Classicalization of primordial perturbations by Continuous Spontaneous Localization, a quantum collapse model, after PLANCK and BICEP2

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Inflationary Paradigm: Success

- Solves Big Bang pathologies
- Generates primordial perturbations → seeds for large scale structures, CMB anisotropy
- Predicts (Single field models):

1. Almost scale invariant scalar power spectrum: $n_s = 0.9635 \pm 0.0094$
2. Almost Gaussian distribution of primordial perturbations : $f_{NL} < 2.7 \pm 5.8$
3. Consistency relation : $r = -8n_T$

Lingering conceptual issues

A. Trans-Planckian issue: Largest observable modes were below Planck length during inflation

- Solutions : Alternatives to inflation

B. Quantum to Classical transition of primordial perturbations: origin of perturbations are quantum but observed structures are classical

- Solutions :

1. Does not modify the basic mechanism of QM : Decoherence
2. Modifies basic mechanism of QM : Collapse models [Continuous Spontaneous Localization (CSL) Model]

Continous Spontaneous Localization

- Modifies Schrödinger equation by adding non-linear stochastic terms :

$$d\psi_t = \left[-\frac{i}{\hbar} H dt + \frac{\sqrt{\gamma}}{m_0} \int d\mathbf{x} (M(\mathbf{x}) - \langle M(\mathbf{x}) \rangle_t) dW_t(\mathbf{x}) - \frac{\gamma}{2m_0^2} \int d\mathbf{x} \, (M(\mathbf{x}) - \langle M(\mathbf{x}) \rangle_t)^2 dt \right] \psi_t$$

- Non-linear terms break the superposition of wave functions

- Amplification Mechanism

$$\gamma(m) = \gamma_0 \left(\frac{m}{m_N} \right)^\beta, \qquad \gamma(m) = n^2 \gamma_0 \left(\frac{m}{m_N} \right)^\beta$$

- Hamiltonian not conserved due to non-Hermitian evolution → Non-conservation of energy

$$\langle E \rangle = \frac{3\gamma\alpha\hbar^2}{4m} t$$

Schrödinger picture of inflation

- Scalar perturbations in terms of Mukhanov-Sasaki variable

$$\zeta(\tau, \mathbf{x}) = a\left[\delta\varphi^{\mathrm{gi}} + \varphi_0' \frac{\Phi_B}{\mathcal{H}}\right]$$

- Quantum state wavefunctional satisfy functional Schrödinger equation

$$i\frac{\partial \Psi_{\mathbf{k}}^{\mathrm{R,I}}}{\partial \tau} = \hat{\mathcal{H}}_{\mathbf{k}}^{\mathrm{R,I}} \Psi_{\mathbf{k}}^{\mathrm{R,I}}$$

- Hamiltonian that of harmonic oscillator

$$\hat{\mathcal{H}}_{\mathbf{k}}^{\mathrm{R,I}} = -\frac{1}{2}\frac{\partial^2}{\partial\left(\zeta_{\mathbf{k}}^{\mathrm{R,I}}\right)^2} + \frac{1}{2}\omega^2\left(\zeta_{\mathbf{k}}^{\mathrm{R,I}}\right)^2, \qquad \omega^2 \equiv k^2 - \frac{a''}{a}$$

- Solution of functional Schrödinger equation is a functional Gaussian State

$$\Psi_{\mathbf{k}}^{\mathrm{R,I}}\left[\tau, \zeta_{\mathbf{k}}^{\mathrm{R,I}}\right] = \sqrt{N_k(\tau)}\exp\left(-\frac{\Omega_k(\tau)}{2}\left(\zeta_{\mathbf{k}}^{\mathrm{R,I}}\right)^2\right)$$

Wigner Function & Squeezing

- Wigner function recognises the correlation between position (field) and its momentum (conjugate to field)

$$\mathcal{W}\left(\zeta_{\mathbf{k}}^{\mathrm{R}},\zeta_{\mathbf{k}}^{\mathrm{I}},p_{\mathbf{k}}^{\mathrm{R}},p_{\mathbf{k}}^{\mathrm{I}}\right) = \frac{1}{(2\pi)^2}\int dxdy\Psi^*\left(\zeta_{\mathbf{k}}^{\mathrm{R}}-\frac{x}{2},\zeta_{\mathbf{k}}^{\mathrm{I}}-\frac{y}{2}\right)e^{-ip_{\mathbf{k}}^{\mathrm{R}}x-ip_{\mathbf{k}}^{\mathrm{I}}y}\Psi\left(\zeta_{\mathbf{k}}^{\mathrm{R}}+\frac{x}{2},\zeta_{\mathbf{k}}^{\mathrm{I}}+\frac{y}{2}\right)$$

