirtanen[412], van den Bergh[475], and Abell[11]. Only Zwicky[520] continued to deny the existence of superclusters. Zwicky[524] thought that the apparent non-uniform distribution of clusters was due to the obscuration effects of inter-galactic and IC dust. He eventually discovered a supercluster himself[526] (no.20 in Zucca et al.[510]'s catalogue), but refused to call it a supercluster. Zwicky's point of view was however very different from Hubble's. Zwicky thought galaxy clusters to be much larger than usually accepted, almost reaching to the sizes of superclusters. Clusters, he wrote in 1952, *"fill the universe just as the bubbles fill a volume of suds"*. For these reasons, Abell[14] thought that Zwicky's opposition to the idea of superclusters was purely semantic.

In a series of papers, Neyman, Scott, Shane & Swanson[320,321,322,408] addressed the issue of galaxy clustering by applying mathematical models to the Lick galaxy counts of Shane & Wirtanen[412], and were the first to compare the observed galaxy distribution to synthetic images of the Universe[408].

The introduction of new techniques and new ideas pushed the search for clusters to higher redshifts. Baum[53] pointed out that clusters at redshifts $\sim$ 0.5 could be most easily detected

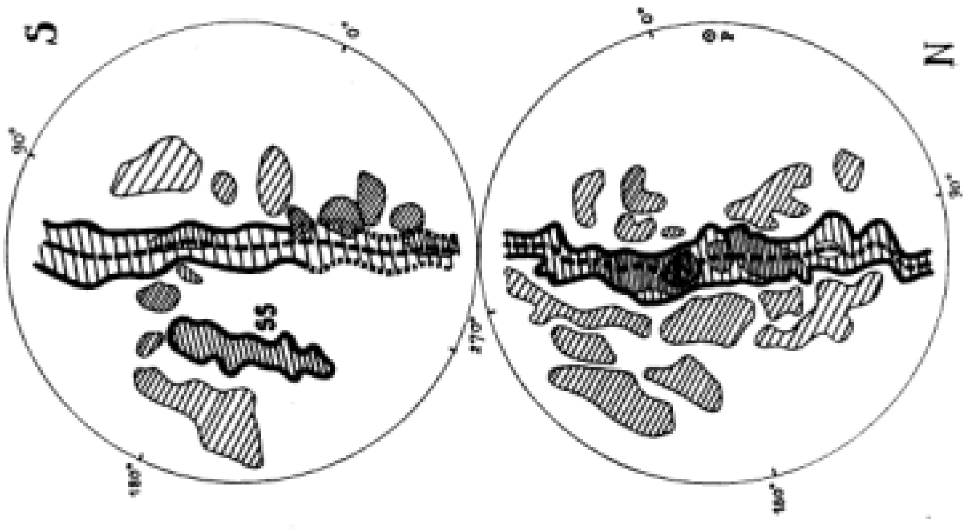

Figure 3: Clustering of the nebulæ in the southern and northern emisphere giving evidence of the Local Supercluster. The density of the shading gives in a qualitative way an idea of the nebular density – from de Vaucouleurs (1953).

by moving redwards the observing waveband. Minkowski[307] speculated that collisions between galaxies could produce radio-emission; since collisions should be frequent in dense environments, he suggested that clusters could be found around radio-galaxies. In 1960 he applied this idea to the region around 3C295, and found a system of galaxies at a redshift $\simeq$ 0.44–0.46. 3C295 held the record of the highest redshift cluster for a long time[d].

Meanwhile, the search for nearby galaxy clusters had become systematic. The time of serendipitous discoveries was long gone, and in 1957 Herzog, Wild & Zwicky[216] announced the construction of a *Catalogue of Galaxies and Clusters of Galaxies*[527], that upon completion would contain $\sim$ 10000 clusters. Their announcement came just one year before the publication of Abell's catalogue[7], but the final *CGCG* was to be published only in 1967.

Abell's paper, *The distribution of rich clusters of galaxies*, is a milestone in the history of science with galaxy clusters[e]. The very fact that *Abell cluster* has become a synonymous with *rich cluster* tells us a lot about the importance of this paper.

Abell's 2712 clusters were selected on red *POSS* plates because he realized the advantage of the red band over the blue band for the identification of distant clusters. Abell's radius was subjectively chosen by looking at the projected overdensities of clusters, and yet is close to the cluster gravitational radius[73]. Abell's subjective selection criteria were extremely well chosen, and even the background subtraction was quite accurate.

Abell's paper was much more than a catalogue of clusters. He was the first to show that the distribution of cluster richnesses – which is broadly related to the mass distribution – is very steep. He knew that his cluster sample was incomplete at the low richness end, and for this reason he defined a statistical subsample of the richest 1682 clusters. As a matter of fact, he wrote

> *"during the course of the plate inspections, many thousands of clusters and groups*

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[d] 3C295 later became one of the two clusters where Butcher & Oemler[85] found evidence for an increased fraction of blue galaxies.

[e] Abell's paper was just *"a portion of a thesis submitted in partial fulfillment of the requirements for the Ph.D. degree"* – though requirements, no doubt!

> *of galaxies were recognized which were not catalogued because they obviously were not sufficiently rich to insure their essentially complete identification. Thus neither the statistical sample of clusters nor a subjective impression indicates a maximum in the* $N(n)$ *versus* n *relation."*

We better remember this statement when commenting upon the results of modern optical cluster surveys[286,362] (see also LOBO, these proceedings).

The publication of Abell's catalogue opened a new era in the investigation of galaxy clusters. All of a sudden, researchers had a catalogue of clusters, and they could start look at them as a population, rather than as individual objects. The first volume of Zwicky et al.[527]'s *Catalogue of Galaxies and Clusters of Galaxies* was published only a few years later, but it did not exert such a large influence on the study of clusters. The main problem with the *CGCG*, as immediately pointed out by Abell[12], was that the sizes of Zwicky's clusters were distance-dependent, since they were defined within the isopleth contour that represents twice the field density. The *CGCG* could then not be used as a statistical homogeneous cluster catalogue, and most researchers preferred to base their analysis on Abell's catalogue (and they still do).

The first critical examination of Abell's and Zwicky's catalogues was done by Reaves[370]. Abell's statistical subsample was shown to be ~ 85 % complete, while the completeness of the full Abell catalogue is only ~ 40 %, similar to that of the Zwicky catalogue. Reaves' estimates were based on how frequently a given cluster detected on one plate was missing on another plate where it should have been seen. His conclusions are quite close to those obtained by Lucey[284] and Briel & Henry[79] several years after.

In the following years, there was an increase and an improvement in the classification of clusters, along these five main research lines:

- Finer classifications: Bautz & Morgan[55] and Rood & Sastry[387] invented finer cluster classification schemes, to supersede the traditional regular–irregular cluster classification. Oemler[331] classified clusters according to their galaxy morphological content, and suggested a relationship between a cluster compactness and its galaxy morphological mix.

- Redshift determinations: Noonan[327,328] published lists of cluster redshifts (138 in 1973, and four times as many in 1981).

- Southern clusters: Klemola[258], Snow[426], Rose[391], Duus & Newell[149] provided lists of hundreds of clusters in the southern hemisphere.

- Poor galaxy systems: de Vaucouleurs[137] published a list of 55 groups of galaxies, based on his *Reference Catalogue*[135]. Another list of 174 groups was published by Holmberg[230]. Shakbazyan & Petrosyan[411] published a catalogue of *Compact groups of compact galaxies,* followed by Rose's catalogue of compact groups in 1977[392]. Turner & Gott[466] provided the first complete catalogue of galaxy groups. Morgan et al.[311] and Albert et al.[24] identified poor clusters dominated by giant elliptical at their centre.

- Automated search for clusters: in 1976 MacGillivray et al.[290] inaugurated the automated search for galaxy clusters. Clusters were identified in galaxy catalogues built using the *COSMOS* automatic plate-measuring machine.

In 1973, Karachentseva[249] published a *Catalogue of isolated galaxies.* Clustered galaxies have become the rule, isolated galaxies the exception, to such a point that two years later de Vaucouleurs could ask: *"Are there isolated galaxies?"*

In 1971 Meekins et al.[299] and Gursky et al.[201] detected extended X-ray emission from the Coma cluster (see Fig. fig-hgcomax). Little by little, optical catalogues of galaxy clusters would give way to X-ray catalogues. Initially there were just lists of optical counterparts for a few

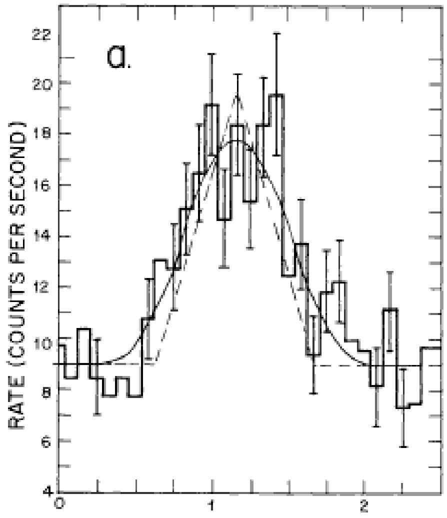

Figure 4: Counting rates per degrees (relative azimuth on x-axis) in the Coma cluster. The solid line indicates a fit with an extended source, the dashed line the expected response to a point source. From Gursky et al. (1971).

X-ray sources (e.g. Melnick & Quintana[301]), but soon after extensive X-ray surveys of hundreds of Abell clusters were published (see, e.g., Ulmer et al.[470]).

2.3 Superclusters and voids

In his milestone paper, Abell[7] also demonstrated the existence of *"clusters of clusters"* in 3 dimensions. Abell used his magnitude-based cluster distance estimates to establish that the average size of superclusters is $\simeq$ 60 Mpc. He rejected Zwicky's hypothesis of IC dust by showing that regions of the sky devoid of intermediate-distance clusters were nevertheless occupied by even more distant clusters. Ten years after, Reaves[369] was able to set an upper limit of 0.1 magnitudes to the extinction by IC dust, based on the colour vs. redshift relation for galaxies in cluster fields. Despite Abell's and Reaves' results, Bogart & Wagoner[74] in 1973 still invoked IC dust as the origin of an apparent cluster–cluster anti-correlation.

In 1962 Abell[12] published the first list of (seventeen) superclusters. He noted that the existence of superclusters was to be taken into account when estimating the probability of chance projection effects in a cluster catalogue, thus anticipating the ideas of Lucey[284]. A few years later, Abell & Seligman[20] showed that superclusters could be easily identified even in Zwicky's *CGCG*[527].

A step further towards establishing the reality and properties of the Local Supercluster, was done by de Vaucouleurs[137]. He considered the distribution of 55 nearby groups. By noting that 85 % of all nearby galaxies are in groups, he suggested that superclusters may well overlap and fill all the space available. He correctly argued that the observational samples had not yet reached to the distance of homogeneity, thus making it meaningless any attempt to estimate the mean density of the Universe. The concept of the Large Scale Structure of the Universe was taking his first steps.

Despite this observational progress, the reality of superclusters remained an open issue. Peebles and collaborators published papers arguing both against[508] and in favour of the existence[207] of superclusters. Peebles' final word came in 1974, with the development of a mathematical tool that was to stay with cosmologists ever since: the covariance function[352]. By showing that the covariance function is a simple power law over a very large distance range, he concluded that there was no physical division between groups and clusters, nor between clusters and superclusters.

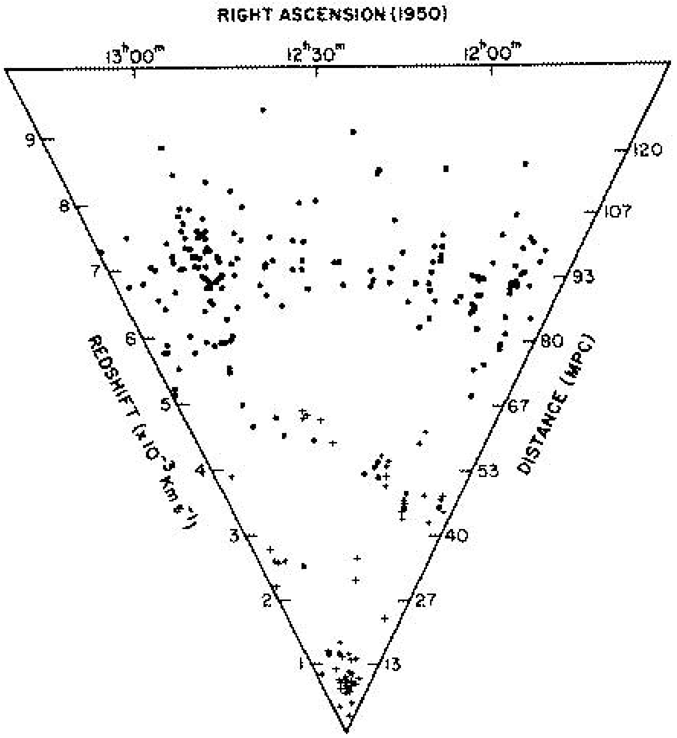

Figure 5: The wedge diagram of the Coma supercluster; crosses indicate galaxies that would be too faint to be detected if they were at the distance of the Coma cluster – from Gregory & Thompson (1978)

Zwicky continued to reject all evidences in favour of the existence of superclusters. He thought that IC dust could account for irregularities of the clusters distribution. Zwicky's hypothesis was finally falsified by Reaves[370] in 1974. Reaves showed that intermediate-distance clusters are less often seen behind nearby clusters than very distant ones. Correctly, he attributed this to the difficulty of distinguishing clusters in projection when they are not well separated along the line of sight, and the two cluster luminosity functions peak at a similar magnitude.

Fritz Zwicky did not live long enough to read Reaves' paper. He died on Feb. 8th 1974, just a few days before his 76th birthday.

After Zwicky's death the reality of superclusters was no longer questioned. A major breakthrough in this topic came with the extensive redshift surveys of Chincarini, Gregory, Rood, Tarenghi, Thompson & Tifft[459,195,454,449,109,448,196], that drew the 3-dimensional structures of the Coma – see Fig.5 –, Hercules, Hydra-Centaurus, Perseus and Pisces superclusters. Cluster-connecting filaments and voids were identified. The emerging picture was thus summarized by Abell[16]:

> *"The picture that suggests itself is that of a large inhomogeneity or region of space containing galaxies, groups, and clusters, in which what is commonly called the Coma cluster is simply a dense concentration, rather like an urban center in a large metropolitan area"*

In 1978 Jôeveer et al.[242] described Perseus and other eight superclusters, and noted that the majority of clusters of galaxies form chains. Einasto et al.[152] pointed out that the large scale structure of the Universe resembles cells, with galaxies and galaxy clusters concentrated towards cell walls, whereas the spatial density of galaxies inside cells is very low. In 1981 Kirshner et al.[257] found the million Mpc^3 Boötes void, that Bahcall & Soneira[49] showed to be associated with the Hercules supercluster and the CorBor extension.

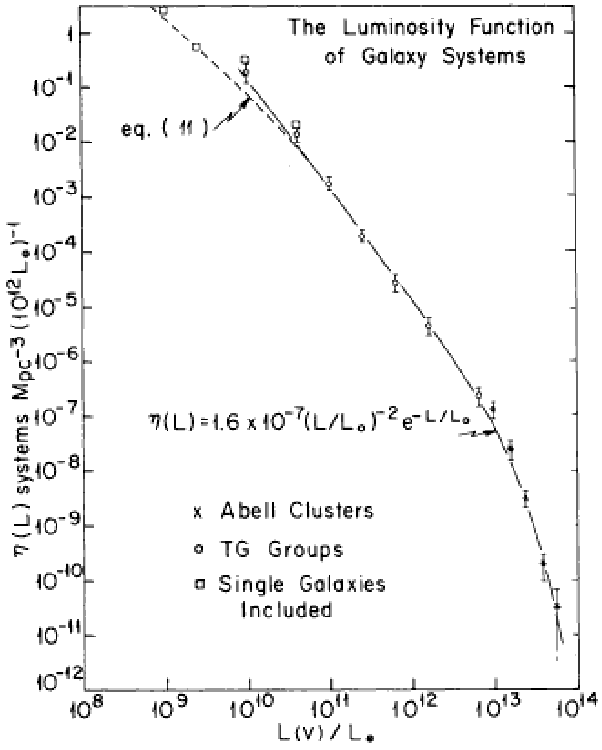

Figure 6: The luminosity function of all galaxy systems. The solid line represents the best fitting curve. From Bahcall (1979a).

Numerical simulations were keeping abreast of observations: in 1979 Aarseth et al.[4] were able to produce 3-dimensional plots of the galaxy distribution where the recently discovered huge voids were quite evident[f].

2.4 Clusters and the Large Scale Structure of the Universe

The huge observational effort of the seventies made it possible to evaluate the distribution functions of cluster properties. At the end of the 70's Chincarini[105] established the relation between cluster luminosities and their richness classes. One year later, based on similar relations, Neta Bahcall[43,44] produced the first optical – see Fig. 6 – and X-ray luminosity functions of galaxy systems, ranging six decades in luminosity. Subsequent studies, based on larger datasets, confirmed the validity of Bahcall's determinations (see, e.g., McKee et al.[298], Hintzen et al.[219] and Abramopoulous & Ku[21]). A preliminary attempt to produce the virial mass function of clusters was done by Struble & Bludman[438], but their sample was incomplete and biased at the low-mass end. The first unbiased estimates of the cluster mass function[46,70] would only come in 1993, 14 years later.

In 1982 Davis et al.[122] produced the first wide-angle galaxy redshift survey, not dominated by the Local Supercluster. The authors hoped that their survey *"would begin to approximate a fair sample volume of the universe"*. Maybe the first CfA survey was no so *"fair"* after all, but Davis et al.[122]'s description of the galaxy distribution was fairly correct. The galaxy distribution, they wrote, *"is frothy, characterized by large filamentary superclusters of up to 45 Mpc in extent, and corresponding large holes devoid of galaxies"*.

