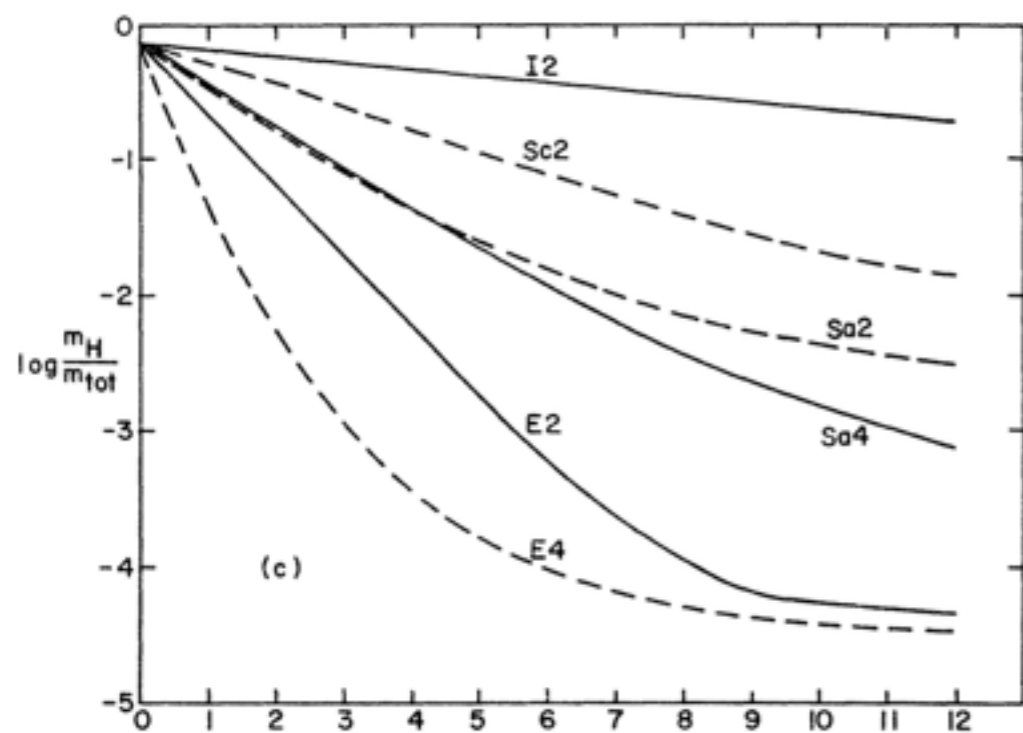


JARLE BRINCHMANN (IA CAUP PORTO/LEIDEN)

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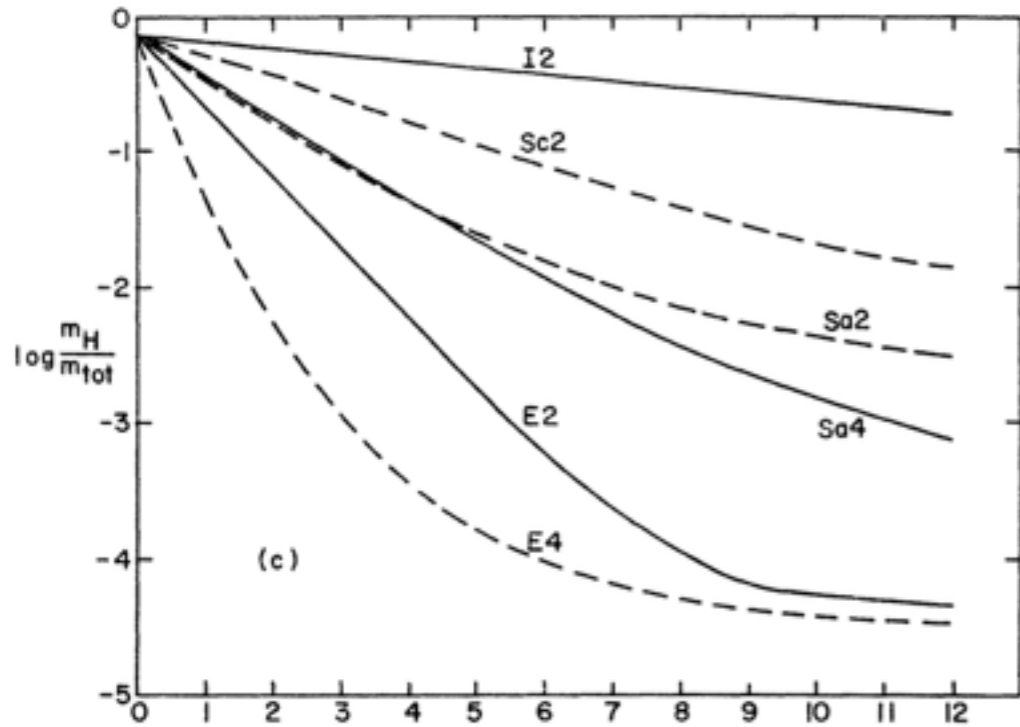
**FROM AN OBSERVED SED TO  
PHYSICAL PARAMETERS**

# THE NEED FOR BETTER STATISTICAL TECHNIQUES



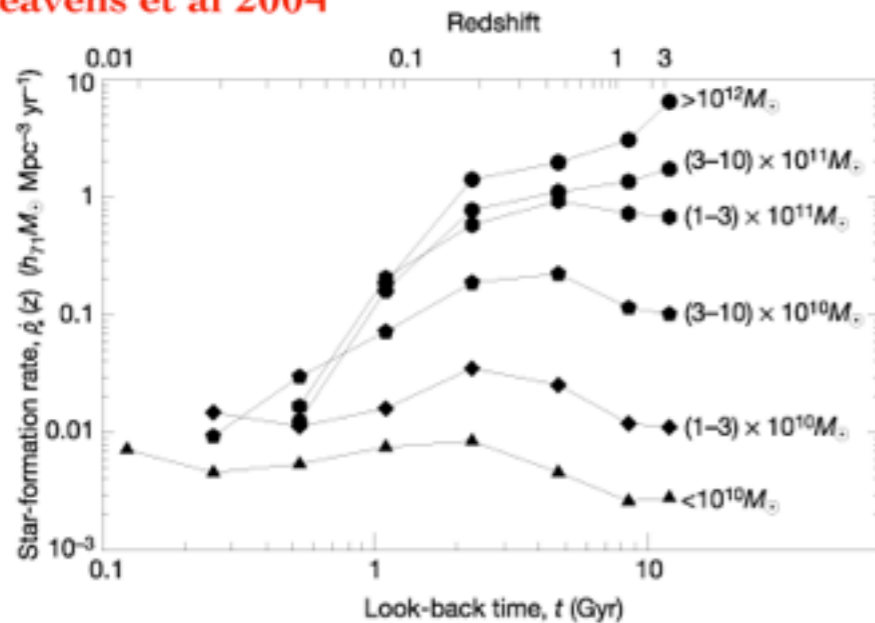
Tinsley's (1968) "star formation history" diagram.

# THE NEED FOR BETTER STATISTICAL TECHNIQUES



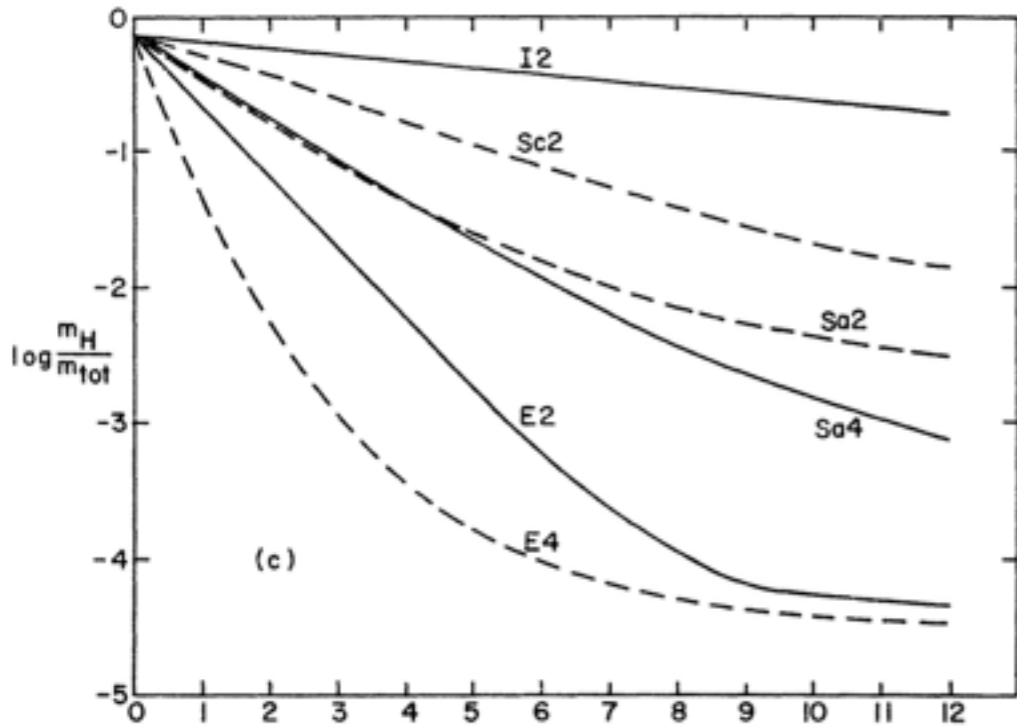
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Heavens et al 2004

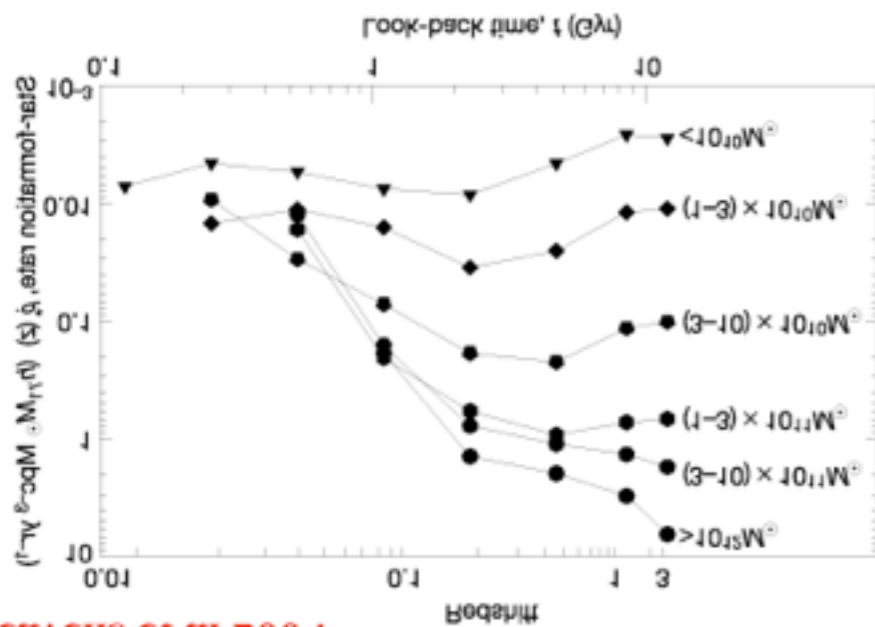


Heavens et al's (2004) paper on the star formation history of nearby galaxies from SDSS.

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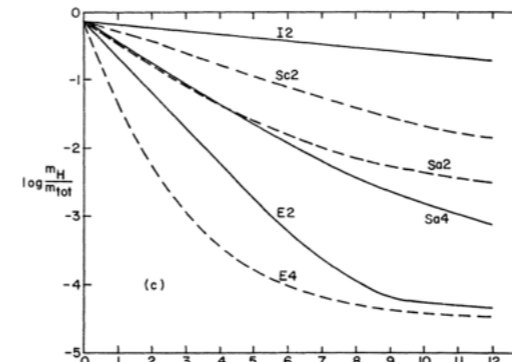


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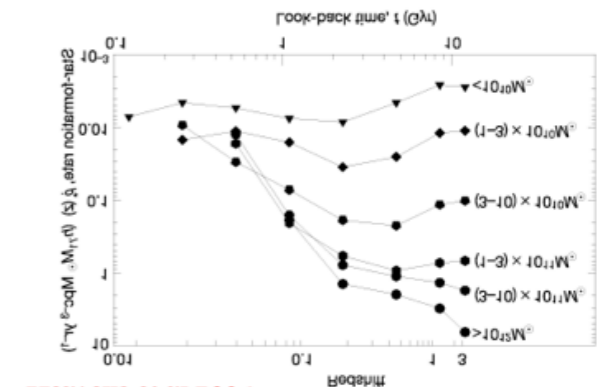


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# THE NEED FOR BETTER STATISTICAL TECHNIQUES



Tinsley (1968)



F005 La 19 29 29 29 29 29

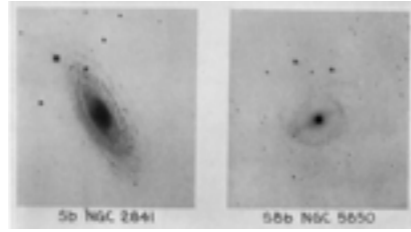
Heavens et al (2004)

Some parameters are robustly constrained - most methods should give similar results, when using the same assumptions.

However we do not necessarily know how well we do.

# Morphology:

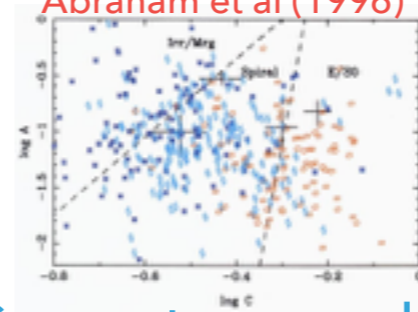
Hubble (1926)



Visual classification



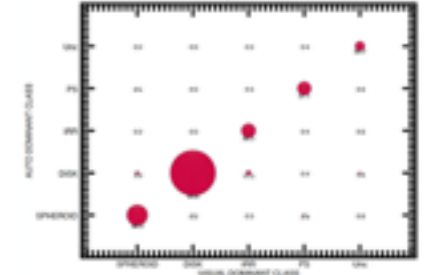
Abraham et al (1996)



Computer morphology



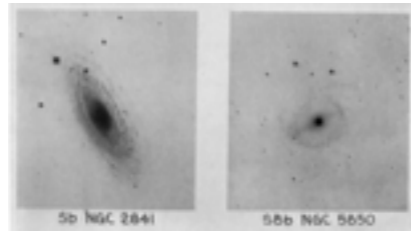
Huertas-Company et al (2015)



Deep learning/SVMs/  
Neural networks

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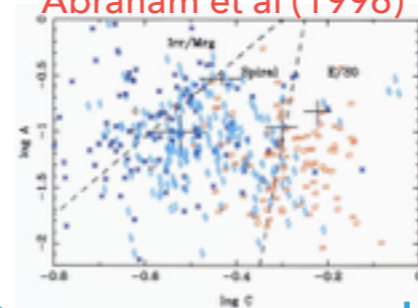
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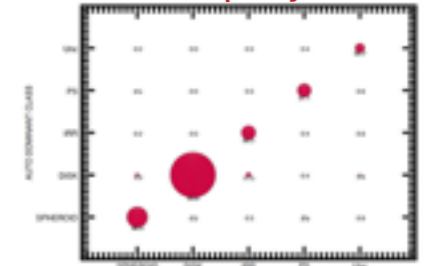
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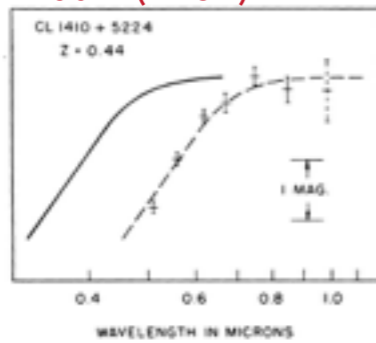
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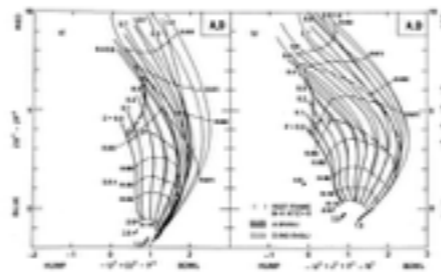
Deep learning/SVMs/  
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# Photo-zs:

Baum (1962)



Koo (1985)



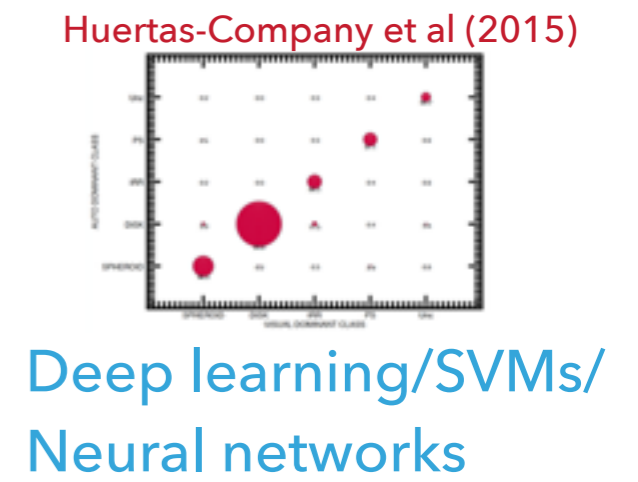
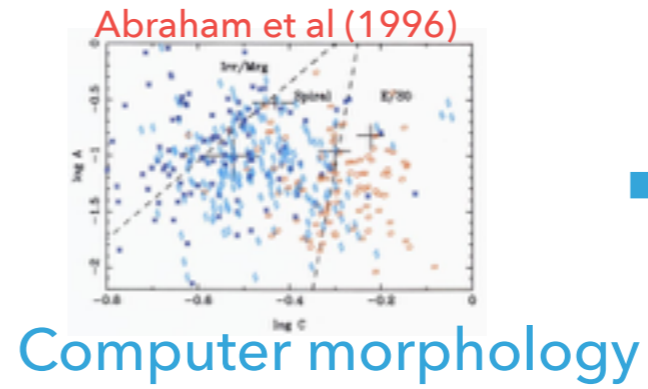
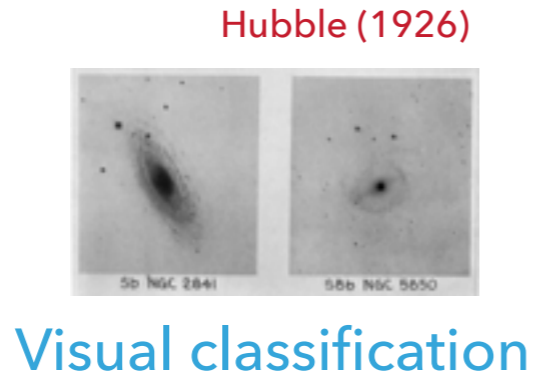
Template fits (Loh & Spillar 1986)

Linear regression (Connolly et al 1995)

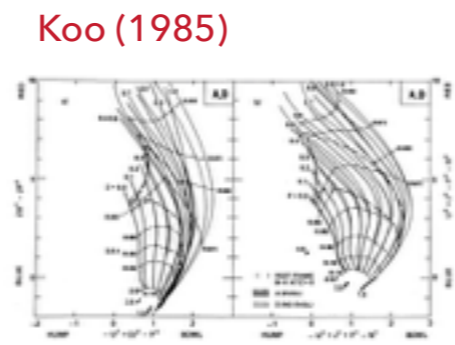
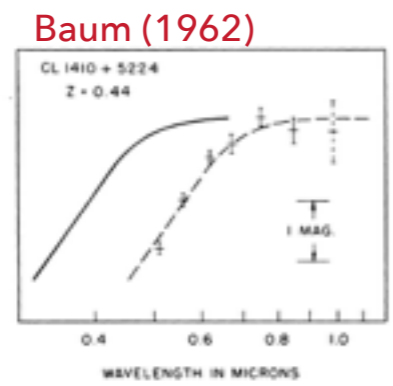
Neural networks (Collister & Lahav 2004)

Random forests, Gaussian process regression, Support Vector Machines, +++

# Morphology:

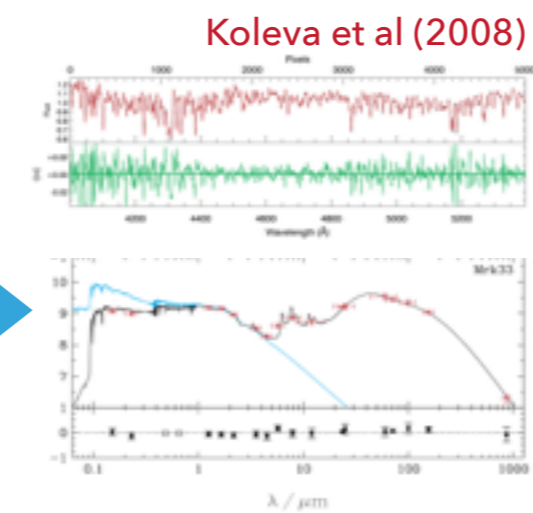
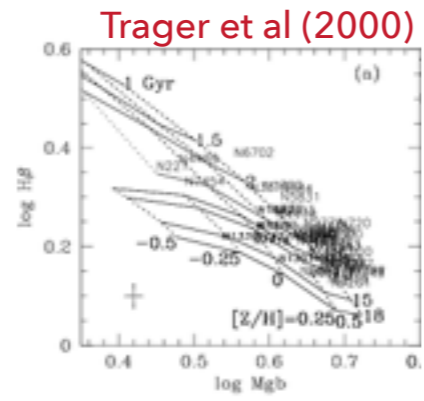


# Photo-zs:



- Template fits (Loh & Spillar 1986)
- Linear regression (Connolly et al 1995)
- Random forests, Gaussian process regression, Support Vector Machines, +++

# SED fitting:



- Min  $\chi^2$
- Grid-based Bayesian
- MCMC Bayesian

da Cunha et al (2008)



# THE NEED FOR BETTER STATISTICAL APPROACHES

How well do we know what we know?

