

# The common denominator of cosmological attractors

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## Scalar potentials

Planck is pointing towards plateau-like potentials:

$$\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\partial\mu)^2 - V(\mu) \right]$$

Plateau at infinite / finite distance, with (inverse) polynomial / exp fall-off.



## Kinetic formulation

Redefinition to trivial potential:

$$\frac{1}{2} R - \frac{1}{2} \left( \frac{d\mu}{dt} \right)^2 - \frac{1}{2} \omega^2 \mu^2 - \rho_0^2$$

Simplest case of alpha-attractors

Plateau in potential implies a singularity in kinetic term! Behaviour close to singularity is crucial.

## Inflationary predictions

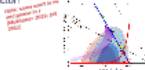
Behaviour at  $N=60$  determined by leading pole in Einstein-frame kinetic term:

$$K_E = \frac{1}{2} + \dots$$

Independent of subleading terms in K and fully independent of V, robustness of attractor!

$$n_s = 1 - \frac{2}{n-1}$$

$$r = \beta \left( \frac{2}{n-1} \right)^2$$



## Non-minimal coupling

Jordan frame formulation:

$$\frac{1}{2} \Omega(\phi) R - \frac{1}{2} (\partial\phi)^2 - \frac{\lambda}{\xi^2} (\Omega - 1)^2$$

Higgs inflation:  $\Omega = 1 + \xi \phi^2$

Universal attractor:  $\Omega = 1 + \xi \phi^n$

Induced inflation:  $\Omega = \xi \phi^2$

Stability:  $\xi > 0$   
 Jordan frame:  $\xi > 0$   
 Einstein frame:  $\xi > 0$   
 Induced inflation:  $\xi > 0$

## Higgs inflation: $\Omega = 1 + \xi \phi^2$

Reformulation leads to quadratic potential plus

$$K_E = \frac{1}{2} \frac{1}{\xi^2} + \frac{1}{2} \frac{1}{\xi^2} \frac{1}{(\Omega - 1)^2} = \frac{1}{2} \left( 1 + \frac{1}{\xi^2} \right) + \dots$$

Infinite coupling limit:

Stability model = a pure pole

Large coupling: increases residue and hence r

Small coupling: subleading terms important



## Universal attractor: $\Omega = 1 + \xi \phi^n$

Reformulation leads to quadratic potential plus

$$K_E = \frac{1}{2} \frac{1}{\xi^2} + \frac{1}{2} \frac{1}{\xi^2} \frac{1}{(\Omega - 1)^2} = \frac{1}{2} \left( 1 + \frac{1}{\xi^2} \right) + \dots$$

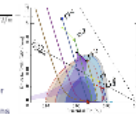
Infinite coupling:

same pole

suppressed poles

Order-zero coupling: other pole takes over

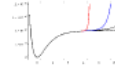
Small coupling: subleading corrections



## Generic deformations

Setting  $\xi \sim 10^3$  for power spectrum amplitude yields at least 50 flat e-folds.

Percent-level power less for larger N.



Single parameter that sets:

- spectral index

- tensor-to-scalar ratio

- power-law normalization

- number of flat e-folds

Roest, de Putter '13

## Induced inflation: $\Omega = \xi \phi^2$

Reformulation leads to quadratic potential plus

$$K_E = \frac{1}{2} \left( 1 + \frac{1}{\xi^2} \right) + \dots$$

Two contributions feeding into r:

1) positive offset from Jordan to Einstein frame

2) second contribution due to Jordan kinetic term

- Coupling can be negative, but only in Einstein frame!

- Jordan frame imposes a lower bound  $r > 0.003$ .

- Conformal value of predicts zero tensors.

- Equivalent to alpha-attractors with  $n = 1 + \frac{1}{\xi^2}$

## Summary

plateau inflation = pole inflation **alpha-attr. EE**

$n_s$  and  $r$  determined by order and residue of leading pole

Cosmological attractors with non-minimal coupling stem from a pole of order two:

- natural permittive value of  $r$

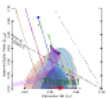
- different contributions to coeff

- lower bound  $r > 0.003$  from

Jordan frame

- relation between amplitude and if e-folds

- relation to alpha-attractors



# The common denominator of cosmological attractors

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30th Institut d'astrophysique de Paris Colloquium