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[f] In the discussion following Aarseth's talk[3], Peebles referred to Aarseth's plots as *"propaganda films"* and deemed it *"very dangerous to compare them too closely to the real Universe"*.

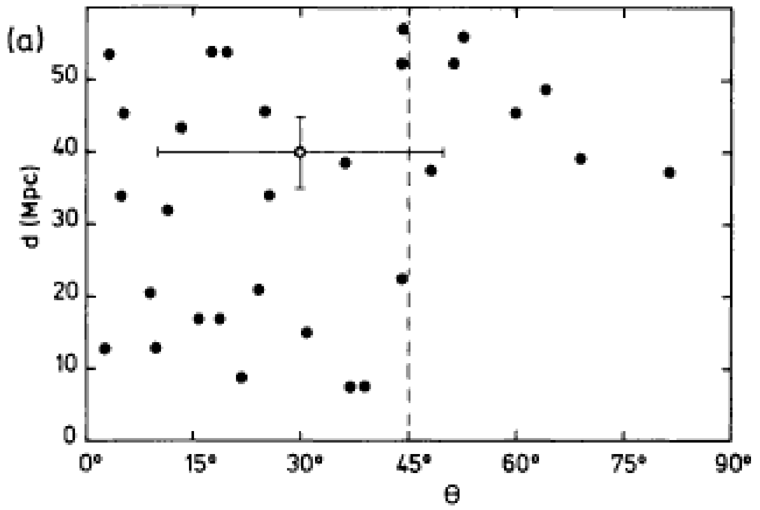

Figure 7: The difference between the cluster position angle and the position angle defined by the direction to the closest neighbouring cluster (x-axis), vs. the spatial distance to the closest neighbour (y-axis). From Binggeli (1982).

A major output of the first CfA survey was Huchra & Geller[235]'s catalogue of groups of galaxies. For the first time, groups were identified in 3-dimensions, as volume-density enhancements in the distribution of galaxies. Of the 176 catalogued groups, 74 were identified for the first time[174]. In those years, another famous catalogue of groups was created, Hickson[217]'s catalogue of 100 compact groups.

Meanwhile, astronomers started to use galaxy clusters as tracers of the Large Scale Structure of the Universe. Binggeli[62] showed the existence of cluster alignments on scales up to 45 Mpc – see Fig.fig-bbalign. The cluster correlation function was computed by Bahcall & Soneira[50] and Klypin & Kopylov[260], and shown to extend to 200 Mpc. Other useful tracers of the Large Scale Structure were found to be voids (Sharp[420]) and Lyman-α absorbers, which Oort[336] used for the first time to shed light on the clustering at very high redshift ($z > 2$).

In 1983 Abell[17] revised the properties of superclusters and suggested that they constitute the end of the clustering hierarchy, since their separations are comparable to their sizes, so that superclusters are interconnected. Shortly before his death, occurred on October 7th 1983, Abell[18] (together with Corwin) announced the preparation of the southern extension of his catalogue, a work that would keep busy his collaborators for six more years[19]. Abell's original catalogue was however to remain unsurpassed for the quality of the cluster richness estimates (see Girardi et al.[182]).

3 The cluster components

3.1 The morphology-density relation

It was probably Harold Shapley[413] in 1926 the first to explicitly refer to the different galaxy content of the Virgo and the Coma cluster, Coma being dominated by *"spheroidal"* galaxy types[g]. However, Shapley thought that with increasing resolution many apparently featureless spheroidals would turn out to be real spirals. Ten years after, in *The Realm of the Nebulæ*, Hubble first hinted at the existence of a morphology–density relation:

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[g] It was only in 1923 that Reynolds[372] pointed out the existence of many *"globular or ovoid"* nebulæ, distinctly different from spirals.

> *"There are some indications of a correlation between characteristic type and compactness, the density of the cluster diminishing as the most frequent type advances along the sequence of classification"*

Hubble also noted the *"dominance of late typed among isolated nebulæ in the general field"*. The morphology-density relation was immediately regarded as fundamental, to such a point that Tombaugh[461], in 1937, thought that a galaxy overdensity dominated by spirals could not be a real cluster. In the same year, Tombaugh noted that cluster ellipticals are more centrally concentrated than cluster spirals. In 1942 Zwicky[515] showed that S0s in Virgo are distributed like ellipticals and unlike spirals.

In 1960 van den Bergh[473] first noted the existence of a correlation between morphology and *local* galaxy density. By examining the Ursa Major and Virgo clusters, he noted that

> *"there is some indication that the nebular population type is related to the surface density of galaxies"*

In those years, de Vaucouleurs[131,132] (see also Abell[12]) suggested that spirals and ellipticals in Virgo have different distributions simply because they belong to different clusters. The morphology-density relation was thus reduced to a mere projection effect. An even more extreme view was taken by Neyman et al.[323] who maintained that the observed scarcity of spirals in clusters with respect to the field could be understood as *"a difference in the difficulty of observations"*!

In 1965 an extreme case of morphological segregation was discovered. Morgan & Lesh[312] noted that many clusters are centrally dominated by *"supergiant galaxies"*, that they called cDs. These galaxies were shown to live in the densest cluster environment only. Not only are cDs lacking in the field, but also in poor clusters and groups. In fact, the central dominant galaxies of the poor clusters classified by Morgan et al.[311], were later shown to lack the characteristic extended envelope of cDs (Thuan & Romanishin[458]).

In the 70's the number of available galaxy redshifts increased considerably, finally allowing a more reliable identification of cluster members. Rood et al.[385] were then able to identify 16 spirals as members of the Coma cluster. The idea that rich clusters are dominated by ellipticals and S0s was so firmly established that Rood et al.'s was considered a *"striking"* result.

In 1974 Oemler[331] published his seminal paper *The systematic properties of clusters of galaxies. I. Photometry of 15 clusters.* He noted that the morphological segregation in clusters depends on the cluster content. The morphology-density relation was interpreted as a relation between the morphological content of a cluster and its compactness. Oemler constructed galaxy number density profiles by type, and noticed a decreasing space density of spirals towards the cluster centres, except in spiral-rich clusters. He also noticed that spirals in cD-clusters have a shallower density profile than ellipticals at large radii. However, he could not notice any difference between the density profiles of S0s and ellipticals.

A year later, Gregory[194] showed that the fraction of spirals indeed increases with the distance from the Coma cluster centre. He wrote:

> *"The increase in relative numbers of spiral and irregular galaxies with radial distance seems incontestable. The effect is so strong as to be obvious to the eye on a casual inspection of the Sky Survey"*

Melnick & Sargent[302] confirmed Gregory's finding in other six X-ray bright clusters.

This tendency for ellipticals to be more clustered than spirals was shown by Davis & Geller[121] not to be restricted to clusters. They applied the 2-point correlation function to the Uppsala catalogue to show that morphological segregation exists on scales up to 6 Mpc. Four years earlier, in 1972, Takase[445] had already pointed out a colour segregation of galaxies on the scale of the Local Supercluster.

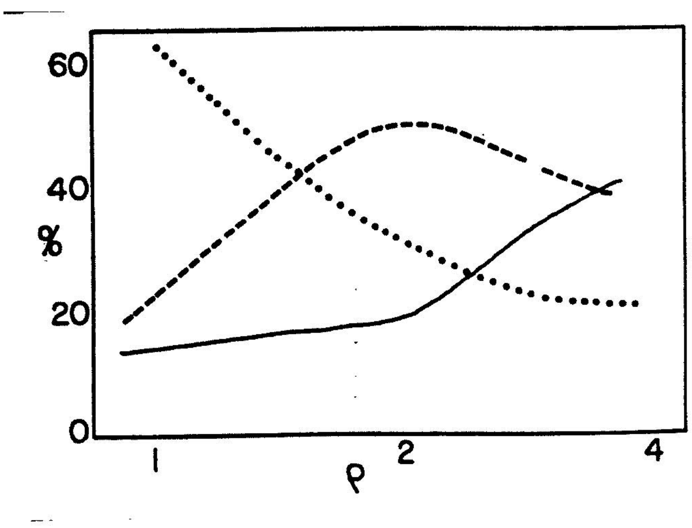

Figure 8: The variation of galaxy population with the mean density of clusters. Solid-line: ellipticals; dashed-line: S0s; dotted-line: spirals. From Oemler (1977).

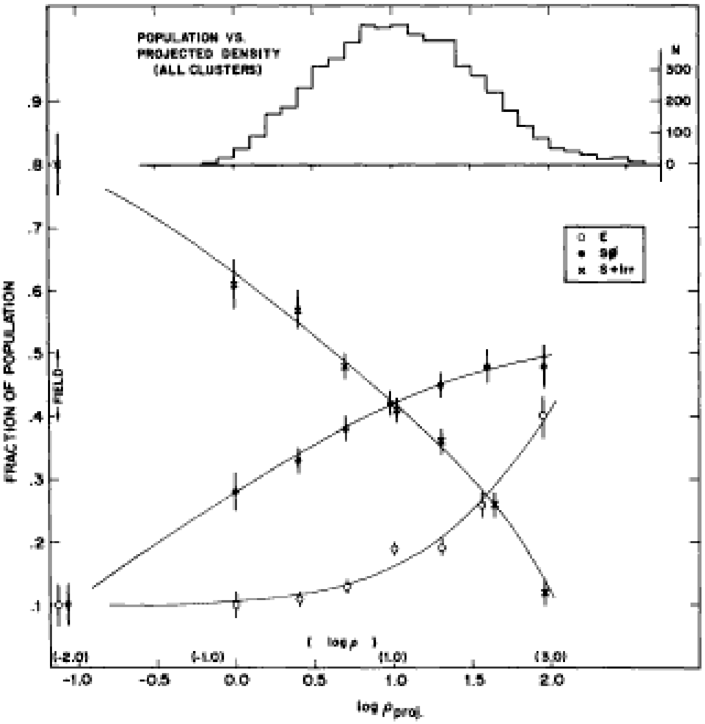

Figure 9: The fraction of E, S0, and S+I galaxies as a function of the logarithm of the projected density. The upper histogram shows the number distribution of the galaxies over the bins of projected density. From Dressler (1980a).

In 1977 Oemler[333] wrote that *"density is the physical significant parameter in determining the galaxy population of a cluster."* Figure 3 of his paper – here reproduced in Fig. 8 – is qualitatively very similar to Figure 4 in the 1980 paper of Dressler[142] – here reproduced in Fig. 9. Both figures show the fractional variation of spirals, S0s and ellipticals as a function of the cluster density. However, Oemler's density is the *mean cluster density,* and Dressler's density is the *local density* around each galaxy. Anyway, Oemler wrote (but did not show) that the same morphology-density relation was also verified individually in clusters dominated by early-type galaxies. The same year, even a spiral-rich cluster (Abell 262) was found to display a *"striking"* morphological segregation (Moss & Dickens[313]).

Times were mature for Alan Dressler's milestone paper, *Galaxy morphology in rich clusters: implications for the formation and evolution of galaxies*[142], published in 1980, and based on the evergreen *Catalog of morphological types in 55 rich clusters of galaxies*[143]. Dressler pointed out that: i) regular as well as irregular clusters display the same morphology-density relation; ii) it is not the radial distance, but the local density, the basic parameter which determines the morphology mix. Dressler's conclusions are still controversial nowadays (see, e.g. Sanromà & Salvador-Solé[397]), and it is possible that both global cluster properties *and* the local galaxy environment may play a role in determining the galaxy morphology[453].

In the two following years, Bhavsar[60] and de Souza[126] extended Dressler's morphology-density relation into the low galaxy density regime, through the analysis of loose groups.

3.2 Luminosity segregation

The idea that clusters form by gravitational clustering of field galaxies led Zwicky[513] (and others) to suggest that cluster galaxies are more massive than average, making their mutual gravitational attraction stronger. The most massive galaxies would cluster first, forming the cluster core, and other galaxies would follow. Assuming proportionality between a galaxy luminosity and its mass, Zwicky then thought that luminosity segregation must exist in clusters. Between 1942 and 1951 he found some evidence for it in Virgo[515], and in Coma[519]. At the same time he noted that also dwarf galaxies are clustered[h], an evidence later confirmed by Reaves[366] and Hodge[220].

In the sixties, Reaves[368] and Rood & Turnrose[388] showed that dwarf galaxies are less clustered than giant galaxies – see Fig. 10. Not much later, Rood[381] and Rood & Abell[384] noted that the bright peak in the luminosity function of Coma galaxies (first described by Shapley[416] in 1934), is not present in the outer regions of the cluster. This was interpreted as evidence for an excess of bright galaxies in the cluster core, i.e. luminosity segregation.

Oemler[331] noted an increase of the mean radius of cluster galaxies with galaxy magnitudes, another evidence for luminosity segregation, which was not seen, however, in spiral-rich clusters.

Capelato et al.[91] examined in detail the luminosity segregation in Coma, showing that it concerns the most luminous galaxies in a range of about 2 magnitudes. They also enlightened the role of the central cD in destroying the evidence of luminosity segregation through cannibalism, as originally suggested by Dressler[140].

Luminosity segregation also had opponents, like Noonan[326], Bahcall[40], and Sarazin[398], who suggested the evidence for luminosity segregation to be spurious, and mostly due to poor background subtraction. Recent analyses[69,73], based on cluster members only, show that luminosity segregation is indeed limited to the very bright galaxies only, $M_R < -22.6$.

3.3 Kinematical segregation

The issue of kinematical segregation also dates back to the 30's. Smith[425] pointed out that there was no evidence for bright and faint galaxies in the Virgo cluster to have different velocity

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[h] Reaves noted that the main problems for the identification of dwarf galaxies were their low surface brightness, and the fact that these galaxies *"resemble water spots and certain common emulsion defects"*.

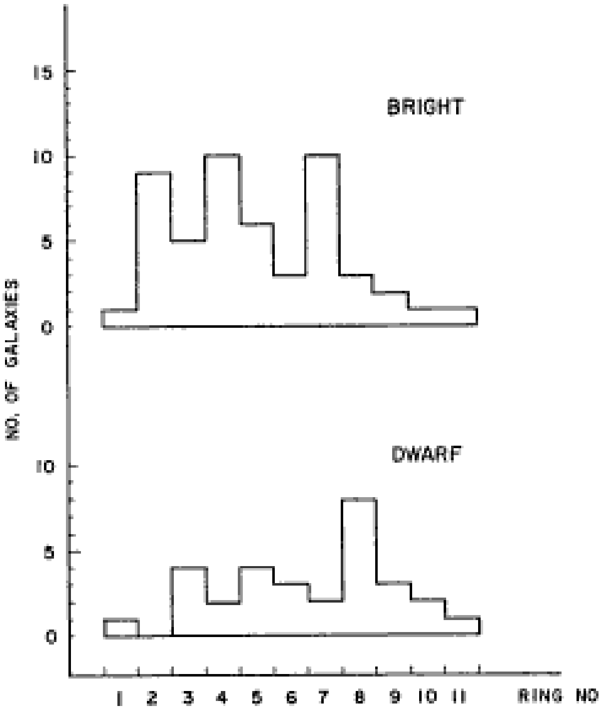

Figure 10: The radial distribution of bright and dwarf galaxies in the Coma cluster. From Rood & Turnrose (1968).

distributions, and so did Zwicky[513] for galaxies in the Coma cluster. The first evidence for kinematical segregation of cluster galaxies came from Holmberg[225] who, as early as in 1940, noticed that Virgo spirals had a larger velocity dispersion than Virgo ellipticals, thus anticipating Tammann[446]'s result.

Chandrasekhar[103]'s paper on dynamical friction showed how the more massive galaxies in a cluster could decelerate with respect to the less massive galaxies. However, a huge observational effort was needed before a clear evidence for kinematical segregation was established. In 1960, only 50 redshifts were known for galaxies in the Coma cluster, each obtained through $\simeq$ 2 hours exposures[297], leading Mayall to complain that the *"current rate of less than 10 velocities per year is impracticably slow"*.

In 1964, Zwicky & Humason[529] had obtained 42 galaxy redshfits in the cluster Abell 194. They claimed that the 21 brightest galaxies had a higher velocity dispersion than the 21 faintest. Reanalyzing their data with a biweight estimator[56] proves their result was correct. In fact, there is a difference of 200 km/s between the velocity dispersions of the bright and faint samples, and this is significant at the $\sim$ 95 % level. The conclusions of Zwicky & Humason were confirmed 13 years later by Chincarini & Rood[108], on a slightly larger sample of 57 redshifts for cluster members. Meanwhile, in 1972 Rood et al.[385] had shown the velocity dispersion of bright galaxies in the Coma cluster core to be as low as 231 km/s.

In the same year, Tammann[446] put Holmberg's early result on solid bases, by analyzing a sample of 122 Virgo cluster members with available velocities. Tammann showed that the velocity dispersion of Virgo spirals was 40 % higher than that of ellipticals and S0s. Tammann's result was extended by Moss & Dickens[313] to clusters in general. Moss & Dickens showed that the velocity distribution of ellipticals and S0s is broader than that of spirals not only in Virgo, but also in Abell 194, 262, and 1367 – see Fig. 11. Kent & Gunn[252] later found the same effect in Coma.