How can we fully exploit large datasets?

**But: We need the right ingredients!**

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An estimate is insufficient - we need reliable confidence/credible intervals

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Rigorous analysis exploring parameter space properly.

A deep understanding of our photon (typically) gathering process.

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- Rigorous analysis exploring parameter space properly.

- A deep understanding of our photon (typically) gathering process.

## How can we fully exploit large datasets?

Lots of data  $\neq$  easily lots of (good) science

**Needs:**

- Fast algorithms, low (or calibratable) bias

## But: We need the right ingredients!

# THE IMPORTANCE OF THE RIGHT INGREDIENTS

Advanced statistical tools are great - but:

Any chain (of reasoning) is only as good as its weakest link

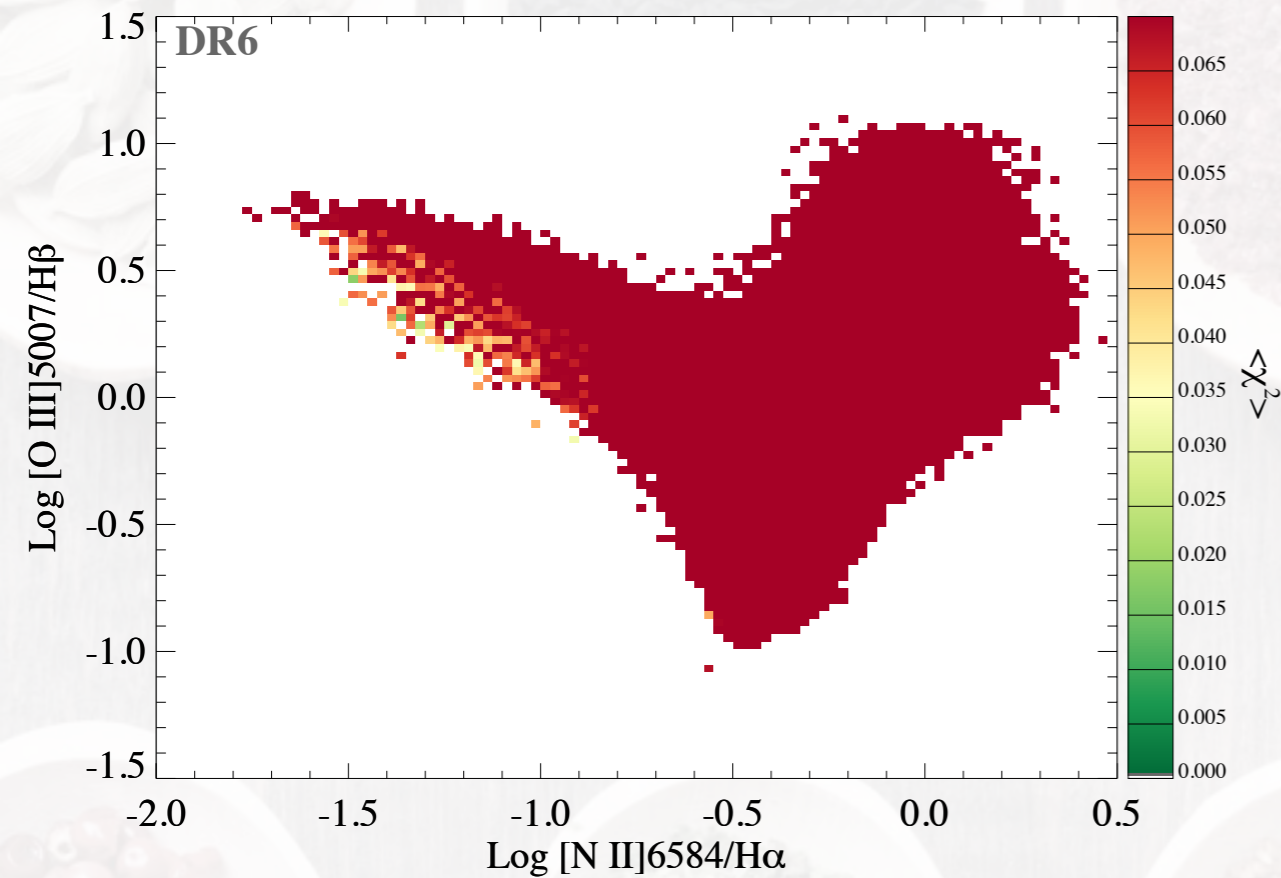
The good news:

Lot of excellent work in the last 10-15 years: Massive progress

**BUT: This remains a key uncertainty in much of our work:**

**Binaries, stellar rotation,  $\alpha$ -enhancement, IMF, far UV spectra, turbulence, ...**

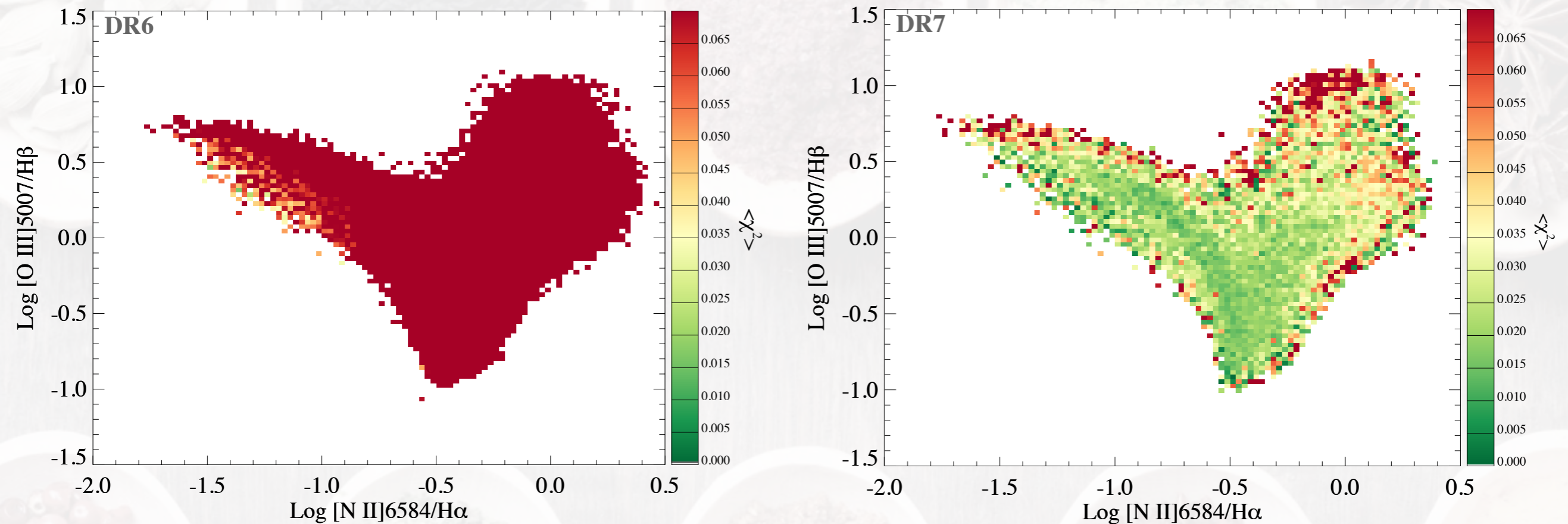
# THE IMPORTANCE OF THE RIGHT INGREDIENTS



The quality of fit to the continuum in  $\sim 200,000$  SDSS galaxies.

Moving to a better basis set for the continuum fit (in this case MILES spectra), made much more of a difference than changes in the algorithm used for fitting.

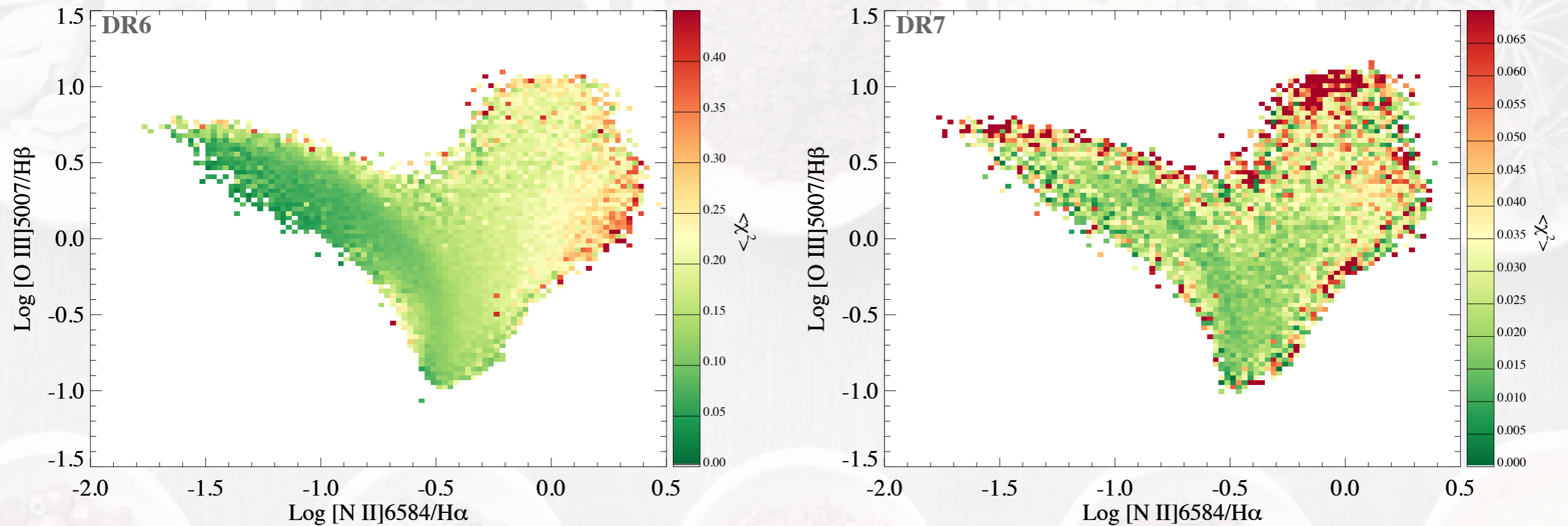
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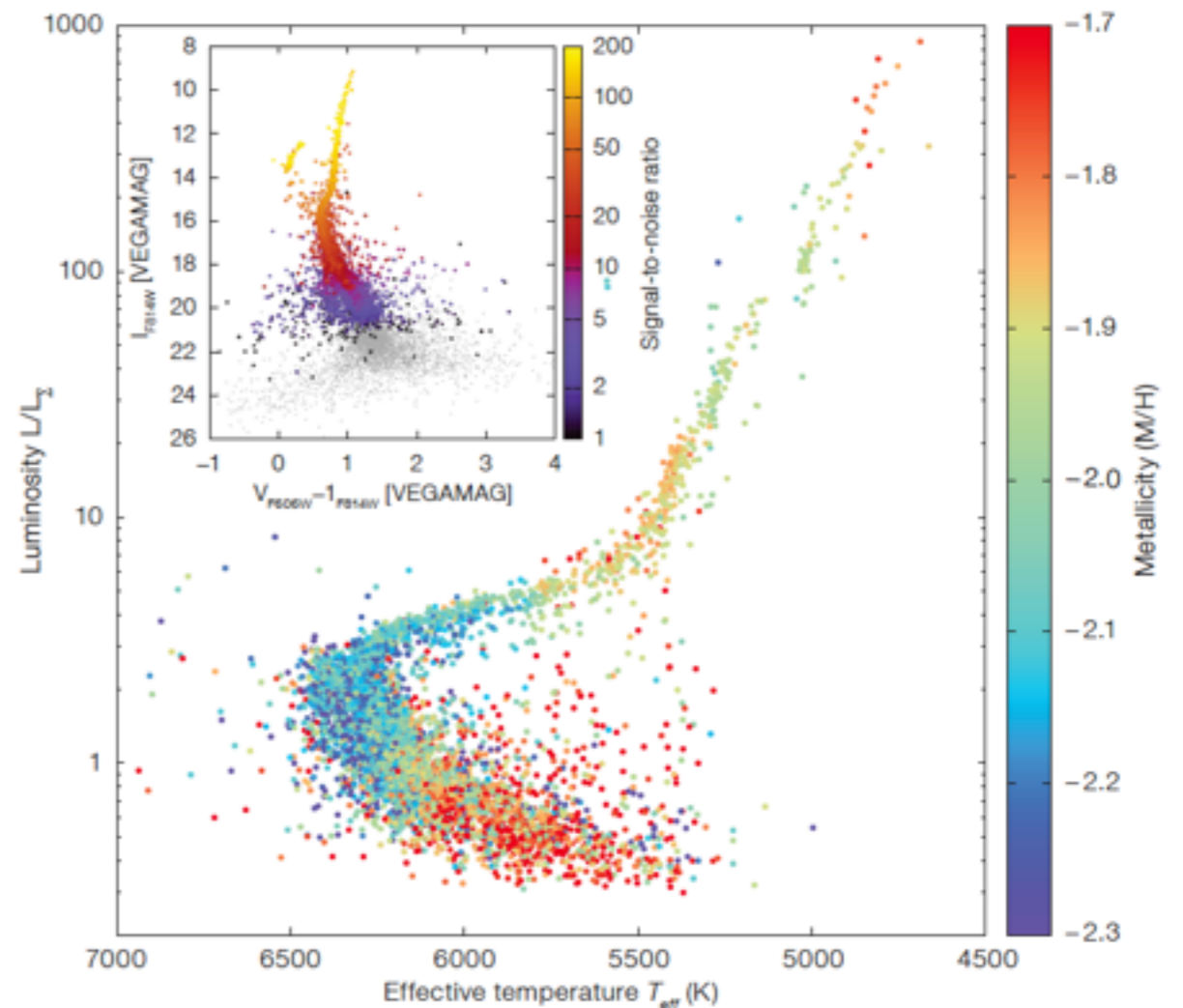
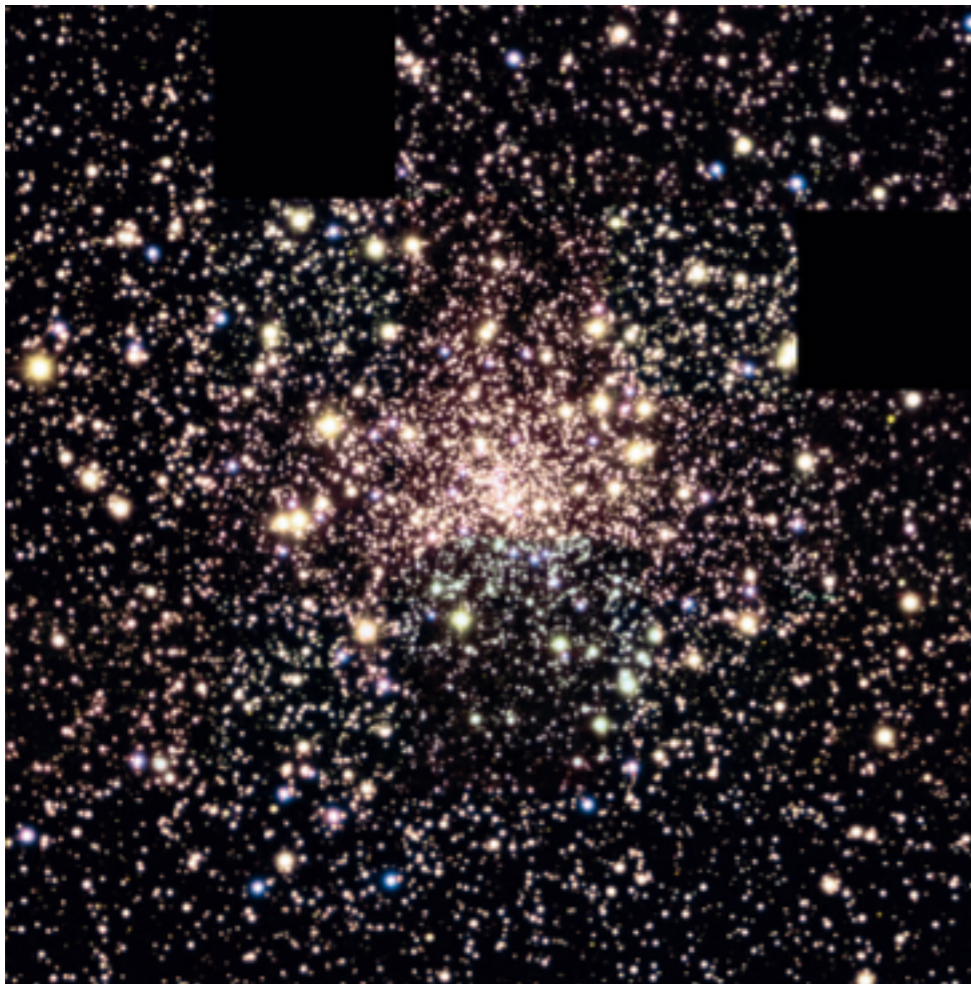
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# CHECKING THE INGREDIENTS

Classical: Milky Way globulars - e.g. Maraston et al (2003); Barber et al (2014)

Next step?: IFU spectroscopy of stellar systems & HST CMDs

## Kamann et al (2016) – A Stellar Census in NGC 6397 with MUSE



Preliminary results (MSc thesis Pietrow): SED modelling consistent with CMD



# FULL-SPECTRUM FITTING

The move from index-index plots has led to a proliferation of methods to fit the full spectrum of galaxies:

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LePhare, GalMC, CIGALE, MagPhys, BayeSED, FAST, HyperZ, GP-CV, Annz, Stable-GP, EAZY, BPZ, ++++++

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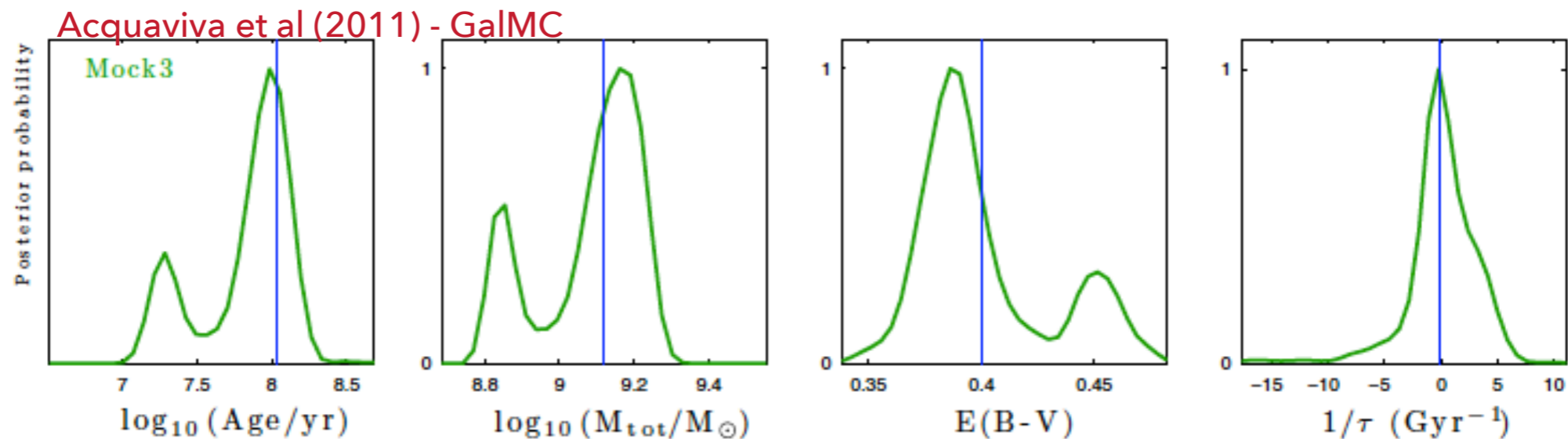
These have used a range of methods: Principal Component Analysis, Non-negative least squares, non-negative matrix factorisation, bounded value least-squares, MCMC +  $\chi^2$ , constrained minimisation with generalised CV etc.

But beyond these optimisation methods the main progression has been from minimum  $\chi^2$  to more rigorous Bayesian approaches.

# THE APPEAL OF BAYESIAN APPROACHES

$$\text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Model evidence}}$$

- ✓ Need to explicitly state (many) assumptions
- ✓ Complex posterior distributions can be handled
- ✓ Efficient MCMC methods exist to sample high-dimensional problems
- ✓ Potentially flexible framework to handle systematic uncertainties (SFHs?)



