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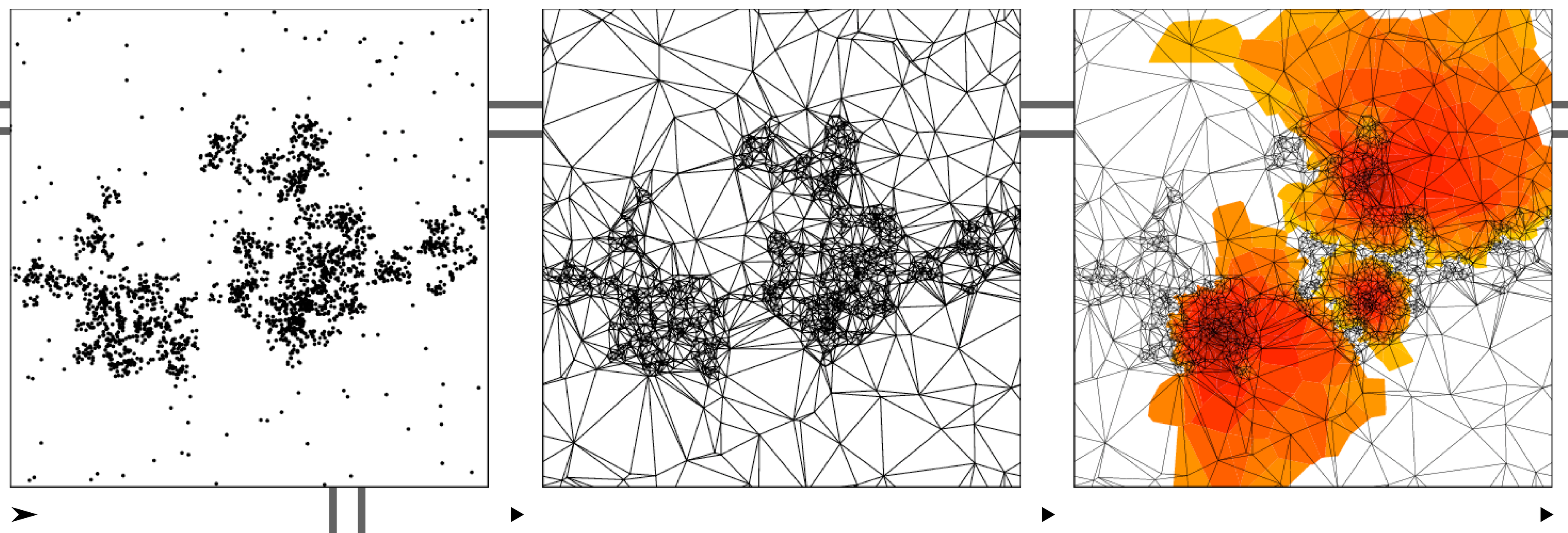
# SimpleX *Using unstructured grids to simulate radiative transfer during the EoR, allowing sources at all gridpoints and explicit tracing of recombined photons.*

**SimpleX** is a general method for solving transport problems. Our version is specialized for the problem of radiative transfer by considering this as a photon transport problem. The method has the extraordinary property that it doesn't scale with the number of sources because of its use of unstructured grids. These are **Delaunay tessellations** based on a point sampling $n_p$ of the underlying density field $n$. In $d$ dimensions we have for the length $L$ in the grid and the mean free path $\lambda$:

$$\langle L \rangle \propto n_p^{-d} \quad \text{and} \quad \lambda \propto n^{-1}$$

If we sample the $d$th power of the density field, $n_p \propto n^d$, we have $\langle L \rangle \propto \lambda$. In this way we can couple all physics to the grid and the movement of photons reduces to jumps between neighbouring gridpoints. The basic principle has been shown to work! Here we present results on the detailed properties of the method.




