The maximum radius of habitable planets

Yann Alibert[1,2]

[1]Center for Space and Habitability & Physikalisches Institut, Universität Bern,
CH-3012 Bern, Switzerland
[2]Observatoire de Besançon, 41 avenue de l'Observatoire, 25000 Besançon, France

Introduction

- The conditions that a planet must fulfill to be habitable are not precisely known. However, it is comparatively easier to define conditions under which a planet is very likely not habitable.
- It is known, at least in the case of the Earth, but also likely for many planets, that **the presence of the Carbon cycle** (see e.g., Kasting 2010, Pierrehumbert 2010) **is key for habitability**. Indeed, on Earth, the C-cycle acts as a very important temperature stabilizing process and buffers the surface temperature at values close to those allowing liquid water. This is especially important since the luminosity of a star increases with time and, without any stabilization process, it could be difficult to maintain liquid water at a surface of any planet during an appreciable amount of time.
- **The stabilization effect of the C-cycle comes from both the dependence of silicate weathering on temperature** and the strong greenhouse effect of CO_2. Silicate weathering requires in turn a reaction between CO_2 dissolved in oceans, or directly with CO_2 present in the atmosphere,vand rocks from the planetary mantle. This, in turn, is **only possible if there is a physical interface between liquid water (or atmosphere) and rocks**.
- Water-rich planets (or ocean planets) are covered by a global ocean. **If the amount of water is large enough, the pressure at the bottom of the global ocean is so large that a layer of high pressure ice (ice VII) appears**. This effectively prevents any contact between silicates and liquid water (and therefore atmospheric CO_2 that could be dissolved in the ocean), and suppresses silicate weathering. **As a result, the C-cycle and its stabilization effect cannot exist**. As a consequence, planets with a high enough water content, and therefore a high pressure ice layer at the bottom of a global ocean, are **not habitable**.
- **For a given planetary mass, a large radius** implies either the presence of a lot of water, or the presence of a massive gaseous envelope, or both. In the first case, habitability is hindered by the presence of a high pressure ice layer at the bottom of the global ocean. In the second case, the temperature and pressure at the bottom of the atmosphere are too large to allow for the presence of liquid water. In both cases, the planet is therefore likely **not habitable**.

Models

Planet internal structure

- We compute the internal structure of planets that consists of five layers: a core, an inner mantle, an outer mantle, a water layer, and a gas envelope. The model is similar to the one of Sotin et al. (2007).
- The core is made of Fe (for simplicity, we do not include any inner/outer core dichotomy and do not include the effect of the presence of a volatile like S in the core)
- The inner mantle is made of perovskite ($MgSiO_3/FeSiO_3$) and wustite (MgO/FeO)
- The outer mantle is made of olivine (Mg_2SiO_4/Fe_2SiO_4) and entstatite ($Mg_2Si_2O_6/Fe_2Si_2O_6$).
- The water layer is made of pure water.
- The planetary envelope is made of perfect gas of a given mean molecular weight.

Structure equations

We solve the standard internal structure equations:

$$\frac{dr}{dP}=\frac{1}{\rho g} \qquad \frac{dm}{dP}=\frac{4\pi r^2}{g} \qquad \frac{dT}{dP}=\frac{T}{P}\nabla_{\rm ad} \tag{1}$$

where P is the pressure, r the radius, m the mass interior to radius r, g the gravity, ρ the density given by the equation of state (see below), T the temperature, and $\nabla_{\rm ad}=(d\ln T/d\ln P)_{\rm ad}$ the adiabatic gradient. The equations are solved, using the pressure as an independent variable, for each layer separately, and altogether provide the internal structure of the planet.