- Can be time-consuming
- Can be appear misleadingly "rigorous"

# THE LIKELIHOOD

Standard approach:

$$f^{\text{obs}}(\lambda) = f^{\text{true}}(\lambda) + \epsilon \quad \& \quad \epsilon \sim N(0, \sigma^2)$$

so

$$\log L_j = \sum_i \frac{[f(\lambda_i) - M_j(\lambda_i)]^2}{\sigma_i^2}$$

but this supposes that we know the variance - if it is estimated from the data or uncertain, this is not correct and you need a Student t-distribution:

$$L(x_i | \nu, \sigma_i) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\pi\nu\sigma_i}} \left(1 + \frac{1}{\nu} \left(\frac{x_i}{\sigma_i}\right)^2\right)^{-\frac{\nu+1}{2}}$$

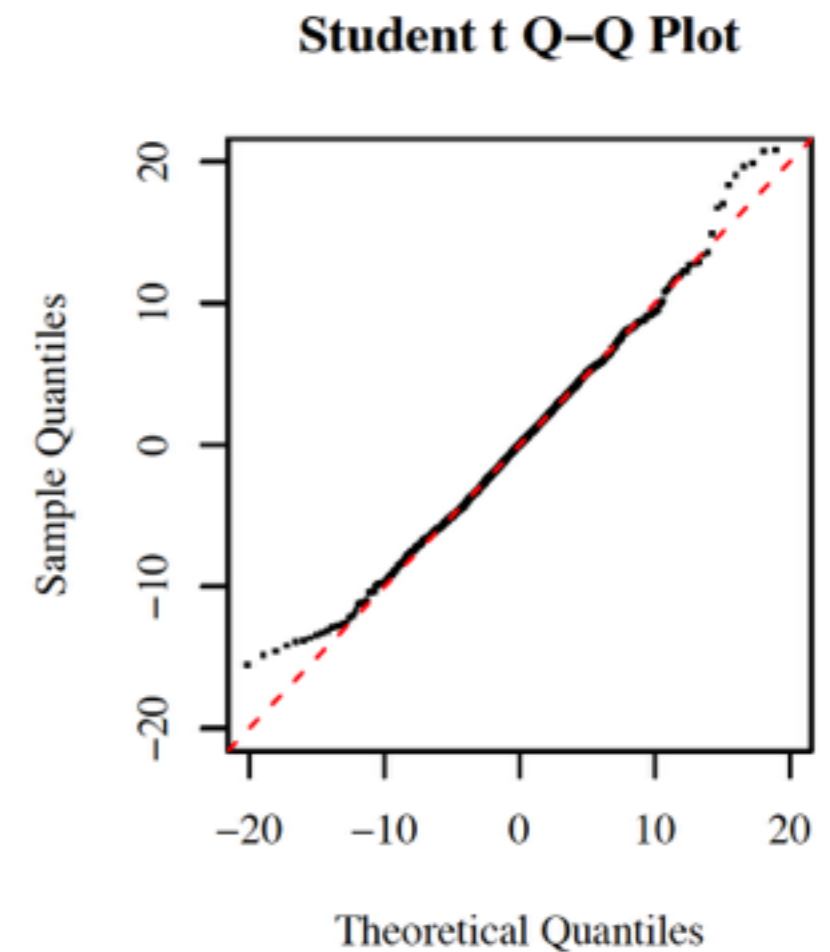
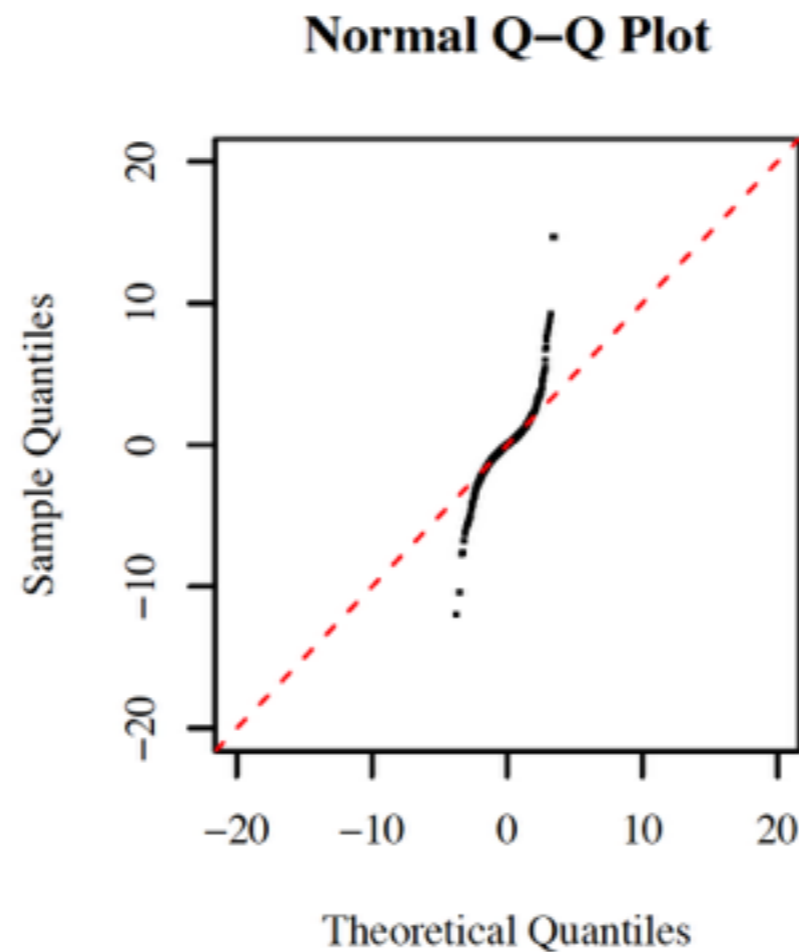
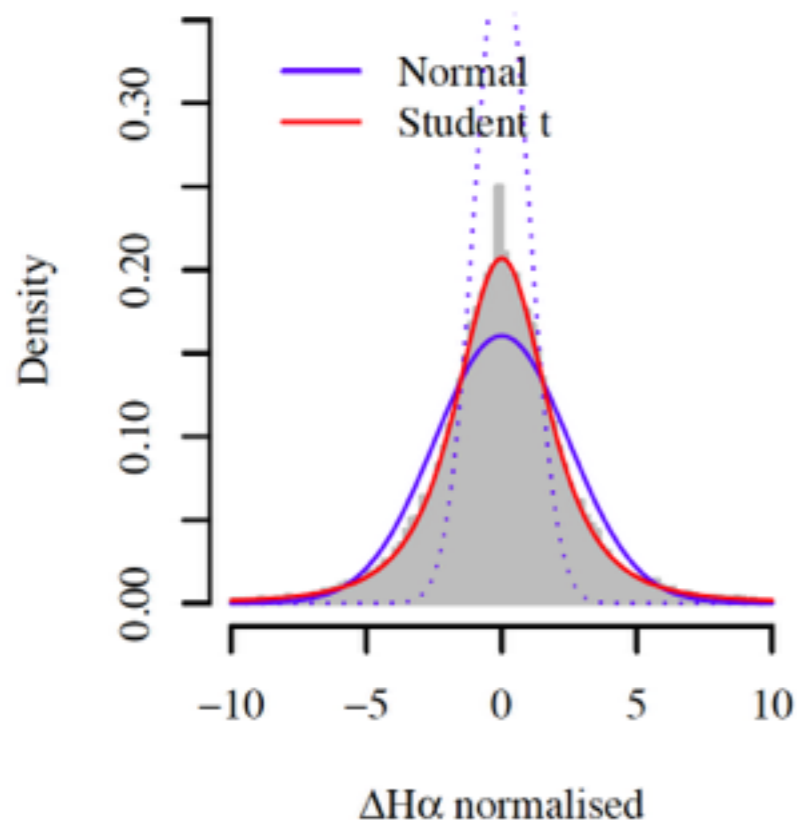
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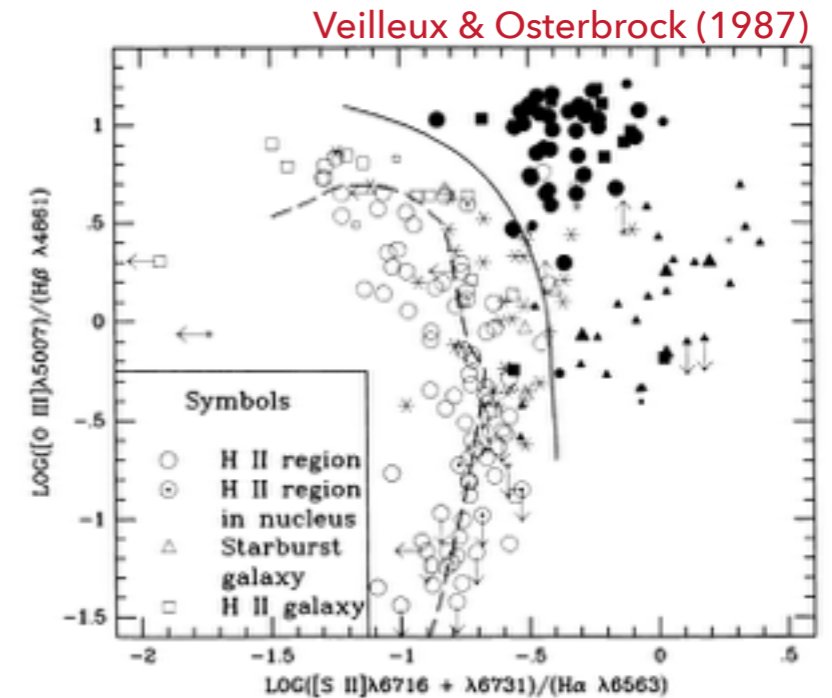
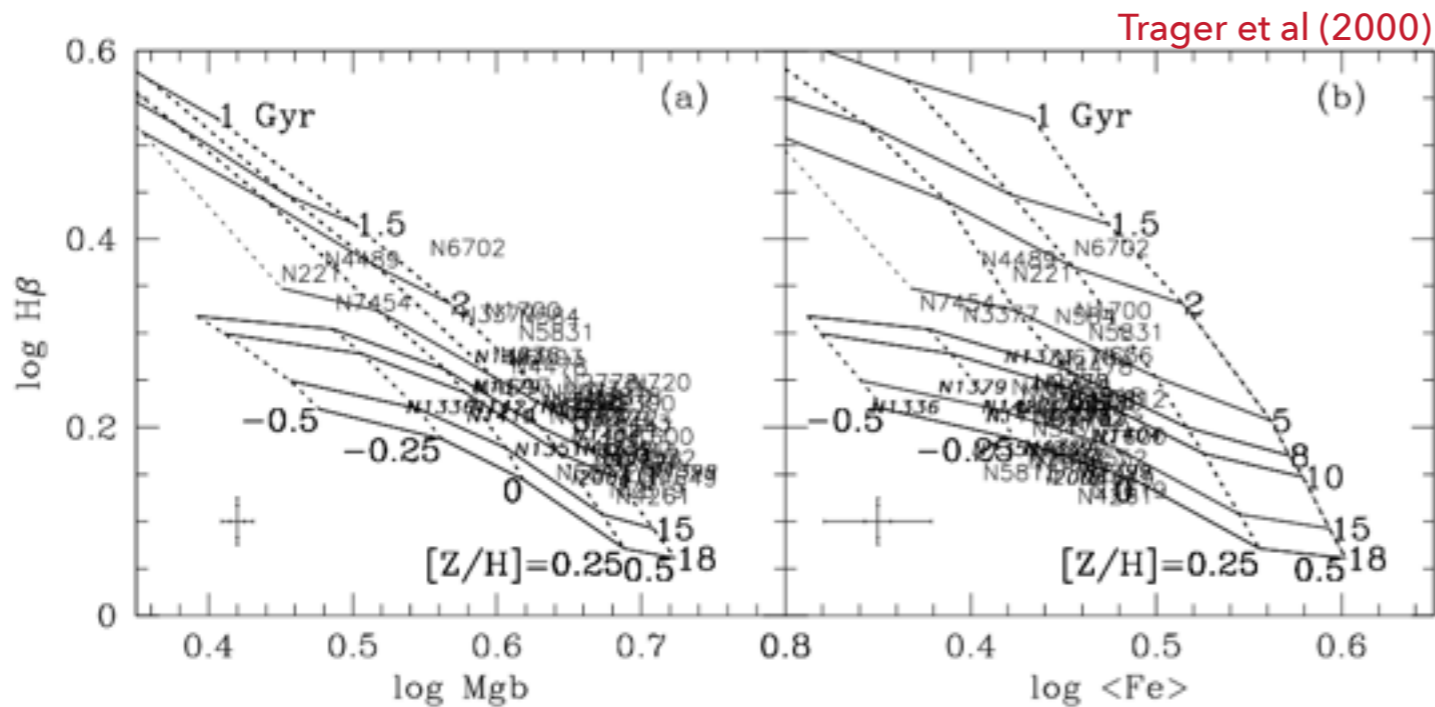
$$f^{\text{obs}}(\lambda) = f^{\text{true}}(\lambda) + \epsilon \quad \& \quad \epsilon \sim N(0, \sigma^2)$$

A real concern:



$$x_i = f(\lambda_i) - M_j(\lambda_i)$$

# INDEX-INDEX & DIAGNOSTIC DIAGRAMS



Are these now a waste of time?

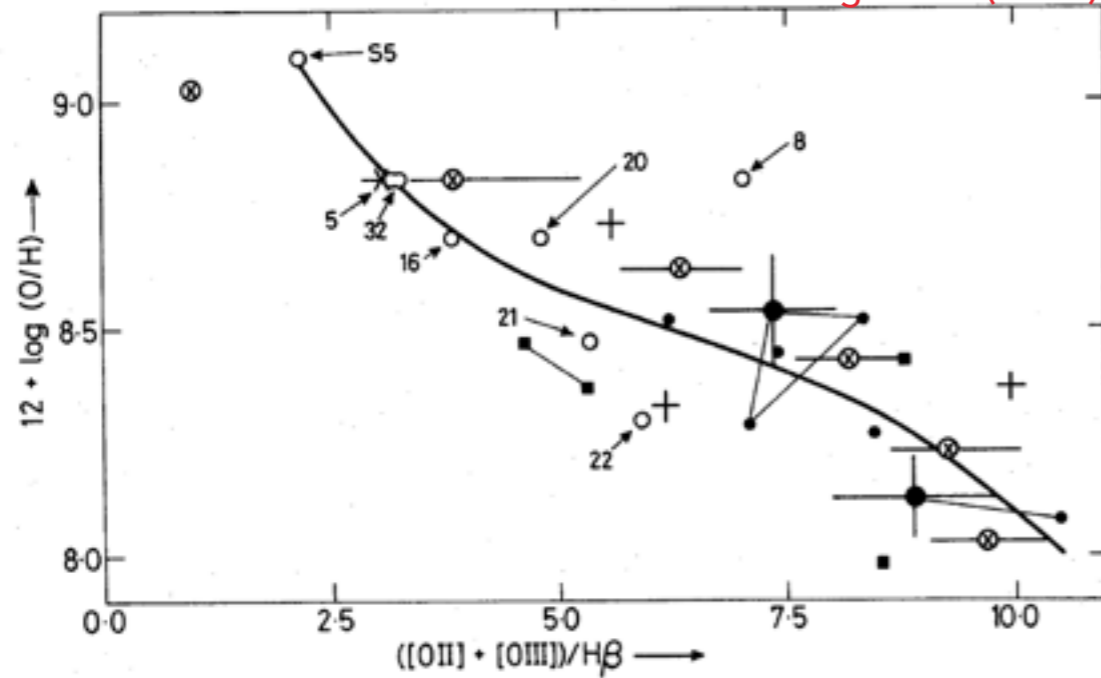
Certainly not - they provide physical insight, usually much more than a triangle plot of posterior samples from an MCMC chain.

**EMISSION LINES**



# INFERRING IONISED GAS PROPERTIES

Pagel et al (1979)



While the theoretical models have advanced, many studies today use analysis techniques similar to this.

$$12 + \log O/H = 7.056 + 0.767R_{23} + 0.602R_{23}^2 - O_{32} (0.29 + 0.332R_{23} - 0.331R_{23}^2)$$

$$\log[N II]/[O II] = 1106.8660 - 532.1545112 + \log O/H +$$

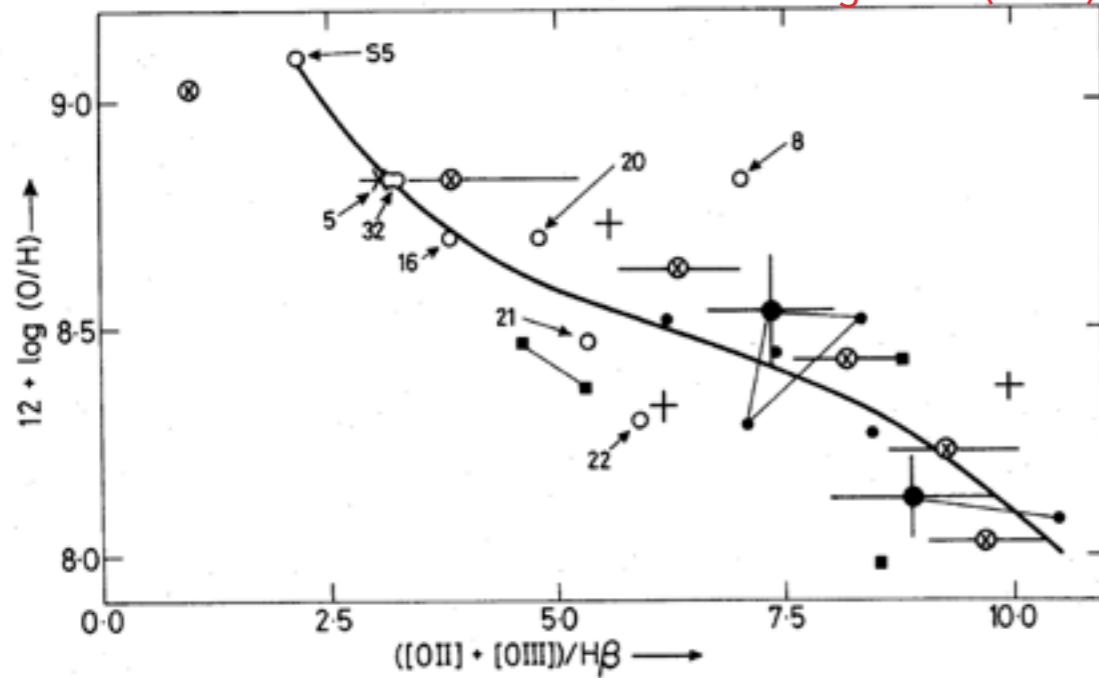
$$96.3732612 + \log O/H^2 - 7.810612312 + \log O/H^3 + 0.2392824712 + \log O/H^4$$

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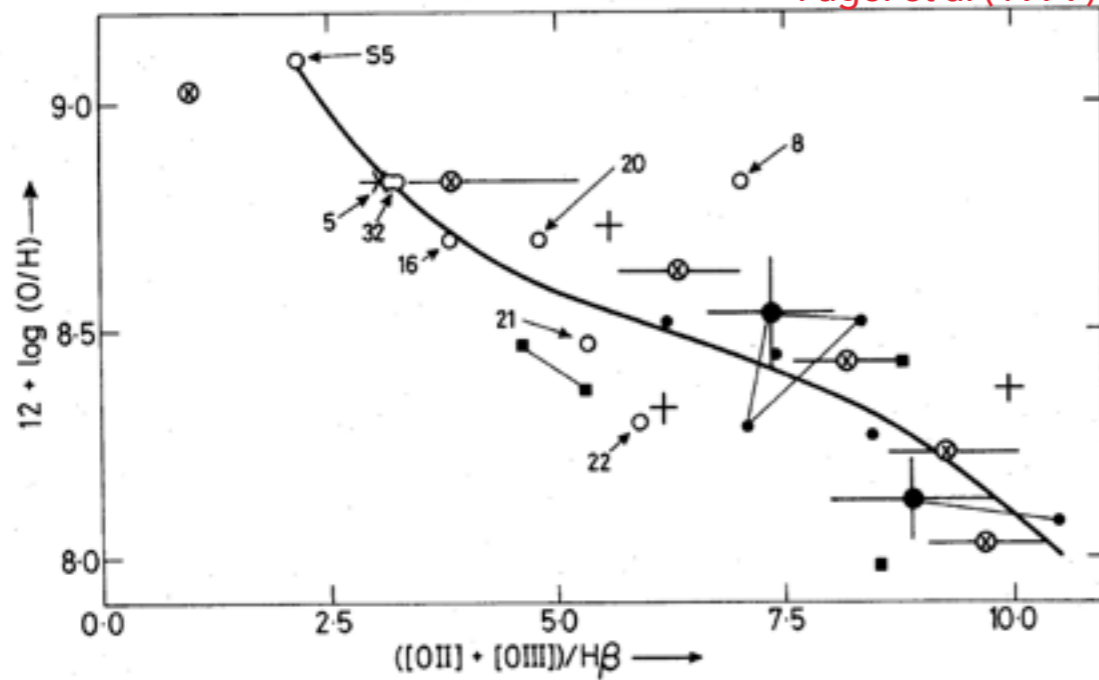
Models



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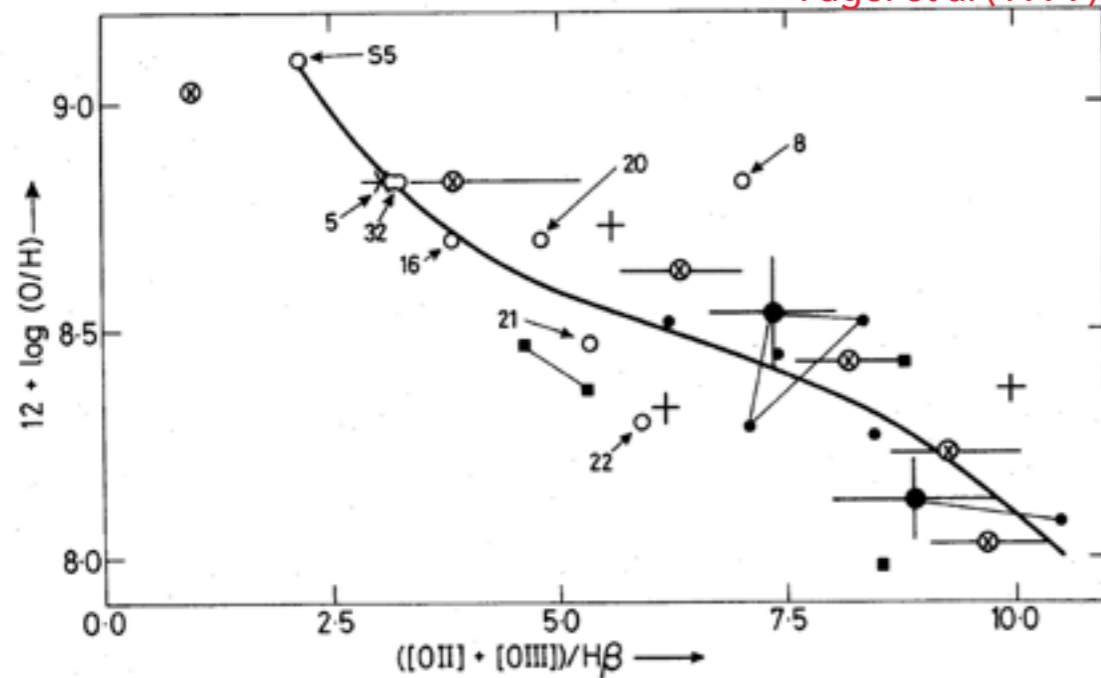
## “EMPIRICAL METHODS”:

Atomic data  $\longrightarrow$  suitable data (e.g. [O III]4363)  $\longrightarrow$   $O/H$

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“STRONG-LINE METHODS”

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O/H = f(line ratios)

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O/H

**THESE METHODS CONTAIN IMPLICIT PRIORS**

O/H = f(line ratios)



# INFERRING IONISED GAS PROPERTIES

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But they still work well - sometimes

Analogous to Bell & de Jong (2001) M/L as a function of colour method.