## Background picture

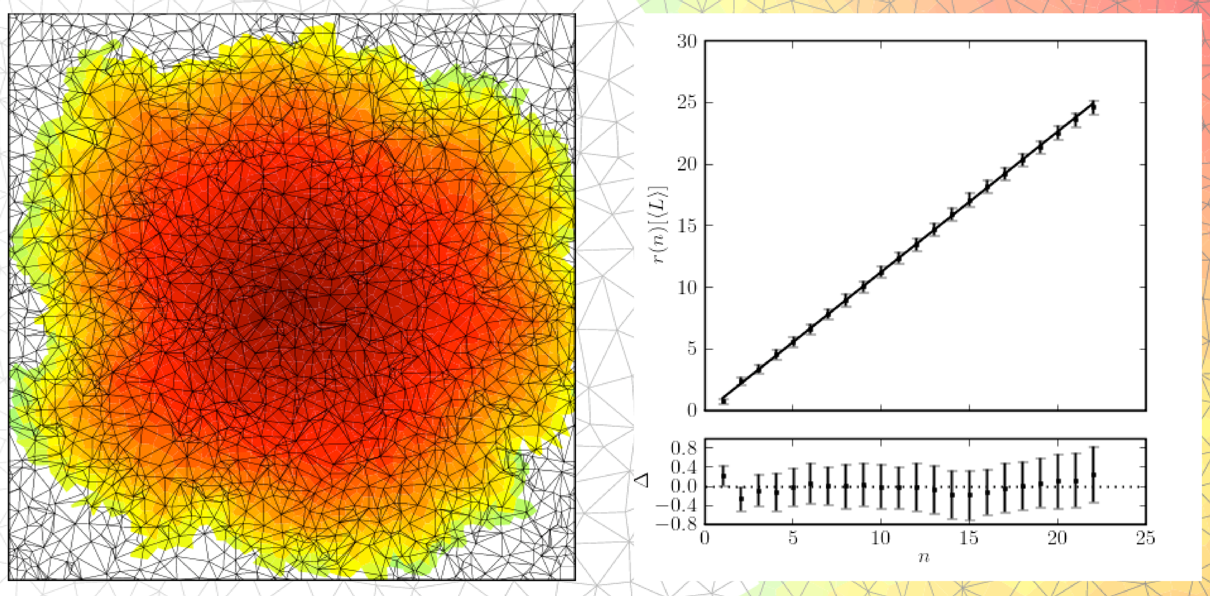
By coupling all physics to the underlying grid, we can very efficiently calculate the transport of photons through a background density field. The transport reduces to moving photons from gridpoint to gridpoint. Therefore the method doesn't scale with the number of sources. The difficulty is that we need to have a very detailed understanding of the properties of the grid, to separate physical effects from effects inherent in the method.

The background picture shows a nice example of this. It consists of a grid based on three seperate Poisson point processes. One process fills the whole simulation box with 10000 points. Along each diagonal another 5000 points were placed in a narrow box. Then the Delaunay tessellation after two *Lloyd-iterations* was set up and 40 sources with varying source strength were randomly placed along the diagonals. We simulate only the diffusion of photons over the grid. No physics is included. Effectively we are simulating a case with no absorption.

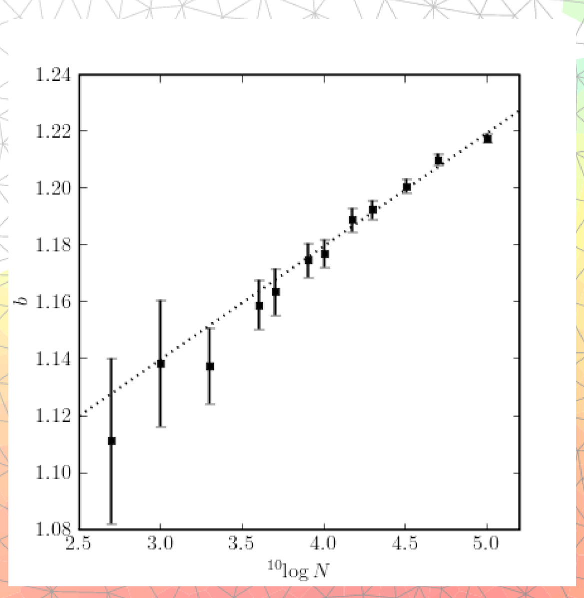
In this situation photons should distribute isotropically and the wavefront should have the same velocity everywhere. Clearly this is not the case. There are large gaps in the wavefronts at the filaments. The presence of more points causes photons to take longer to pass . It is not clear yet how to seperate this inherent effect from true absorption.

## Diffusion speed

Let's do a very simple test. We start with a uniform (Poissonian) background, put a single source in the center and let the photons diffuse. One would expect the wavefront to travel on average a distance $\langle L \rangle$. Unfortunately the distribution of edgelengths has a very long tail. The result is that the wavefront travels outward faster. Worse is that the effect depends on the number of gridpoints. If there are more gridpoints the tail becomes better 'filled' and the wavefront will travel even faster.