$$= \frac{1}{\pi^2}e^{-\mathrm{Re}\,\Omega_k\left(\zeta_{\mathbf{k}}^{\mathrm{R}^2}+\zeta_{\mathbf{k}}^{\mathrm{I}^2}\right)}e^{-\frac{\left(p_{\mathbf{k}}^{\mathrm{R}}+\mathrm{Im}\,\Omega_k\zeta_{\mathbf{k}}^{\mathrm{R}}\right)^2}{\mathrm{Re}\,\Omega_k}}e^{-\frac{\left(p_{\mathbf{k}}^{\mathrm{I}}+\mathrm{Im}\,\Omega_k\zeta_{\mathbf{k}}^{\mathrm{I}}\right)^2}{\mathrm{Re}\,\Omega_k}}$$

- During inflation ⟶ on superhorizon scales $\mathrm{Re}\,\Omega_k \to 0$

$$\mathcal{W}\left(\zeta_{\mathbf{k}}^{\mathrm{R}},\zeta_{\mathbf{k}}^{\mathrm{I}},p_{\mathbf{k}}^{\mathrm{R}},p_{\mathbf{k}}^{\mathrm{I}}\right) \to \frac{\mathrm{Re}\,\Omega_k}{\pi}e^{-\mathrm{Re}\,\Omega_k\left(\zeta_{\mathbf{k}}^{\mathrm{R}^2}+\zeta_{\mathbf{k}}^{\mathrm{I}^2}\right)}\delta\left(p_{\mathbf{k}}^{\mathrm{R}}\right)\delta\left(p_{\mathbf{k}}^{\mathrm{I}}\right)$$

- Highly squeezed in momentum direction and spread in field

- Observation shows classicality in field direction

↓

Expect `collapse models' to squeeze the modes in field direction

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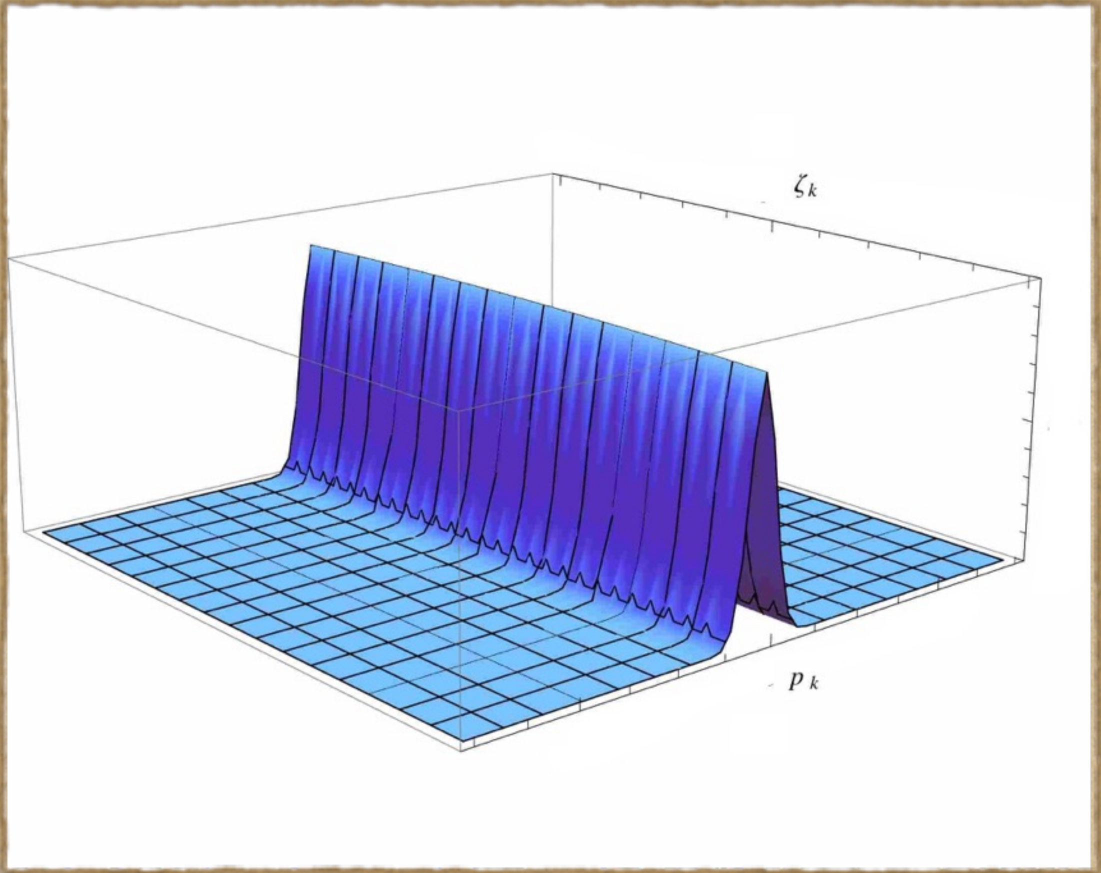
$$\mathrm{Re}\,\Omega_k \to \infty$$