**Problems:**

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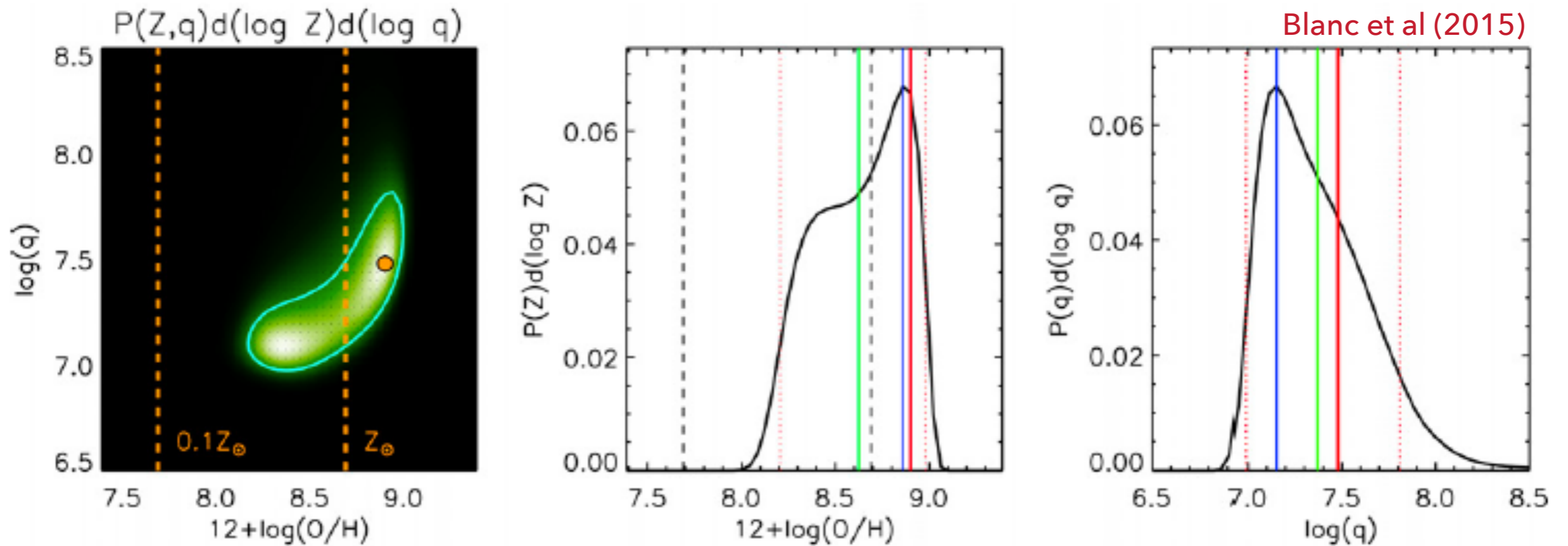
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E.g.: pyqz (Dopita et al 2013), IZI (Blanc et al (2015), HII-CHI-mistry (Perez-Montero 2014), MPA-JHU code (Brinchmann et al 2004; Tremonti et al 2004).



# EXAMPLE: IZI - SINGLE HII REGION



Note in particular the **double-peaked** nature of the PDFs & the **correlation** between ionisation parameter ( $q$ ) and metallicity ( $12 + \log O/H$ ).

## HANDLING OF PDFS

PDFs for e.g. O/H are often bimodal/complex in shape - this requires some care:

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**Bimodality:**

$$\beta = \frac{\text{skewness}^2 + 1}{\text{kurtosis}}$$

**Entropy:**

$$S(P) = \int P(x) \ln P(x) dx$$

**Quantiles:**

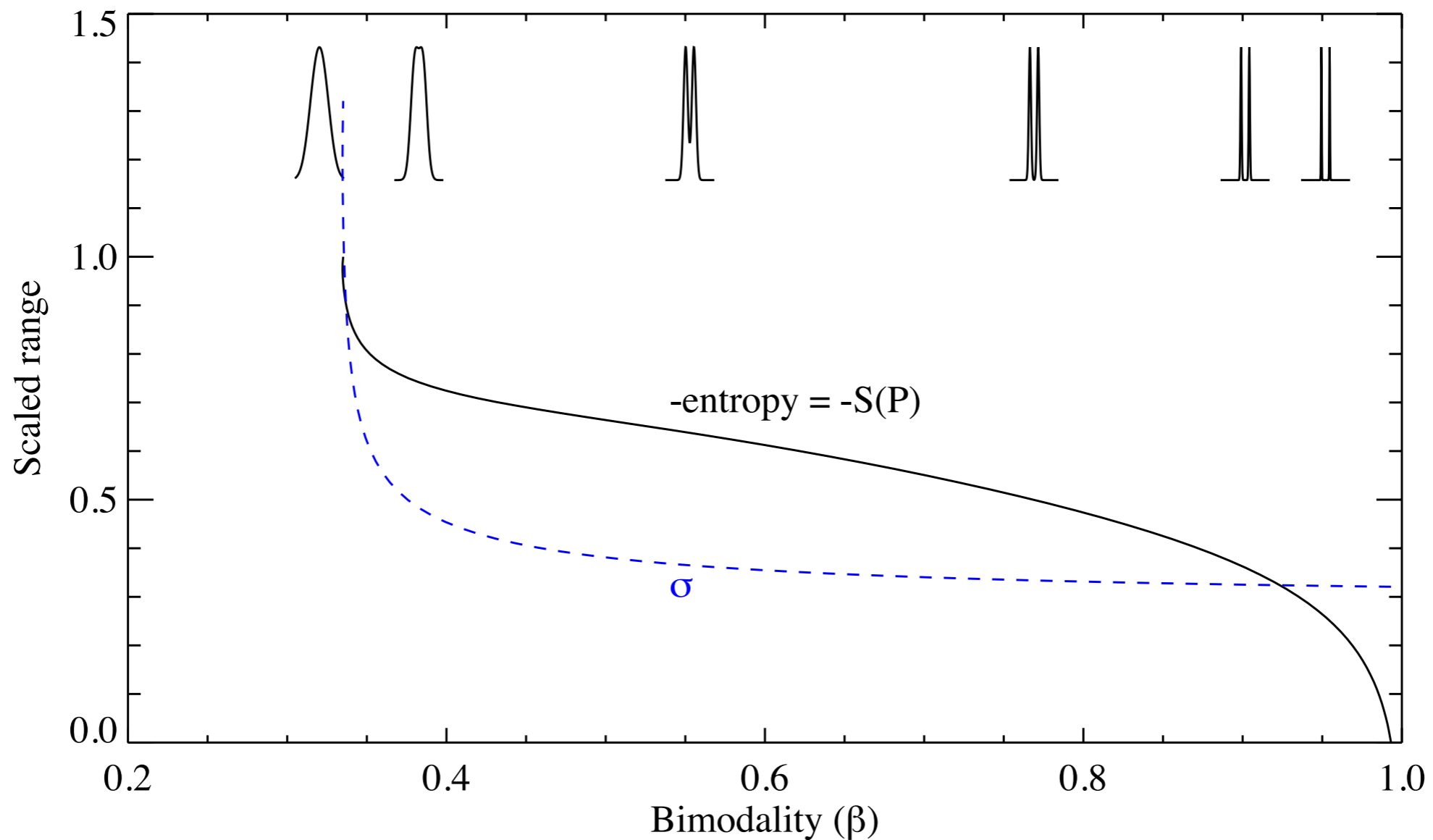
$$\left\{ x_q : q = \int^{x_q} P(t) dt \right\}$$

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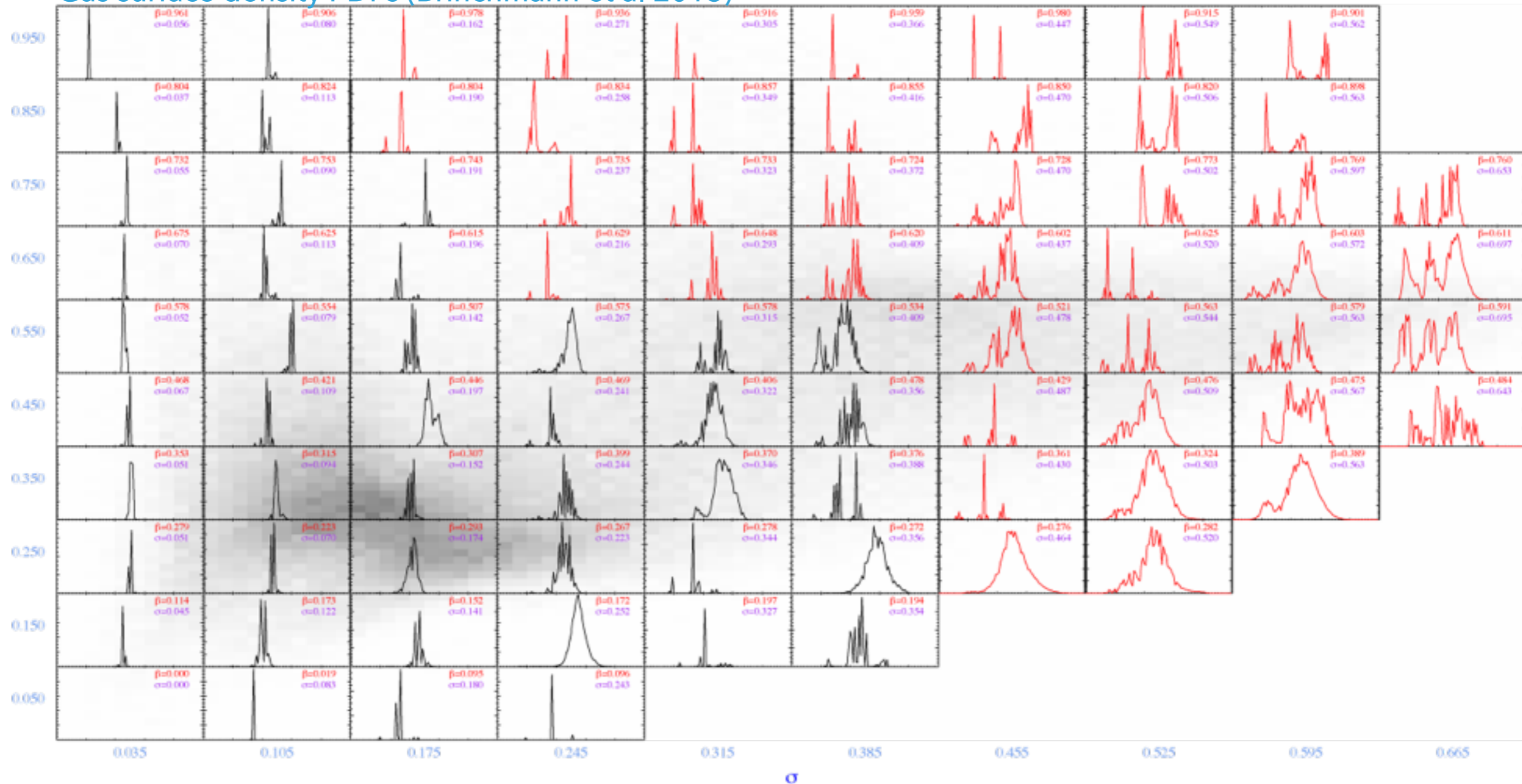
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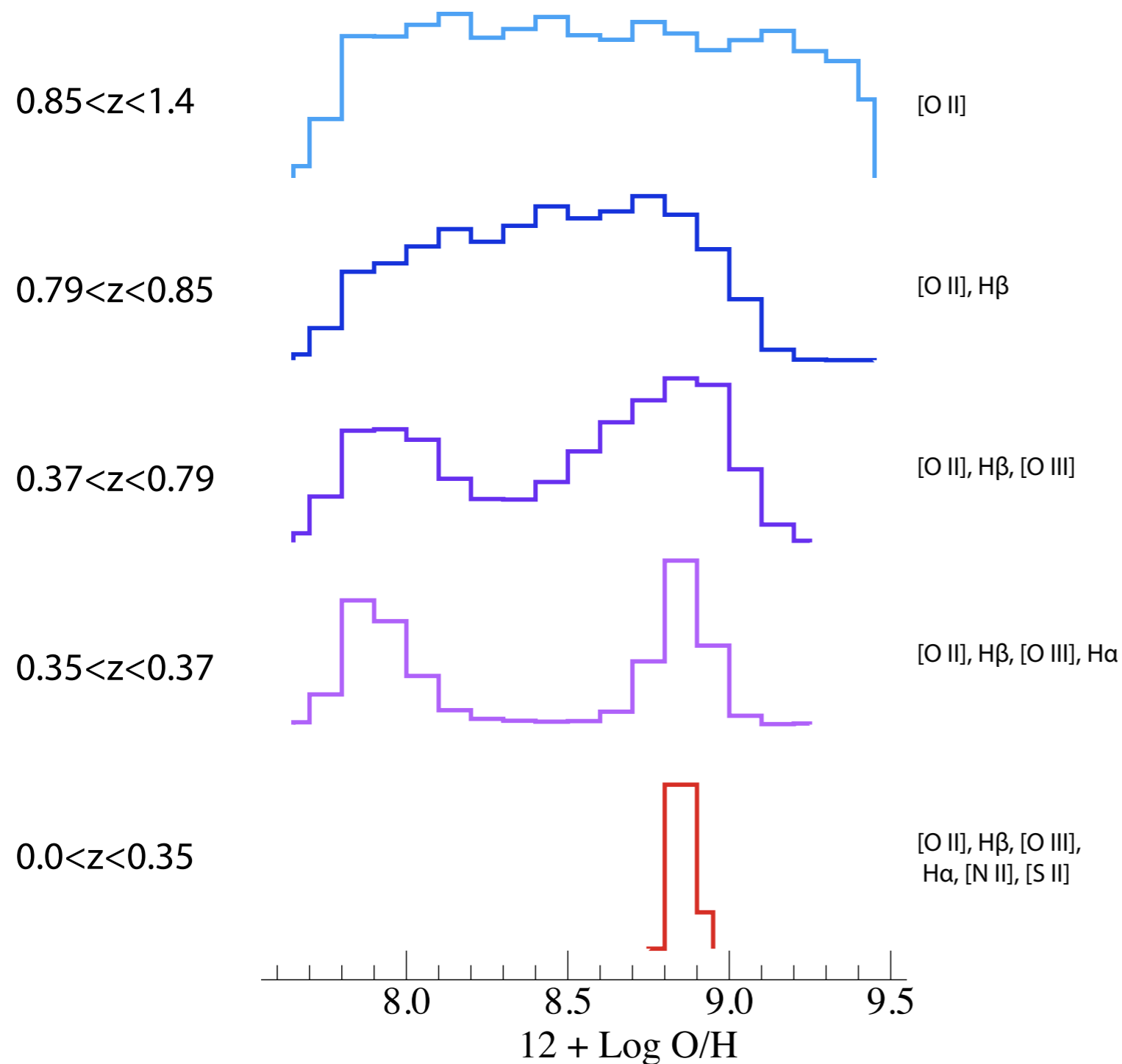
$$\beta = \frac{\text{skewness}^2 + 1}{\text{kurtosis}}$$

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Gas surface density PDFs (Brinchmann et al 2013)



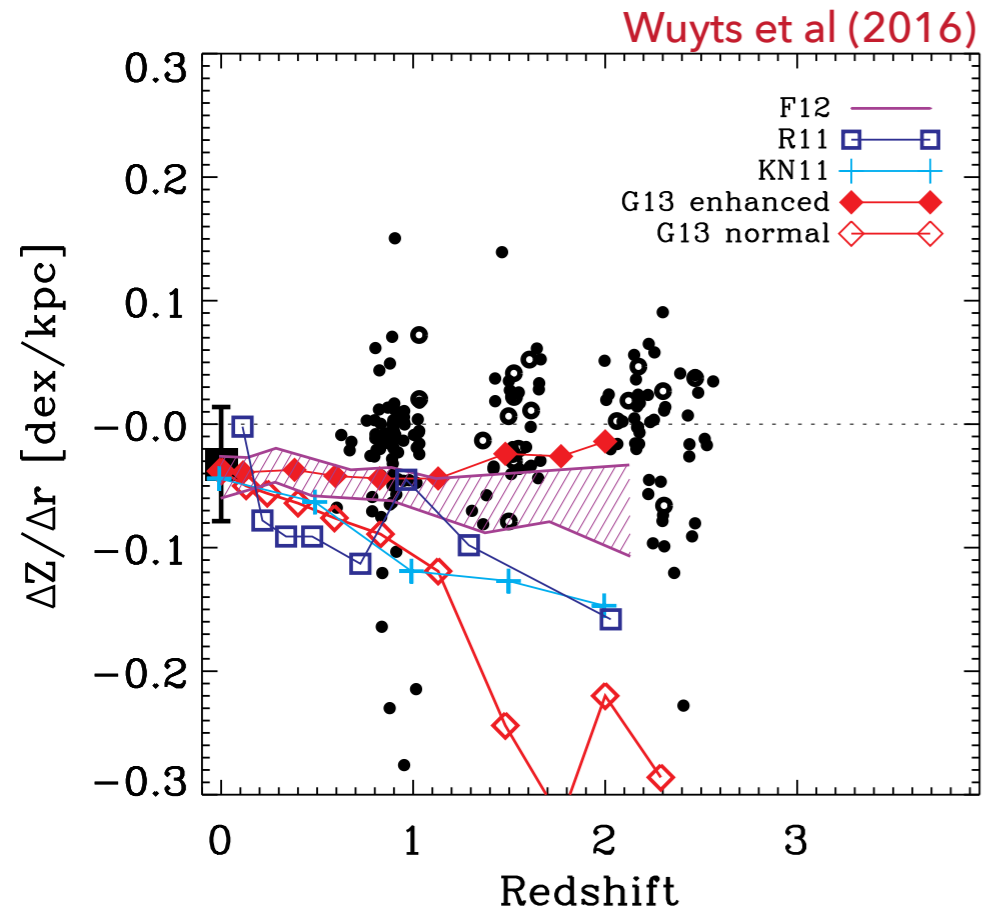
# SO IS A BAYESIAN ANALYSIS THE WAY FORWARD?



It might be - but priors might have to be carefully chosen to get useful results.