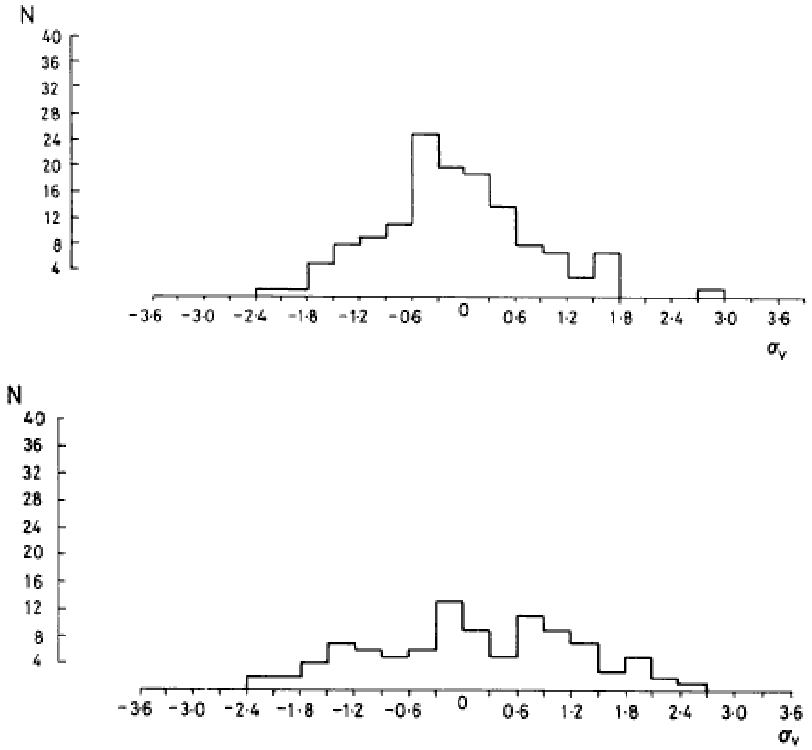

Figure 11: The combined velocity distribution of ellipticals and S0s (top panel) and spirals (bottom panel) in five clusters. From Moss & Dickens (1977).

Struble[437] considered 13 galaxy clusters, each with at least 30 galaxy redshifts, up to a maximum of 325 in Coma. Using the variance-ratio test he showed that there was no evidence for kinematical segregation with luminosity, except in Coma. Since Abell 194 was among the clusters he considered, his result was at odds with those of Zwicky & Humason[529] and Chincarini & Rood[108]. Struble noticed that several clusters have a lower velocity dispersion in their cores, and interpreted it as a product of cannibalism and/or dynamical friction, a scenario that still holds[22].

Thanks to the huge observational effort of the 70's, in 1980 there were more than 800 Virgo cluster galaxies with available redshifts. Using this sample, Hoffman et al.[222] constructed the velocity dispersion profile of the Virgo cluster, for spirals and early-type galaxies separately. Not only the velocity dispersion of spirals was confirmed to be higher than that of ellipticals and S0s, but also the shapes of the velocity dispersion profiles were different. By looking at Figure 9 in Hoffman et al.'s paper – here reproduced in Fig. 12 –, we can notice that the velocity dispersion profile of spirals is significantly steeper than that of early-type galaxies. It almost took 20 years to extend the validity of such a result to clusters in general (Adami et al.[22]).

3.4 Star formation in cluster galaxies

The first to notice the small spread of the colours of cluster galaxies was Baade[35] in the 30's. Such a small spread was related to the predominance of ellipticals and S0s among cluster galaxies, and the existence of a tight colour-magnitude relation, discovered by Baum[54] – see Fig. 13 – and de Vaucouleurs[133] around 1960, and refined by Visvanathan & Sandage[487] in 1977. Recently, Stanford et al.[431] confirmed the validity of the colour-magnitude relation also for distant clusters ($z \simeq 0.9$). They also showed that the relation is one between the mass and the metallicity of galaxies. The tightness of the colour-magnitude relation and its mild evolution with

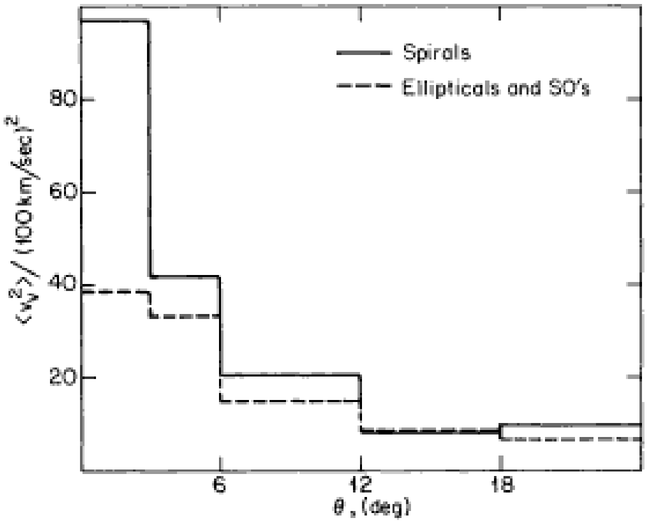

Figure 12: The velocity dispersion profiles for ellipticals and S0s (dashed line) and spirals (solid line) in the Virgo cluster. From Hoffman et al. (1980).

redshift indicate that most cluster ellipticals (and S0s) have formed at high redshifts, and they evolve passively through the aging of their (old) stellar populations (see, e.g., DICKINSON, these proceedings).

As far as cluster spirals are concerned, it was Erik Holmberg[228], in 1958, the first to notice that Virgo spirals are redder than field spirals. His result was confirmed by Chester & Roberts[104], Davies & Lewis[120] and van den Bergh[478] around 1970, and later interpreted[479,251] as a decreased star formation rate in cluster spirals.

In 1973, Davies & Lewis[120] analyzed the HI-content of 25 Virgo galaxies and showed it to be 60 % lower than in field galaxies, on average. Three years later, van den Bergh[479] coined the term *"anemic spirals"* to indicate a class of galaxies with intermediate characteristics between normal spirals and S0s. He attributed their anemic appearance to a reduced star formation rate, probably a result of their HI-deficiency. A reduced star formation rate could also naturally explain the redder colours of Virgo spirals, an interpretation later supported by Kennicutt[251].

In following years, Davies & Lewis' result was generalized to other clusters by Sullivan and collaborators[440,439], Giovanelli et al.[181], Chincarini et al.[106], and Giovanelli & Haynes[180]. These authors also showed that HI-deficient galaxies preferentially occur in high-density regions, i.e. the rich cluster cores. A recent update on this topic can be found in SOLANES (these proceedings).

3.5 Density and velocity dispersion profiles

It was Zwicky[516], in 1942, the first to propose an analytical form for the spatial distribution of galaxies in clusters, i.e. Emden's model for a bounded isothermal gas sphere – see Fig. 14. In 1954, Shane & Wirtanen[412] found that the surface brightness profile of galaxy clusters could also be fitted with the distribution function proposed by de Vaucouleurs[127] as a fit to the surface brightness profile of elliptical galaxies. As a matter of fact, the similarity of the profiles of ellipticals and the Coma cluster had already been noted by Zwicky[513] in 1937. In 1962 Abell[12] pointed out that equally good fits could be obtained using distribution formulæ different from Emden's. The fact that Emden's model fit the data well could not be taken as evidence that clusters are isothermal spheres. One year later, as to support Abell's conclusions, King[254] published his empirical density law for star clusters which proved very successful in describing cluster density

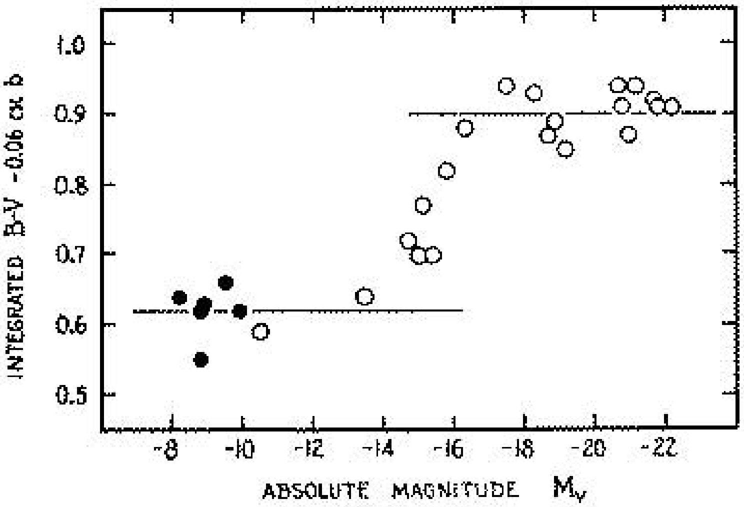

Figure 13: Intrinsic colour indices of old stellar systems as a function of their absolute magnitudes. The circles represent elliptical galaxies, and the dots globular clusters.

profiles as well.

One of the assumptions of all these models, spherical symmetry, was called into question when Matthews et al.[295] and Sastry[400] noted that the major axis of the central giant galaxy was aligned with the galaxy distribution in cD clusters, thus anticipating the results of Carter & Metcalfe[97] and Binggeli[62]. Moreover, The NE–SW elongation of the Coma cluster was remarked upon by Bahcall[40], Schipper & King[403], and Thompson & Gregory[454]. Things complicated even further when Sharov[419], Omer et al.[335], and Clark[110] found evidence for secondary peaks in the density profiles of several clusters. Their findings were later confirmed by Oemler[331]. In 1978 Dressler[140] proposed subclustering as an explanation for irregularities in the density profiles.

Another assumption of Zwicky's model was isothermality, an hypothesis supported by an early plot of the Virgo galaxy velocities vs. clustercentric distances (Smith[425]). The validity of Zwicky's assumption was shattered in 1960 by Mayall[297]'s diagram of velocities vs. radii for 50 galaxies in the Coma cluster. In this diagram – here reproduced in Fig. 15 – one could clearly see a decrease of the velocity dispersion with radius. A similar trend was later found by Karachentsev[247] for the Virgo cluster. On the other hand, Zwicky & Humason[529] found a flat velocity dispersion profile in Abell 194.

In 1971 Chincarini & Rood[107] showed the Perseus cluster to have a decreasing velocity dispersion profile, and one year later Rood et al.[385] confirmed Mayall[297]'s early suggestion that the Coma cluster velocity dispersion profile is a decreasing function of the clustercentric distance. These early measurements of the Perseus and Coma velocity dispersion profile, were later refined by Kent & Sargent[253] and, respectively, Kent & Gunn[252], who confirmed deviation from isothermality. The velocity dispersion profiles of galaxy clusters were classified into four different types by Struble[437]. He showed that isothermal profiles are not a common feature of all clusters.

Density and velocity dispersion profiles have now been obtained for the different galaxy populations[22,73]. Velocity dispersion profiles are certainly not isothermal, and are different for different galaxy populations[73], so that the global velocity dispersion profile of a cluster changes according to its galaxy morphological mix. So far, no analytical model has been proposed for the cluster velocity dispersion profile. Recently Navarro et al.[318] have proposed a new analytical model for the cluster density profiles, which is now extremely popular. Consistency has been found between this new model and the data, but, once more, other models provide equally good fits to the data[93].

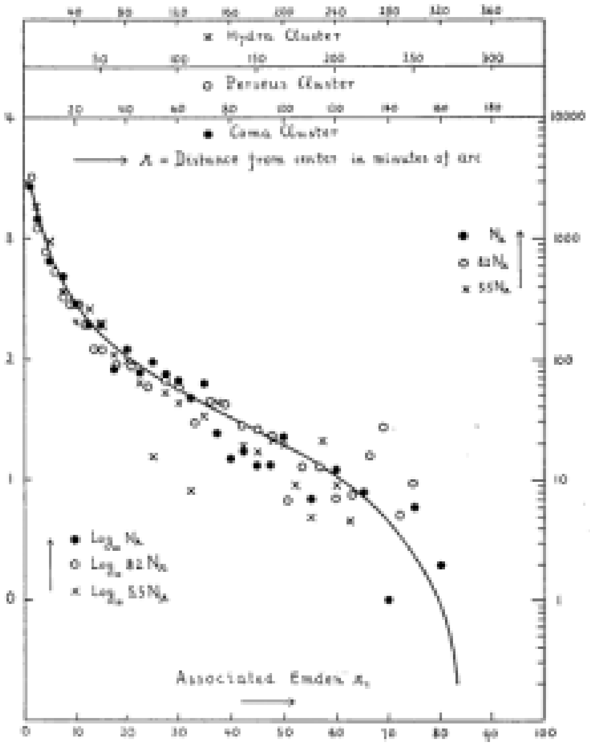

Figure 14: The number of nebulæ per square degree vs. the distance from the cluster centre and the best fit Emden model (solid line). From Zwicky (1942b).

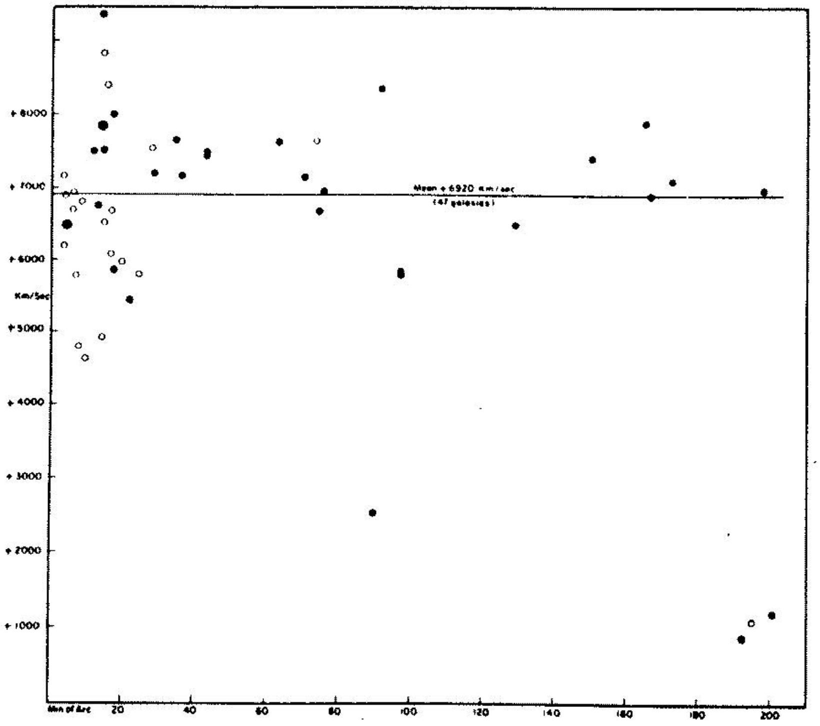

Figure 15: The velocities of galaxies in the Coma region vs. clustercentric distances. The distances range from 0 to 200', the velocities from 0 to 9000 km/s. From Mayall (1960).

3.6 The hot IC gas

It was Limber[280] in 1959 the first to suggest that diffuse gas must be present among galaxies, and clusters be filled with a hot IC diffuse gas component. He argued that galaxy formation from gas cannot be 100 % efficient, and some gas must be lost from galaxies through collisions. The first detection of an X-ray source associated with a cluster of galaxies came from Byram et al.[88], in 1966. They detected M 87, the central giant galaxy of the Virgo cluster. In the same year, Boldt et al.[75] claimed detection of the Coma cluster in X-ray. It took just one year to Friedman & Byram[170] to show that Boldt et al.'s detection was spurious. However, Boldt et al.'s spurious result inspired Felten et al.[162]'s correct theoretical estimate. Felten et al. estimated that a thermalized diffuse gas in the Coma cluster should have a temperature $\simeq 7 \times 10^7$ K, and would therefore emit in the X-ray via thermal bremsstrahlung.

In 1971, Cavaliere et al.[100] suggested that many extragalactic X-ray sources are probably associated with clusters of galaxies. The same year, the extended X-ray emission from the Coma IC gas was detected, by Meekins et al.[299], with observations from an *Aerobee* 150 rocket, and, independently, by Gursky et al.[201], with the *Uhuru* satellite. Thanks to *Uhuru* many more clusters were X-ray detected, and as early as in 1972, Gursky et al.[202] suggested that

> *"most, if not all, rich clusters include an X-ray emission region of large size and of net luminosity* 10^{43}–10^{44} *erg* s^{-1}*"*

A first indication about the nature of the diffuse cluster X-ray emission came from Solinger & Tucker[427] in 1972, with an early indication of a correlation between the X-ray luminosities of clusters and the velocity dispersions of their member galaxies. Such a correlation is naturally expected if the gas is thermalized, in equilibrium with the cluster gravitational potential, and the emission mechanism is thermal bremsstrahlung. This correlation was later improved by Cooke & Maccagni[111].

Always in 1972, Syunyaev & Zel'dovich[442] proposed *The observation of relic radiation as a test of the nature of X-ray radiation from the clusters of galaxies.* Immediately after, an over-enthusiast Parijsky[347] gave a start to a series of spurious detections of the *Syunyaev–Zel'dovich effect.* Other early controversial detections were claimed by Gull & Northover[198], Lake & Partridge[265,266], Birkinshaw et al.[65,64], all regarded with much scepticism by theorists (Gould & Rephaeli[193], Tarter[450]). White & Silk[497] noted that the combined X-ray and microwave observations of Abell 576 would have implied an improbable value for the Hubble constant of $\simeq 1.5$ km s^{-1} Mpc^{-1}!

There has been an impressive observational progress in this field over the last decade. Nowadays, the rate of reliable Syunyaev–Zel'dovich detections of clusters is very high, and techniques allow Syunyaev–Zel'dovich imaging of galaxy clusters (see CARLSTROM, these proceedings).

In 1973, Lea et al.[276] analysed the distribution of the IC gas and showed the gas to be less centrally concentrated than galaxies. Their model of the IC gas distribution was the first of a long series[272,197,48], among which the β-model of Cavaliere & Fusco-Femiano[98,99] proved the most successful. Lea et al.[276]'s result was confirmed by Bahcall[41], and by Gorenstein et al.[186], who estimated the slope of the galaxy number density profile in Coma to be twice the slope of the gas density profile. Bahcall[41] also showed that the peak of the diffuse X-ray emission coincides with the centre of the galaxy distribution, or with the position of the cD galaxy.

Bahcall[41] started a systematic comparison of optical and X-ray cluster properties. She found richer clusters to be more likely associated with X-ray sources, and cD-type clusters to have higher X-ray luminosities. On the other hand, she confirmed Kellogg et al.[250]'s result that clusters of a given richness class span a wide range of X-ray luminosities. Later, she found a relation between the fraction of spirals in clusters and the X-ray luminosity[42].