## THE PRIMORDIAL UNIVERSE AFTER PLANCK

From Monday December 15<sup>th</sup> to Friday December 19<sup>th</sup>, 2014

*[(Galante,) Kallosh, Linde, Roest '13, '14]*

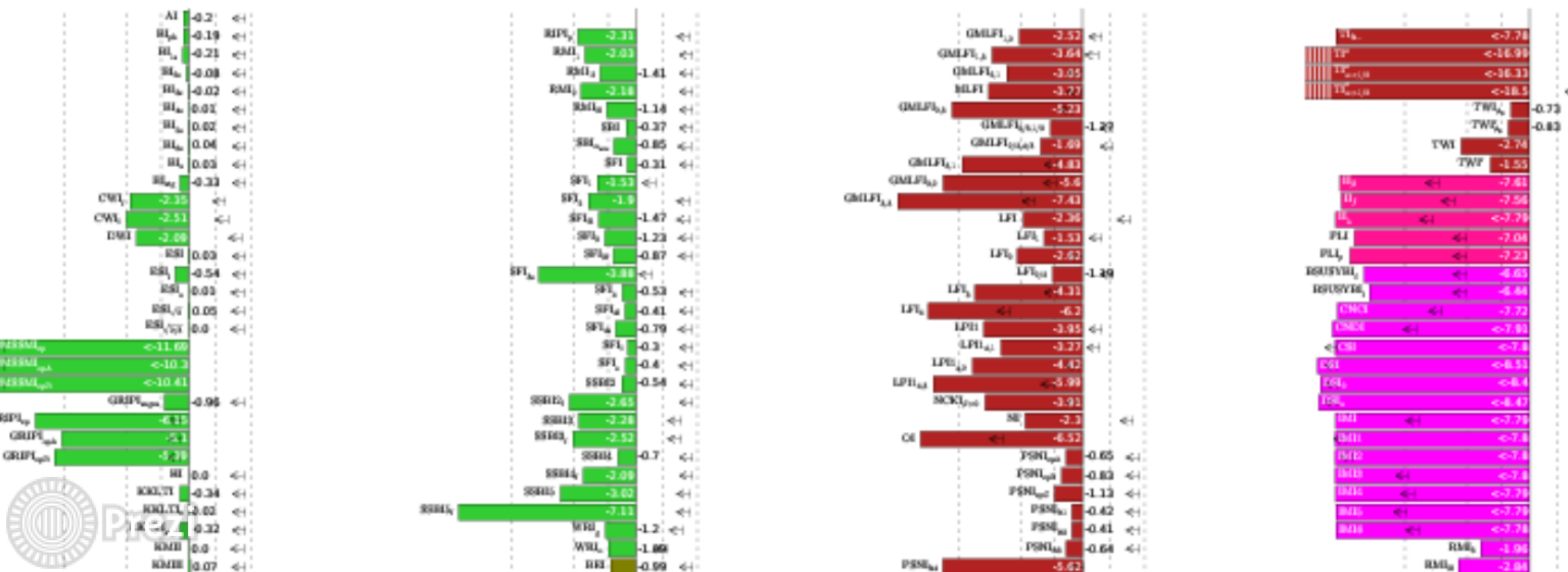
# The Best Inflationary Models After Planck

Jerome Martin, Christophe Ringeval, Roberto Trotta, Vincent Vennin

(Submitted on 12 Dec 2013 (v1), last revised 3 Jun 2014 (this version, v3))

We compute the Bayesian evidence and complexity of 193 slow-roll single-field models of inflation using the Planck 2013 Cosmic Microwave Background data, with the aim of establishing which models are favoured from a Bayesian perspective. Our calculations employ a new numerical pipeline interfacing an inflationary effective likelihood with the slow-roll library ASPIC and the nested sampling algorithm MULTINEST. The models considered represent a complete and systematic scan of the entire landscape of inflationary scenarios proposed so far. Our analysis singles out the most probable models (from an Occam's razor point of view) that are compatible with Planck data, while ruling out with very strong evidence 34% of the models considered. We identify 26% of the models that are favoured by the Bayesian evidence, corresponding to 15 different potential shapes. If the Bayesian complexity is included in the analysis, only 9% of the models are preferred, corresponding to only 9 different potential shapes. These shapes are all of the plateau type.