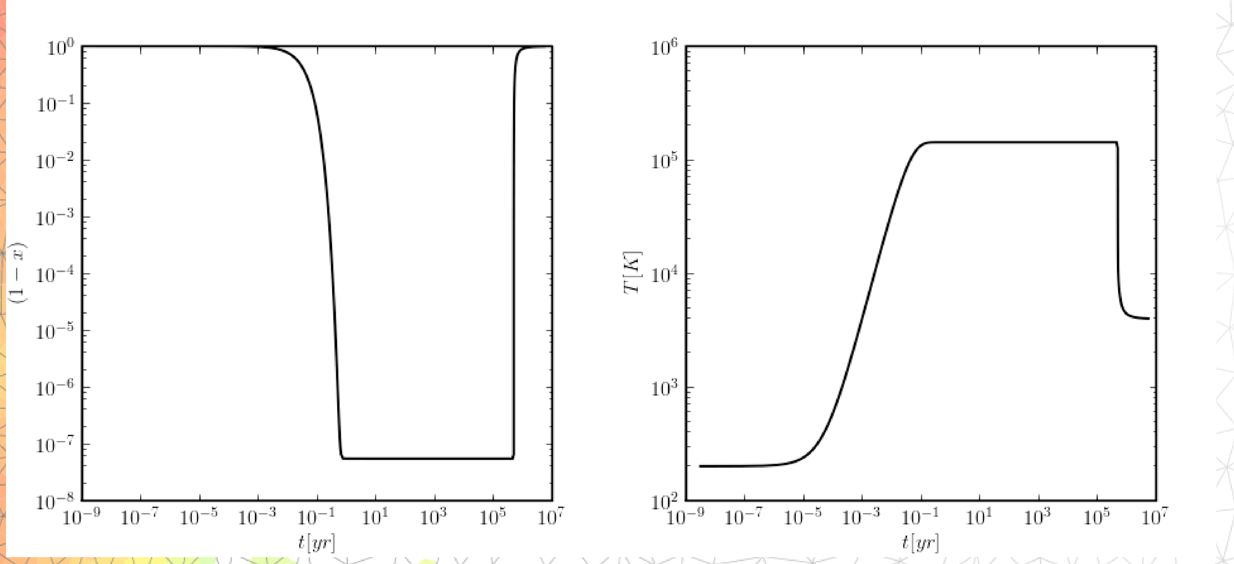


The results of the experiment with 2000 Poissonian points after 18 iterations. The top left figure shows the wavefront. The top right figure shows the radius of the wavefront in units of $\langle L \rangle$. The radius is determined as the mean of the maximum and minimum radius of the wavefront. The slope $b$ is $1.14 \pm 0.01$ . The errorbars show the maximum and minimum radius at each iteration. The bottom figure shows the dependency of the slope on the number of gridpoints $N$.



## Additional Physics

The first version of Simple*X* only knows about Hydrogen ionization and recombination at the ionization frequency. We are trying to add in more physics by allowing more frequencies and more particles (eg Helium). Adding more frequencies is very difficult, because the mean free paths are very different. By now a dynamical tessellation code is available, which will help tremendously in dealing with more frequency bands. An attempt to deal with cooling- and heatingprocesses was also undertaken. For this we try to refine the case 0 experiment of the Code Comparison Project[2]