Wolff et al.[503] were possibly the first to record a deviation of the X-ray surface brightness distribution from spherical symmetry. They showed the X-ray emission of Perseus to be elongated

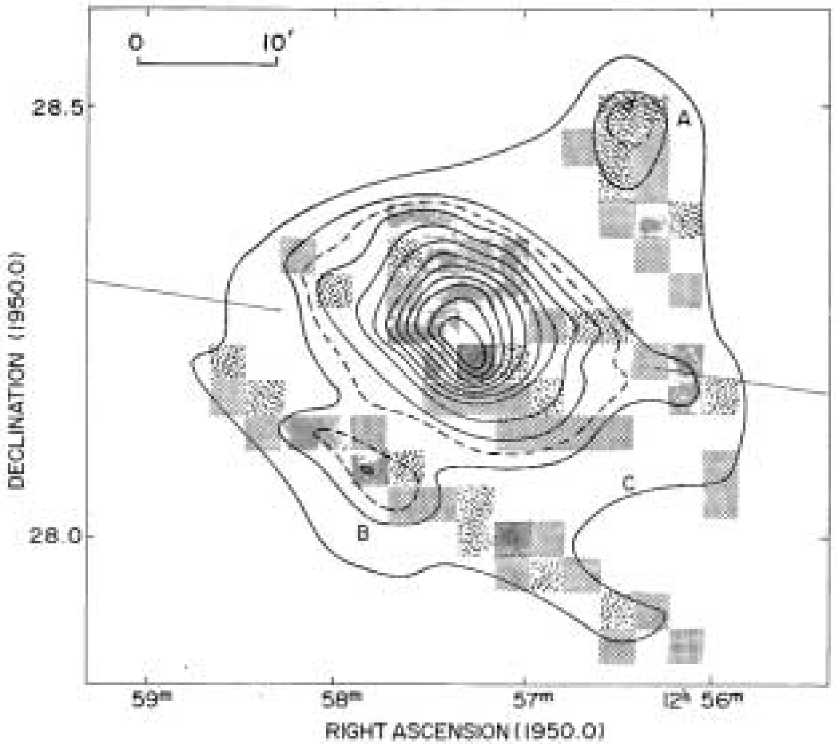

Figure 16: The Coma cluster X-ray brightness distribution, according to two different reconstruction algorithms (contours and boxes). The straight line is the major axis of the galaxy luminosity distribution. From Johnson et al. (1979).

along the E–W direction, like the galaxy distribution. Some years later, in 1979, Gorenstein et al.[186], and Johnson et al.[243] found a good correspondence between the shape of the X-ray emission and the galaxy distribution in Coma – see Fig. 16. The *Einstein IPC* observations of Jones et al.[244] finally revealed all the complex cluster X-ray morphologies. The close correspondence between the X-ray emission and the galaxy distribution was interpreted by Gioia et al.[179] as evidence for equilibrium of both the IC gas and the cluster galaxies in the cluster gravitational potential.

The thermal bremsstrahlung interpretation received further support by the lack of detection of hard (>20 keV) X-ray emission from Coma and Perseus by Scheepmaker et al.[401]'s balloon-borne X-ray experiment. The thermal origin of the X-ray emission was finally demonstrated in 1976 and 1977, with the *Ariel V* detection of the 7 keV Iron line in Perseus and Centaurus by Mitchell et al.[309] and Mitchell & Culhane[308] (see Fig. 17), and with the analogous *OSO 8* detections in Virgo, Perseus and Coma, by Serlemitsos et al.[410]. In 1977, 30 clusters had been identified as X-ray sources, 10 of them with extended emission[116]. Mitchell et al.[310] and Mushotzky et al.[315] produced the first relations between the X-ray temperatures and velocity dispersions of eight, and, respectively, 13 clusters. With much scatter, these relations looked however consistent with $T_X \propto \sigma_v^2$ (where T_X is the X-ray temperature and σ_v the galaxy velocity dispersion), as expected if the X-ray emission was produced by an IC gas in equilibrium with the gravitational potential traced by cluster galaxies.

In 1980, Schwartz et al.[404,405] detected X-ray emission from poor clusters and compact groups, at temperatures consistent with the low velocity dispersions of their member galaxies. The nature of the X-ray emission from poor galaxy systems is still debated. Both the contribution of individual galaxies to the total emission and Supernova heating must be considered (see PONMAN, these proceedings).

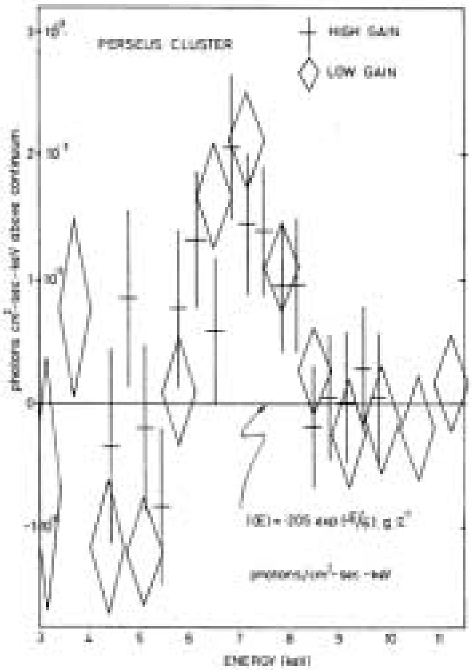

Figure 17: The deviation of the flux as a function of energy from the flux predicted by the best fitting single temperature continuum in the Perseus cluster. The Iron line feature is evident at around 7 keV. From Mitchell et al. (1976).

3.7 Radio components

The idea that clusters could be associated with extragalactic radio-sources dates back to 1960. At that time, it was generally thought that galaxy-galaxy interactions and merging were a pre-requisite for radio-source activity in galaxies. Spitzer & Baade[430]'s work had shown that collisions must be frequent among cluster galaxies. It was then quite natural to suggest that extragalactic radio-sources could be associated with galaxy clusters (Minkowski[307]). Rogstad et al.[377] however pointed out that radio-galaxies in clusters are often associated with cDs. Ko[261] estimated an average of only one bright radio-galaxy per cluster.

In their search for clusters of galaxies around radio-sources, Bahcall et al.[38] and Bahcall & Bahcall[37] found evidence for significant galaxy clustering around quasars at $z \sim 0.1$–0.2. In those years (the early 70's) the importance of this discovery was that it provided evidence for a common origin of the galaxy and the quasar redshifts. If the galaxy redshifts were cosmological, so were the redshifts of quasars. Rózyczka[394] extended the quasar–cluster association up to redshifts $z \sim 0.5$. In 1980 Stockton et al.[435] showed that while giant radio-galaxies are often found in clusters, quasars live in intermediate density environments, like galaxy groups.

A class of radio-sources that are exclusively found in clusters are the head-tail radio-sources. Immediately after the IC gas discovery by Meekins et al.[299] and Gursky et al.[201], Miley et al.[304] were able to model this peculiar radio morphology in terms of radio-trails of galaxies moving through the dense IC gas.

In 1959 Large et al.[267] detected the extended radio-source Coma C at 408 MHz, in the direction of the Coma cluster. Willson[498] showed Coma C to be a wide 40 arcmin diffuse emission, not originating from the integrated emission of individual galaxies. If located at the distance of the Coma cluster, the size of Coma C corresponds to 1.2 Mpc. For this reason, Willson named it *"the halo"*.

In those days, Coma was still considered as the typical cluster. However, it was soon clear that clusters with radio-halos are rare. Hanisch et al.[205,203] could list only four clusters with detected radio-halos, and Jaffe & Rudnick[241]'s extensive search for radio-halos in 32 clusters did not detect any. Eventually, two other cluster radio-halos were discovered in those years, by Harris & Miley[206] and Roland et al.[379].

Cluster radio-halos were as difficult to model, as they were to find. A first attempt was done by Jaffe[240], who suggested that the radio-halo could be created from the leakage of electrons out of radio-galaxies, but the model could not really account for the wide distribution of the radio-emission. Roland[378] proposed an *in situ* acceleration of relativistic electrons by magnetic field fluctuations generated in the wakes of moving galaxies. A hint to the nature of radio-halos came from their rarity. In 1979 Smith et al.[423] remarked that both Coma and Abell 2319 (two radio-halo clusters) have too high an X-ray temperature for their velocity dispersion. Three years later, Hanisch[204] and Vestrand[483] noted that the rare clusters harbouring a radio-halo have many other similar properties. These are: anomalous high X-ray temperatures for their galaxy velocity dispersions, low spiral contents, intermediate Bautz-Morgan types, large X-ray core-radii, smooth X-ray distributions, without the central peak typical of cD clusters. Hanisch and Vestrand suggested that the presence of a radio-halo could be related to a short-lived dynamical configuration, thus anticipating modern scenarios (see, e.g., FERETTI, these proceedings).

4 Structure

4.1 Subclustering

The uneven internal structures of clusters were recognized quite early on. By looking at Wolf[500]'s plot of the galaxy distribution in Coma it is easy to spot the south-western subcluster dominated by NGC 4839 – see Fig. 18. This was re-discovered by Shane & Wirtanen[412] in 1954, more than half a century later. The subcluster is clearly visible in their Plates no.303 and no.1613 – here reproduced in Fig. 19 –, and the authors suggested it could be a distant cluster seen in projection in the Coma cluster region. Shane & Wirtanen[412] classified clusters in two broad classes: regular Coma-like and irregular Virgo-like clusters. The uneven structure of the Virgo cluster had of course been noticed very early (e.g. Zwicky[513]). However, it is remarkable that subclustering in the prototype regular cluster was also noticed very early, but apparently ignored until being re-discovered in the X-ray[495]. A telling example is that of Oemler[332]. In 1976 he remarked that the giant galaxy NGC 4839 was quite an exception in his class, because there was not *"any evidence of clustering of galaxies around NGC 4839"*!

The first systematic analyses of subclustering in galaxy clusters date back to the early 60's. Sydney van den Bergh[474,476] analyzed the distribution of velocity differences among pairs of galaxies in the Virgo and Coma clusters. He compared the observed distributions to those obtained from azimuthal scramblings of the data-sets – see Fig. 20 – and found evidence for subclustering in both clusters, on ~ 0.1 Mpc scales: *"Taken at face value, this result implies that subclustering occurs in the Coma cluster."* Abell et al.[13] analyzed eight clusters and found evidence for subclustering in six of them, but not in Coma. However, Abell[14] remarked that accounting for the presence of subclusters could not remove the mass discrepancy problem (see § 4.2).

In 1973, Bahcall[40] first noticed the existence of substructures around the two central dominant galaxies of Coma, NGC 4874 and NGC 4889. Her result was later confirmed by Rood[382], and refined, many years later, by Perea et al.[357], Fitchett & Webster[163], and Mellier et al.[300]. Bahcall also suggested that these subclusters should be detectable as X-ray sources, independent from the cluster itself, a suggestion confirmed by Vikhlinin et al.[485] 21 years later.

According to Dressler[140], another evidence for subclustering was given by the secondary peaks detected in the density profiles of several clusters[419,335,110].

Subclusters became theoretically appealing after White[491]'s n-body simulations showed that

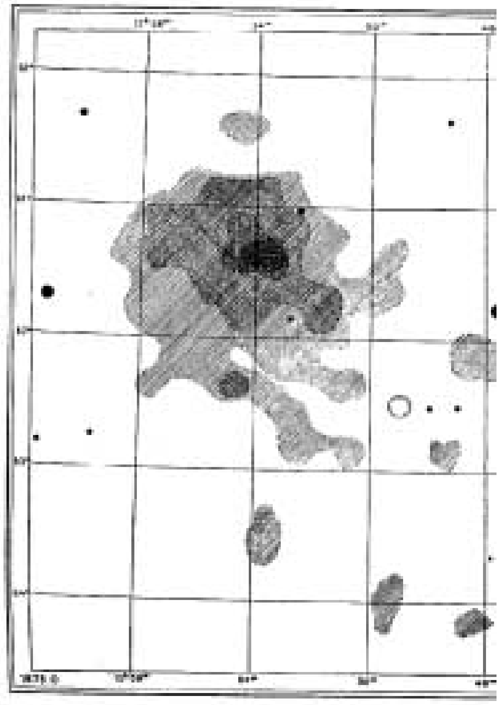

Figure 18: The density of nebulae in the region of Coma, according to Wolf (1901). Note the south-western extension (north is up, east is to the left). Every grid element is 28'×60'.

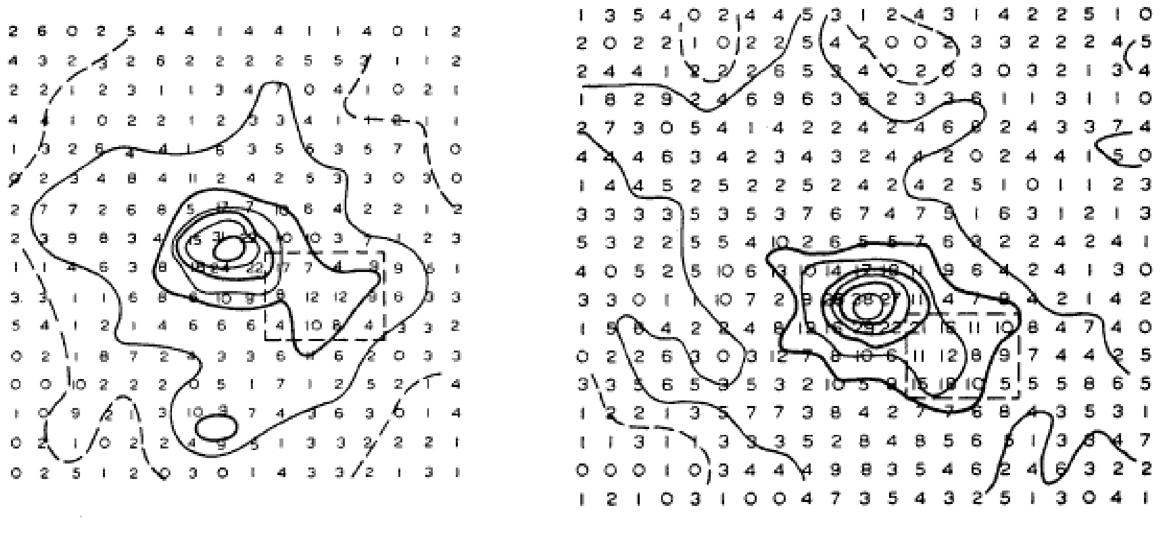

Figure 19: Contour maps of the Coma cluster of nebulæ, based on smoothed counts by 10' squares. Plate n.303 is on the left and no.1613 is on the right. From Shane & Wirtanen (1954).

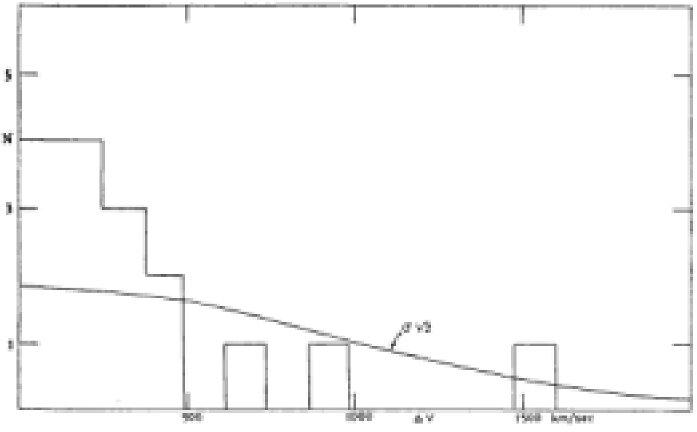

Figure 20: The observed distribution of velocity differences of pairs of galaxies in Virgo with separation smaller than 10', compared to the expected distribution for optical pairs. From van den Bergh (1960b).

"clusters form by the progressive amalgamation of an inhomogeneous system of subclusters".

Thanks to the increasing angular resolution of X-ray observations, subclusters started to be found also in this band. In 1979 Gorenstein et al.[186] attributed the granularity in the Coma cluster X-ray emission to subclustering, and a hint of the south-western subcluster could already be seen in Johnson et al.[243]'s X-ray map of Coma. A major breakthrough came with the *Einstein IPC* images of Jones et al.[244]. They showed that the X-ray morphologies of clusters, far from being smooth and spherically symmetric, were quite often irregular and clumpy. Subclustering was a common feature of galaxy clusters!

In 1982 Geller & Beers[173] draw density-contour maps of the galaxy distributions in 65 clusters and identified subclusters in 40 % of them. The techniques for the detection of subclusters have considerably improved in more recent years, but subsequent works have roughly confirmed this fraction[147,155]. With gravitational lensing techniques it is now possible to look for sub-condensations directly in the mass distribution, and the existence of dark subcluster has been suggested (see Kneib, these proceedings).

4.2 Mass

In the 30's Hubble & Humason, aiming at a high-redshift extension of the velocity–distance relationship, measured several velocities for galaxies in clusters. In 1931, they[234] provided the first estimates of the velocity dispersions in four clusters of galaxies[i]. Hubble & Humason noted that the velocity range spanned by Coma galaxies was larger than in other clusters (Virgo, Pegasus, Pisces). This was a first hint of the relation between richness and velocity dispersion that Bahcall[45] later established in 1981. Hubble[233]'s early estimate of the cluster velocity dispersion was $\simeq$ 700 km/s – see Fig. 21, from Smith[425] –, a value remarkably close to modern estimates[182]. Zwicky[512,513] immediately saw the great potentiality of Hubble & Humason's data, and used them for deriving the mass of the Coma cluster, via the application of the virial theorem[j]. Smith[425] followed Zwicky and derived the virial mass of the Virgo cluster.

Zwicky[513]'s milestone paper: *On the Masses of Nebulae and of Clusters of Nebulae*, published in 1937, is an exceptional work. In that paper, Zwicky correctly noticed that the masses of nebulæ, derived from rotation curves, are underestimated. By assuming, *"as a first approximation"*,

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[i] Hubble & Humason were interested in cluster velocity dispersions because they wanted to estimate the uncertainties in the cluster mean velocities, which were relevant to the velocity–distance relationship.