Equation of State

- For the inner mantle, the EOS is given by the Mie-Gruneisen-Debye formulation:

$$P=P(\rho,T_0)+\Delta P \tag{2}$$

$$\Delta P=\gamma\rho\,(E(T)-E(T_0)) \tag{3}$$

$$\begin{aligned} P(\rho,T_0)=\tfrac{3}{2}K_0\left[\left(\tfrac{\rho}{\rho_0}\right)^{7/3}-\left(\tfrac{\rho}{\rho_0}\right)^{5/3}\right] \\ \times\left(1-\tfrac{3}{4}\,(4-K_0')\left[\left(\tfrac{\rho}{\rho_0}\right)^{2/3}-1\right]\right)\end{aligned} \tag{4}$$

$$E=\frac{9n}{M_{\rm Mol}}P\left(\frac{T}{\theta_{\rm D}}\right)^3\int_0^{\theta_{\rm D}/T}\frac{x^3e^x}{(e^x-1)}dx \tag{5}$$

where n is the number of atoms in the considered compound. The Debye temperature $\theta_{\rm D}$ is given by

$$\theta_{\rm D}=\theta_{\rm D,0}\left(\frac{\rho}{\rho_0}\right)^{\gamma} \tag{6}$$

and γ is given by $\gamma=\gamma_0\left(\frac{\rho}{\rho_0}\right)^{-q}$.

- For the outer mantle and the liquid water layer, the EOS is given by the Birch-Murnaghan of third order formulation:

$$\begin{aligned} P=\tfrac{3}{2}K^0_{T,0}\left[\left(\tfrac{\rho}{\rho_{T,0}}\right)^{7/3}-\left(\tfrac{\rho}{\rho_{T,0}}\right)^{5/3}\right] \\ \times\left(1-\tfrac{3}{4}\,(4-K_0')\left[\left(\tfrac{\rho}{\rho_{T,0}}\right)^{2/3}-1\right]\right)\end{aligned} \tag{7}$$

where $K^0_{T,0}=K_0+a_P(T-T_0)$, $\rho_{T,0}$ is given by

$$\rho_{T,0}=\rho_0\exp\left(\int_{T0}^{T}\alpha(x,0)dx\right) \tag{8}$$

and $\alpha(T,0)=a_T+b_TT-c_T/T^2$.

- We use the EOS derived by Belonoshko (2010) for pure Fe, which is similar to the Mie-Gruneisen-Debye EOS, but has a different thermal pressure term:

$$\begin{aligned} P=\tfrac{3}{2}K^0_{T,0}\left[\left(\tfrac{\rho}{\rho_0}\right)^{7/3}-\left(\tfrac{\rho}{\rho_0}\right)^{5/3}\right] \\ \times\left(1-\tfrac{3}{4}\,(4-K_0')\left[\left(\tfrac{\rho}{\rho_0}\right)^{2/3}-1\right]\right) \\ +3R\gamma(T-T_0)\times M/\rho\end{aligned} \tag{9}$$

where γ has the same definition as for the Mie-Gruneisen-Debye EOS.

- The temperature profile is adiabatic.

