A Bayesian approach to filter design: The Biparametric Scale Adaptive Filter for the detection of compact sources

Marcos López-Caniego [1]
Diego Herranz [2]
Jose Luis Sanz [1]
Belen Barreiro [1]

[1] Instituto de Física de Cantabria, Santander, Spain
[2] Istituto di Scienze e Technologie dell'Informazione, CNR, Pisa, Italy

Outline of the talk:

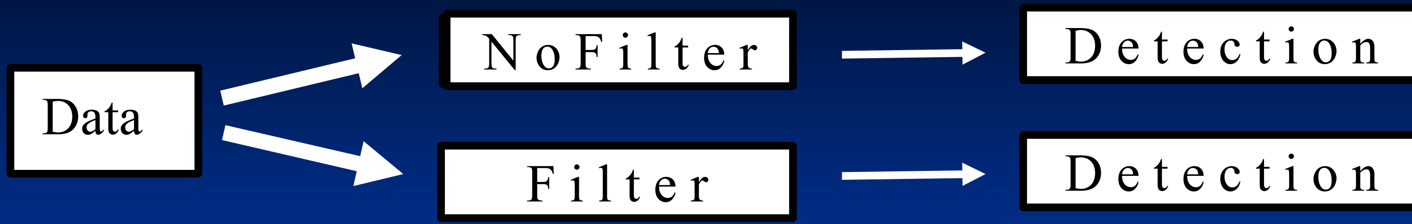
- Introduction
- The Filter: Biparametric Scale Adaptive Filter
- The Detector
- Theoretical results
- Simulations
- Conclusions

1. Introduction

- Detection of compact signals embedded in a background
  - extragalactic point sources (EPS) in μwave frequencies.
- CMB maps: mixture of components such as CMB, Galactic dust, synchrotron, free-free, compact sources, etc.
  - Difficult to separate EPS due to their unknown frequency dependence.
- Common approach: **filtering and thresholding**

In this approach: filtering and detection are independent

To filter or not to filter…



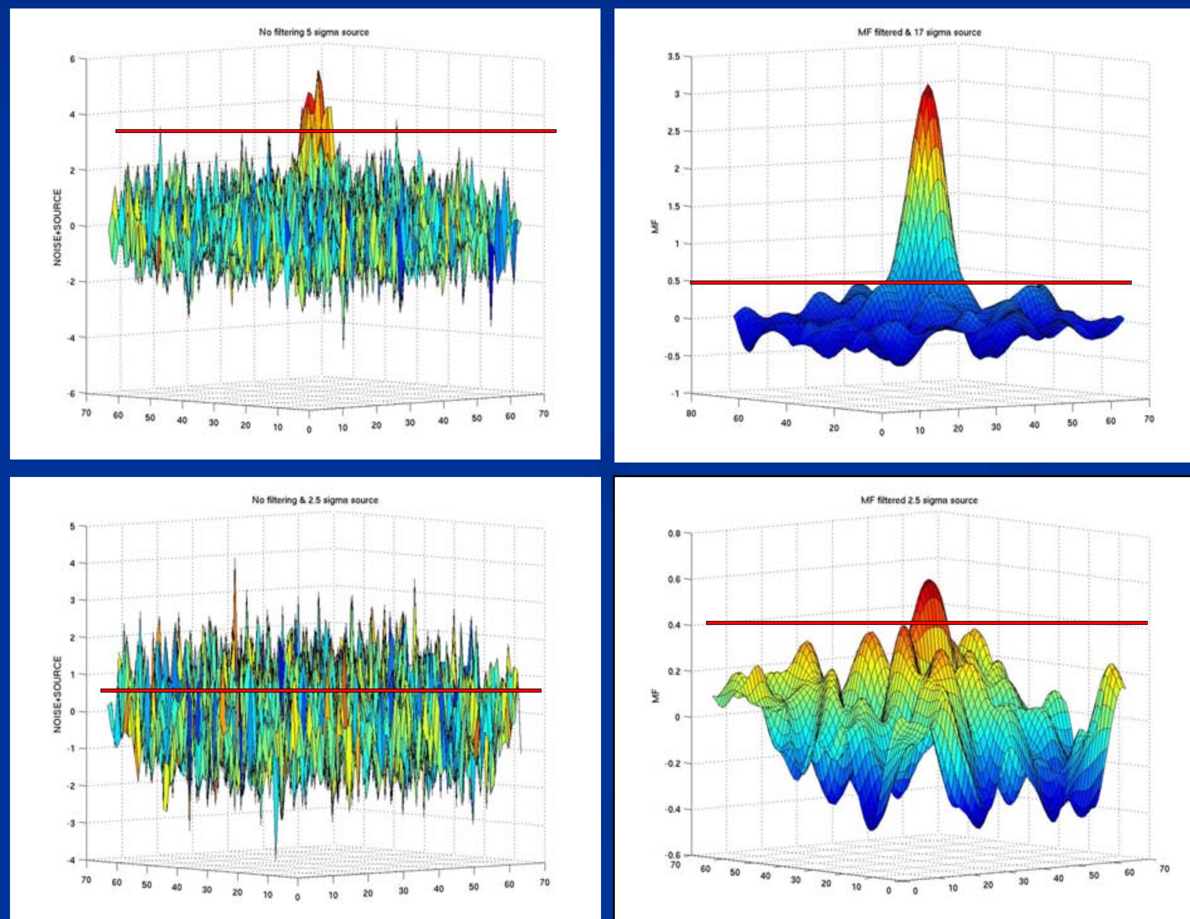
No filtering may be ok for bright sources…