[j] The virial theorem had been first used in astronomy by Poincaré in 1911.

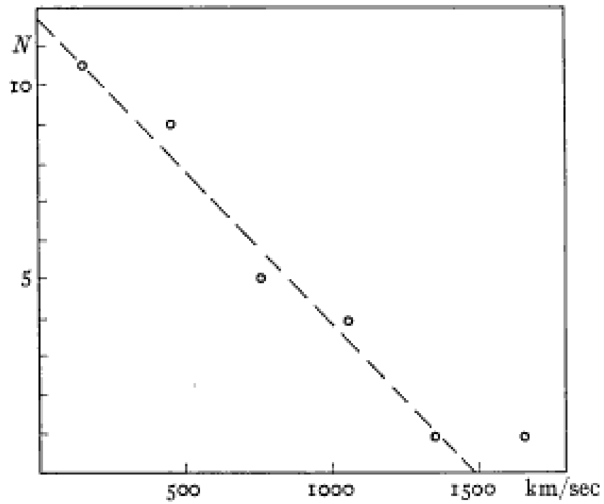

Figure 21: The distribution of velocities of Virgo cluster galaxies. From Smith (1936).

that clusters of nebulæ are stationary systems, and using the virial theorem, he derived a very conservative estimate of the Coma cluster mass. This implied a cluster mass-to-light ratio of 68 $M_\odot/L_\odot$ (after conversion to a modern value of the Hubble constant). Zwicky had discovered the *missing mass* problem.

His discovery relied very much on the hypothesis of cluster stability. In support of his hypothesis, Zwicky noted that galaxies in the field have a much lower velocity dispersion than galaxies in clusters. This indicated that field galaxies could not origin from cluster disruption, or they would have much higher velocities than observed. In this context, Zwicky implicitly criticized the work of Smith[425], and emphasized the danger of applying the virial theorem to irregular systems of galaxies, which are not likely to be stable systems. Because of the possible biases inherent to the virial mass estimates, Zwicky suggested to use gravitational lensing as the *"simplest and most accurate mass determination"*. He was half a century in advance of observations[289,428]!

Smith[425]'s paper essentially followed in the steps of Zwicky[512], but was published one year before the English version of Zwicky[513]'s paper, and not surprisingly Hubble[233] quoted Smith and not Zwicky (although Zwicky was quoted by Smith himself). Hubble remarked that galaxy mass estimates were likely to be lower limits, while virial theorem estimates of cluster masses were likely to be upper limits, so that eventually the two might come into agreement. As a matter of fact, Zwicky's and Smith's estimates of the Coma and, respectively, Virgo masses, were quite correct, or, if anything, too low (Zwicky having tried to be conservative). Anyway, a straightforward application of the virial theorem was not without problems. In 1959 Limber[280] obtained a more general expression for the virial theorem, in order to account for the possible presence of diffuse IC matter. Much later Nezhinskii & Osipkov[324] showed that the uncertainties in the virial mass estimates are much larger than generally assumed if the diffuse matter is not distributed like galaxies, and dominates the potential. However, as it turned out, the cluster virial mass estimates were essentially correct, and it was the galaxy mass estimates which had to be revised upwards.

Holmberg[225] was possibly the first to criticize Zwicky's dark mass hypothesis, that he considered an *"unlikely assumption"*. He attributed the high velocity dispersion of cluster galaxies to the presence of a large number of galaxies on hyperbolic orbits, i.e. interlopers. In 1954 Schwarzschild[406] tried to get rid of *"interlopers"* to improve the estimate of the Coma cluster velocity dispersion. After eliminating many supposed interlopers from the Coma cluster sample

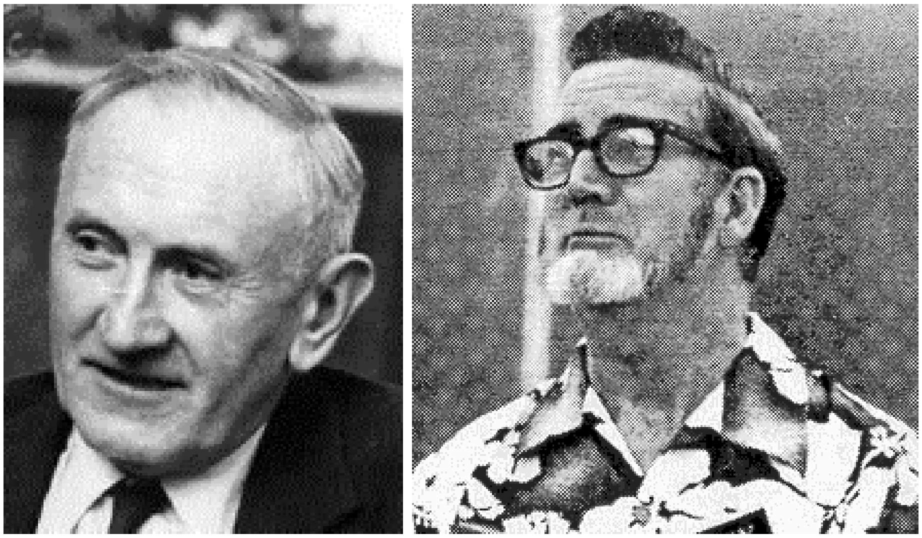

Figure 22: Portraits of Fritz Zwicky (left) and George O. Abell.

(far too many, in fact) he came to the wrong estimate of 630 km/s for the velocity dispersion of the Coma cluster. Some years later Abell[11] pointed out that the existence of superclusters enhances the probability of projection effects, leading to overestimate the cluster velocity dispersions. In 1977 Yahil & Vidal[507] devised a method for getting rid of interlopers in galaxy clusters that remained in use until recently[182].

Schwarzschild's estimate was too low, yet not enough to solve the discrepancy between the mass-to-light ratios of clusters and those of individual galaxies, or galaxy pairs. Page[345] had just found that galaxy pairs have a much lower mass-to-light ratios than clusters. Of course, estimating the masses of galaxy pairs was not simpler than estimating the masses of clusters[k], as Limber[281] pointed out. Despite the intrinsic uncertainties due to poorly controlled selection biases, Page's work strongly influenced the astronomical community, leading to a diffuse scepticism towards the cluster mass estimates. Interestingly, however, the nearest galaxy pair (M 31 and the Milky Way) was shown in those years to display the same missing mass problem of clusters (Kahn & Woltjer[246]). The mass estimate of Kahn & Woltjer relied on the simple assumption that M 31 and the Milky Way are on a bound orbit. Apparently, Kahn & Woltjer were unaware of Zwicky's and Smith's results on the mass of galaxy clusters.

Around 1960, Ambartsumian[28,29] reversed Zwicky's hypothesis on the stability of clusters. According to Ambartsumian, the large velocity dispersions of clusters indicate they have positive total energy, i.e. they are disintegrating, and missing mass is not needed. In those years astronomers were discovering the wild world of radio-galaxies, with their jets, suggestive of a mechanism to emit matter out of galaxies. Similarly, interacting galaxies looked to many as the result of a fragmentation process rather than the result of encounters. Somewhat later, Noerdlinger[325] invoked quasars as the source of the energy leading to the cluster disruption. Ambartsumian's hypothesis became quite popular in the astronomical community because

> *"unless one is prepared to make wild hypotheses outside the realm of verification by direct observation* [...] *the 'hidden-mass' hypothesis must be ruled out"* (de Vaucouleurs[130])

The stability of groups and irregular clusters started to be questioned. Zwicky[517,518] insisted on the stability of clusters, even the Cancer cluster, which Bothun et al.[78] much later proved to be just *"an unbound collection of groups"*. On the other hand, the Burbidge's[83] suggested

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[k] The work of Page required 165 hours of observations!

that the Hercules cluster was just an unbound collection of groups, but in fact it is not, it is only rich in substructures[66]. de Vaucouleurs[130,131,132] suggested that groups might result from random encounters of unbound field galaxies. He also provided marginal evidence that Virgo was not a single dynamical unit, but two different clusters seen in projection. His hypothesis was turned down first by Kowal[263] who used Supernovæ to estimate the distances of Virgo galaxies, and then by Sandage & Tammann[396] who used a much larger sample of Virgo galaxy velocities. Finally Helou et al.[210] closed this issue by determining the relative distances of galaxies in Virgo with the Tully-Fisher relation[465].

At variance with irregular clusters and small groups, the stability of Coma was never in question, given the high degree of symmetry and regularity of this cluster. This implied that the Coma cluster contains a large quantity of unseen mass, and so *"why should not the others?"* (Burbidge & Sargent[84]). Abell[14] used the cluster virial mass estimates to provide an estimate of the mean density of the Universe, $\Omega_0 \simeq 0.1$.

A possible solution to the missing mass problem was to revise the estimates of cluster velocity dispersions. Internal subclustering was known to be a potential source of error in the velocity dispersion estimates[l]. However, subclustering in Coma took long to be recognized, and Abell[14] pointed out that the correction for subclustering, while important, was nevertheless too small to get rid of the missing mass (Ozernoy & Reinhardt[342] later came to the same conclusion). Godfredsen[184] and Holmberg[229] suggested that the cluster velocity dispersion estimates were boosted up by large errors in the galaxy velocities. Their hypothesis was rejected by de Vaucouleurs & de Vaucouleurs[134] and, later, by Kirshner[256], who found a similar mass discrepancy in groups, despite a considerable improved determination of galaxy velocities. Finally, Rood[381] pointed out that an *a-priori* assumption of isotropic galaxy orbits could lead to overestimate a cluster velocity dispersion, if these orbits were instead mainly radial.

In the early 60's Burbidge & Burbidge[83,81] and Limber[281] advanced the major argument in favour of the stability of galaxy clusters. If clusters have positive energy, the time-scale for their disruption is very short. Clusters must therefore be young systems. However, clusters are populated by ellipticals, which are old galaxies, as inferred from their stellar populations. This argument seemed ironclad, yet many astronomers still preferred to question the old age of ellipticals (and the models of stellar evolution), rather than accepting the existence of dark matter (see, e.g., Neyman et al.[319])!

After 1965 the growing evidence for dark matter in single galaxies started to change the situation. As early as in 1939 Babcock[36] had shown that the rotation curve of M 31, as measured in the optical, was still raising at the last measured point. But the observational evidence for non-Keplerian galaxies rotation curves really came from radio-observations. In 1965 Seielstad & Whiteoak[409] noted that the turn-over radii of the galaxy rotation curves were larger when measured in the radio than when measured in the optical. More 21cm measurements accumulated, in particular through the work of Roberts[375] and Roberts & Rots[376]. In 1969 Vorontsov-Velyaminov[489] argued that the 21cm measurements indicated flat rotation curves for galaxies and Freeman[168] and Lewis[278] suggested that this implied an increasing mass-to-light ratio with radius. Arp & Bertola[32] and de Vaucouleurs[136] argued for a high mass of the giant elliptical M 87, a suggestion later confirmed by Fabricant et al.[159]. Hunt & Sciama[239] suggested that the brighter galaxies may have X-ray coronæ, a prediction later confirmed by Mathews[292]. In 1973, Ostriker & Peebles[339] argued for the need of a massive halo to stabilize the spiral disks.

Progress was also being made in the dynamical modeling of galaxy systems. In 1970 Allen[27] derived a velocity-independent distance for NGC 7320, based on the hydrogen-mass to optical-luminosity ratio. He found that this galaxy lies at a different distance from other galaxies of the Stephan's quintet, thus reducing the mass discrepancy in this system. On the other hand, the

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[l] A detailed account of the topic of subclustering is given in § 4.1.

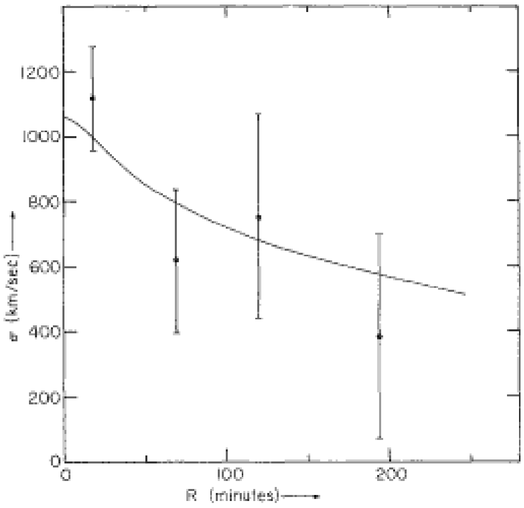

Figure 23: The Coma cluster velocity dispersion profile. A model with isotropic galaxy orbits is also plotted. From Rood et al. (1972).

n-body simulations of Aarseth & Saslaw[6] indicated that the group masses were underestimated by the use of the virial theorem, thus anticipating the conclusions of Tully[464], and Giuricin et al.[183]. A few years later, Geller & Peebles[175] obtained a robust statistical estimate of the masses of groups, and showed that interlopers cannot cause the whole of the mass discrepancy problem. Gott et al.[192] and Turner & Sargent[468] however argued that only a fraction of all groups are bound, and of these, very few are virialized.

In 1966 Aarseth[2]'s simulations had established that a cluster in equilibrium should be characterized by a Gaussian distribution of galaxy velocities. Six years later Rood et al.[385] proved the velocity distribution of galaxies in the Coma cluster to be Gaussian, lending support to the idea that the Coma cluster was a stable dynamical system. Using a larger data-set, they confirmed Mayall[297]'s earlier suggestion that the velocity dispersion of Coma decreases with increasing radius. Previously, a similar trend in the Virgo cluster had been explained by Karachentsev[247] as an indication of the expansion of the cluster. Rood et al. instead correctly pointed out that the decreasing velocity dispersion profile was due to the finiteness of the cluster. They fitted the profile with a model where galaxies on isotropic orbits trace the mass distribution – see Fig. 23.

Despite the observational and theoretical progress, still in the early 70's the general feeling of the astronomical community about the dark matter issue was quite negative. As an example, here are Chincarini & Rood[107]'s conclusions from their 1971 paper on the dynamics of the Perseus cluster:

> *"We are not inclined to admit this possibility of adequate intergalactic mass in the cluster* [. . .] *The large 'mass' of the Perseus cluster therefore is explained with difficulty if the cluster is bound, and may suggest instability"*

Another telling example is the obituary of Fritz Zwicky, written by Cecilia Payne-Goposchkin[348] in 1974. Many of Zwicky's major contributions to astrophysics were mentioned, but not the discovery of dark matter.

I do not know how Zwicky managed to change astronomers' minds from Heaven. It is a fact, however, that only a few months after his death, Einasto et al.[153] and Ostriker et al.[340] published two papers that catalyzed a paradigm change in favour of the existence of dark matter in the Universe. Einasto et al. and, independently, Ostriker et al. summarized the evidence supporting the existence of galaxy dark halos, and argued that the mass-to-light ratio increases with scale, from galaxies to galaxy clusters. Despite some residual criticism from Burbidge[82], the existence of dark matter became rapidly accepted, to such a point that in 1980 Jim Gunn[199] claimed that *"observations now leaves little doubt of its presence."*

The paradigm had changed, and dark matter rapidly became a very popular subject in astronomy. Many different determinations of the galaxy system masses reached very similar conclusions. Peebles[353] developed the *"cosmic virial theorem"* and performed the first analysis of the peculiar velocity field in the Local Supercluster[354]. Davis et al.[123] followed in his steps a few years later. Capelato et al.[92,89,90] developed their "Multi-Mass Model" which accounted for a distribution of the masses of cluster galaxies. Ozernoy & Reinhardt[343] and, independently, Valtonen & Byrd[471] developed a binary model for Coma, later shown to be inconsistent with the X-ray and optical data by Tanaka et al.[447] and The & White[452], respectively. Bahcall & Tremaine[39] invented the *"projected mass estimator"*, as an alternative to the virial theorem. In 1982 Kent & Gunn[252] analyzed the phase-space distribution of galaxies in Coma, and found that an isotropic mass-follows-light model was the best fit to the data, thus confirming Rood et al.[385]'s result. On the other hand, Bailey[51], using the same data, showed that many other dynamical models were equally acceptable, and the cluster mass was poorly constrained. One year later, Kent & Sargent[253] found that radial orbits were needed to model the dynamics of another cluster, Perseus. Beers et al.[57], following in the steps of Kahn & Woltjer[246], applied a two-body dynamical analysis to the double cluster Abell 98. In 1980 Lucey et al.[285] showed Centaurus to be another example of a double cluster.

The virial mass estimates of galaxy clusters received a definitive confirmation through the gravitational lensing analyses (see, e.g., Fort & Mellier[167]), just as predicted by a visionary Fritz Zwicky some 60 years earlier. New methods of cluster mass determinations are reviewed by GELLER (these proceedings).

4.3 Luminosity

The first studies on the luminosity function (LF, hereafter) of cluster galaxies aimed at determining the population of cluster galaxies, and, in particular, if dwarf galaxies were clustered like bright galaxies. When Zwicky[512] discovered the missing mass problem, it became very important to evaluate the total cluster luminosity, in order to understand how much of the missing mass could be accounted for by galaxies fainter than the highest observed magnitude, or by diffuse IC light.

In 1931, Carpenter[95] analyzed the LF of the newly discovered Cancer cluster, and noted that it was a steeply rising function at faint magnitudes, with no maximum. Hubble & Humason[234] and Hubble[233], on the other hand, advocated for a LF with a maximum around the magnitude $\simeq 17$. Such a maximum was also noted by Baade[34] in Ursa Major, and by Shapley[416] in Coma, but only in the inner region, while the LF seemed to increase to fainter magnitudes in the surrounding regions. Such a phenomenology was later confirmed by Rood & Abell[384], and reproduced by White[491]'s numerical simulations. White explained the difference between the inner and outer LFs as an effect of dynamical friction and merging, leading to an excess of bright galaxies in the core – see Fig. 24. Recently, the non-monotonous behaviour of the Coma LF has been reconsidered[456,67].