True when comparing different redshifts, but also when comparing *within* a galaxy

# METALLICITY GRADIENTS DETERMINATION



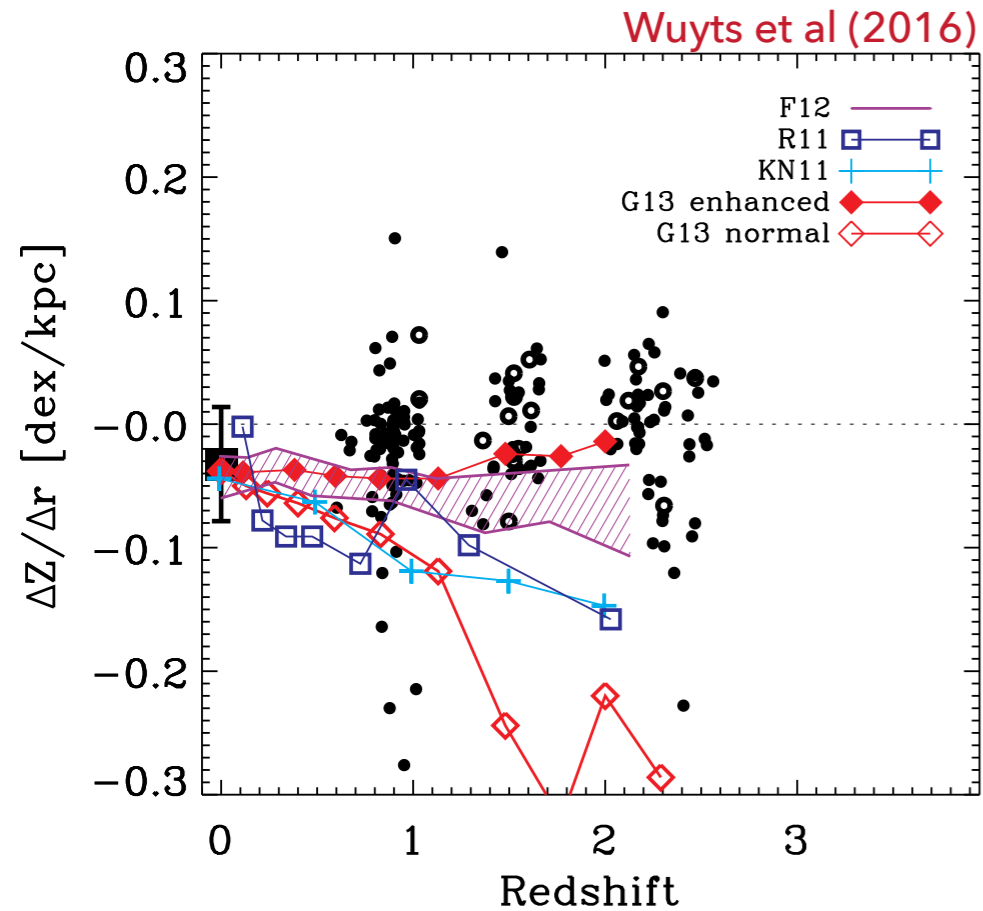
One approach (e.g. KMOS-3D, Wuyts et al 2016):

- strong lines close in wavelength
- beam smearing correction from models

Works well - but hard to quantify uncertainties in a rigorous way.



# METALLICITY GRADIENTS DETERMINATION



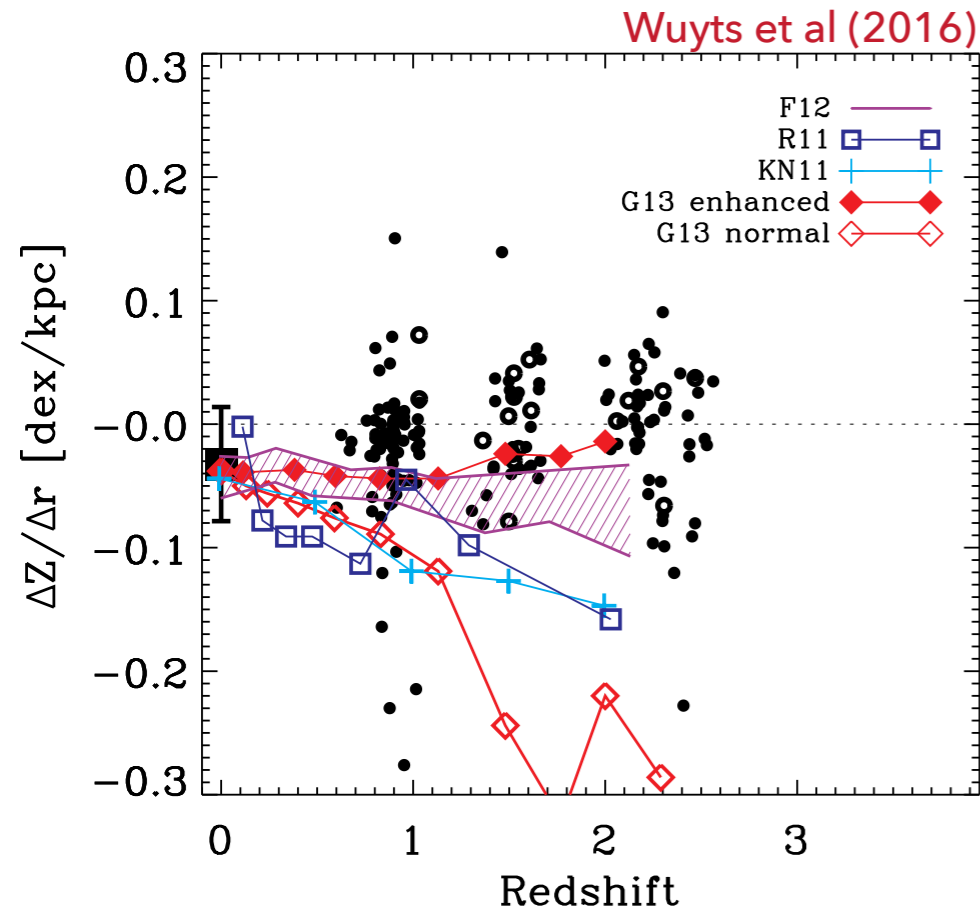
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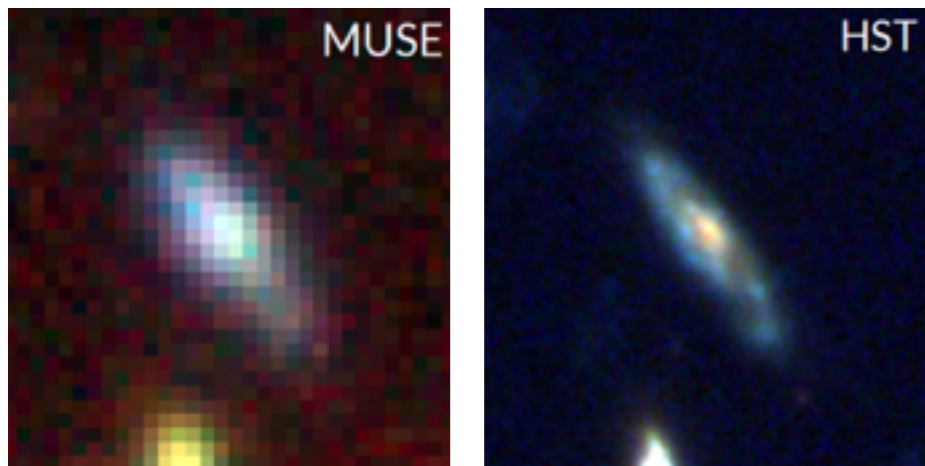
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With MUSE:

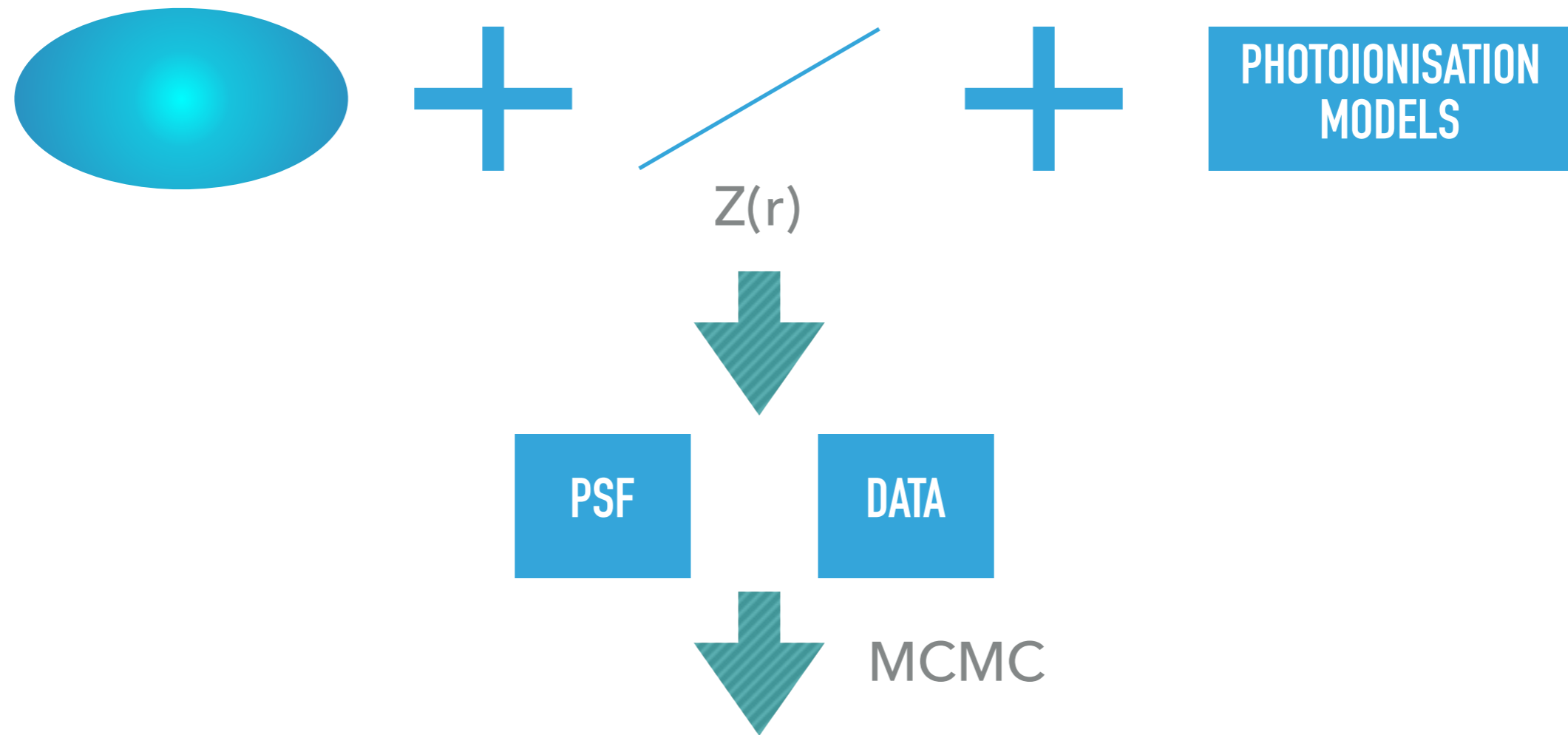
- a range of lines, weaker in the outskirts
- different sets of lines at different redshifts
- wavelength dependent PSF

# METALLICITY GRADIENTS DETERMINATION



David Carton

IFU data: need to exploit spatial correlations



Metallicity gradient, central metallicity

# METALLICITY GRADIENTS DETERMINATION

IFU data: need to exploit spatial correlations

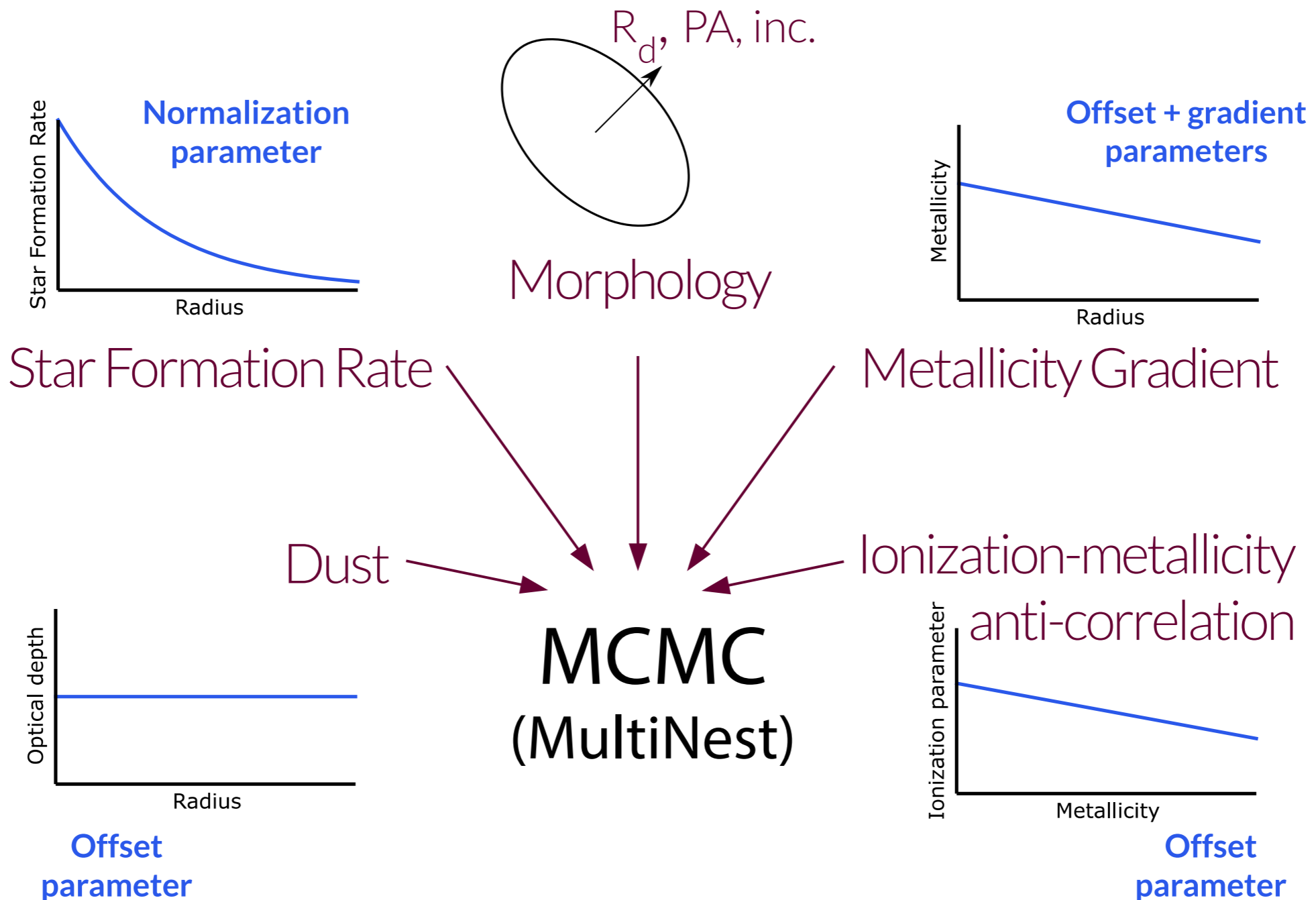
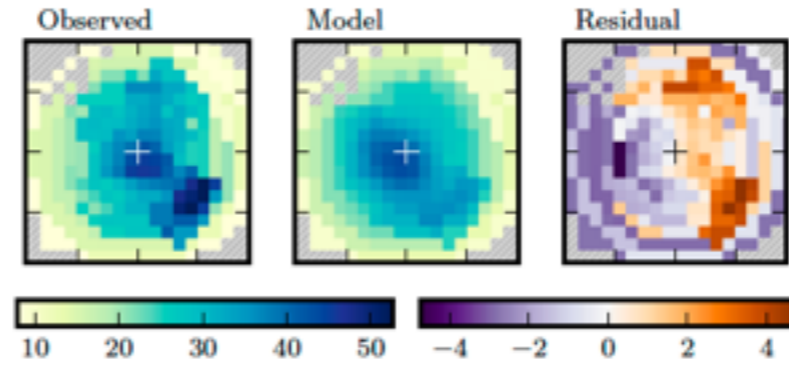
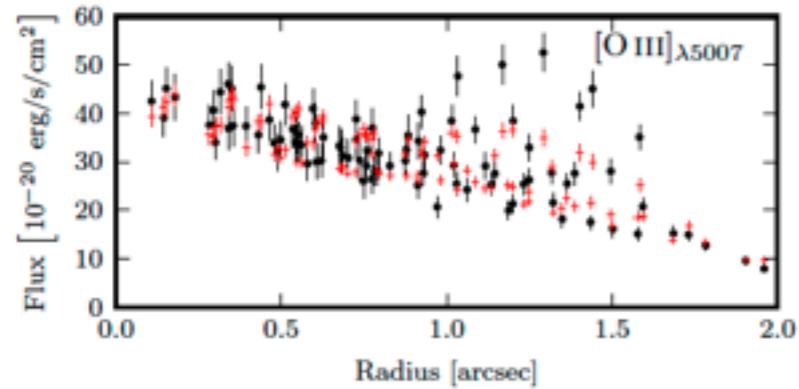
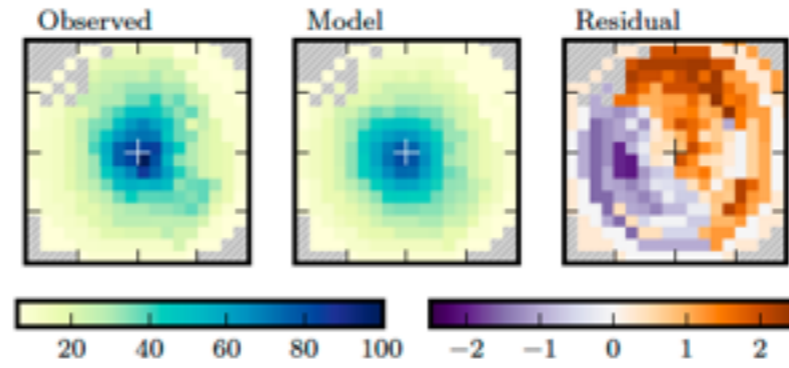
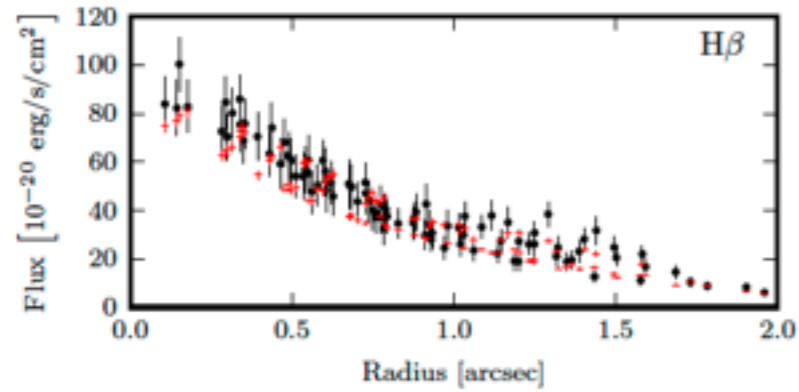
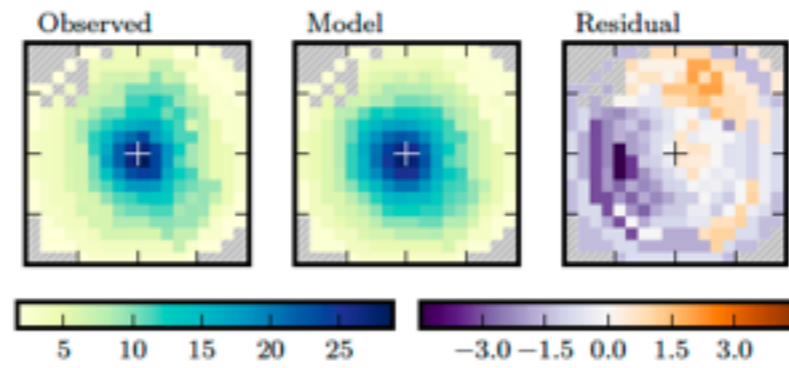
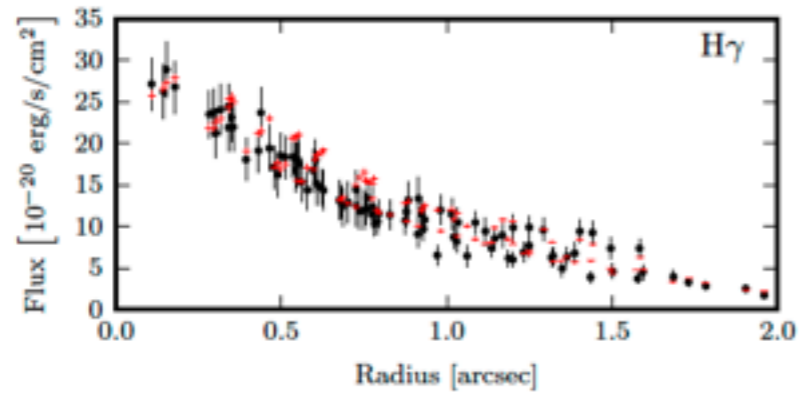
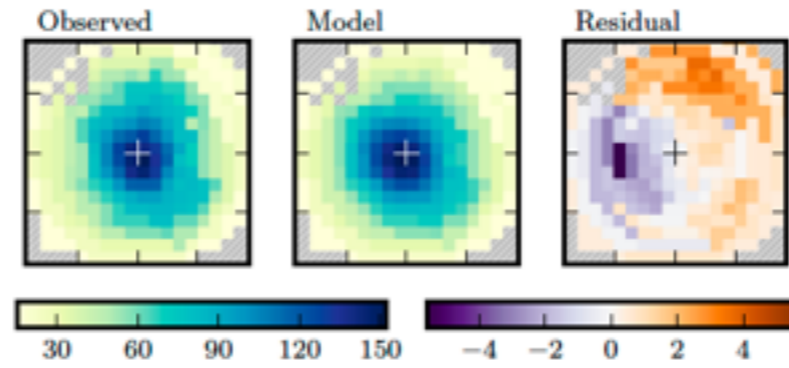
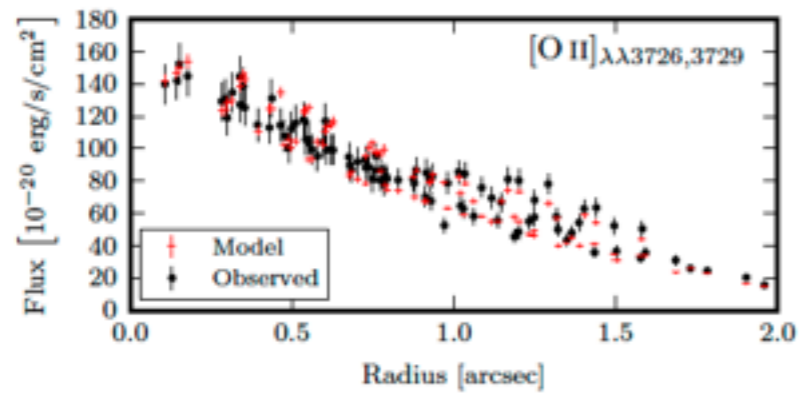
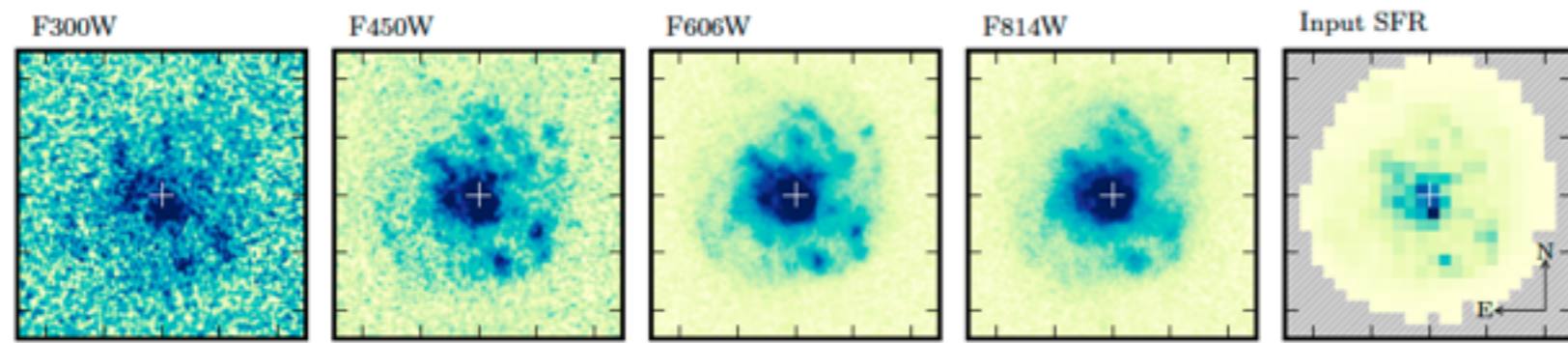
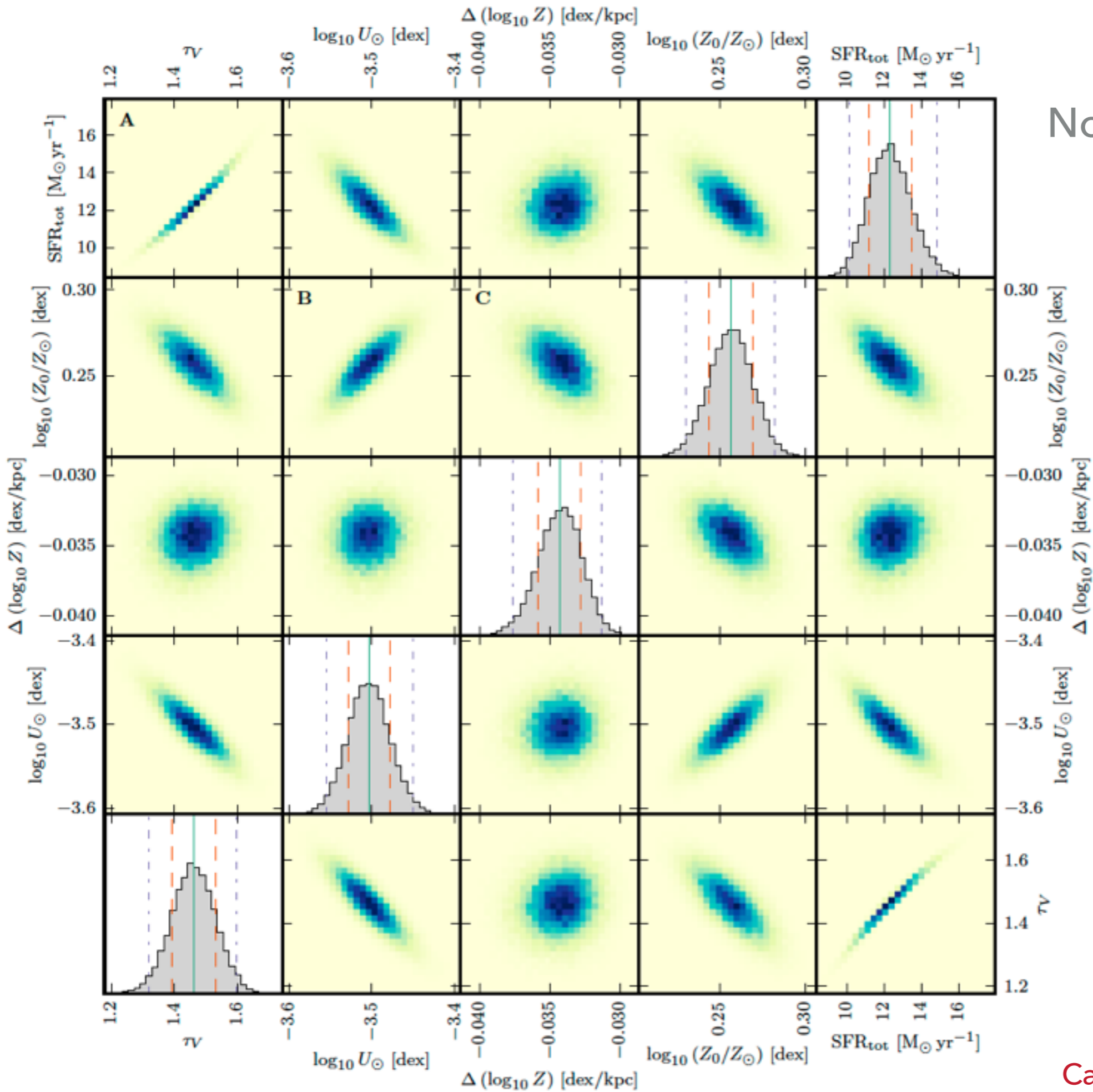