3σ before filtering
17σ after filtering

But, what if the sources are weak?

$\sigma_{sf} = 1$
0.5σ before filtering

$\sigma_f = 0.17$
3σ after filtering



- Other filtering based
  - Mexican Hat wavelet (Cayón et al. 2000, Vielva et al. 2001)
  - Matched Filter (Tegmark and Oliveira-Costa 2002)
  - Adaptive Top Hat Filter (Chiang et al. 2002)
  - Scale-adaptive Filter (Sanz et al. 2001).

It is not clear which one, if any, is optimal

- Our approach:
  - filtering and detection are not independent.
  - the goal of filtering is to transform the data in such a way that the detector performs better.
  - Criterion is:

Find a combination of optimal filter and detector such that the number of detections is maximum for a fixed number of spurious sources.

2. The Filters:

- Biparametric Scale Adaptive filter (BSAF)

$$\tilde{\Psi} \propto x^{\gamma} e^{-\frac{1}{2}x^2}\left(1 + cx^2\right), \quad x \equiv \alpha q R$$

  - BSAF => combination of MF + MHW for $\gamma = 0$.
  - Conditions to obtain it:
    - $<w(R_0,0)>=A$ unbiased estimator of the amplitude
    - The variance of $w(R_0,0)$ has a minimum at R_0
    - $w(R_0,b)$ has a maximum in the filtered image at $b=0$
    - Power spectrum $P(q)=Dq^{-\gamma}$, γ spectral index of background.
  - The BSAF has two free parameters **c** and $\boldsymbol{\alpha}$,
    - **c:** arbitrary parameter
    - α: $\alpha > 0$, modifies the filtering scale R.

- Filtering at other scales than R can improve detections
  - MHW (Vielva et al. 2001)
  - MF (López-Caniego et al. 2004)

- Following this idea, we introduce α in the other filters.

  - The Filters:

MF - $$\tilde{\Psi} \propto x^{\gamma} e^{-\frac{1}{2}x^2}$$

SAF - $$\tilde{\Psi} \propto x^{\gamma} e^{-\frac{1}{2}x^2} \left(1 + \frac{t}{m^2} x^2\right)$$

MHW - $$\tilde{\Psi} \propto x^2 e^{-\frac{1}{2}x^2}$$

BSAF - $$\tilde{\Psi} \propto x^{\gamma} e^{-\frac{1}{2}x^2} \left(1 + cx^2\right)$$

$$x \equiv \alpha q R$$

$$m = \frac{1+\gamma}{2}$$

$$t = \frac{1-\gamma}{2}$$

3. The Detector

- Optimality & Concept of detection.
  - Was there a signal at the input of the receiver?
  - Not obvious => Signal corrupted by the "**noise**"
- The ***detector:*** use the information in terms of pdf's

H_0: null hypothesis => **background** alone

H_1: alternate hypothesis => **background + signal**

- The ***detector*** divides the space R in 2:

R_*: H_0 is rejected → A signal is present

R_-: H_0 is accepted → No signal is present

R_* is the region of acceptance

- A simple detector is ***thresholding***. *T*he space R is defined by the objects above/below an arbitrary threshold, nσ.

high threshold → small number of detections

low threshold → many detections with a high probability of spurious.