In 1951 Zwicky[519] denied the existence of a maximum in the Coma cluster LF. He advocated for a LF rising all the way down the faintest magnitudes reached by observations. This was in agreement with Holmberg[227]'s recent analysis of the LF of the M 81 and M 101 groups, which indicated a considerable fraction of dwarf galaxies. As a matter of fact, the large fraction of dwarf galaxies in the Local Group was already known in the 30's, and clearly at odds with Hubble's Gaussian LF. In the late 50's dwarf galaxies were also found in Virgo (Reaves[366,367]) and Fornax (Hodge[220,221]).

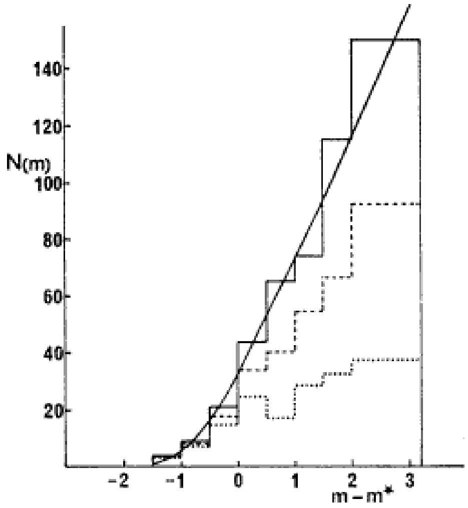

Figure 24: Luminosity function for a cluster numerical model. The solid histogram is the overall luminosity function, and the smooth curve is the Schechter function from which it is derived. The other two histograms correspond to luminosity functions constructed using only particles within 3.9 Mpc (dashed line) and 1 Mpc (dotted line). From White (1976a).

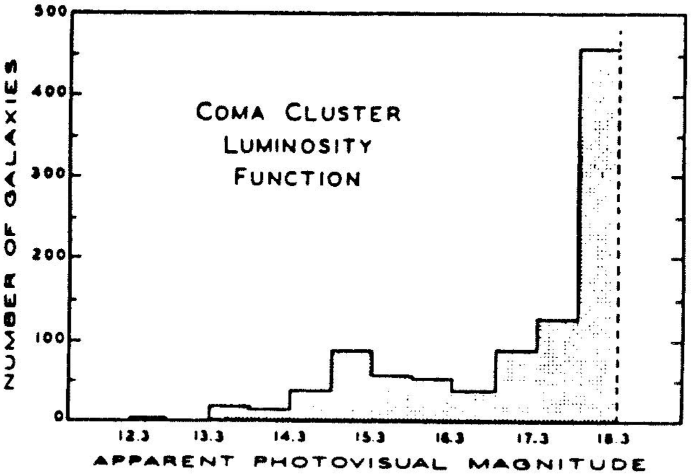

Figure 25: Abell's estimate of the differential LF of Coma galaxies. From Sky & Telescope (1959).

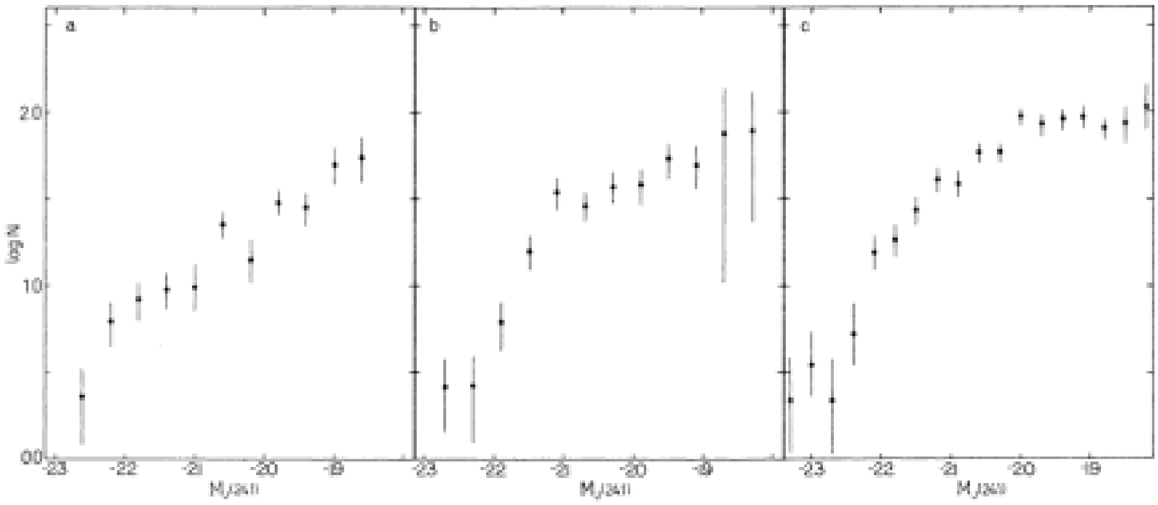

Figure 26: Composite differential luminosity functions for spiral-rich (panel a), spiral-poor (panel b), cD-clusters (panel c). From Oemler (1974).

In 1959 Abell[8,9,10] showed that the cluster LF increased down to a photovisual magnitude of 19.2, despite a secondary maximum around magnitude 15 – see Fig. 25. Two years later Abell[12,13] analyzed several cluster LFs, and confirmed Zwicky's view of a LF steeply raising down to very faint magnitudes. However, Abell noted the existence of a particular magnitude where the LF changes slope, in disagreement with Zwicky[529], who did not consider the LF secondary maximum to be statistically significant. Abell also explained the apparent Gaussian shape of Hubble's LF as a result of a selection effect.

In 1952 Zwicky[521] first claimed the detection of IC light in Coma. Twenty years later, his finding was confirmed by Welch & Sastry[490]. de Vaucouleurs& de Vaucouleurs[138] showed that most IC light was due to the extended halos of the two central dominant galaxies. They estimated that the IC light accounts for less than 40 % of the total cluster luminosity. Mattila[296] and, independently, Thuan & Kormendy[457] remarked that the blue colour of this IC light suggested it could be originated in dwarf galaxies. Rood et al.[385] had previously estimated that dwarf galaxies could contribute at most 15 % of the total cluster light.

In 1974, Austin & Peach[33] found a secondary maximum in the LF of Abell 1413. This was the second cluster, after Coma, to show a non-monotonous behaviour of its LF. However, three major works put the LF irregularities into oblivion. First, Oemler[331] insisted upon the similarity of the LFs of clusters of different type. However, this is not apparent from Figure 11 in his paper – here reproduced in Fig. 26. Possibly Oemler overlooked differences among the observed LFs, in order to emphasize the overall remarkable similarity with the theoretical mass function recently worked out by Press & Schechter[363]. In their paper, Press & Schechter compared their model to Oemler's LF for Coma, and explained Abell's exponential cut-off magnitude $M^\star$ as a characteristic feature of the *"self-similar gravitational condensation"* model. Finally, in 1976, Schechter[402] condensed the results of Oemler and Press & Schechter. He built a composite LF from Oemler's data for 13 clusters, and show it to be consistent with a soon-to-be famous *"analytic expression for the luminosity function for galaxies"* – see Fig. 27.

Schechter's universal LF was readily accepted, probably because it was not purely phenomenological, like the previous ones of Zwicky[525] and Abell[12], but based on Press & Schechter's physical model. Several authors[264,467] stressed the similarities of the LFs of different clusters and groups. Nonetheless, the numerical simulations of Simon White[491] indicated that an evolution of the LF in clusters was expected, because of dynamical friction and merging – see Fig. 24. In

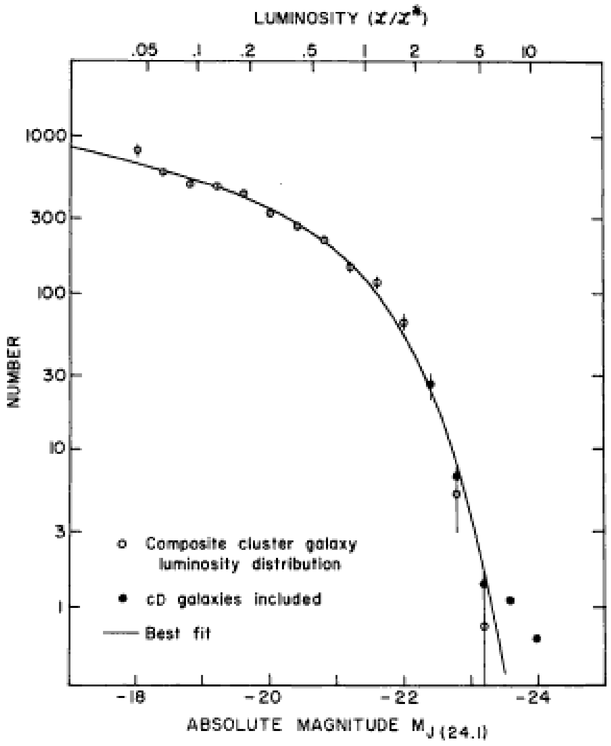

Figure 27: Best fit of Schechter's analytic expression to Oemler's observed composite cluster luminosity distribution. From Schechter (1976).

the discussion following a talk of Ostriker[337] White remarked upon the similarity of his results and the recent determination of the Coma LF by Godwin & Peach[185]. In 1980, Thompson & Gregory[455] remarked that Schechter's analytic form can fit the LF of all cluster galaxies, but not the LFs of separate morphological classes. In particular, they noted that Hubble's Gaussian LF could provide a good fit to the LF of bright cluster ellipticals. Since different clusters have different fractions of ellipticals, their LFs should be different. The idea of an universal LF was being shattered. Later works[395,456,30] confirmed Thompson & Gregory's result. The universality of the LF may still hold within each morphological class separately (e.g. Krupp[264], Andreon[30]).

In 1977 Abell[16] claimed evidence for a steepening of the Coma LF beyond magnitude 17.5. A few years later, Heiligman & Turner[209] noted on the contrary a lack of faint galaxies in compact groups. These two papers anticipated the current discussion on the faint end of the cluster LF[58,125,282,23], which seems to be quite steeper than the field LF[511,283] (see Andreon, Ulmer, these proceedings). If true, this difference can be explained in the context of Cavaliere et al.[101]'s model for the evolution of galaxies in clusters, a model supported by the observations of Wilson et al.[499].

4.4 On the nature of the dark matter in clusters

The early papers by Zwicky[512,513] and Smith[425] did not convince the astronomical community that dark matter existed in clusters. Most astronomers favoured the alternative hypothesis, cluster instability, until the 21cm measurements proved the galaxy rotation curves to be non-Keplerian (see § 4.2 and the excellent reviews of Sidney van den Bergh[481,482]). However, many astronomers took the dark matter hypothesis very seriously and tried to elucidate its nature.

When Zwicky[513] discovered the missing matter problem, his first reaction was to question the validity of Newton's gravitational law. Later he turned his attention to possible forms

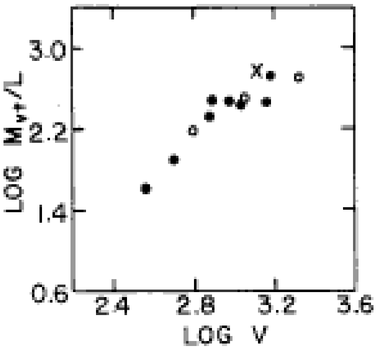

Figure 28: The logarithm of the mass-to-light ratios for rich clusters vs. the logarithm of their velocity dispersions. From Rood (1974b).

of dark matter, that could also provide IC obscuration and thus explain the non-uniform sky distribution of clusters[520,522,524]. During the 50's he could not find much observational evidence for a significant quantity of IC matter[521], so he[524] again suggested abandoning the general theory of relativity.

In 1956 Heeschen[208], motivated by Stone[436]'s theoretical argument, searched for and detected HI emission from Coma. The detected emission implied a mass of $\simeq 5 \times 10^{13}$ solar masses. Heeschen's detection was however shown to be spurious by Muller[314], three years later. From a theoretical point of view, Limber[280] noted that clusters are likely to contain IC gas, because the galaxy formation process is unlikely to be 100 % efficient, and because galaxy-galaxy collisions sweep gas out of galaxies. He pointed out that if this IC gas remained undetected at 21 cm, it could be hot and ionized. Extensive searches for intergalactic material by Zwicky & Humason[528] did not prove very successful. In 1961 the total amount of IC cold gas was constrained by Penzias[356] to be less than a tenth of the virial mass in the Pegasus I cluster. Penzias remarked that the integrated 21 cm emission from individual cluster galaxies could well account for the total cluster emission.

Dark matter was also searched for in the form of diffuse optical luminosity[521,490,296,457] and dwarf galaxies[227,366,220,221,385], but these components were found to account for less than half the total cluster luminosity (see § 4.3).

In 1960 de Vaucouleurs[130] summarized the observational situation by noting that the missing mass could neither consist of cold HI, nor of dust, nor of diffuse (optical) luminosity. The total mass of all these components only makes a small fraction of the total cluster mass, and therefore *"a large number of essentially dark bodies must also be assumed."* If the existing observations could not establish the nature of dark matter, they were anyway not in conflict with the hypothesis that only a small fraction of the matter in the Universe is in bright galaxies (Layzer[271]).

The idea that galaxies have dark halos gained a hold upon the astronomical community in a very short time, from 1969 to 1974, through the work of Arp & Bertola[32], de Vaucouleurs[136], Vorontsov-Velyaminov[489], Freeman[168], Lewis[278], Ostriker & Peebles[339], Einasto et al.[153] and Ostriker et al.[340] (see § 4.2). Rood[380] had already demonstrated in 1965 that not all cluster dark matter can be attached to galaxies, or relaxation processes could produce much more energy equipartition (and hence, luminosity segregation) than observed (Rood's early finding was later confirmed by White[493]). Moreover, Lecar[277] pointed out that galaxies in clusters should loose their halos via tidal stripping.

The idea of a scale-dependent mass-to-light ratio took a step forward through the works of Zwicky & Humason[529], Karachentsev[248], Rood et al.[386], Rood[383] – see Fig. 28 –, Ostriker et al.[340], Einasto et al.[153], Bahcall[45] and Davis et al.[123]. The mass-to-light ratio seemed to increase from galaxies to groups and to rich clusters. This evidence seemed to indicate that the dark matter does not follow the distribution of bright galaxies. Dressler[140] however noted that including the IC gas mass would reduce the mass discrepancy in galaxy clusters, and destroy the evidence for a scale dependence of the mass-to-light ratio. BAHCALL (these proceedings) has recently shown that the mass-to-blue light ratio increases with scale up to the size of galaxy clusters, but not beyond. The trend can be explained as an age effect (galaxies in high density environments form earlier, so that their blue luminosity fades earlier).

The apparently different distribution of dark matter and bright galaxies strengthened the idea that the dark matter consists of diffuse gas. A significant amount of IC HI had been ruled out by observations. Astronomers then started looking for ionized gas. In 1967 Woolf[504] put the first constraints on diffuse ionized gas, by looking at Hα and Hβ emission from clusters. He concluded that if the cluster dark matter was in the form of ionized gas, the temperature of this gas had to be below 10^6 K. Three years later, Turnrose & Rood[469] confirmed Woolf's estimate, using X-ray data. When diffuse X-ray cluster emission was detected (Meekins et al.[299], Gursky et al.[201]) it was immediately clear that the IC gas could not account for all the cluster missing mass.

It was at this point that astronomers really started to grope in the dark. In 1969, van den Bergh[477] considered massive collapsed objects (of 10^8–$10^{12} M_\odot$ each) as dark matter candidates, but ruled them out on the basis of the limited tidal distortion of galaxies in Virgo. Peebles[351] suggested frozen HI snowballs as dark matter candidates, a possibility never really ruled out (see, e.g., Wright et al.[505]). Another form of baryonic dark matter was proposed by Tarter & Silk[451] (M8 dwarf stars), but they also frankly remarked that *"nothing better"* could be said on this topic than had already said thirty years earlier by Zwicky. A scaled-down version of Tarter & Silk's dark-matter candidates were Napier & Guthrie[317]'s $10^{-2} M_\odot$ *"black dwarfs"*. Tarter & Silk's dark matter candidates were later suggested by Sarazin & O'Connell[399] to be the end-products of the cooling flows onto cD galaxies (see § 5.4). In 1981 Gott[187] proposed a gravitational lensing experiment to detect an hypothetical huge population of low-mass stars in galaxy halos, thus anticipating the recent *AGAPE*[31], *EROS*[25] and *MACHO*[25] projects.

Baryons as candidate for dark matter are however ruled out by the theory of primordial nucleosynthesis (see, e.g., Cavaliere et al.[102]), and therefore more exotic dark-matter candidates were proposed. Here is a short list of them: a variable G (Lewis[279]); MOdified Newtonian Dynamics (Milgrom[305]); vacuum strings (Vilenkin[486]); magnetic monopoles (Hoyle[231]); heavy neutrinos (Cowsik & McClelland[115], Szalay & Marx[444], Doroshkevich et al.[139]) – eventually unstable (Sciama[407]); gravitinos (Pagels & Primack[346]), axions (Stecker & Shafi[432]), and cold dark matter in general (Bond et al.[76]).