## Bayesian Evidences $\ln(\mathcal{E}/\mathcal{E}_{HI})$ and $\ln(\mathcal{L}_{\max}/\mathcal{E}_{HI})$

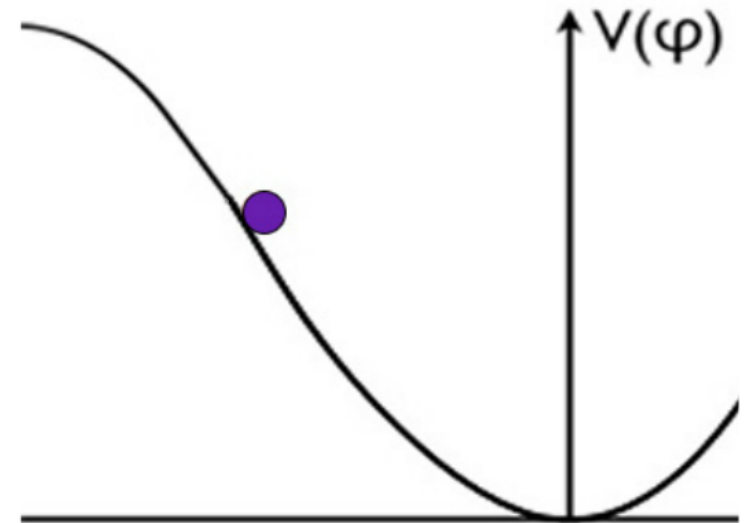


## Scalar potentials

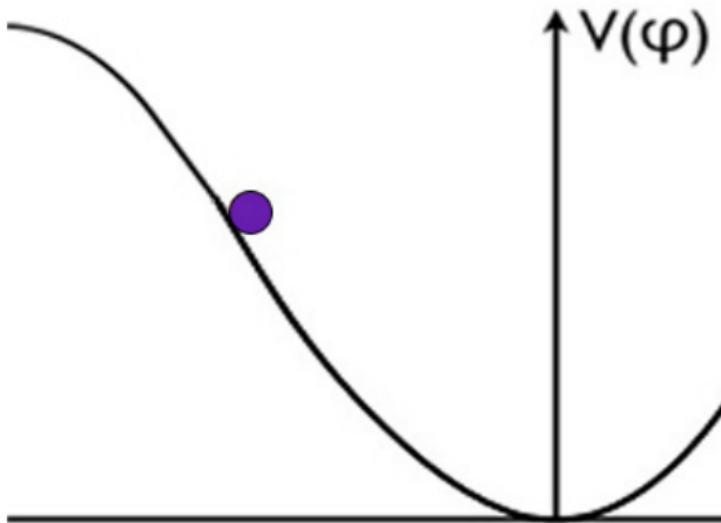
Planck is pointing towards plateau-like potentials:

$$\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\partial\varphi)^2 - V(\varphi) \right]$$

Plateau at infinite / finite distance, with (inverse) polynomial / exp fall-off.



## Kinetic formulation



Redefinition to trivial potential:

$$\frac{1}{2}R - \frac{1}{2} \left( \frac{\partial \varphi}{\partial \rho} \right)^2 (\partial \rho)^2 - \frac{1}{2} m^2 (\rho_0 - \rho)^2$$

*Similar to V in terms of Hubble!*

Plateau in potential implies a singularity in kinetic term!  
Behaviour close to singularity is crucial.

# Inflationary predictions

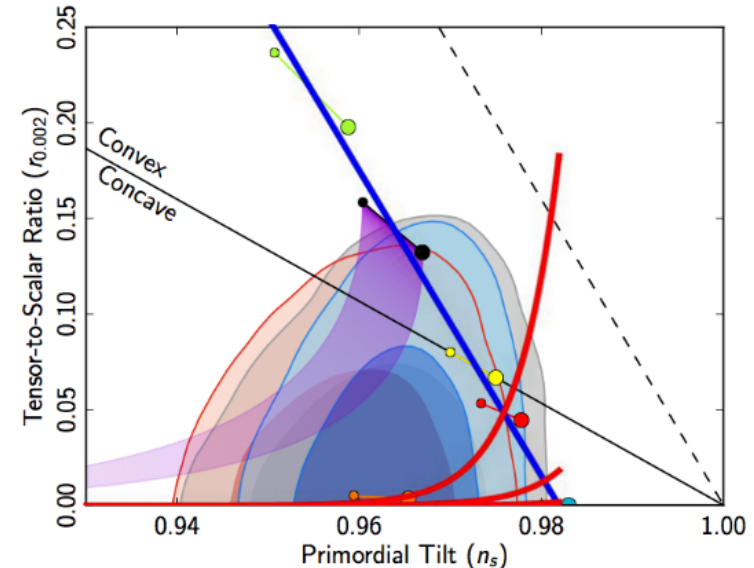
Behaviour at  $N=60$  determined by leading pole in Einstein-frame kinetic term:

$$K_E = \frac{a_p}{\rho^p} + \dots$$

Independent of subleading terms in  $K$  and fully independent of  $V$ :  
robustness of attractor!