CSL-like modification with constant γ

- Modify functional Schrödinger equation with `CSL-like' terms

$$d\Psi_{\mathbf{k}}^{\mathrm{R,I}} = \left[-i\hat{\mathcal{H}}_{\mathbf{k}}^{\mathrm{R,I}} d\tau + \sqrt{\gamma}\left(\hat{\zeta}_{\mathbf{k}}^{\mathrm{R,I}} - \left\langle \hat{\zeta}_{\mathbf{k}}^{\mathrm{R,I}} \right\rangle\right) dW_\tau - \frac{\gamma}{2}\left(\hat{\zeta}_{\mathbf{k}}^{\mathrm{R,I}} - \left\langle \hat{\zeta}_{\mathbf{k}}^{\mathrm{R,I}} \right\rangle\right)^2 d\tau \right]$$

- Frequency of the Harmonic Oscillator Hamiltonian becomes time dependent and complex

$$\omega^2 = k^2 - 2i\gamma - \frac{a''}{a}$$

- Smaller modes $(2\gamma \ll k^2)$

$$\mathrm{Re}\,\Omega_k \approx 2k(-k\tau)^2 \;\to\; 0, \qquad \mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{16\pi^2 \epsilon M_{\mathrm{Pl}}^2}$$

- Wigner function not affected by γ
- Squeezing in momentum direction (can't explain classicality)
- Power spectrum scale-independent (good for observation)

- Larger modes $(2\gamma \gg k^2)$

$$\mathrm{Re}\,\Omega_k \approx \frac{2\gamma}{k}(-k\tau) \;\to\; 0, \qquad \mathcal{P}_{\mathcal{R}}(k) = \frac{H^2 k^3}{16\pi^2 \epsilon M_{\mathrm{Pl}}^2 \gamma k_0} e^{-\Delta N}$$

- Wigner function affected by γ (which we wanted !!)
- Squeezing in momentum direction (can't explain classicality)
- Power spectrum scale-dependent (bad for observation)

$Jerome Martin, Vincent Vennin, Patrick Peter (Phys. Rev. D$86(2012)103524)

Modification by scale-dependent γ

- Modes behave more classically as they start crossing the horizon
- γ should discriminate between different modes according to their physical length scales ⟶ grow stronger as a mode starts crossing the horizon during inflation
- γ should be a function of time

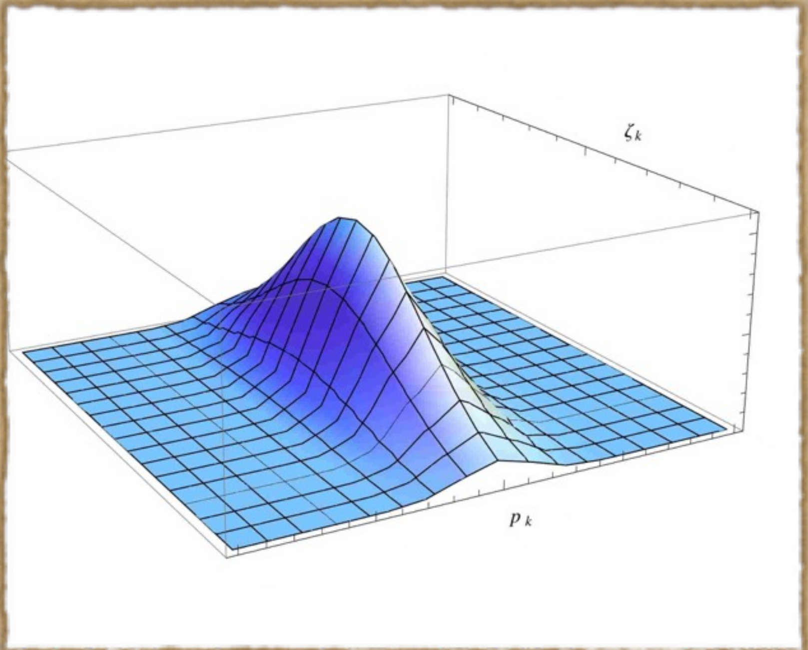
$$\gamma = \frac{\gamma_0(k)}{(-k\tau)^\alpha}\,, \qquad 0 < \alpha < 2$$

on superhorizon scales

$$\mathrm{Re}\,\Omega_k \approx \frac{k}{2}(-k\tau)^{1-\alpha}\left(\frac{2\gamma_0(k)}{k^2}\right)$$