Are there any detectors that include more information than thresholding?

It will be shown the importance of introducing the curvature of the maxima in the detector because allows to distinguise between maxima of the background and maxima of the background + source..

- Our approach:
  - calculate the region of acceptance
  - look for maxima
  - apply the detector

- The detector: obtained with the Neyman-Person rule taking into account the balance two types of errors:

Type I: Spurious detection → α is the probability of accepting an event when a signal was not present.

Type II: False dismissal → $1-\beta$ is the probability of rejecting an event when a signal was present.

The acceptance region R_* giving the highest number density of detections n^*, for a given number density of spurious n^*_b, is the region

$$L(\nu,\kappa) \equiv \frac{n(\nu,\kappa)}{n_b(\nu,\kappa)} \geq L_*$$

If $L \geq L^*$ => signal is present
If $L < L^*$ => signal is absent

Background

- 1D background represented by a Gaussian random field ξ(x) with average value <ξ(x)>=0 and power spectrum P(q).
- The distribution of maxima of the background (Rice 1954).
- The expected number of maxima

$$n_b(\nu,\kappa)=\frac{n_b\kappa}{\sqrt{2\pi(1-\rho^2)}}e^{-\frac{\nu^2+\kappa^2-2\rho\nu\kappa}{2(1-\rho^2)}},\ \nu\equiv\frac{\xi}{\sigma_o},\ \kappa\equiv\frac{\xi''}{\sigma_2},\ \rho\equiv\frac{\sigma_1{}^2}{\sigma_o\sigma_2}.$$

Local Source

- The expected number density of maxima given a gaussian source of amplitude A is given by (Barreiro et al. 2003):

$$n(\nu,\kappa\,|\,\nu_s)=\frac{n_b\kappa}{\sqrt{2\pi(1-\rho^2)}}e^{-\frac{(\nu-\nu_s)^2+(\kappa-\kappa_s)^2-2\rho(\nu-\nu_s)(\kappa-\kappa_s)}{2(1-\rho^2)}}$$

Real and spurious detections

- In any region R_*: the number density of spurious and real detections:

$$n_b^* = \int_{R_*} d\upsilon \, d\kappa \, n_b(\upsilon, \kappa)$$

$$n^* = \int_{R_*} d\upsilon \, d\kappa \, n(\upsilon, \kappa), \qquad n(\upsilon, \kappa) = \int_0^\infty d\upsilon_s \boxed{\mathrm{p}(\upsilon_s)} \, n(\upsilon, \kappa \mid \upsilon_s)$$

- R_* is given by the detector $L \geq L^*$, or equivalently by $\varphi \geq \varphi^*$, where

$$\boxed{\varphi(\upsilon, \kappa) \equiv C_1 \nu + C_2 \kappa} \quad \begin{cases} C_1 = \dfrac{1 - \rho y_s}{1 - \rho^2} \quad C_2 = \dfrac{y_s - \rho}{1 - \rho^2} \\ y_s \equiv \dfrac{\kappa_s}{\nu_s} \quad \rho \equiv \dfrac{\sigma_1^2}{\sigma_0 \sigma_2} \end{cases}$$

The detector is linear and independent of the pdf's

4. Numerical results

- Application to the detection of compact sources characterized by a Gaussian profile in backgrounds $q^{-\gamma}$.
  - We want to detect very weak sources [0,1.5] before fitering
  - We use two *a priori* distributions of sources
    - Uniform Distribution
      - [0,2]σ after filtering