$$n_s = 1 - \frac{p}{p-1} \frac{1}{N}$$
$$r = \# \left( \frac{a_p}{N^p} \right) \frac{1}{p-1}$$

Note: same coeff in  $n_s$   
and power in  $r$   
[Mukhanov 2013; DR  
2013]



## Non-minimal coupling

Jordan frame formulation:

$$\frac{1}{2}\Omega(\phi)R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{\xi^2}(\Omega - 1)^2$$

Always leads to  
quadratic  
potential in pole  
coordinate!

Higgs inflation:  $\Omega = 1 + \xi\phi^2$

Universal attractor:  $\Omega = 1 + \xi\phi^n$

Induced inflation:  $\Omega = \xi\phi^2$

[Salopek, Bond,  
Bardeen '89, Bezrukov,  
Shaposhnikov '07]

[Kallosh, Linde, DR '13]

[Giudice, Lee '14]

**Higgs inflation:**  $\Omega = 1 + \xi\phi^2$

Reformulation leads to quadratic potential plus

$$K_E = \frac{3}{2} \frac{1}{\rho^2} + \frac{1}{4\xi} \frac{1}{(\rho_0 - \rho)\rho^2} = \frac{3}{2} \left(1 + \frac{1}{6\xi}\right) \frac{1}{\rho^2} + \dots$$

Infinite coupling limit:

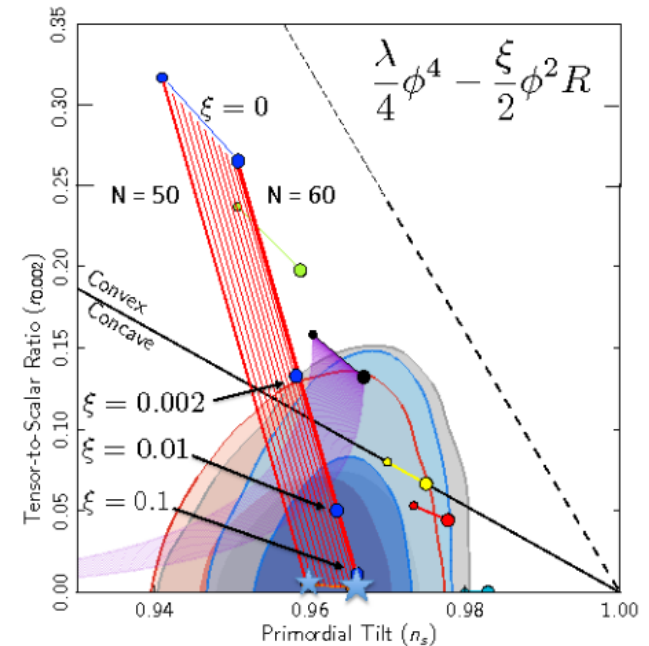
Starobinsky model = a pure pole

Large coupling:

increases residue and hence  $r$

Small coupling:

subleading terms important



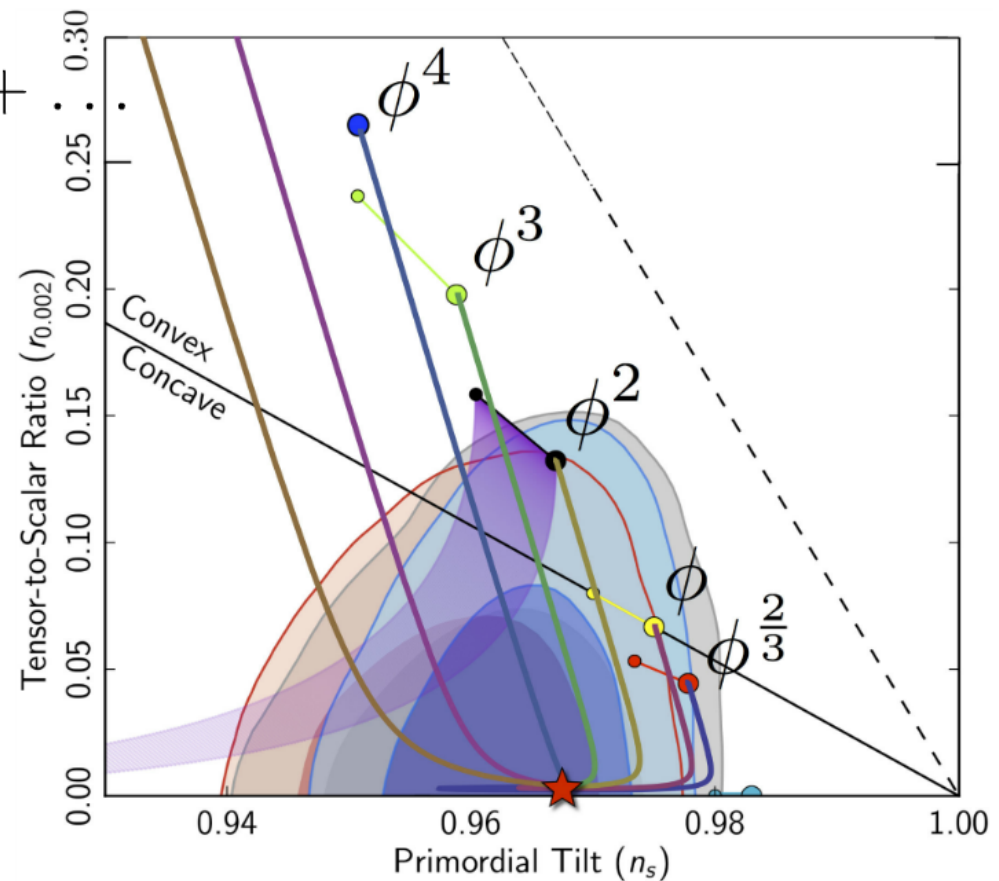


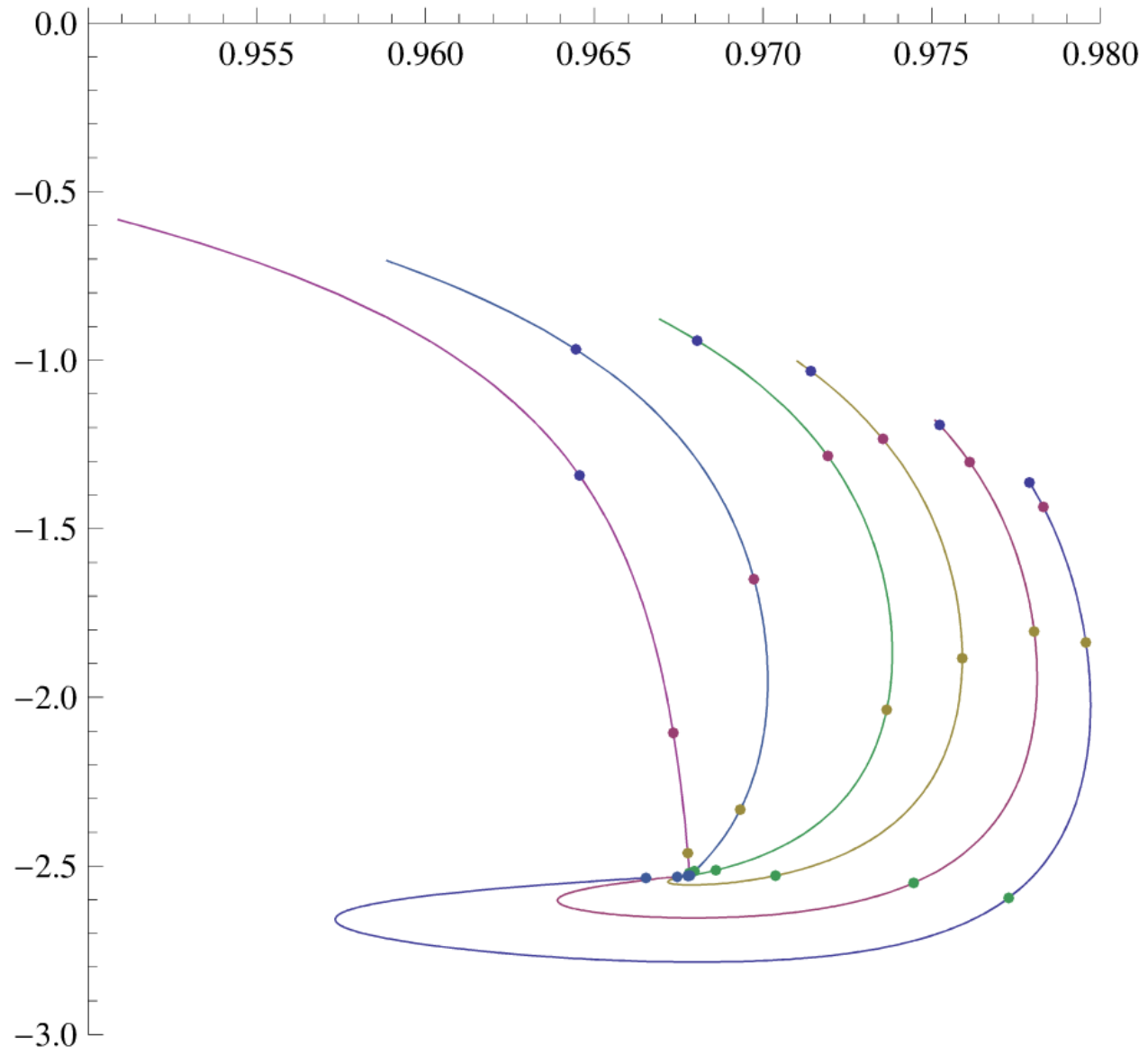
**Universal attractor:**  $\Omega = 1 + \xi\phi^n$