Recent observations of the cosmic microwave background (de Bernardis et al.[124]) have added considerable constraints on the nature of the missing mass, which is now thought to consist of a mixture of cold dark matter and dark energy (in the form of a cosmological constant or quintessence, see, e.g., Bahcall et al.[47]). It is nevertheless wise to close this section with a statement of Alan Dressler[140]:

> *"The answer to the mass discrepancy problem awaits more data and more inspiration, not necessarily in that order."*

5 Evolution

5.1 The formation and evolution of galaxy clustering

The question of the origin of clusters of galaxies was addressed as soon as the extragalactic nature of nebulæ was established. In 1927 Lundmark[287] suggested that clusters could form through many subsequent galaxy–galaxy encounters. These encounters would lead to a loss of the orbital energy of the galaxies, which would then form a bound system. Nine years later, the theory had not progressed much, and Hubble[233] was unable to be very specific on this topic:

> *"condensations in the general field may have produced the clusters, or the evaporation of clusters may have populated the general field."*

The different velocity dispersions of cluster and field nebulæ led however Zwicky[513] to reject the latter of Hubble's possibilities. He also considered extremely unlikely that the rich, regular, centrally concentrated clusters could be just an effect of geometrical chance alignments of galaxies along the line-of-sight. His favourite scenario was that of Lundmark[287]. By requiring the mass of cluster galaxies to be higher than the mass of field galaxies, gravitational clustering could be made more efficient. The large cluster virial masses obviously seem to support this view. Despite the large masses implied for cluster galaxies, Zwicky however realized that the formation of great clusters by subsequent capture of field galaxies would take a very long time, much larger than the age of the Universe. In 1943, by using Chandrasekhar[103]'s theory of dynamical friction, Tuberg[463] indeed estimated the cluster relaxation timescale to be 10^{11}–10^{12} years, i.e. orders of magnitude larger than the estimated age of the Universe at that time.

In 1941, Erik Holmberg[226], a supporter of the capture theory for cluster formation, published his remarkable paper *A study of encounters between laboratory models of stellar systems by a new integration procedure.* Two decades before the first n-body numerical simulation of von Hoerner[488], Holmberg ideated an ingenious device to simulate galaxy–galaxy encounters. His idea was bright and simple: gravitation was replaced by light in his model. The mass elements (37 per stellar system, set in circular annuli on a plane) were represented by light-bulbs, the candle power being proportional to mass. By modulating the bulb candle power with the distance from the centre of the system of bulbs, Holmberg was able to simulate a given density profile. The two stellar systems were given a certain approach velocity, and were also set in rotation. All measurements were performed on a plane surface, so that the simulation was 2-dimensional. The acceleration on a given element was measured by integrating the light at the position of that element with a photocell. The light bulb was then moved accordingly.

Holmberg's results were very interesting. By looking at Figure 4 in his paper – here reproduced in Fig. 29 –, we can see clear evidence for the tidal features that Toomre & Toomre's[462] n-body simulations reproduced only 30 years later. Holmberg however mistook tidal features for spiral arms in the process of formation. Moreover, even if *"in favorable cases, captures may occur"*, the experiment essentially ruled out the capture theory for cluster formation (which was Holmberg's favourite scenario).

In 1952 and 1956 Zwicky[520,523] remarked upon the similarity of distant and nearby clusters. This lack of evolution seemed difficult to reconcile with an expanding Universe. Zwicky was trying to rule out Hubble's expanding Universe, because its short age was clearly inconsistent with the long dynamical timescales he thought necessary to build the rich regular galaxy clusters. Detecting the evolution of the cluster number density was to prove very difficult. Some observational evidence in this sense was claimed by Just[245] in 1959, by Paal[344] in 1964, and by Rowan-Robinson[393] eight years later. Rowan-Robinson however warned against possible selection effects that could have biased his result. The preferential selection of the richest clusters as spectroscopic targets was shown by Reaves[370] to account for the evidence for evolution. Anyway, these first attempts opened the way to modern investigations of the cluster number density evolution (see, e.g., Borgani et al.[77]).

In 1961 van Albada[472] performed a numerical integration of a model for the cluster evolution, and draw the first modern scenario of cluster formation:

> *"Clusters can be formed by gravitational amplification of statistical density fluctuations in an initial homogeneous field of galaxies"*

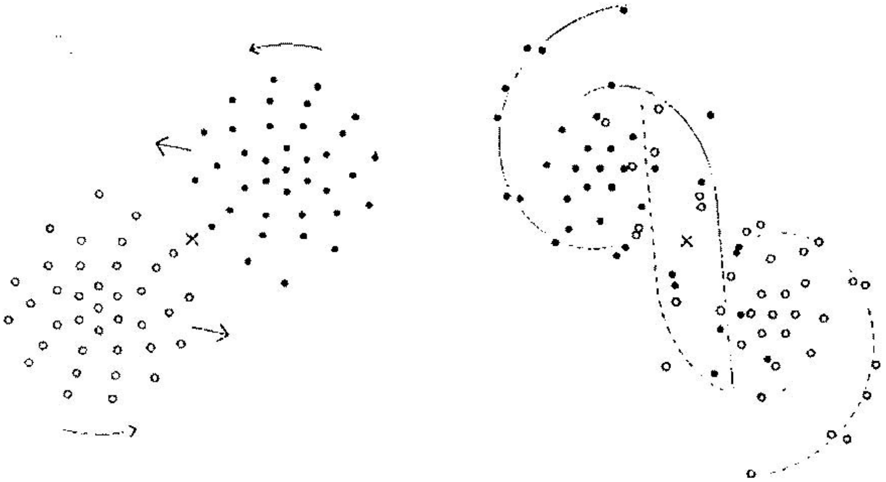

Figure 29: Results of the simulation of a collision between two nebulæ. Left panel: two disky galaxies approaching. Right panel: after the collision. From Holmberg (1941).

In 1963 Aarseth[1] performed the first of a long series of n-body simulations of galaxy (or stellar) clusters. His first simulations contained at most 100 point-masses. Twenty years later, thanks to the advances in computer technology, Miller[306] could run a 10^5-body simulation. The increase rate of n in n-body simulations over the last thirty years is described in MOORE (these proceedings).

One year later, Hénon[211] performed numerical computations of the dynamical mixing in spherically symmetric clusters. He noted that phase-mixing rapidly leads to a steady-state configuration after the initial system contraction. He prepared the field to Lynden-Bell[288]'s milestone paper *Statistical mechanics of violent relaxation in stellar systems,* published in 1967. Lynden-Bell showed that:

> *"the violently changing gravitational field of a newly formed galaxy is effective in changing the statistics of stellar orbits* [which] *in the relevant limit* [...] *becomes Maxwell's distribution but with temperature proportional to mass".*

Lynden-Bell showed that the predicted density distribution approached the modified isothermal sphere, or King[255]'s recently published distribution. Violent relaxation removes the need of very long timescales for a cluster to reach stable, relaxed dynamical configurations. Lynden-Bell's results was confirmed by Peebles[350]'s 300-body simulation of a Coma-like cluster. Peebles showed that 10 Gyr suffice to form a rich symmetric cluster – see Fig. 30.

The cluster collapse and subsequent infall of material into clusters were theoretically examined by Gunn & Gott[200] (see also §§ 5.2, 5.3, 5.4). They were probably the first to remark that *"the present is very much the epoch of cluster formation".* Their statement was based on the idea that the many existing irregular clusters were still in a pre-collapse phase. This idea was later developed by Richstone et al.[374], who saw the possibility of constraining the density of the Universe by estimating the fraction of substructured (i.e. irregular) clusters. Oemler[331] also elaborated Gunn & Gott's idea by identifying the irregular clusters with the spiral-rich, and the regular, evolved ones with the cD-type, which *"must have begun as the densest fluctuations in the early Universe".*

Between 1965 and 1975, two opposite scenarios for the formation of structures were developed, mainly by Peebles[349], Peebles & Dicke[355], Silk[421] and Gott & Rees[189], on one side, and Zel'dovich & Syunyaev[509,441,443] on the other side. Peebles and collaborators advocated for a

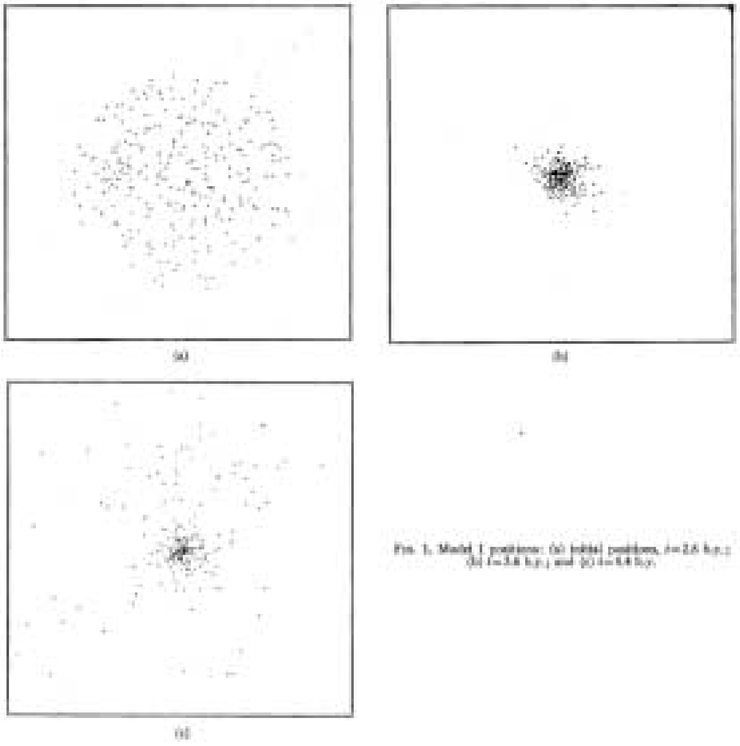

Figure 30: 300-body numerical simulation of the Coma cluster, at three different times (increasing from panel a to c). From Peebles (1970).

hierarchical bottom-up formation of the galaxy structures, while Zel'dovich and collaborators developed a theory for the evolution of large density perturbations leading to a top-down scenario, with the formation of galaxies from fragmentation of *pancakes*. In their original purely baryonic versions, the hierarchical scenario predicted an evolution of structures from isothermal primordial density fluctuations, while in the top-down scenario the primordial fluctuations were adiabatic. The bottom-up scenario was soon proven by Aarseth & Hills[5]'s simulations to be a viable scenario for the formation of a cluster via the merging of separate subclusters. It then received a formidable support from Press & Schechter[363]'s 1974 paper *Formation of galaxies and clusters of galaxies by self-similar gravitational condensation.* Press & Schechter obtained their famous mass function, and compared it with observations, finding *"rather striking agreements"* (see § 4.3). Also the Russian *pancake* theory (with the added ingredient of massive neutrinos – see Klinkhamer & Norman[259]) had many supporters. As an example, Thompson & Gregory[454] argued that Coma is *"a Zel'dovich disk"*. The popularity of the model started to decline in 1983, when Frenk et al.[169] showed it implied a very late formation of galaxies, much too late to reconcile with observations. In the end, a hierarchical structure formation from primordial adiabatic density fluctuations has emerged, a sort of compromise between the two original scenarios, where the Zel'dovich approximation is still valid for describing the initial evolution of structure on large scales, and cold dark matter plays a leading role in shaping the structure of the Universe (Bond et al.[76]).

In 1976, further support to the hierarchical clustering scenario came from White[491]'s 700-body simulations. He showed that *"clusters form by the progressive amalgamation of an inhomogeneous system of subclusters"*. The direction of the final major merger defines the direction of the cluster elongation, and there is no need to invoke cluster rotation or tidal distortions to explain the cluster elongations – see Fig. 31. Following White's result, Forman et al.[165] interpreted the double structure of some X-ray emitting clusters as an evidence for an intermediate

Figure 31: 700-body numerical simulation of a cluster, at four different times. From White (1976a).

evolutionary stage.

In 1978 Fall[160] reproduced the shape of Peebles' covariance function in his 1000-body simulations. The following year, the 4000-body simulations by Aarseth et al.[4] not only confirmed Fall's results, but also reproduced the recently discovered huge voids[459,195,454,449,109,448,196] in the galaxy distribution. Aarseth et al. noted that if the Universe has $\Omega_0 = 1$ *"the clustering is proceeding at the present time"*, while this is not the case if Ω_0 is low. An Ω_0-dependence of the covariance function was noted by Gott et al.[191] and Efstathiou[151] in their n-body simulations. The cellular, filamentary appearance of the structure of the Universe was reproduced in the 10^5-body simulations of Miller[306].

The cluster number density evolution has now become a strong constrain for cosmological theories. Most observational evidence of this kind points to a low-Ω_0 Universe (see BAHCALL, BORGANI, these proceedings), and the extensive ongoing surveys will soon improve the statistics and probe deeper in space (see, in these proceedings, BARTLETT, BÖHRINGER, CARLSTROM, DICKINSON, GAL, GIOIA, JONES, LOBO, SCHUECKER, and ZARITSKY).

5.2 The evolution of galaxies in clusters

The importance of collisions for the evolution of cluster galaxies was understood quite early. Since a cluster of galaxies is a dense environment, *"collisions must necessarily enter as a factor in the evolution of the system"* (Shapley[417], 1935). In 1937 Zwicky[513] imagined that collisions might lead to the disruption of certain types of nebulæ, which could explain why the morphological mix of cluster galaxies is different from the field. The first observational evidence for this effect came only thirty years later, when Reaves[368] found that dwarf galaxies avoid the cluster centres.

In 1943 Chandrasekhar[103] developed his theory of *"dynamical friction"*, *"the systematic decelerating effect of the fluctuating field of force acting on a star in motion"*. Chandrasekhar derived his formula on the basis of the two-body approximation for stellar collisions. More than thirty years later, with the discovery of massive halos around galaxies, Lecar[277] suggested that galaxies gradually settle to the cluster centres by dynamical friction through a sea of tidally-stripped galaxy halos. The validity of Chandrasekhar's formula was confirmed through numerical simulations by White[492,493].

In 1940 Holmberg[225] had remarked that spirals must transform into ellipticals, if clusters form by the capture of field galaxies. Spitzer & Baade[430], in 1951, were the first to suggest collisions as a mechanism to transform a galaxy type into another. They thought that collisions would affect primarily the gas content of a galaxy, and not so much its stellar structure, leading to the formation of irregular galaxies. A year later Zwicky[521] found evidence for intergalactic matter in small galaxy groups, and attributed it to material stripped from galaxies during close encounters. This was confirmed 20 years later by the simulations of Toomre & Toomre[462]. Spitzer & Baade's analysis was revised twice between 1963 and 1965. First Aarseth[1] revised downward Spitzer & Baade's estimate of the number of galaxy-galaxy collisions, as a consequence of the revised

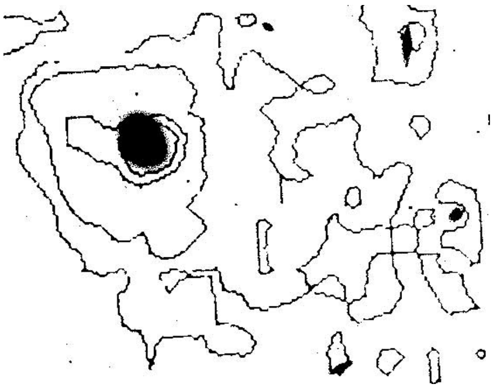

Figure 32: Contours of X-ray emission around the galaxy M 86 in Virgo The extended emission was interpreted as evidence for ram pressure stripping of hot gas from the galaxy. From Forman et al. (1979).

distance scale. Then, Alladin[26] revised upwards Spitzer & Baade's estimate of the internal energy change of a galaxy during a collision.

In 1970 Tinsley[460] developed her theory for the evolution of the spectral energy distribution of galaxies and showed that strong evolutionary corrections were to be expected for the colours of ellipticals, because of the aging of the stellar population[m]. The following year, Oke[334] devised to compare the colours of nearby and distant cluster ellipticals with evolutionary models, and thus infer their (photometric) redshifts.

In 1972 Rood et al.[385] noticed that the Coma cluster S0s were not confined to the cluster core, where collisions were expected to be most effective, and questioned the validity of the collision model for the formation of lenticular galaxies. In the same year, Gunn & Gott[200] and Larson[268] presented two alternative models for the evolution of galaxy morphologies. Gunn & Gott proposed ram pressure stripping of the interstellar gas by the hot IC medium as a mean of transforming spirals into S0s. The first direct observational evidence of such an effect came seven years later, with Forman et al.[166]'s X-ray observations of the Virgo galaxy M 86 – see Fig. 32. Larson, on the other hand, suggested a relation between the morphological type of a galaxy, and the collapse time of the gas during galaxy formation. Galaxies with a short collapse time would have their material used up early, leading to old stellar populations and little gas left (like in ellipticals and S0s). The morphology–density relation could then follow by relating the collapse time to the ambient density. According to Oemler[331], the *"birthrate of elliptical galaxies [...] increases with density relative to the other galaxy types"*, and collisions may be sufficient to transform spirals into S0s but not into ellipticals. Larson's ideas were later developed by Gott & Thuan[190].

In 1975 Biermann & Tinsley[61] remarked upon the similarity of the colours of ellipticals and S0s. This implies that ellipticals and S0s have similar stellar populations, and therefore similar old ages, so that a recent transformation of spirals into S0s is out of question. The issue is certainly not closed, with independent evidences in favour[154] and against[146] an ancient origin of S0s.

In 1976 White[491]'s n-body simulations showed that the formation process of a cluster leads to an increasing ellipticity of galaxy orbits with clustercentric radius, i.e. radial motions are predominant in the outer cluster regions. The observations of Moss & Dickens[313] seemed to confirm White's findings. Moss & Dickens observed that late-type galaxies have a higher velocity dispersion than early-types, and interpreted it as an evidence for an infalling population of field

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[m] As Spinrad[429] noted in 1977, Tinsley's work led to an *"amusing"* conceptual inversion of the classical cosmological quest: instead of comparing the properties of nearby and distant galaxies to constrain the cosmological model, one must adopt a cosmological model in order to constrain the evolution of galaxies.