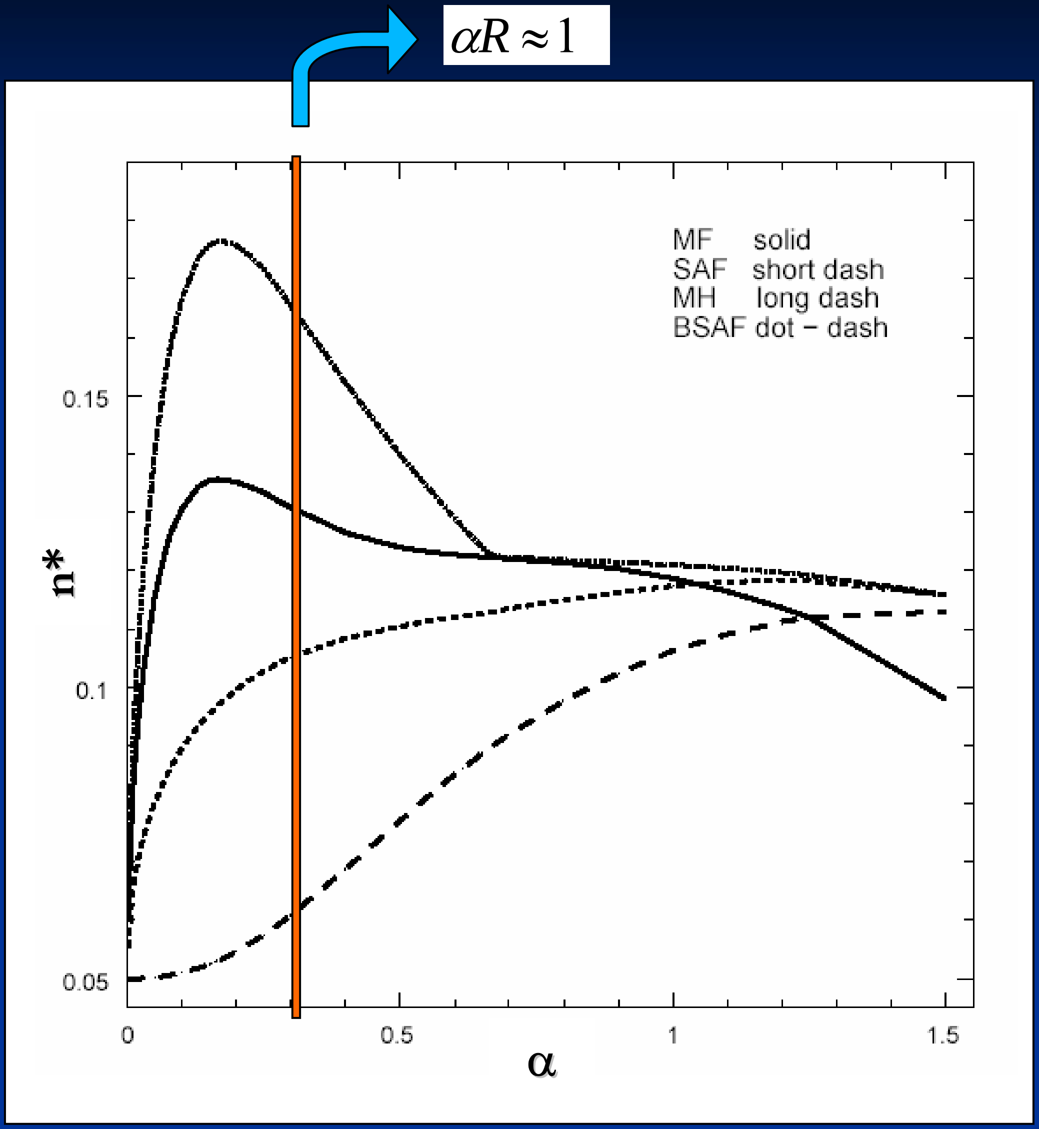
$$p(\upsilon_s) = \frac{1}{\upsilon_c}, \upsilon_s \in [0, \upsilon_c]$$

    - Scale-Free Distribution
      - [0.5,3]σ after filtering

$$p(\upsilon_s) \propto \frac{1}{\upsilon_s^{\beta}}, \upsilon \in [\upsilon_{in}, \upsilon_{fin}]$$