Reformulation leads to quadratic potential plus

$$K_E = \frac{3}{2\rho^2} + \frac{1}{n^2\xi^{2/n}\rho^{1+2/n}} + \dots$$

- Infinite coupling:  
same pole
- Large coupling:  
suppressed poles
- Order-one coupling:  
other pole takes over
- Small coupling:  
subleading corrections



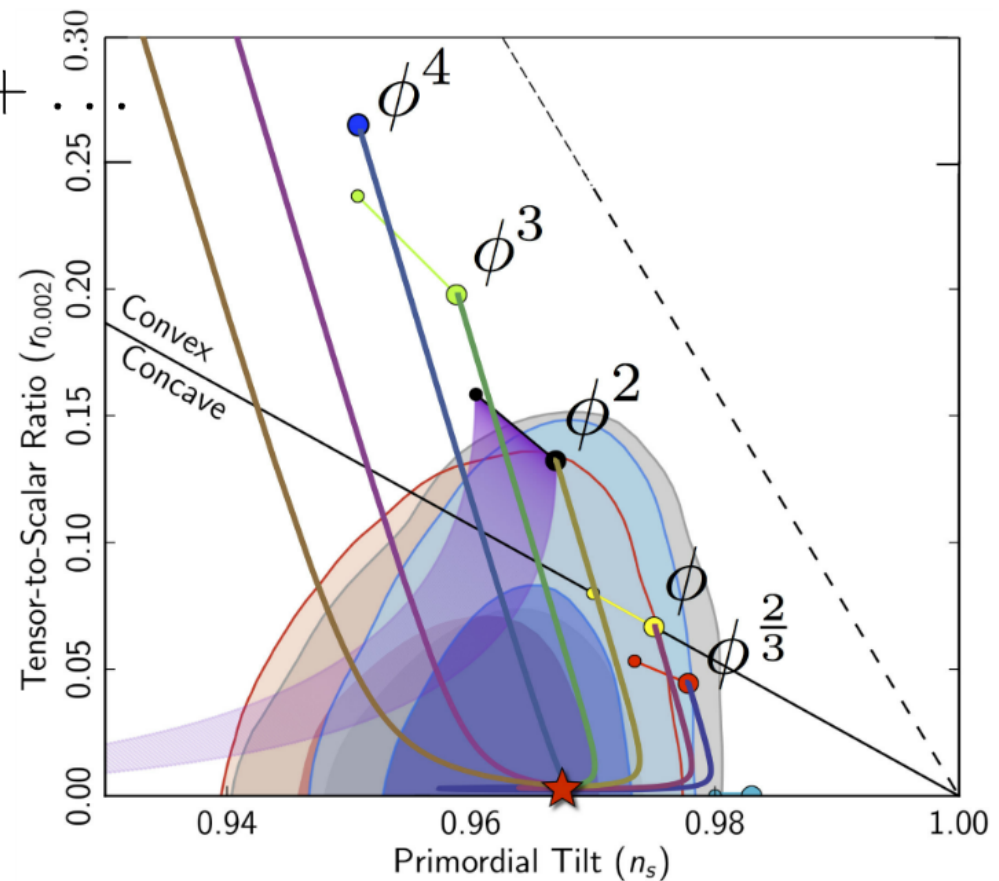


**Universal attractor:**  $\Omega = 1 + \xi\phi^n$

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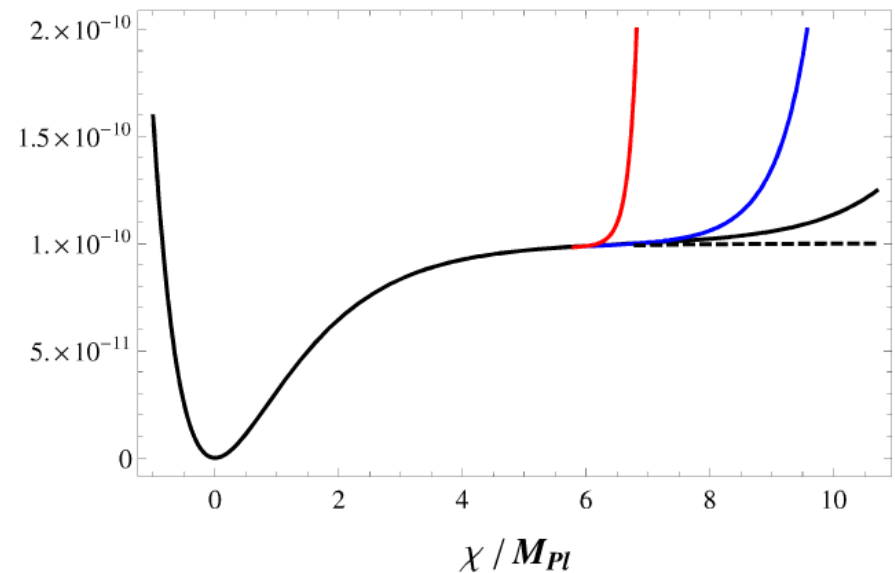


# Generic deformations

Setting  $\xi \sim 10^5$  for power spectrum amplitude yields at least 55 flat e-folds.

Percent-level power loss for larger N.

- Single parameter that sets*
- *spectral index*