Figure from David Carton

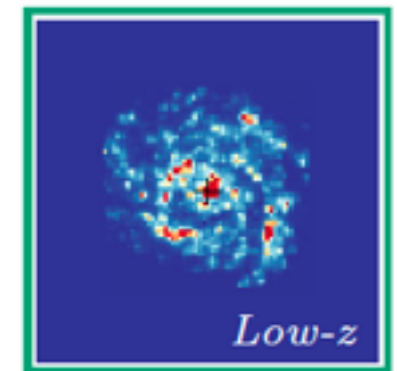
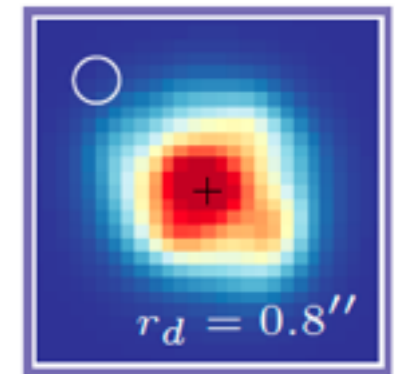
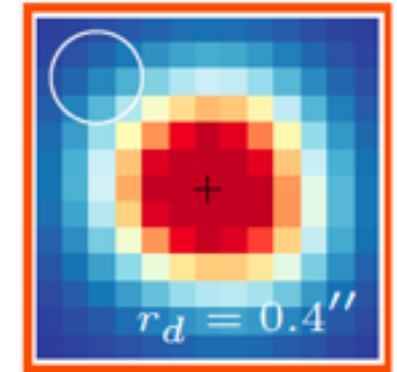
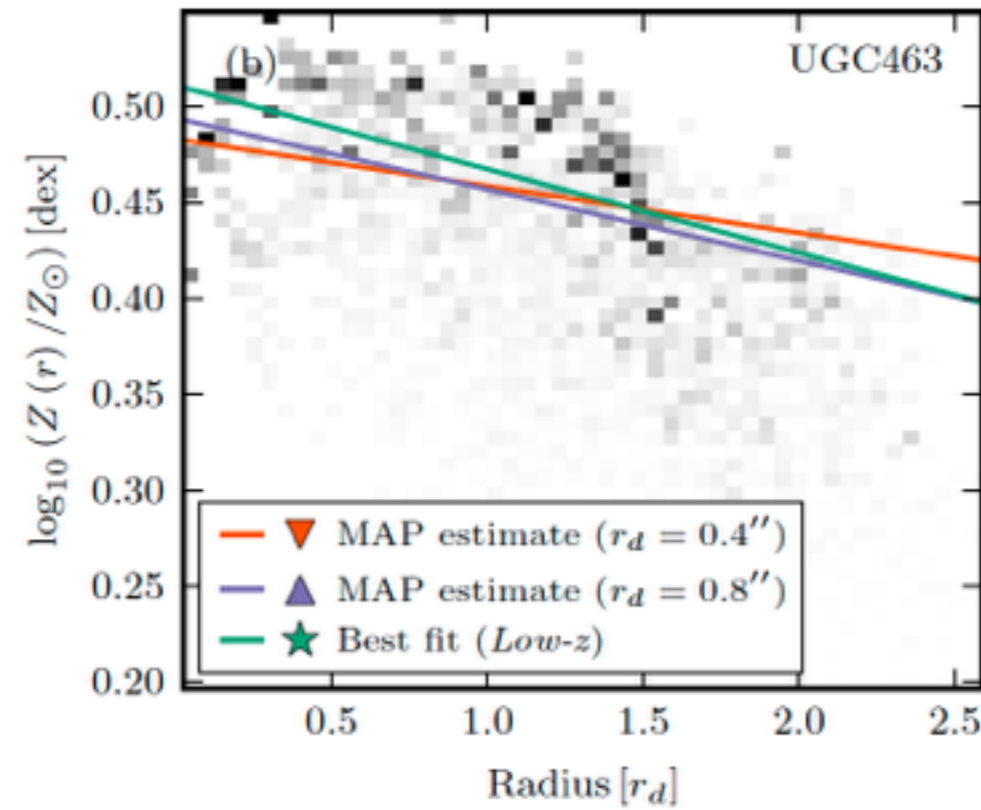
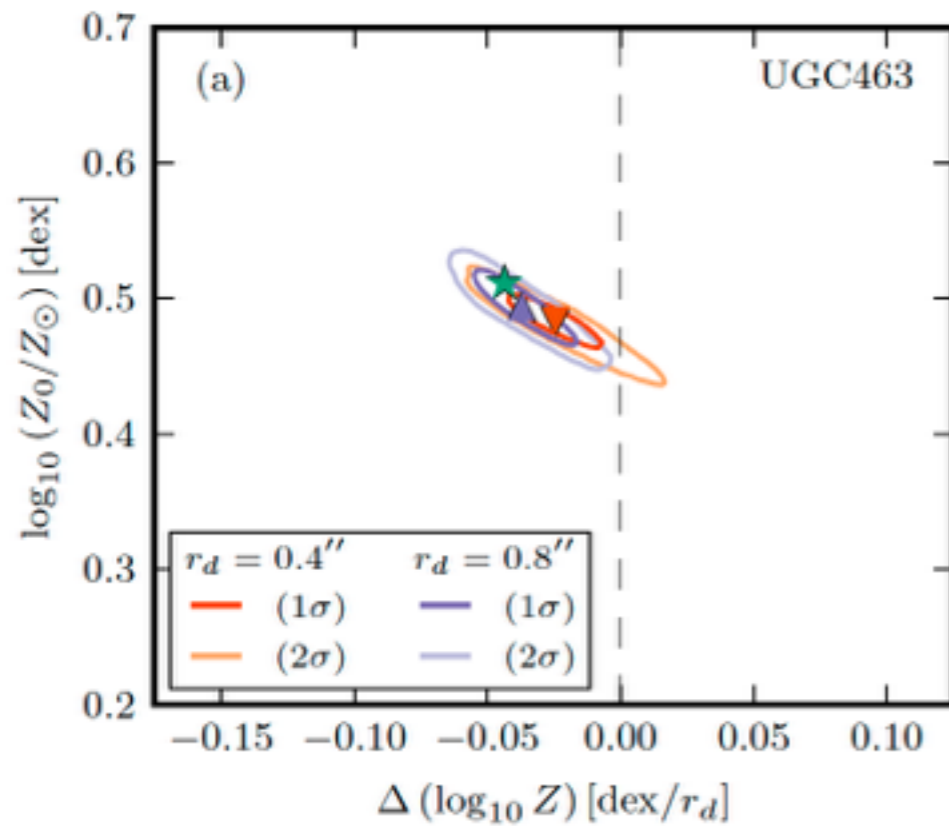




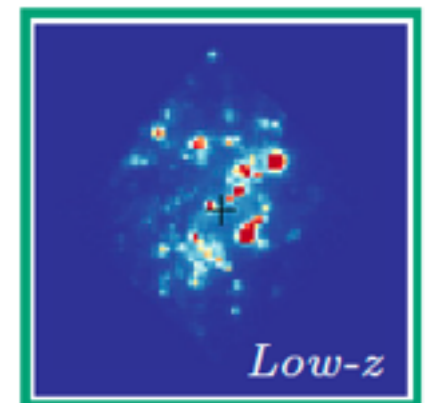
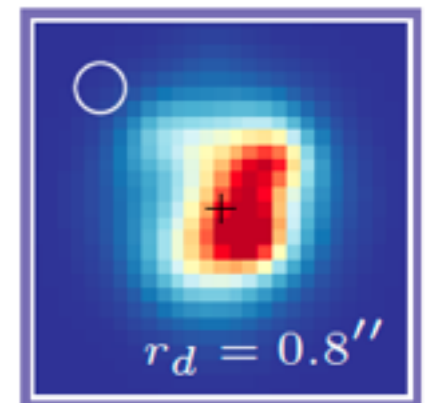
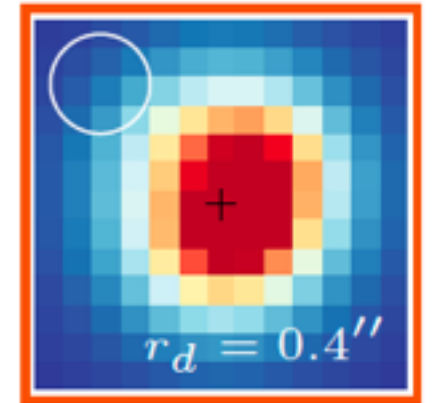
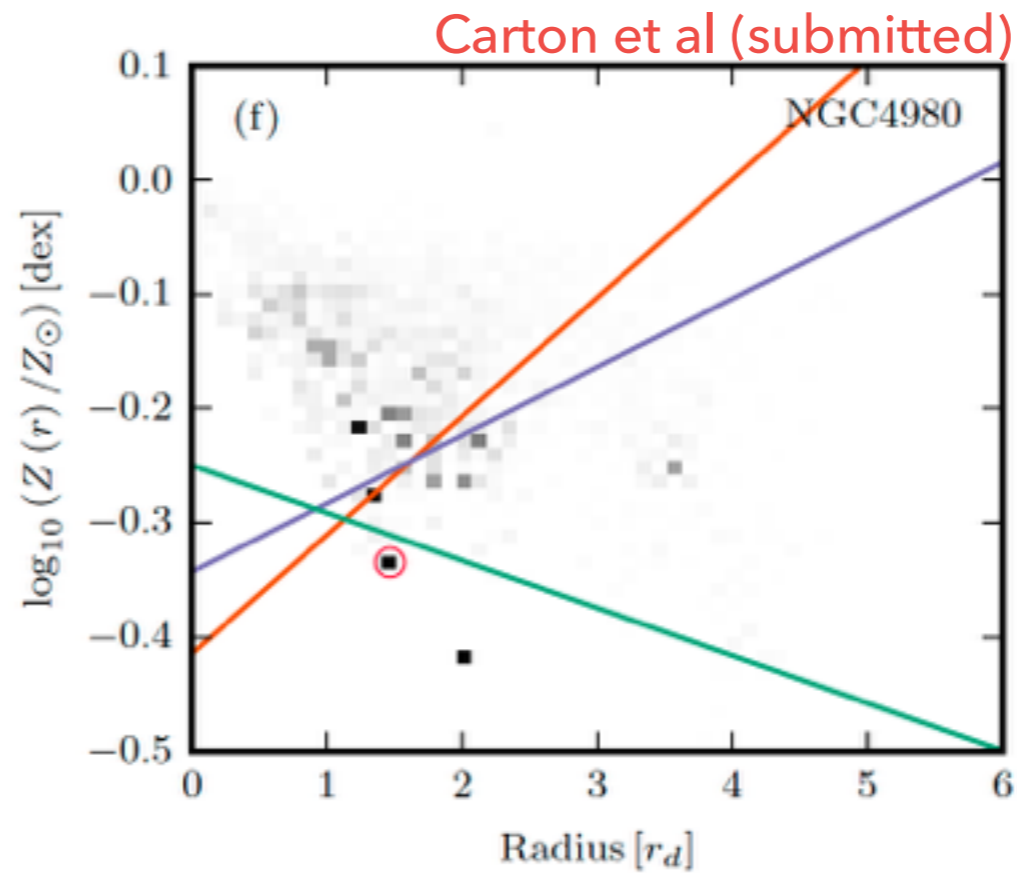
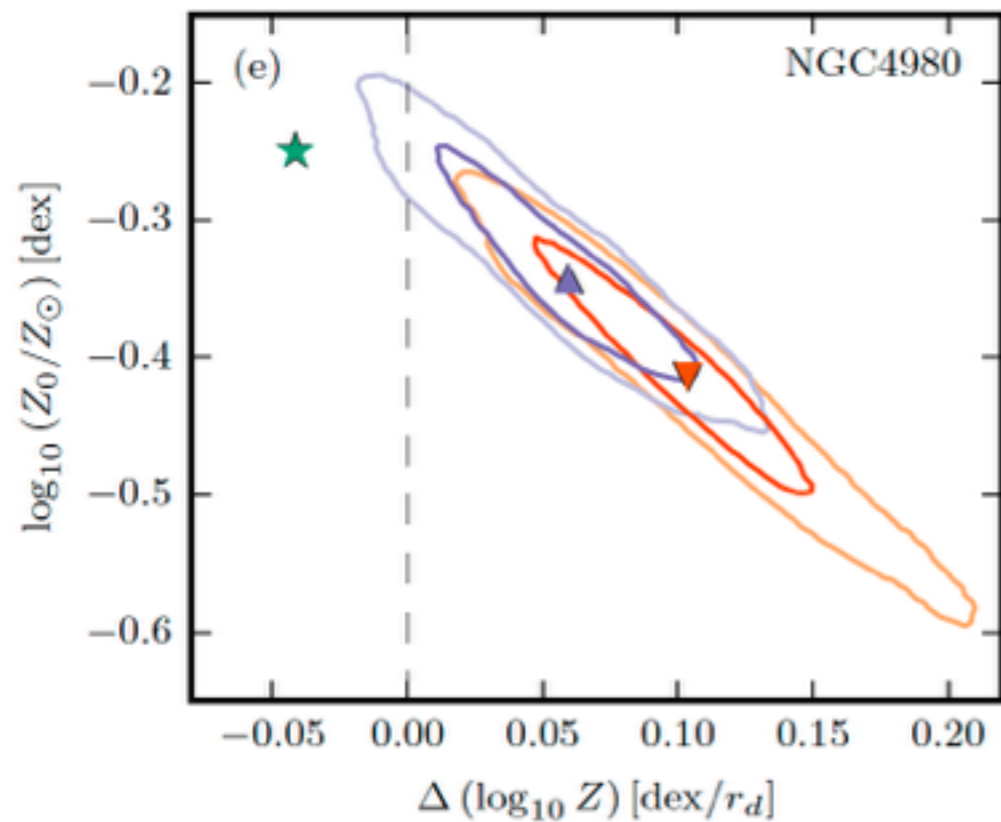
Not always this nice!