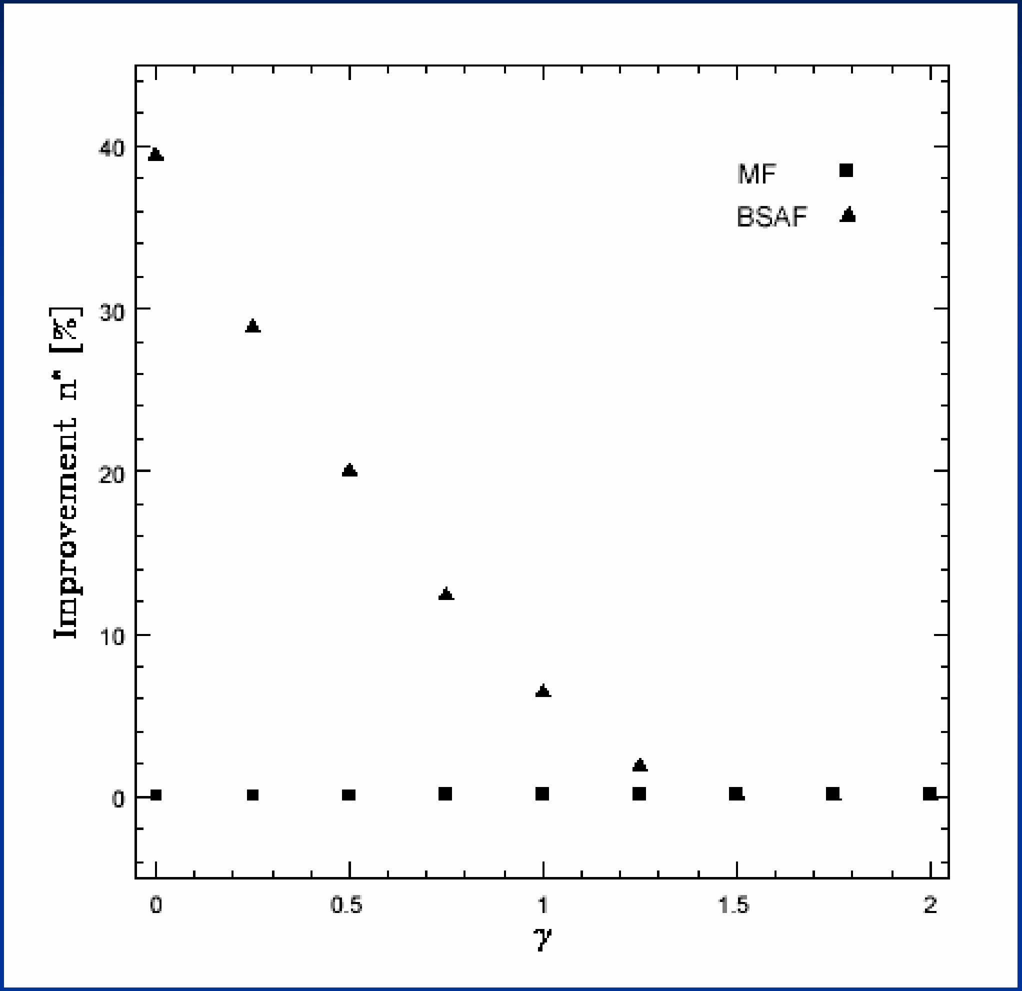
Uniform Distribution [0,2]σ

$\gamma=0$



R	n_b^*	γ	α	c	n^*_{BSAF}	n^*_{MF}	$RD[\%]$
2	0.01	0	0.4	-0.69	0.0860	0.0824	4.4
2	0.03	0	0.4	-0.68	0.1493	0.1311	13.9
		0.5	0.4	-0.59	0.1512	0.1474	2.5
2	0.05	0	0.4	-0.70	0.1900	0.1575	22
		0.5	0.4	-0.59	0.1935	0.1783	9
3	0.01	0	0.3	-0.86	0.0784	0.0658	19.1
3	0.03	0	0.3	-0.86	0.1282	0.1013	26.5
		0.5	0.3	-0.73	0.1242	0.1145	8.4
3	0.05	0	0.3	-0.86	0.1654	0.1186	39.4
		0.5	0.3	-0.75	0.1616	0.1352	19.5
		1	0.4	-0.58	0.1582	0.1487	6.3