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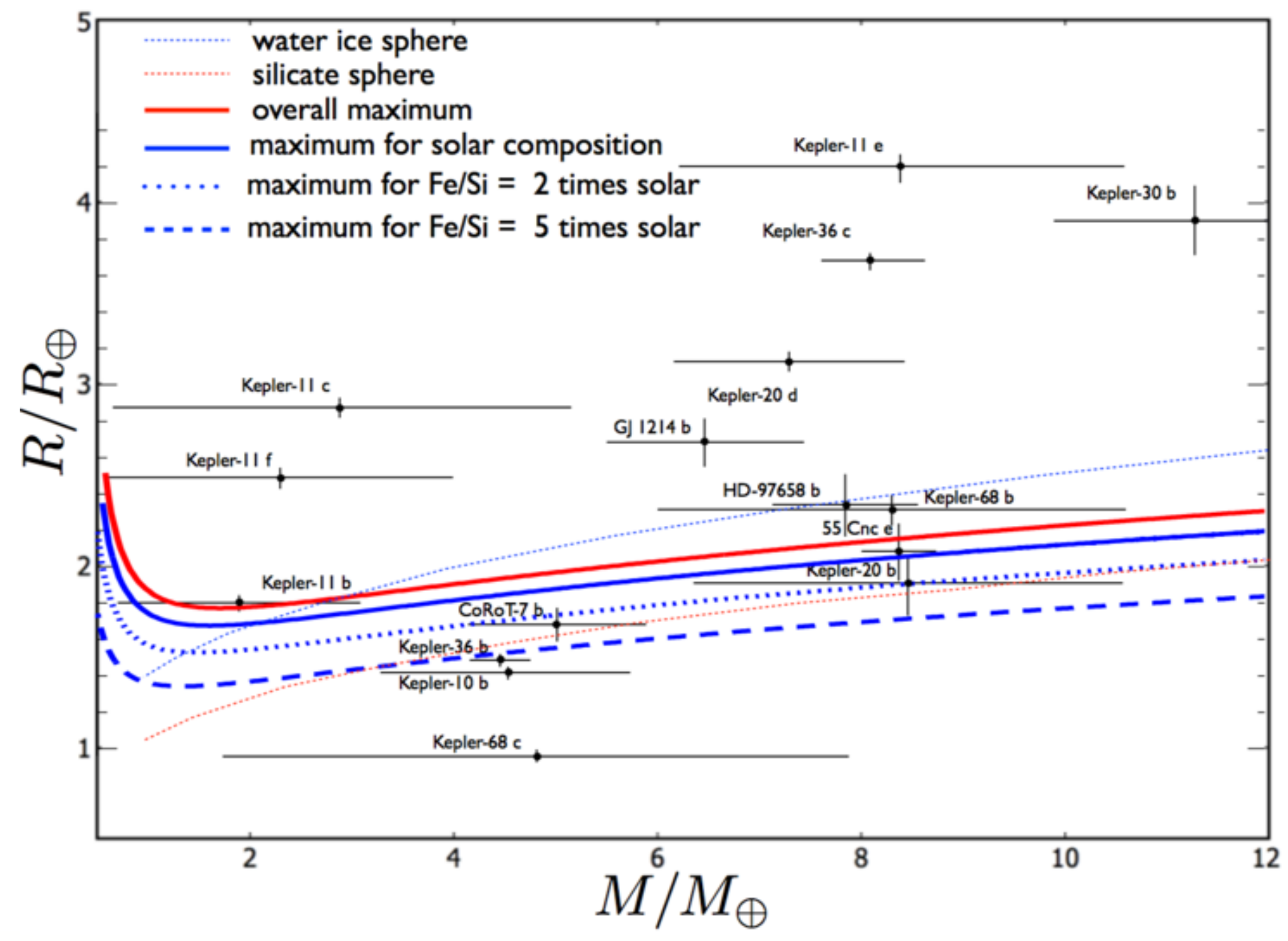

Figure : Mass *versus* maximum radius relationship for different composition of the planetary interior. The heavy red solid line corresponds to an iron-free planet, with an inner mantle made of MgO, and an outer mantle made of $Mg_2Si_2O_6$. The blue lines are computed assuming a solar Mg/Si and a Fe/Si equal to the solar value (solid line), two times the solar value (dotted line), and five times the solar value (dashed line). The mass-radius relationships for a sphere of silicates and a sphere of water are indicated by thin solid lines (red and blue respectively), and are taken from Wagner et al. (2011). The parameters of some transiting planets are taken from exoplanets.org the 2013 September 16.

