Dissipation in resonant systems: Implications of observed orbital configurations

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Resonant/near resonant systems

- What is a resonance between 2 planets?
  - \( P_2/P_1 = p/q \) (\( p, q \) integers)
  - Example: 2/1

- Resonant or near resonant system?
  
  Resonance width depends on \( m_i, e_i \)
Kepler near-resonant planets

- Distribution of period ratio in Kepler data

- Peaks at resonances $\rightarrow$ convergent migration ($P_2/P_1 \downarrow$)

- Peaks slightly shifted to the right ($\rightarrow$ tidal dissipation?)

  (Systems near but outside of resonances)

Lissauer et al. (2011), Fabrycky et al. (2014)
Other possible explanations for the shift:

- protoplanetary disk - planets interactions
  Rein (2012), Baruteau & Papaloizou (2013)

- planetesimals - planets interactions
  Chatterjee & Ford (2015)

- in-situ formation of planets
  Petrovitch, Malhotra, Tremaine (2013), Xie (2014)
Why tidal dissipation?

- Distribution of period ratio close to resonances (2:1 + 3:2)

Delisle, Laskar (2014)
Why tidal dissipation?

- Distribution of period ratio close to resonances (2:1 + 3:2)

\[ \frac{P_2}{P_1} - \frac{p + 1}{p} \]

- KS-tests
  - Close-in vs Farthest: 0.08%
  - Close-in vs Intermediate: 3.5%
  - Intermediate vs Farthest: 10%

Delisle, Laskar (2014)
Analytical model of resonances

- First order resonances (2/1, 3/2, etc.)
  Integrable approximation is straightforward

- Higher order resonances (3/1, 5/2, etc.)
  2 degrees of freedom (not integrable)
  - New simplifying assumption
    \[ \frac{e_1}{e_2} \approx (\frac{e_1}{e_2})_{\text{forced}} \]  (ecc. ratio at resonance center)
  - Integrable pendulum-like approx.
    \[ H = -(I - \delta)^2 + 2R \cos(q\theta) \]
    Delisle, Laskar, Correia, Boué (2012)
    Delisle, Laskar, Correia (2014)
Dissipative evolution in resonance

- Dissipation affects the resonant motion in 2 ways
  - Width change
  - Spiraling of trajectory

- Relative amplitude: \( A = \frac{\text{Amplitude}}{\text{Width}} \)
  - if \( A \downarrow \) Locked in resonance, \( P_2/P_1 \approx p/q \)
  - if \( A \uparrow \) Escape from resonance, \( P_2/P_1 \) no more locked
Migration in protoplanetary disk

- $A \nearrow$ (unstable res.) $\iff \frac{T_{e,1}}{T_{e,2}} < \left(\frac{e_1}{e_2}\right)_{\text{forced}}$

  
  \[ \text{ecc. damping timescales (by disk-planet interactions)} \]

  \[ \rightarrow \text{Escape with } \frac{P_2}{P_1} \searrow \text{ (convergent migration)} \]

- Observed resonant systems
  
  \[ \rightarrow \text{constraints on disk properties} \]
  
  \[ \text{(ex: surface density profile)} \]

Delisle, Correia, Laskar (2015), accepted to A&A
Tidal dissipation

\[ \tau = \frac{T_1}{T_2} \]

\[ \tau_c \approx L \left( \frac{e_1}{e_2} \right)^2 \frac{4+|k_2|(1+L)}{4L-|k_1|(1+L)} \]

\[ \tau_\alpha = \left( \frac{e_1}{e_2} \right)^2 \]

\[ L \approx \frac{m_1}{m_2} \left| \frac{k_1}{k_2} \right|^{1/3} \]

- \( \tau < \tau_c \): Amplitude ↑ → separatrix crossing possible
  - \( \tau < \tau_\alpha \): Diverging
    \[ P_2/P_1 > k_2/k_1 \]
    EXT
  - \( \tau > \tau_\alpha \): Converging
    \[ P_2/P_1 < k_2/k_1 \]
    INT

- \( \tau > \tau_c \): Amplitude ↓ → evolution close to libration center
  - \( q = 1 \): Diverging
    \[ P_2/P_1 > k_2/k_1 \]
    EXT
  - \( q > 1 \): Staying in resonance
    \[ P_2/P_1 \approx k_2/k_1 \]
    RES

Delisle, Laskar, Correia (2014)
Planets b, c close to 3:1 MMR (order 2)

\[ \frac{P_2}{P_1} = 2.97 < 3 \]

Internal circulation (converging)

\[ \tau_\alpha < \tau < \tau_c \]

Delisle, Laskar, Correia (2014)
Who's who? Constraints on planets nature

\[ \Delta t_2 / \Delta t_1 \]

\[ \begin{array}{cccccc}
0 & 500 & 1000 & 1500 & 2000 & 2500 & 3000 \\
\end{array} \]

Initial Amplitude

GJ 163b, c are here
GJ 163b: gaz
GJ 163c: rock

Delisle, Laskar, Correia (2014)
Conclusion

- Classification of outcome of dissipative process in resonance

- Constraints on systems properties from period ratio
  - Disk properties (disk-planet interactions)
  - Planets nature (tidal dissipation efficiency)

- Analytical model
  - Better understanding of these complex process
  - First approximation of constraints
  - Need numerical simulations for precise constraints