BSAF improves the MF 40 %



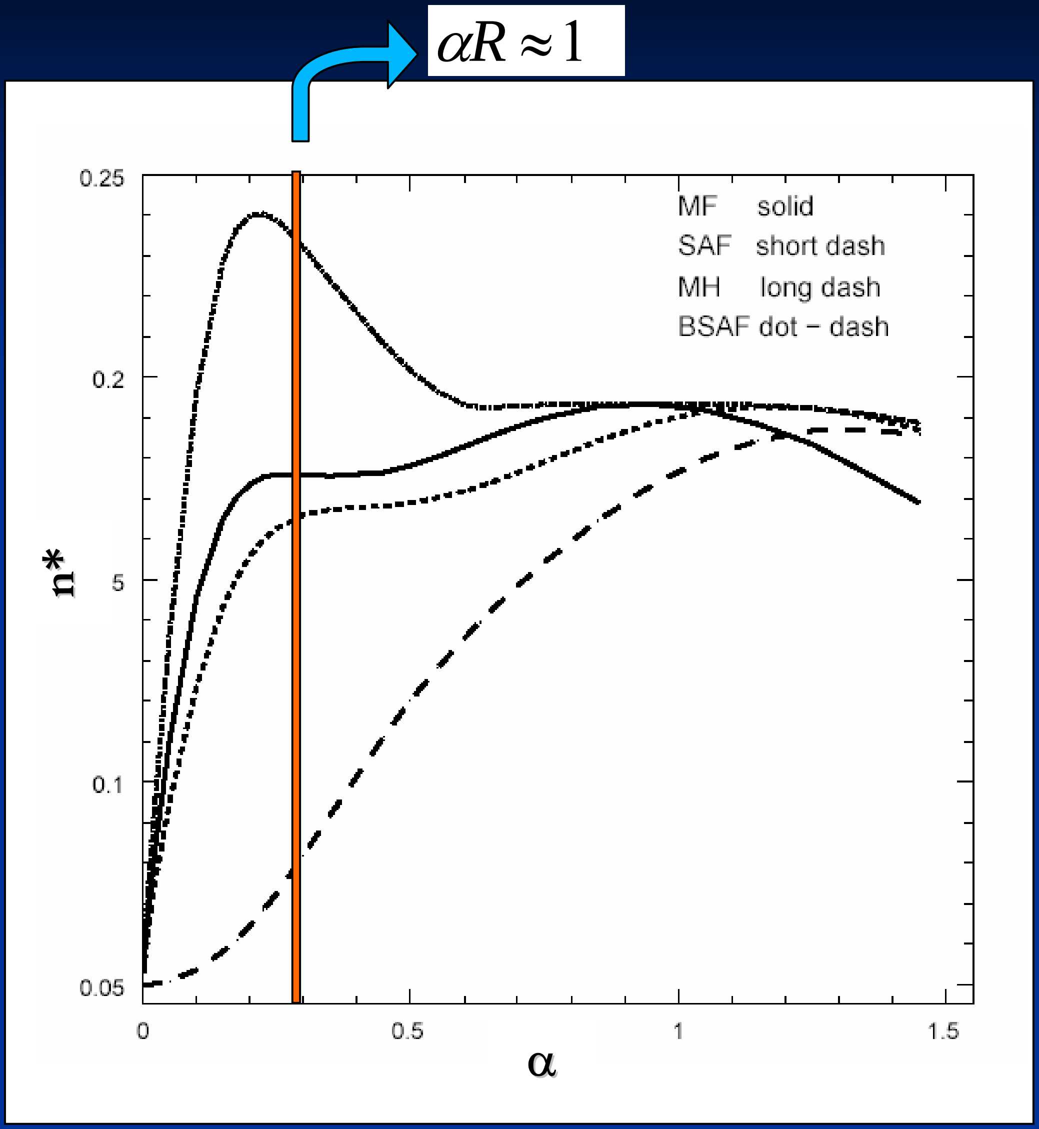
Uniform Distribution [0,2]σ

Improvement BSAF vs. MF

For different values of the spectral index γ

Scale-free Distribution [0.5,3]σ

γ=0



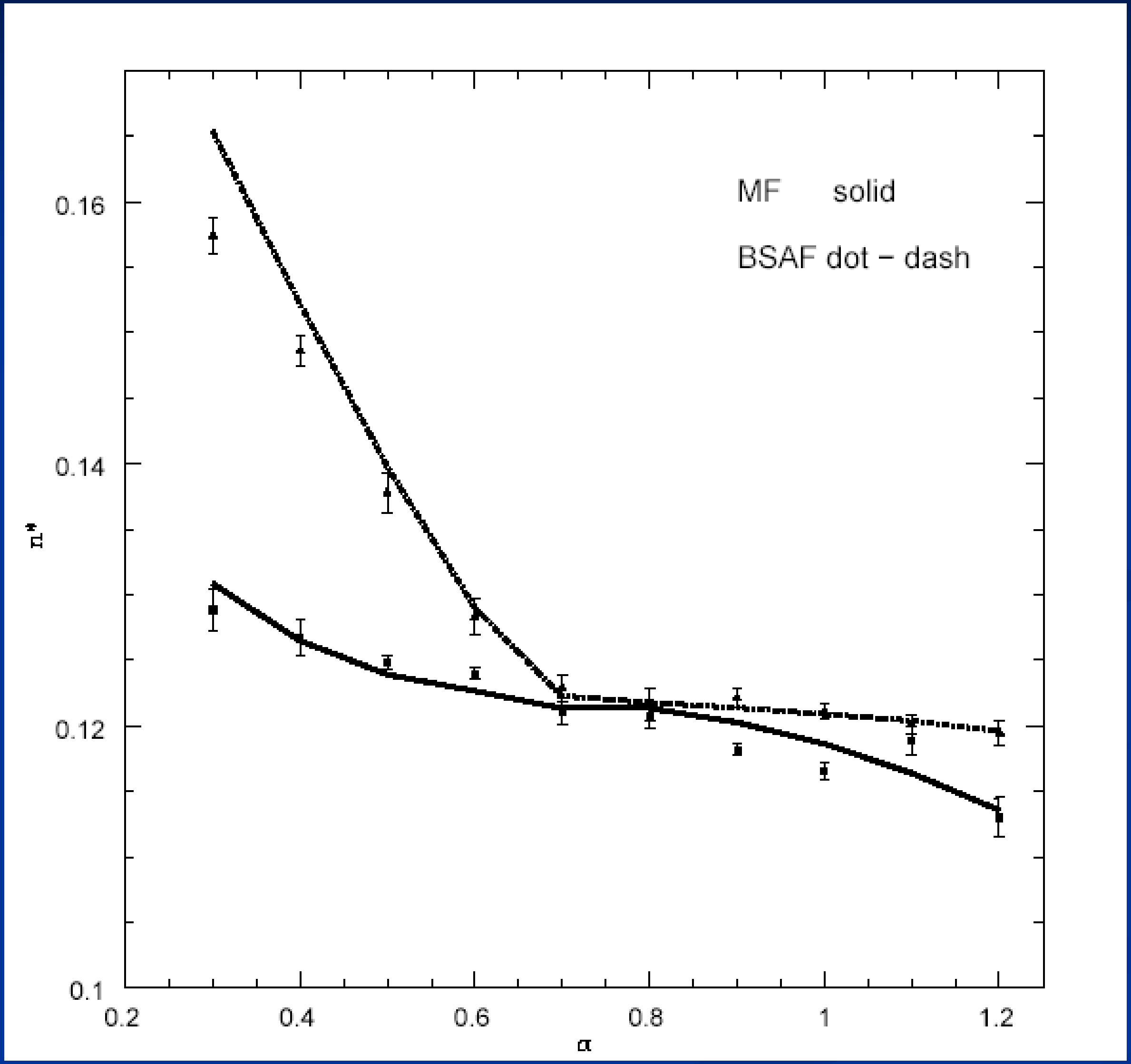
R	n_b^*	γ	α	c	n_{BSAF}^*	n_{MF}^*	$RD[\%]$
2	0.01	0	0.4	-0.66	0.1659	0.1590	8.2
2	0.03	0	0.4	-0.68	0.2376	0.2089	23.2
		0.5	0.4	-0.56	0.2451	0.2432	28
2	0.05	0	0.4	-0.68	0.2772	0.2311	32.8
		0.5	0.4	-0.57	0.2873	0.2705	11.6
3	0.01	0	0.3	-0.83	0.1336	0.1180	13.2
3	0.03	0	0.3	-0.83	0.1975	0.1512	30.6
		0.5	0.3	-0.71	0.1937	0.1767	9.6
3	0.05	0	0.3	-0.81	0.2335	0.1639	42.5
		0.5	0.3	-0.70	0.2321	0.1928	20.4
		1	0.3	-0.62	0.2271	0.2169	4.7

BSAF improves the MF 42 %

5. Simulations

- 1D images with a Gaussian background and white noise P(q)
- Point sources distributed following an uniform distribution.
- Our simulated images:
  - Size: 4096 pixels.
  - White noise dispersion: unity.
  - R=3 pixels for the added source.
- The size of the image is such, that the addition of the source does not modify the previous dispersion in a significant way.
- Each image is filtered with the MF and BSAF.
  - φ_* defines the acceptance region and is obtained from the images.
  - From the observables, A, κ and σ^2_n, we calculate for each peak φ.