  - *tensor-to-scalar ratio*
  - *power normalisation*
  - *number of flat e-folds*



[Broy, DR, Westphal '14]

**Induced inflation:**  $\Omega = \xi \phi^2$

Reformulation leads to quadratic potential plus

$$K_E = \frac{3}{2} \left( 1 + \frac{1}{6\xi} \right) \frac{1}{\rho^2} .$$

Two contributions feeding into  $r$ :

- 1) positive offset from Jordan to Einstein transf.
- 2) second contribution due to Jordan kinetic term

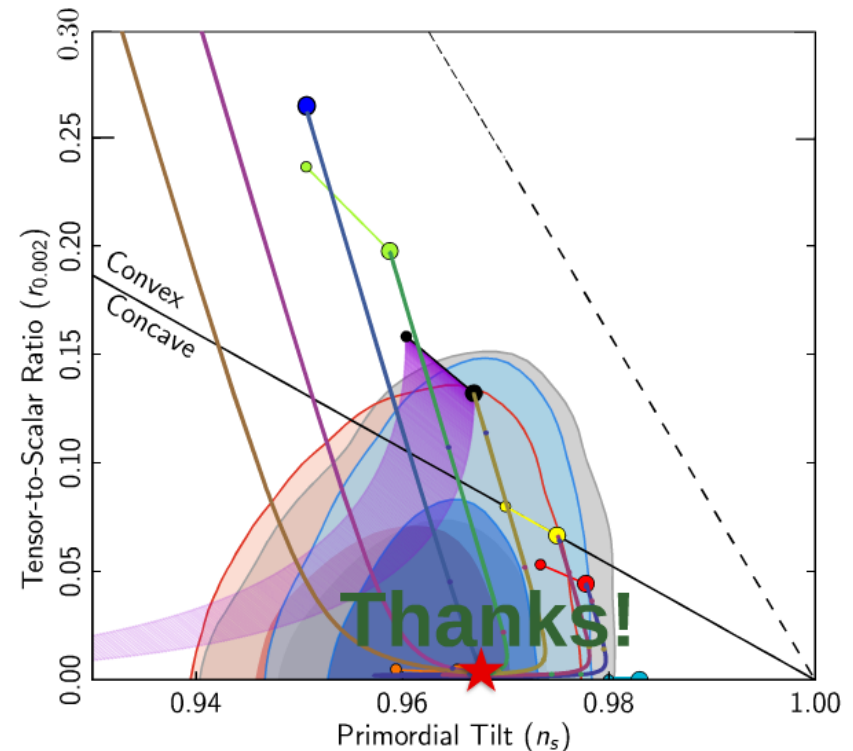
- Coupling can be negative, but only in Einstein frame!
- Jordan frame imposes a lower bound  $r \sim 0.003$ .
- Conformal value predicts zero tensors.
- Equivalent to alpha-attractors with  $\alpha = 1 + \frac{1}{6\xi}$