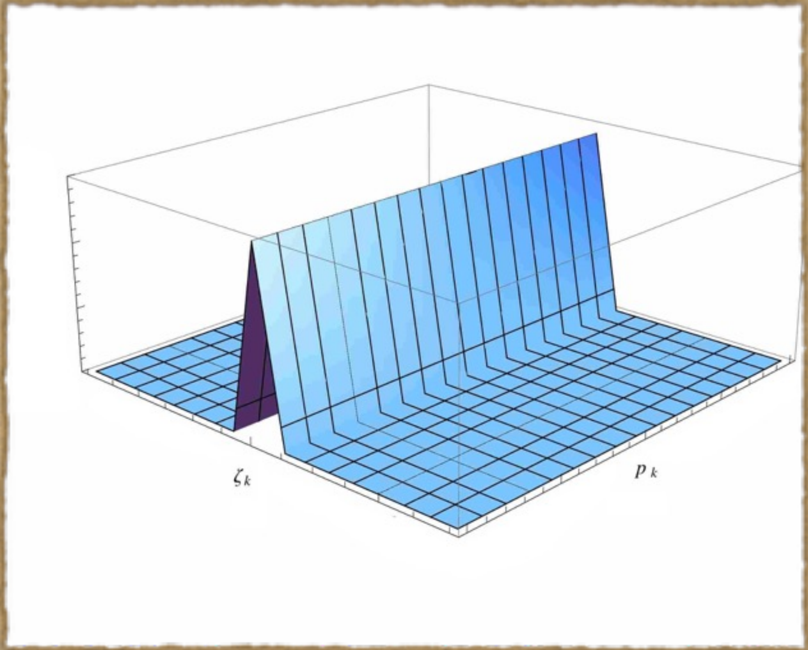
$$0 < \alpha < 1 \quad \longrightarrow \quad \mathrm{Re}\,\Omega_k \to 0$$

No macro-objectification



$$1 < \alpha < 2 \quad \longrightarrow \quad \mathrm{Re}\,\Omega_k \to \infty$$

Macro-objectification occurs



Scale-invariance of power spectrum

- To obtain a scale-invariant power spectrum make γ mode dependent

$$\gamma_0(k) = \tilde{\gamma}_0 \left(\frac{k}{k_0}\right)^{\beta}$$

- The power spectrum becomes

$$\mathcal{P}_{\mathcal{R}}(k) \propto k^{3+\alpha-\beta}$$

- $\beta = 3 + \alpha$ yields scale-invariant power spectrum

Macro-objectification of tensor modes

- Tensor modes $\longrightarrow$ Traceless and transverse part of metric fluctuations $\longrightarrow$ associated with two helicity states $+$ and $\times$
- Each helicity states identical to a massless scalar in de Sitter space
- Previous analysis of scalar perturbations applicable to each helicity states to obtain macro-objectification of tensor modes
- Assumption : CSL-modified dynamics is essentially same for gravitons and inflatons

Observables

- Scalar power spectrum

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{8\pi\epsilon M_{\rm Pl}^2}\frac{k_0^2 H^2}{\tilde{\gamma}_0} e^{-(1+\alpha)\Delta N}\left(\frac{k_*}{k_0}\right)^{3+\alpha-\beta}\left(\frac{k}{k_*}\right)^{3+\alpha-\beta+2\eta-3\epsilon}$$
$$\equiv A_s(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}$$

- Tensor power spectrum

$$\mathcal{P}_h = \frac{2}{\pi^2 M_{\rm Pl}^2}\frac{k_0^2 H^2}{\tilde{\gamma}_0} e^{-(1+\alpha)\Delta N}\left(\frac{k_*}{k_0}\right)^{3+\alpha-\beta}\left(\frac{k}{k_*}\right)^{3+\alpha-\beta-2\epsilon}$$
$$\equiv A_T(k_*)\left(\frac{k}{k_*}\right)^{n_T}$$

- Tensor amplitude depends upon collapse parameter ⟶ Stronger collapse parameter can bring down the scale of inflation

- Scalar spectral index

$$n_s - 1 = \delta + 2\eta - 4\epsilon$$

- δ can be of the order of slow-roll parameters
- Tensor spectral index

$$n_T = \delta - 2\epsilon$$

- Tensor-to-Scalar ratio

$$r = -8n_T + 8\delta$$

- Accurate measurements of r and n_T would be able to distinguish this scenario with the generic one

Summary

- Scale dependent collapse parameter can yield micro-objectification of modes, both scalar and tensor
- Wave-number dependence of collapse parameter yield nearly scale-invariance of power spectrum
- Collapse dynamics changes the consistency relation of single-field model
- Accurate measurement of tensor-to-scalar ratio and tensor spectral index can distinguish this dynamics from the generic scenario


Thank you