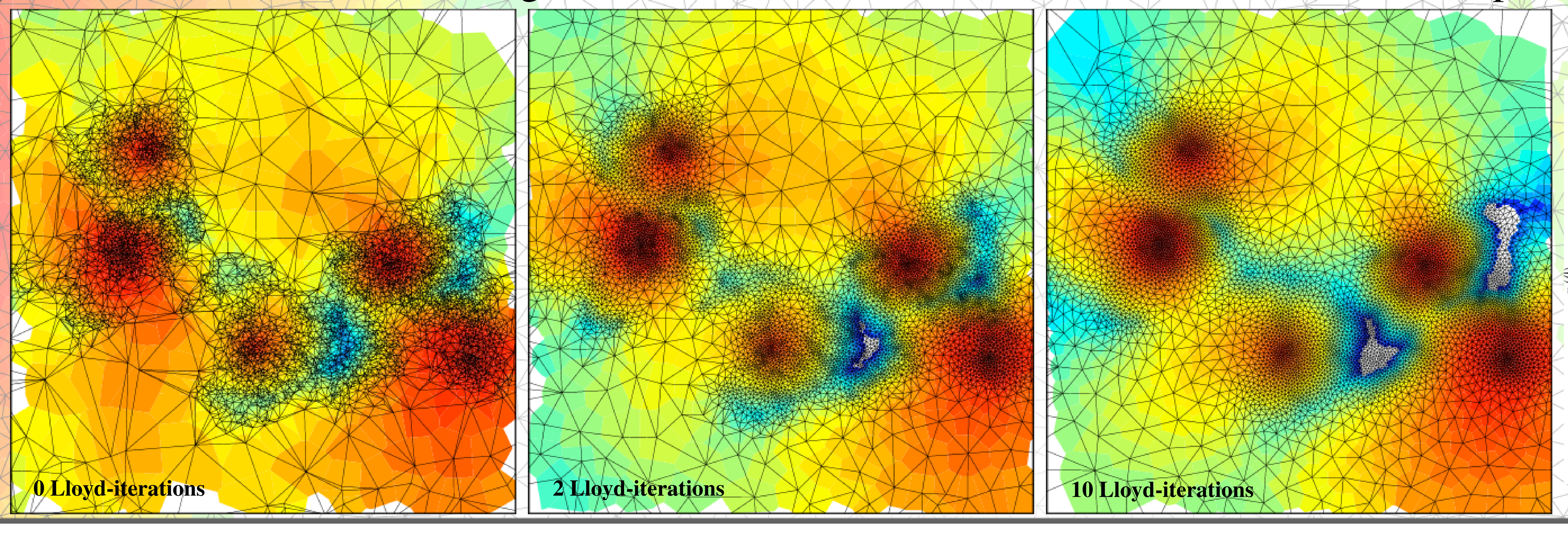
The results for SimpleX for the case 0 experiment of the Code Comparison project after adding a simple estimate for the heating. The method overestimates the temperature slightly. The left figure shows the neutral fraction. The right figure shows the temperature.

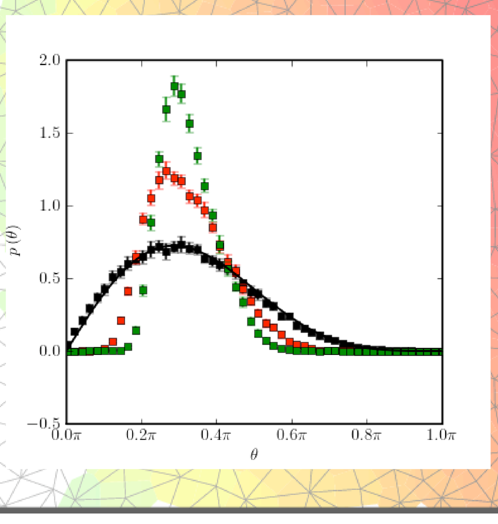
The cooling is given by standard results also used by other radiative transfer codes. The heating per photon is approximated by the average energy above the ionization threshold for a given source spectrum (sofar only Blackbody spectra of a given temperature). This approach overestimates the heating, causing an ionization fraction that is too low. Finding a better way to deal with heating and cooling processes is an important topic of ongoing research.

## Isotropy of the grid

### *Lloyd-iterations*

Besides the distribution of lengths of edges, also the distribution of the angles between outgoing lines at a gridpoint has a large tail. The effect of this is that photons are in many cases not distributed isotropically when they should. We found that we can improve on this by using socalled Lloyd-iterations. These iterations will transform a given Delaunay grid into a Centroidal tessellation. In such a tessellation the gridpoints are precisely at the mass centers of the cells. We show the results of a photon diffusion experiment with four sources after 28 iterations for zero, two and ten Lloyd-iterations and the resulting angular distribution functions. The results show that the grid becomes much better behaved. However, the process also smeers out details and we loose some of the resolution that we got from the unstructured grids. We need to find a criterium that tells us what the optimum is.



The angular distribution function after zero (black), two (red) and ten (green) Lloyd-iterations. The black curve is the theoretical curve. Especially in the first two iterations there is much improvement.

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