$\varphi > \varphi_*$ → # of detections



Simulations

Uniform Distribution

Sources - [0,2]σ

R=3

n^*_b=0.05

ν_c=2

5 sims/point

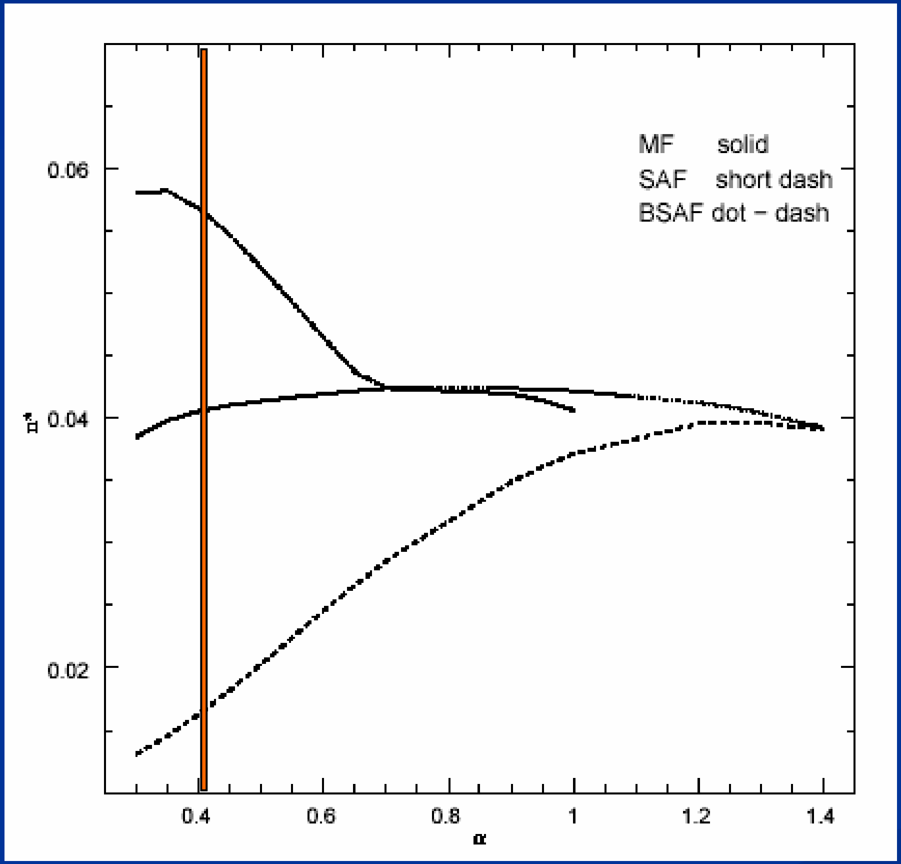
Each simulation:

$50{\cdot}10^3$-$150{\cdot}10^3$

realizations.

Results for 2D case

$\alpha R \approx 1$



Uniform Distribution [0,2] σ

BSAF improves the MF 40 %

R	n_b^*	α	c	n_{BSAF}^*	n_{MF}^*	$RD[\%]$
1.5	0.005	0.5	-0.44	0.0507	0.0484	4.7
	0.01	0.5	-0.46	0.0709	0.0620	14.3
2	0.005	0.4	-0.54	0.0396	0.0335	18.2
	0.01	0.4	-0.54	0.0567	0.0406	39.6
2.5	0.005	0.3	-0.64	0.0320	0.0245	33.3

6. Conclusions

- Filtering based detection criterion:
  - New filter: BSAF. Includes the other filters for white noise.

  - New detector: Bayesian Neyman-Pearson rule with *a priori* information of the source distribution and number densities of maxima that includes amplitude and curvature information.
    - The curvature plays an important role defining the acceptance region.

- BSAF + detector → significant improvement for γ [0,2]
  - White noise: the BSAF improves the standard MF 40%.

- Similar results with two distributions. Tested with simulations.

- 2D: 40 % improvement for white noise.