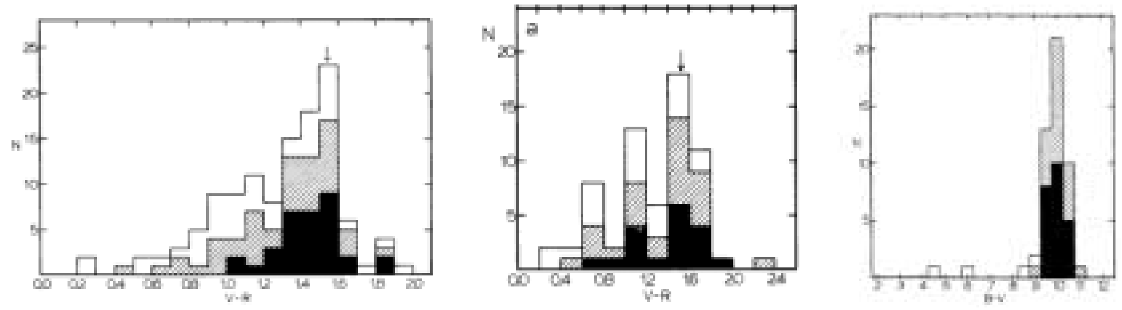

Figure 33: The V-R colour distribution of galaxies in the cluster Cl0024+1654 (left), and in the cluster 3C295 (middle). Different shadings correspond to subsamples of galaxies at different distances from the cluster centres. The B-V distribution of galaxies in the Coma cluster (right). Solid area: ellipticals; hatched area: S0s; remainder: spirals. From Butcher & Oemler (1978a).

galaxies into the clusters. Recently Biviano et al.[71] have shown that emission-line galaxies in clusters are characterized by predominantly radial orbits. A thorough determination of the orbits of different types of cluster galaxies, through the solution of the Jeans equation, is in preparation[72].

White[491,492]'s simulations also showed that a marginal mass segregation can establish in clusters through dynamical friction. Merging of the slowed-down galaxies would then follow in the cluster core, eventually with the formation of a cD galaxy (see § 5.4). Struble[437]'s observation of a low velocity dispersion in the core of some galaxy clusters was taken as supporting evidence for these effects. A few years later Roos & Aarseth[389] re-examined the issue of mass segregation by running n-body simulations of a galaxy system with a Schechter-like distribution of galaxy masses. They noted that segregation establishes in subclusters before these merge to form the final cluster. Segregation is then conserved while the cluster evolves, because tidal stripping predominantly affects the outer regions of subclusters. Such an evolutionary scenario was found to be consistent with Capelato et al.[91]'s observations of luminosity segregation in Coma, and with recent analyses of the Coma cluster structure[300,68].

In 1980 Dressler[142] noted that ram-pressure stripping could not account for the different bulge-to-disk ratios of spirals and S0s. Richstone[373] and Marchant & Shapiro[291] had already shown that collisions of spirals can fatten the galaxy disks, so that Dressler's observation was not a problem in the collision scenario. Farouki & Shapiro[161]'s simulations showed however that also the ram-pressure mechanism would lead to a thickening of the galaxy disks. Finally, in 1982 Nulsen[330] noted that other interaction mechanisms between cluster galaxies and the hot IC gas medium (viscosity, thermal conduction, turbulence) could be even more effective than ram-pressure in stripping gas from galaxies.

In 1980, Larson et al.[270] noted that if star formation continued in galaxy disks at the rate determined in the local Universe, spirals would run out of gas in a relatively short time. Disk replenishment of gas is therefore needed. An early generation of spirals, formed in high density regions, would be characterized by small disks, and such spirals could evolve into nowadays S0s by the loss of their gaseous halos through collisions. According to Roos & Norman[390]'s n-body simulations, ellipticals could instead form via mergers during the early stage of cluster collapse, before the dispersion of galaxy velocities becomes too high.

All these theoretical efforts to determine the evolution of galaxies received a formidable acceleration with the first direct observational evidence for the evolution of the cluster galaxy population. In 1978, Butcher & Oemler[85] published the first of a series of papers on *The evolution of galaxies in clusters*. Their photometric observations of two regular, centrally concentrated, $z \simeq 0.4$ clusters, showed an excess of blue galaxies, as compared to nearby rich clusters – see Fig. 33. Butcher & Oemler[85,86] noted that such a high fraction of blue galaxies was more typical of nearby poor irregular clusters like Hercules. They later confirmed their finding through photometric observations of seven more clusters at redshifts beyond 0.2 (Butcher et al.[87]).

Butcher & Oemler's result was greeted with much scepticism. Even before Butcher & Oemler's paper was published, Baum (in the discussion following a talk of Spinrad[429]) suggested that their result could be due to contamination by field galaxies. Koo[262] imaged another distant cluster, where he did not find evidence for the Butcher-Oemler effect. Mathieu & Spinrad[294] re-examined the fraction of blue galaxies in one of Butcher-Oemler clusters, and showed it to be much lower than originally estimated. Lucey's critical *"assessment of the completeness and correctness of the Abell catalogue"* led him to conclude that the Butcher-Oemler effect was due to an erroneous assignment of cluster membership.

Theorists were however not discouraged by potential observational biases. In the models of Norman & Silk[329] and Himmes & Biermann[218] the IC gas gradually build-up from the gas stripped through collisions of cluster galaxies. Norman & Silk[329] noted that such a gradual built-up of the IC gas can delay the effectiveness of ram-pressure stripping until $z \sim 0.5$. If ram-pressure transforms spirals into S0s, this would explain the excess of spirals in high-redshift clusters. However, Henry et al.[212]'s X-ray observations showed the existence of a dense IC medium in one of the clusters showing the Butcher-Oemler effect.

In 1982, 1983, Dressler & Gunn[144,145] finally performed spectroscopic observations of galaxies in Butcher-Oemler clusters. The fraction of blue galaxies which are cluster members was found to be lower than predicted by Butcher & Oemler, but still higher than in nearby rich clusters. The Butcher-Oemler effect was confirmed.

More than twenty years after the original discovery, the Butcher-Oemler effect is well established (see ELLINGSON, MARGONINER, these proceedings), and a considerable progress has been made in determining the nature of the excess blue galaxies (see, e.g., Poggianti et al.[361]). The physical mechanisms responsible for the evolution of cluster galaxies are not yet determined with certainty, but it is likely that collisions, as initially suggested by Shapley[417], are of fundamental importance (see MOORE, KAUFFMAN, LANZONI, these proceedings).

5.3 The evolution of the IC gas

Many years before its detection, Limber[280] had argued that IC gas must exist because galaxy formation cannot be 100 % efficient, and that it must evolve through the loss of gas from colliding galaxies. The IC gas was eventually detected[299,201] in 1971. In those years, Gott & Gunn[188,200] developed their theory of intergalactic gas infall into clusters. They argued that this infall could generate a hot IC gas through shock heating. They suggested that irregular clusters are seen in a pre-collapse phase, so that their IC gas had not yet reached high temperatures. In this way they hinted at the existence of a class of X-ray faint clusters (which are now being discovered, see Holden et al.[223]). Gunn & Gott[200] also suggested ram pressure as a mean to strip gas from cluster galaxies and enrich the IC medium.

An early gas infall became a common feature of models in which the IC gas is in hydrostatic equilibrium in the cluster gravitational field (Lea[272], Gull & Northover[197], Cavaliere & Fusco-Femiano[98,99]). On the other hand, Yahil & Ostriker[506] developed a theory with an IC gas outflow. They argued that the gas shed from the galaxies would feed an outflow wind from the cluster. Such a radial outflow of the IC gas was soon found to be at odds with the random direction of the cluster galaxy radio-tails (Lea[273]).

In 1973 Lea et al.[276] remarked that since the mass of IC gas is comparable to the total mass in cluster galaxies, not all of the IC gas can originate from cluster galaxies, and most of it must be primordial. On the other hand, Larson & Dinerstein[269] advocated for a galaxy origin of a significant fraction of the IC gas, through supernova explosions and stellar winds. Their model predicted a significant abundance of heavy elements in the IC gas. The hydrodynamic numerical simulations by Lea & De Young[274] indicated that as much as 90 % of the gas can be removed from galaxies moving through the IC gas at transonic speed.

In 1977, Iron was found in the IC gas[309,308,410], proving that at least some of the IC gas had been processed in stars. A purely primordial origin of the IC gas was thus ruled out. As a matter of fact, observations seemed to indicate that the IC Iron mass was larger than could be produced in cluster galaxies. This led Vigroux[484] to suggest an early heavy-element enrichment of the IC gas by a pre-galactic population of massive stars. Fabian & Pringle[158] noted however that the estimates of the total cluster Iron mass were very uncertain, being based on extrapolations from the inner regions. Recently, Gibson & Matteucci[176] have shown that even a large population of dwarf cluster galaxies, as implied by the steep cluster LF, could account for the bulk of the IC gas and metals.

Norman & Silk[329] and Himmes & Biermann[218] developed models for the temporal evolution of the IC gas. An initial amount of IC gas would first originate from galaxies through supernovæ emission. Only then, ram pressure stripping could start. This model was proposed as an explanation of the Butcher-Oemler effect (see § 5.2).

In 1980, White & Silk[497] noted, in disagreement with Gingold & Perrenod[177], that mergers of subclusters can lead to strong heating of the IC gas in the compression region. This was later observed[80].

Cowie & Perrenod[114]'s models indicated a mild evolution of the X-ray cluster luminosity with redshift. Perrenod[358]'s more refined model, now including a cluster gravitational potential varying in time, predicted instead a very strong evolution of the X-ray cluster luminosity, a factor ten from $z \sim 1$ to the present. Perrenod[359] later showed that the evolution rate of the cluster X-ray luminosities was related to the density of the Universe, so that X-ray observations of distant clusters could be used to put useful cosmological constraints.

Perrenod's prediction of a strong evolution in the cluster X-ray properties was first tested observationally by Henry et al.[212]. Unfortunately, the wide range of X-ray luminosities for distant clusters made it impossible to test the model. Two years later, in 1981, Perrenod & Henry[360] argued for an X-ray temperature negative evolution with redshift, based on a limited sample of seven clusters observed at $z > 0.1$. Such an evolution was however not confirmed in other investigations. First, White et al.[496] detected an extremely bright and hot X-ray cluster at $z = 0.54$, then, Henry et al.[213] did not detect any change in the slope of the cluster X-ray luminosity function with redshift.

The first observational evidence for a cosmological evolution of the X-ray cluster properties dates back to 1982. Anticipating the results that were to be published in their entirety by Gioia et al.[178] many years later, Stocke et al.[434] noted that the clusters detected in the flux-limited *Einstein Medium Survey Sample* have a low average X-ray luminosity and a low average redshift, and their total number is half that expected for a uniform distribution of sources. This was interpreted as evidence for a negative evolution of the cluster X-ray luminosity function.

This evolution is now confirmed for the high-luminosity tail of the X-ray clusters only (see MULLIS, these proceedings). The high fraction of hot X-ray clusters at high redshift is now considered to be a strong evidence for a low-Ω_0 Universe (see GIOIA, these proceedings).

5.4 Cooling flows and the evolution of cD galaxies

The phenomenology of cD galaxies was first described in 1964 by Matthews et al.[295]. Eight years later Gunn & Gott[200] and Gallagher & Ostriker[172] proposed two alternative mechanisms for the formation and evolution of these cDs. Gunn & Gott[200] were possibly the first to suggest the existence of a physical link between the IC gas and cD galaxies. They showed that the cooling of IC gas, by thermal bremsstrahlung, would produce a flow of material in the central cluster region, that might accrete onto the cD galaxy. An alternative mechanism for the formation of

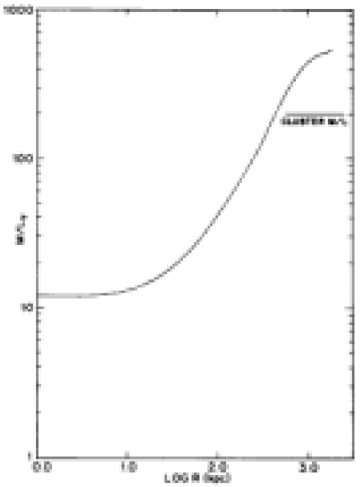

FIG. 6d

Figure 34: The rising mass-to-light ratio with radius in the cD galaxy of the cluster Abell 2029. From Dressler (1979).

cD galaxies was proposed by Gallagher & Ostriker[172] who suggested that the cD might form out of stars stripped from other galaxies. In this case one expects the outer parts of the cD to be in equilibrium with the cluster (rather than the galaxy) gravitational potential. Consistently, Dressler[141]'s observations of the cD in Abell 2029 showed a rapidly growing galaxy velocity dispersion with radius, implying that the mass-to-light ratio of the cD was also rising with distance from the galaxy centre – see Fig. 34. A year later, in 1980, Gallagher et al.[171] showed the envelopes of cDs to be bluer than the mean galaxy colour, again consistent with the tidal debris scenario.

Another popular scenario for the formation of cD galaxies was proposed in the 70's by Ostriker & Tremaine[341], and developed by Ostriker & Hausman[338]. The cD galaxy would grow by cannibalism of its neighbours. This scenario was supported by the n-body simulations of White[492]. White showed that as the cluster evolves, the dynamical friction mechanism can drive galaxies to the centre, and thus favour merging phenomena. Carnevali et al.[94] modelled the evolution of small groups, and showed that the *"merging instability"* leads to the formation of a large central object (they anticipated the discovery of fossil compact groups, see PONMAN, these proceedings). In the 80's the merging scenario for the formation of cD galaxies was re-examined by Roos & Aarseth[389] who concluded for an early creation of cDs via merging in small groups of galaxies, before the cluster formation.

The merging scenario was supported by several observational evidences. Oemler[332] determined the luminosities of cD envelopes and showed them to be correlated with the total luminosities of their parent clusters. Dressler[140] pointed out that the lack of significant luminosity segregation in cD-type clusters was another indication that cD galaxies had cannibalized neighbouring galaxies. Carter & Metcalfe[97] showed the cD major axis to be aligned with the distribution of surrounding galaxies.

The merging scenario for the formation of cDs was shattered in 1978, when White[494]'s simulations showed that merging can produce giant elliptical galaxies, but not the the cD extended halos. In those years, Lea et al.[276], Silk[422], Cowie & Binney[112] and Fabian & Nulsen[156] estimated the cooling time of the IC gas in the dense X-ray emitting clusters to be lower than a Hubble time. Fabian & Nulsen noted that *"slow-moving galaxies in core of X-ray emitting clusters can accrete large quantities of cooling gas"*, and Quintana & Lawrie[365] showed cD galaxies to be characterized by small velocities relative to the cluster mean. This gave new strength to the hypothesis of cD growth via accretion of the cooling IC gas.

A first observational evidence for the existence of cool gas in the cluster centres came in 1979 with the detection of soft X-ray components in the spectrum of the Perseus galaxy NGC 1275 (Mushotzky & Smith[316]). Another observational evidence came with the detection of optical emission-line filaments near the centre of clusters, that were interpreted by Cowie et al.[113] and Fabian et al.[157] as arising from the IC gas cooling down to ~ 10000 K.

Gorenstein et al.[186] remarked upon the different X-ray emissions of the central galaxies in Virgo and Perseus, on one side, and the two dominant galaxies in Coma, on the other side. They correctly pointed out that the difference was related to the lack of cooling flows in the Coma cluster. If NGC 4874 and NGC 4889 were moving through the IC gas, their motion could prevent the formation of a cooling flow (Mathews & Bregman[293]). A significant motion of the two dominant Coma galaxies with respect to the cluster was later discovered[68].

In the 80's Lea et al.[275] and Sarazin & O'Connell[399] noted that the inferred mass deposition rates in cooling flows were much higher than the inferred star formation rates as derived from UV observations (e.g. Bertola et al.[59] for M87). The hypothesis was made[399] that only low-mass stars, characterized by small UV emission, can form in the high-density cooling flow regions.

Cooling flows have since become a major research topic in cluster astrophysics. Two thirds of all clusters contain a cooling flow at their centre. The deposited mass is still unaccounted for, but there exist evidence for X-ray absorption in the centres of cooling-flow clusters, which might be related to the deposited material (see FABIAN, these proceedings). Maybe the active nucleus which is often present in galaxies with cooling flows plays a significant role in re-distributing the accreted material (see MCNAMARA, these proceedings).

6 Conclusions

In the course of time, the concept of cluster of galaxy has significantly evolved. A concentration of nebulæ, maybe galactic objects, like a star cluster, in the early days of the XX century. A remarkable (but relatively rare) concentration of external galaxies, which nevertheless were much smaller than our own, in Hubble's times. Or rather the extreme of a continuous clustered distribution of galaxies, according to Carpenter. A stable, bound dynamical system, with an incredible mass, according to Zwicky. Or instead, a light, rapidly disrupting system, whose explosion was powered by unknown mechanisms operating in the centres of its galaxies, according to Ambartsumian. A galaxy incubator, according to Zel'dovich' top-down scenario, or rather an association of free galaxies, according to Peebles' bottom-up scenario. A dangerous place to live, for spiral galaxies, according to *nurture* scenarios for galaxy evolution. Or maybe a very quiet place, where old ellipticals can passively evolve for billions of years, according to *nature* scenarios for galaxy formation. A knot in the filamentary structure of the Universe, when the Large Scale Structure was finally unveiled by observations in the 80's. A cluster of gas, rather than a cluster of galaxies, in the 90's, when X-ray surveys became more effective in finding high redshift clusters than the traditional optical methods. And now, finally, a cluster of dark matter, a dark cluster, which will be found through the weak lensing surveys (see MELLIER, these proceedings).

If the evolution of clusters is slow (see, e.g., DICKINSON, these proceedings), not so slow is the evolution of science. Moreover, this evolution is often discontinuous and non-monotonic. Zwicky's missing mass was re-discovered in galaxy halos after 40 years; the existence of significant subclustering in clusters was demonstrated in the 60's by van den Bergh, but the irregular X-ray morphologies of clusters came as a surprise to many astronomers. The Local Supercluster was hinted at by J. Herschel in the XIX century, and rediscovered several times before de Vaucouleurs re-affirmed its existence, in the 50's. And many other examples can be found by reading this review. We certainly need to keep track of the evolution of science, or we risk to forget about fundamental results that might take years to be re-discovered. I hope this modest review can be helpful in this respect.

> *"Quello che lei non sa è il vero scopo del nostro lavoro* [...] *È perchè tutto non sia stato inutile, per trasmettere tutto quello che sappiamo ad altri che non sappiamo chi sono nè cosa sanno."*
>
> Italo Calvino, *La memoria del mondo*

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