# When galaxies are well-behaved it works

Carton et al (submitted)



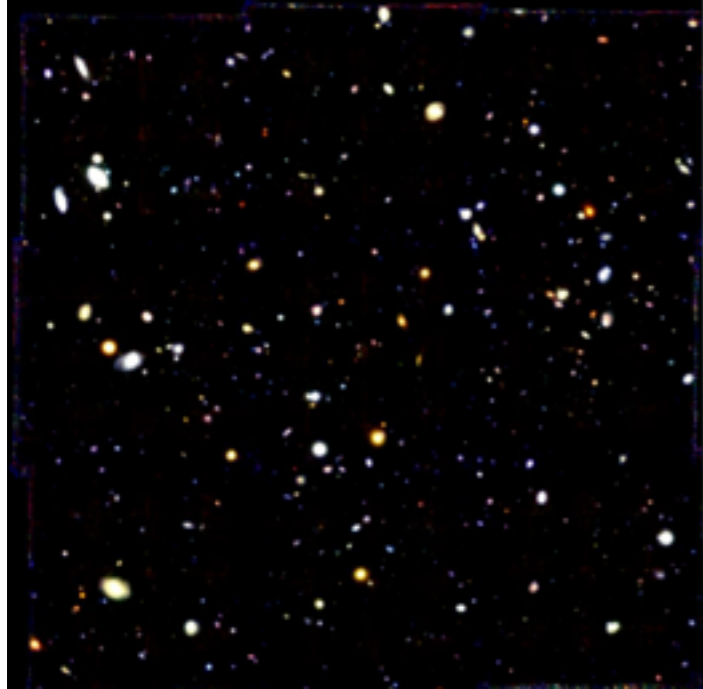
# When they are not - it is a problem



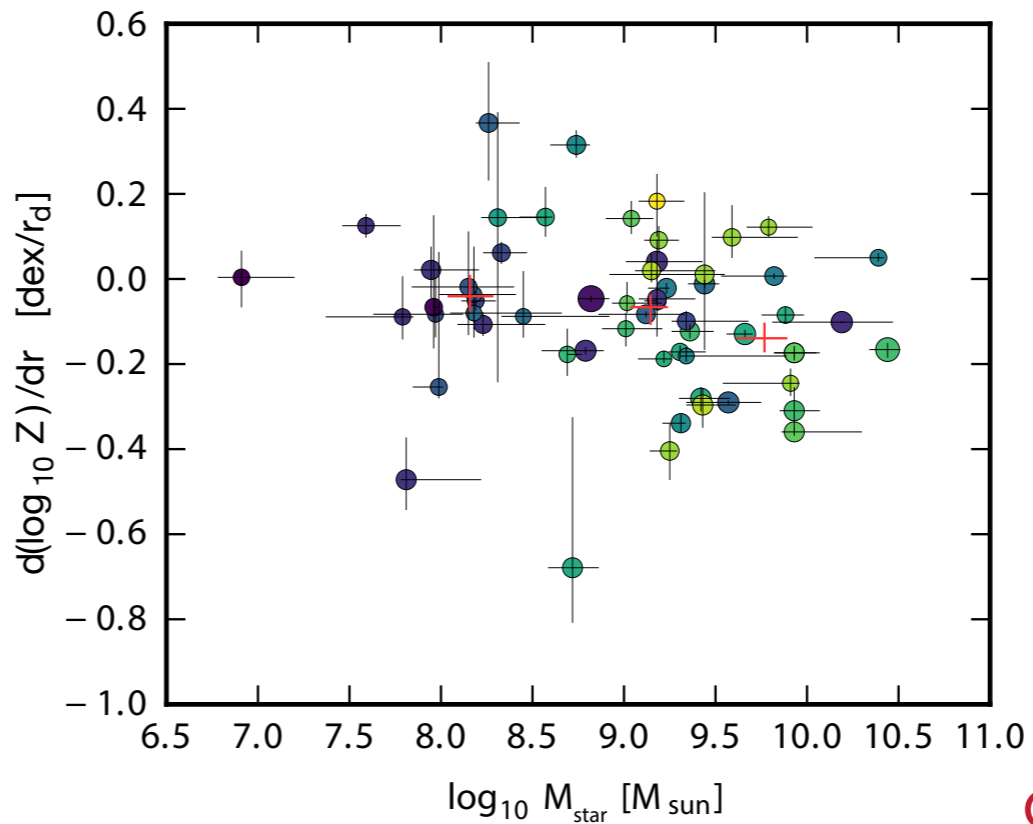
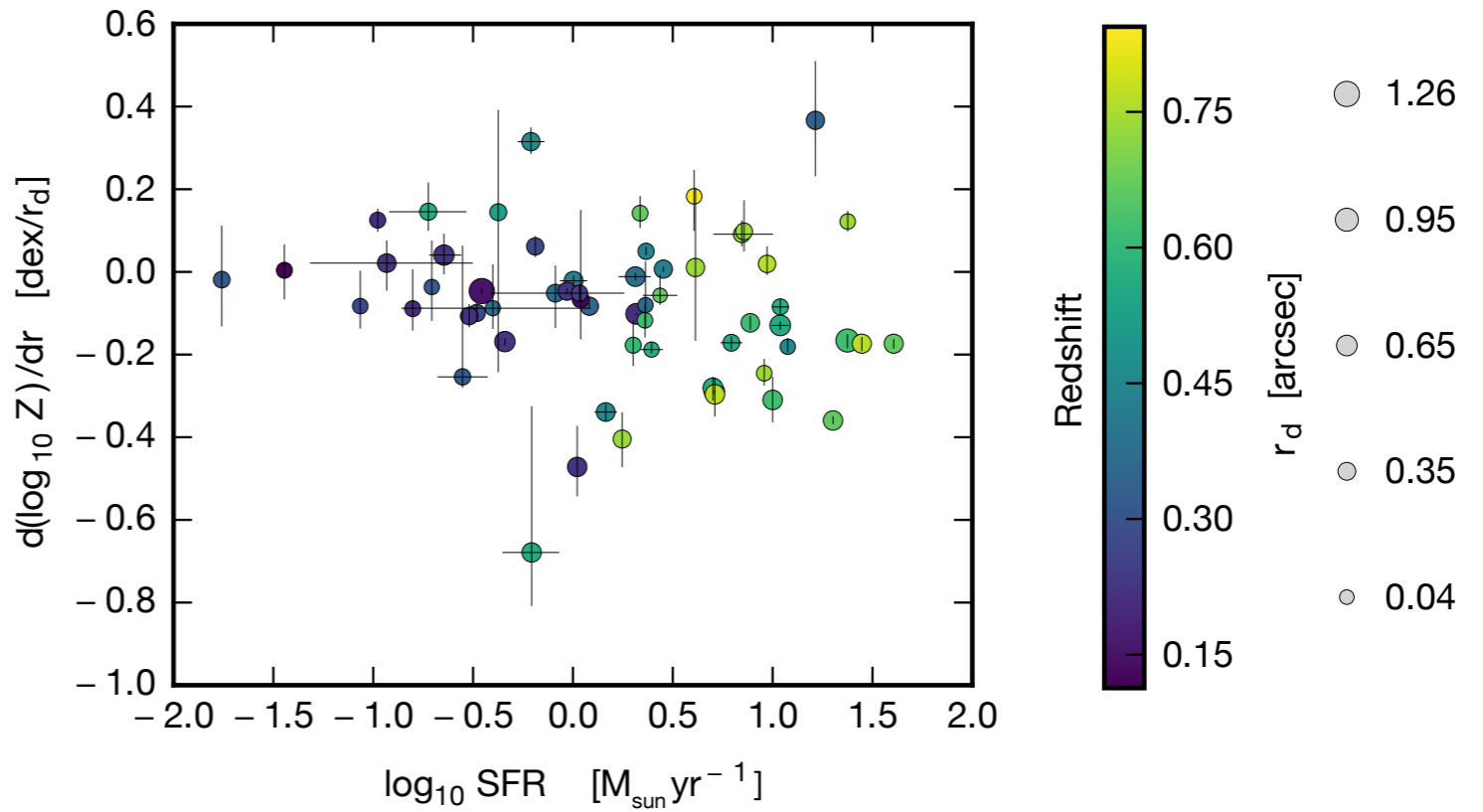
Clumpy structure means comparison of low & high-z metallicity profiles can be problematic.



# APPLICATION: MUSE DEEP FIELDS



3'x3' mosaic in the UDF

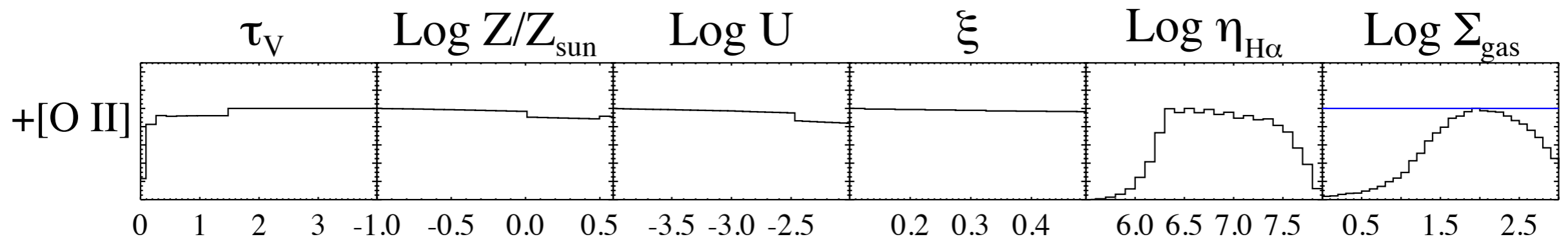


## SUMMARY

- ▶ The basic ingredients are increasingly well known - time for the statistical methodology to take advantage of this.
- ▶ Bayesian modelling, particularly of emission lines, should become the norm for the field over the coming years.
- ▶ With the steady increase in resolved spectroscopy, flexible ways of integrating spatial coherence would be useful extensions to SED/emission line models.
- ▶ A full forward modelling of metallicity profiles allows uncertainties to be well characterised but also shows that low- $z$  and high- $z$  metallicity gradients should be compared with care

**ADDITIONAL  
MATERIAL**

# FITTING EMISSION LINES – A LOT TO LEARN

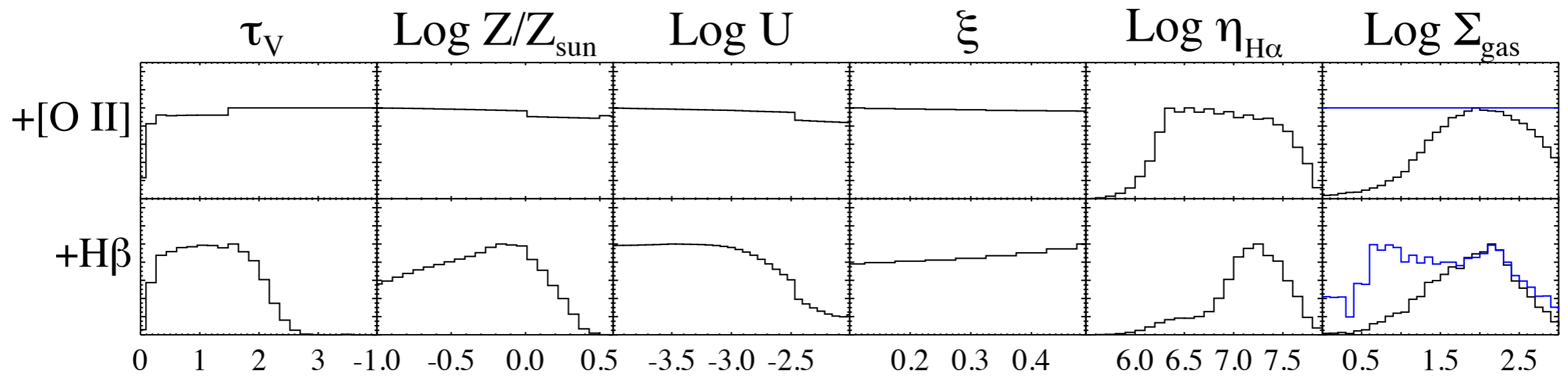


Fit models (Charlot & Longhetti 2001, also Dopita et al (2013) for metallicities):

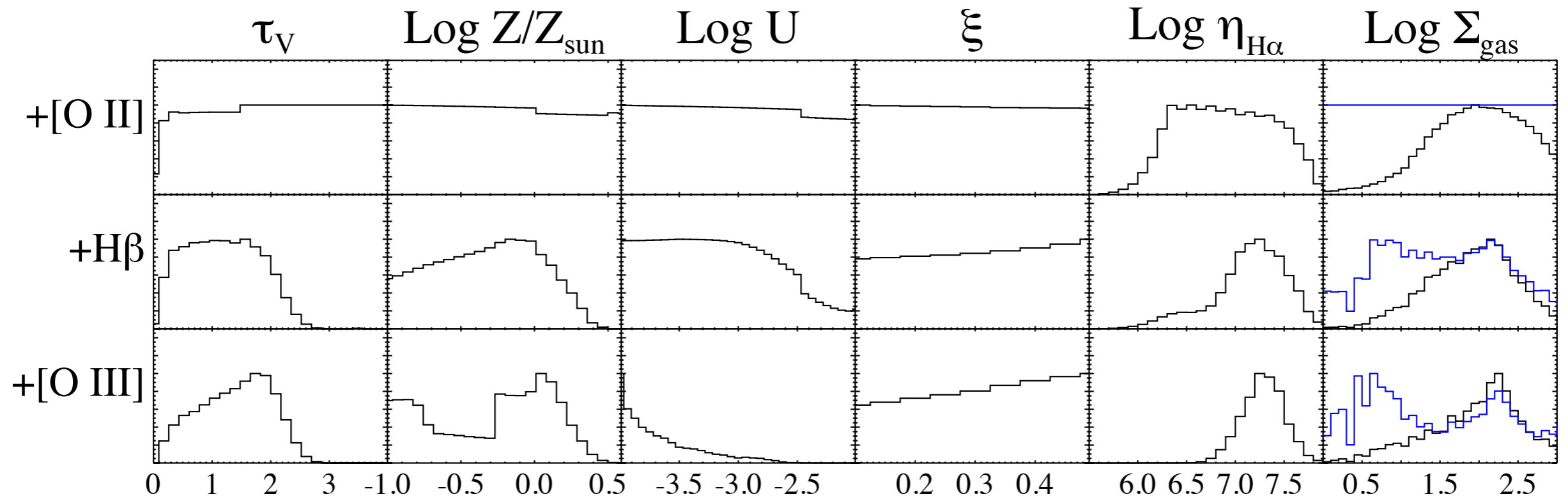
$$\ln P(\mathcal{M}|\{L_i\}) = -\frac{1}{2} \sum_{i \in \{L_i\}} \frac{(f_i - A f_{\mathcal{M}})^2}{\sigma_i^2} + \ln \text{Pr},$$

likelihood
 $f_{\mathcal{M}} = \text{model}$ 
Prior

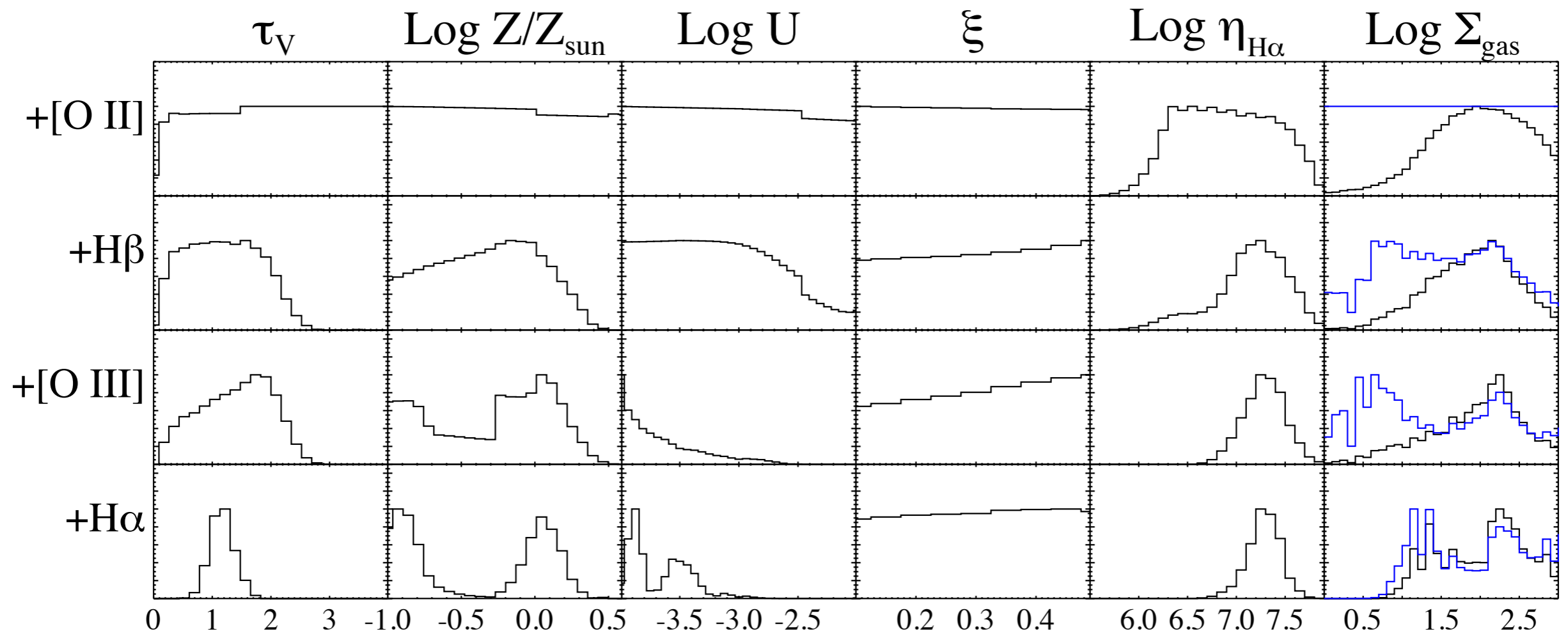
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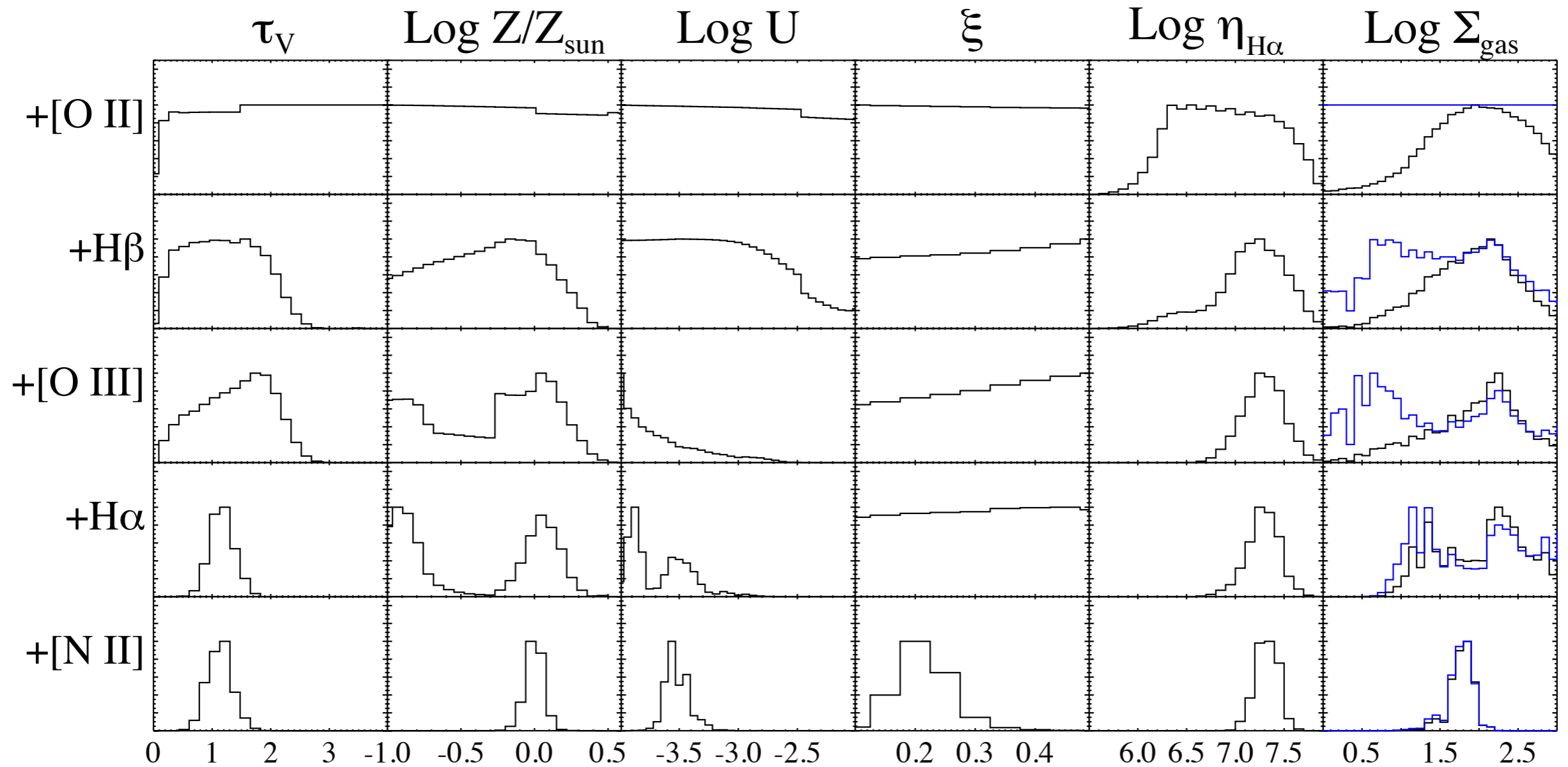
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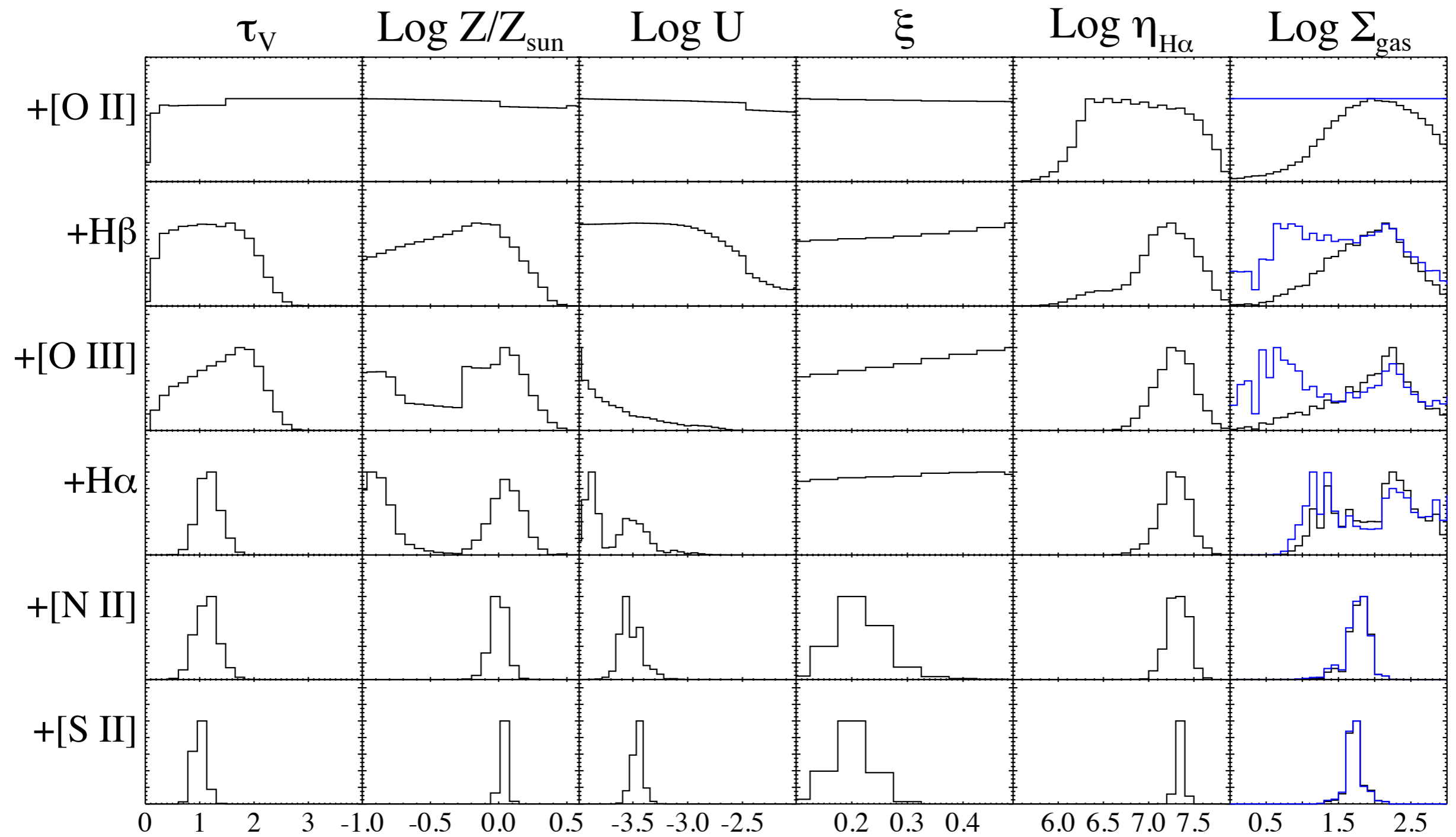