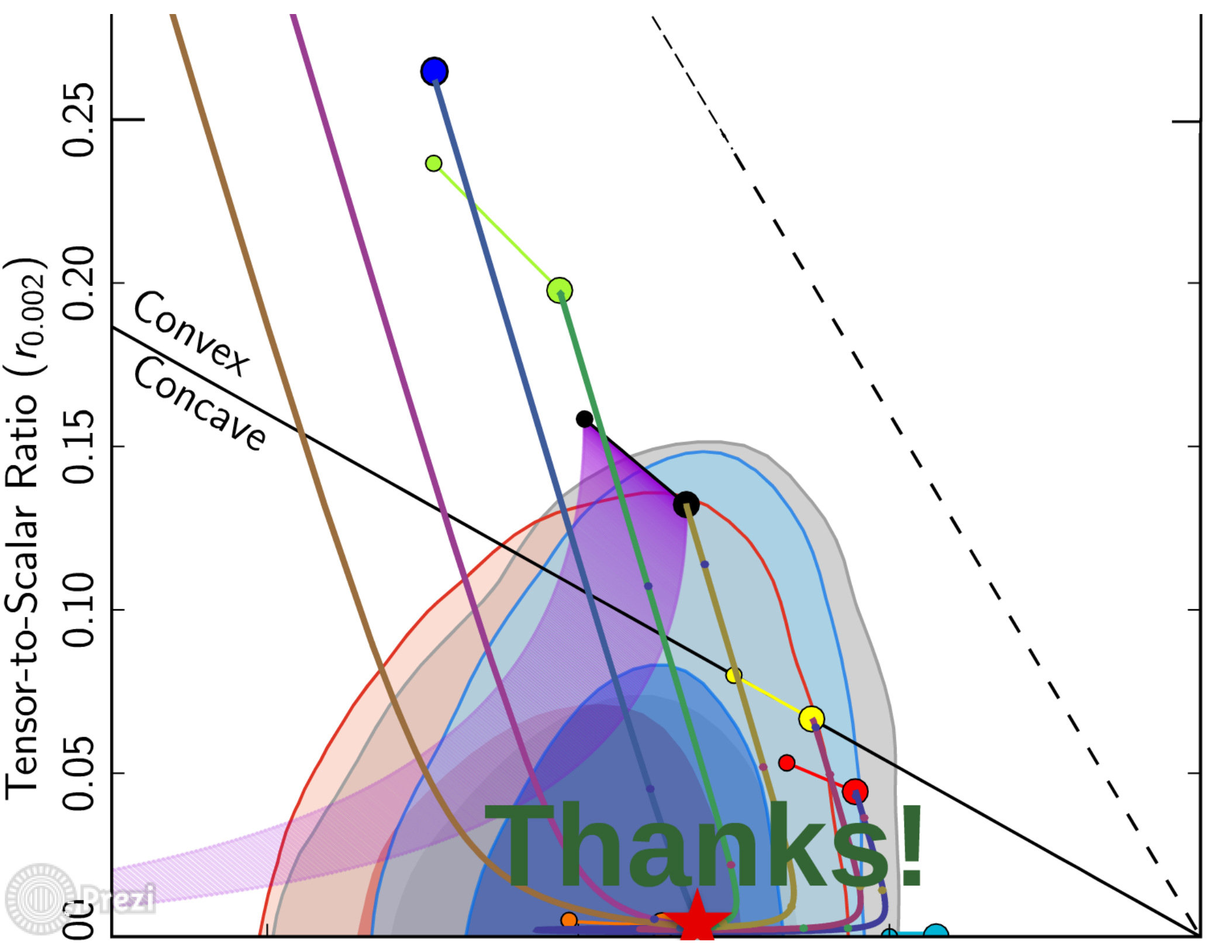
# Summary

plateau inflation = pole inflation **Planck best fit!**  
ns and r determined by order and residue of leading pole

Cosmological attractors with non-minimal coupling stem from a pole of order two:

- natural permille value of r
- different contributions to coeff
- lower bound  $r \sim 0.003$  from Jordan frame
- relation between amplitude and # e-folds
- relation to alpha-attractors





Thanks!