Maximum water fraction

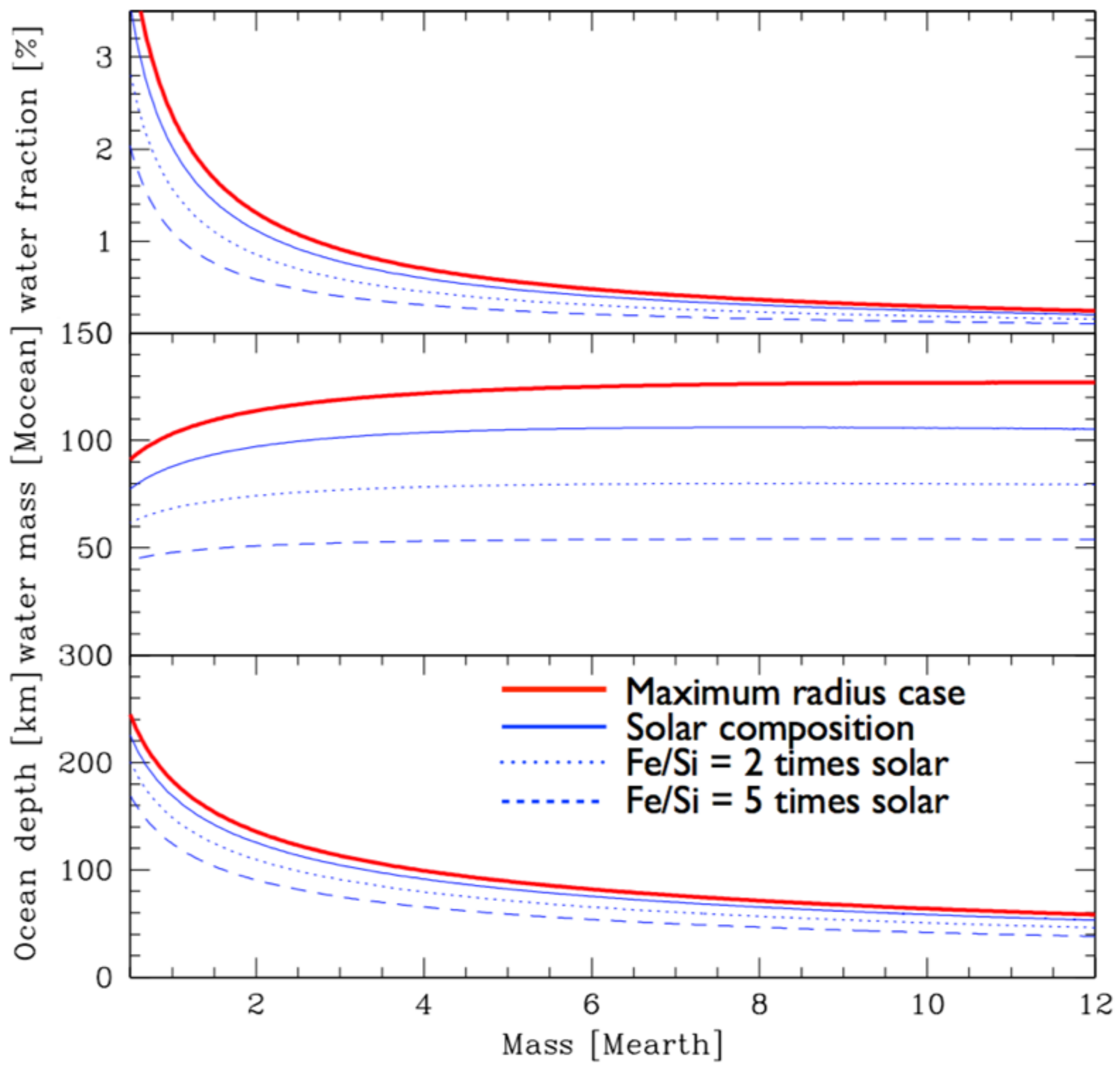

Figure : Characteristic of the maximum ocean as a function of the planetary mass, for different models. The top, middle and bottom panels present respectively the water fraction (considering only the ocean), the ocean mass relative to the ocean mass on Earth ($M_{\rm Ocean}=2.3\ 10^{-4}M_\oplus$), and the ocean depth. The heavy red solid line is in the case of an iron free planet, with an inner mantle made of MgO, outer mantle made of $Mg_2Si_2O_6$. The blue lines are computed assuming a solar Mg/Si, and a Fe/Si equal to the solar value (solid line), two times the solar value (dotted line), and five times the solar value (dashed line).

Conclusions

- **At a given mass, there exist maximum radius above which planets are *not* habitable**.
- Stronger constraints can be derived with assumptions on the bulk composition of planets or by the determination of the mean molecular weight in the envelope.

References

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