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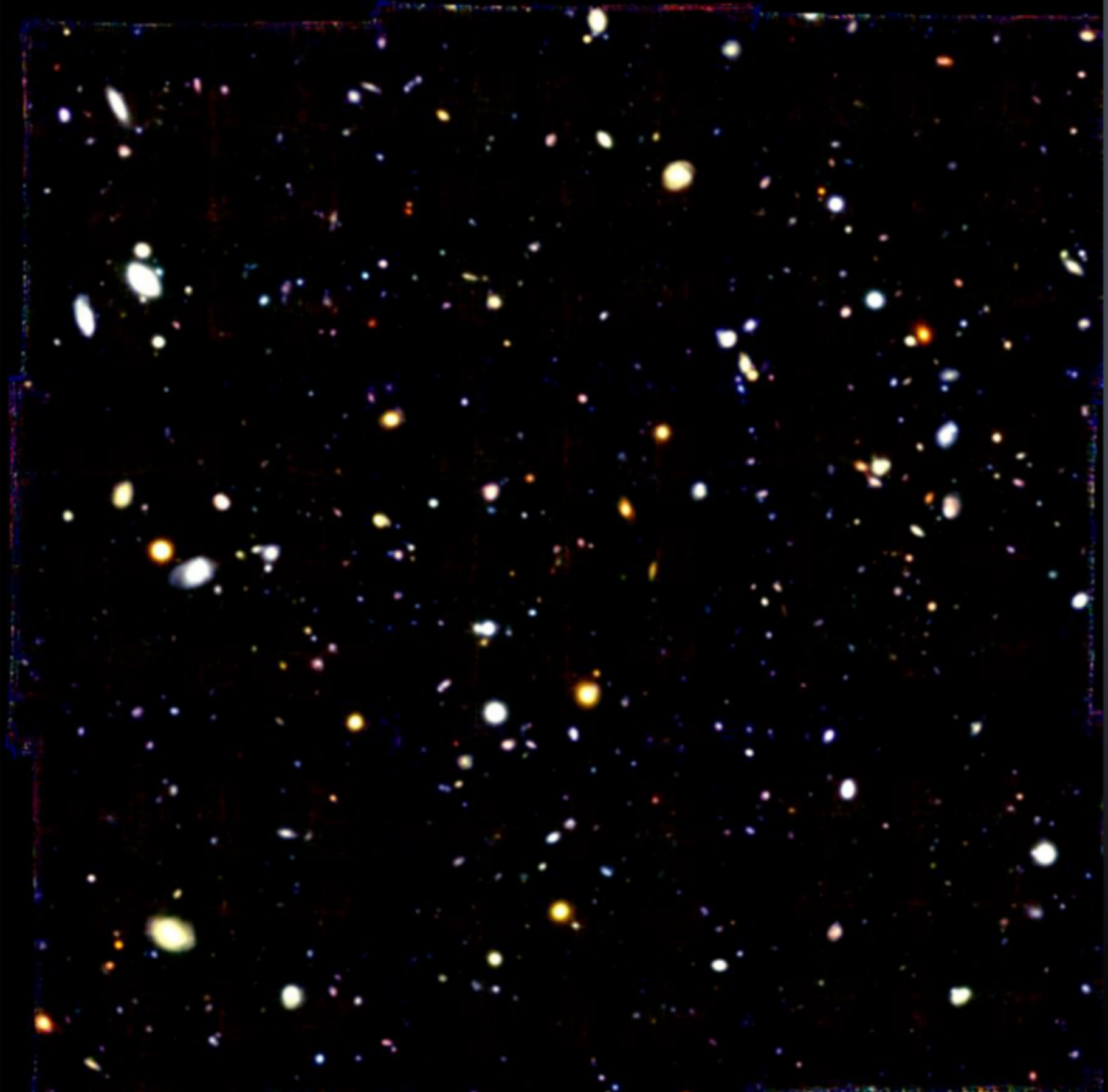




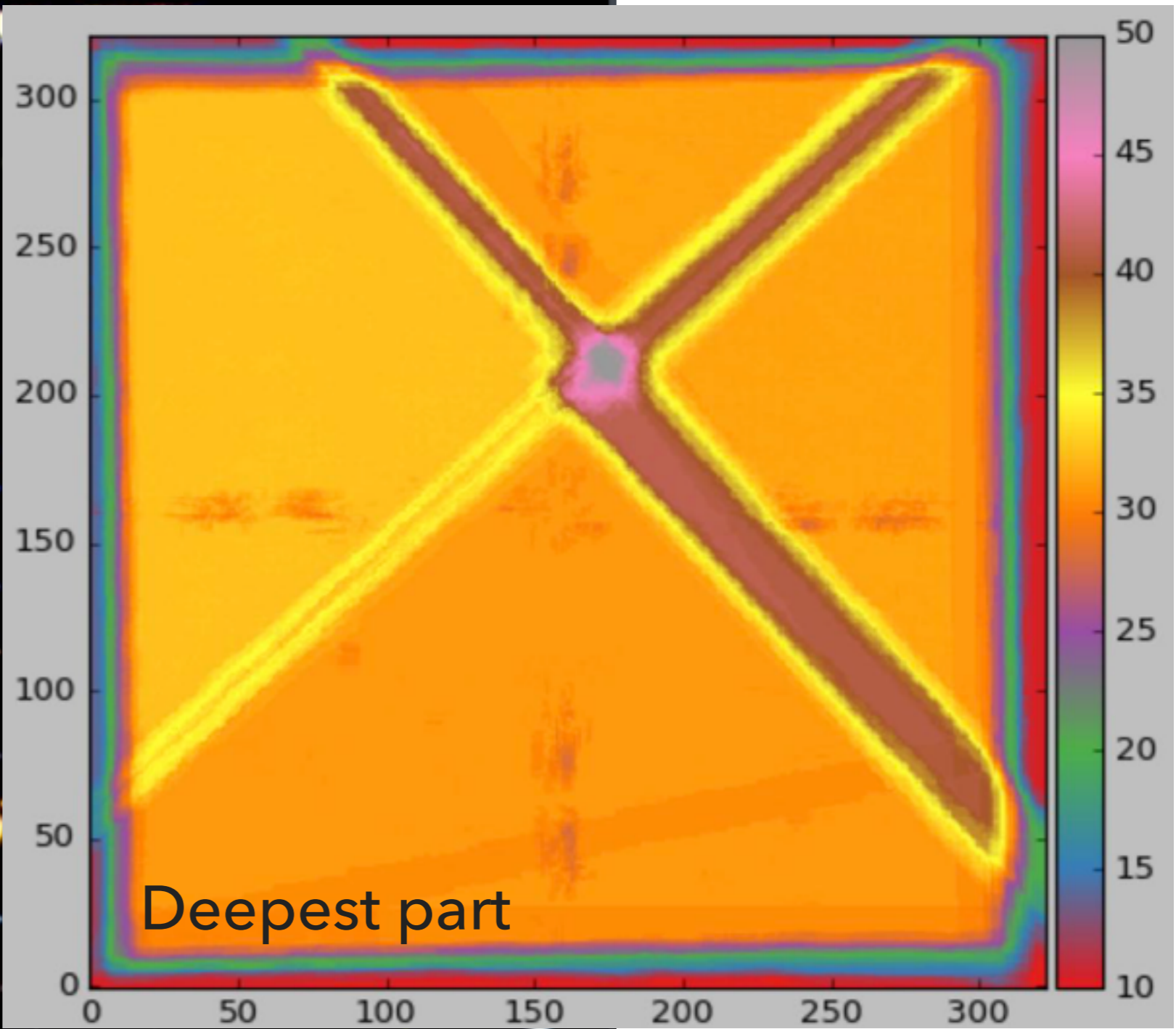
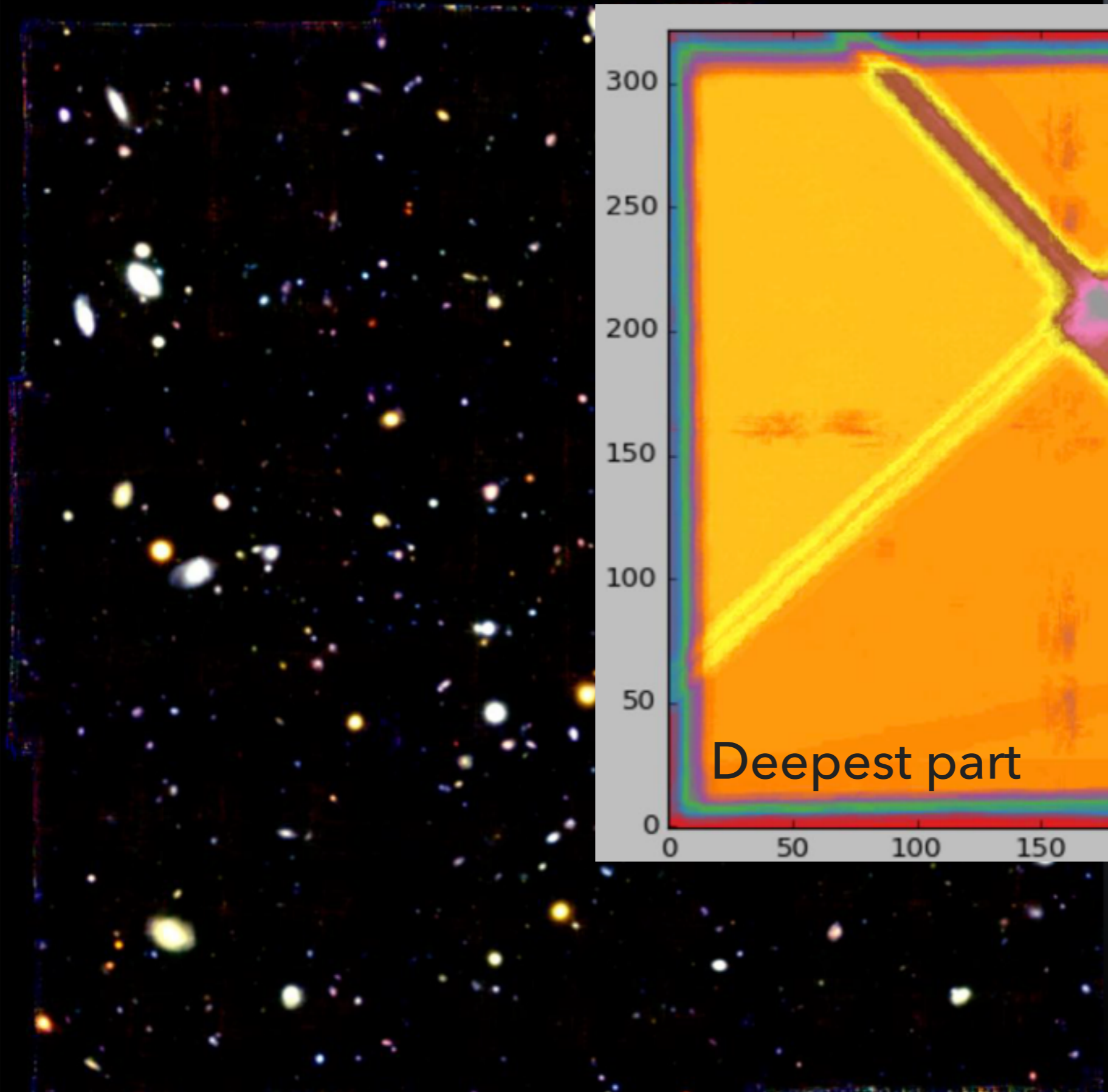
## FITTING EMISSION LINES – A LOT TO LEARN



UDF Mosaic : 3'x3' - >10hr per pointing



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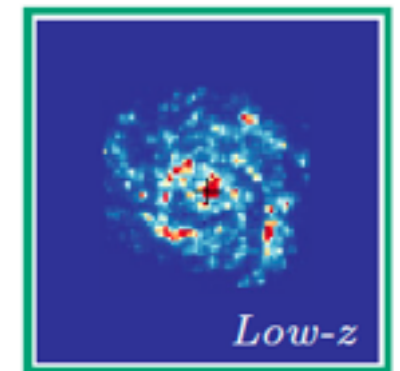
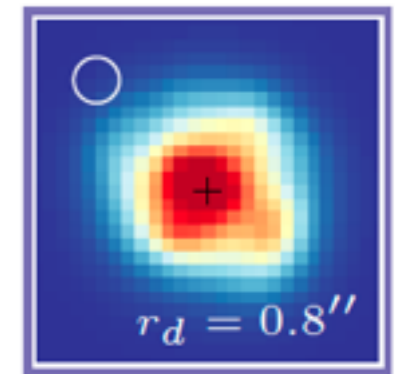
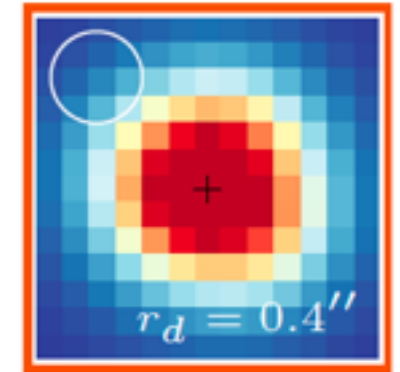
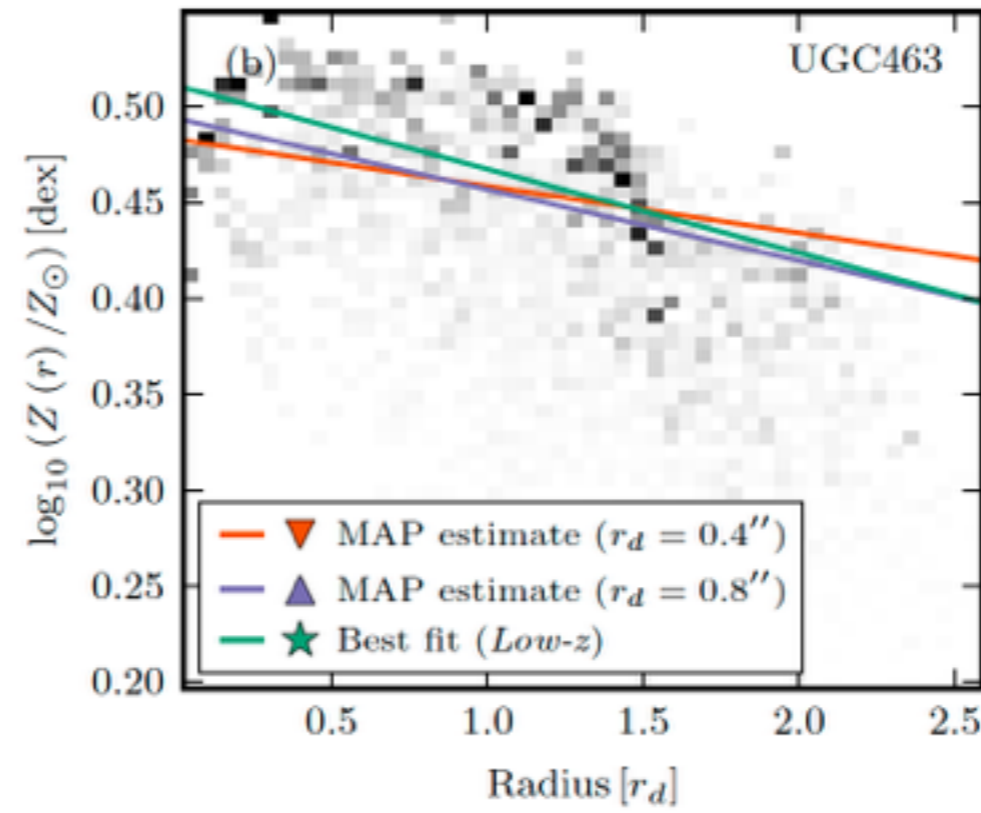
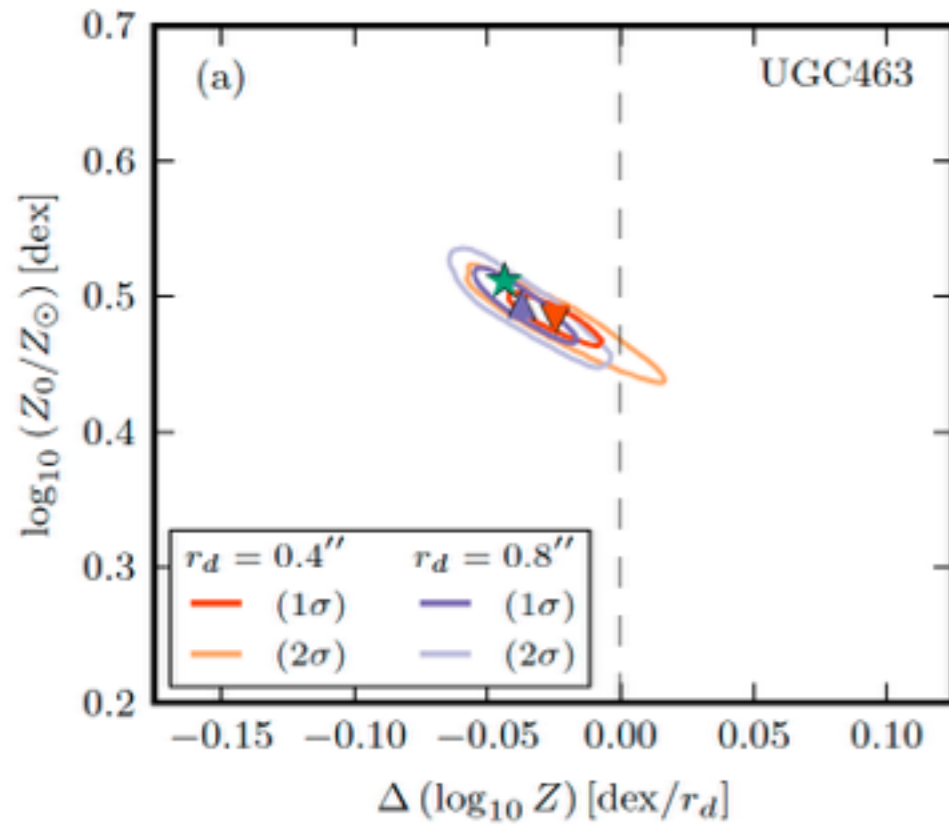
## THE ENEMY: SEEING



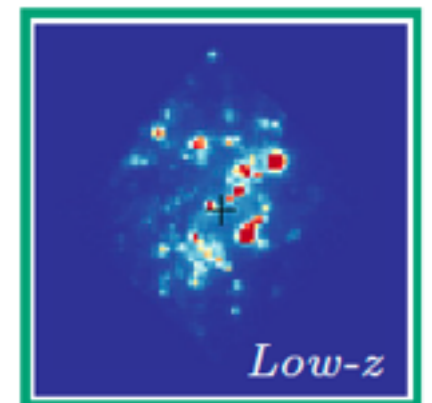
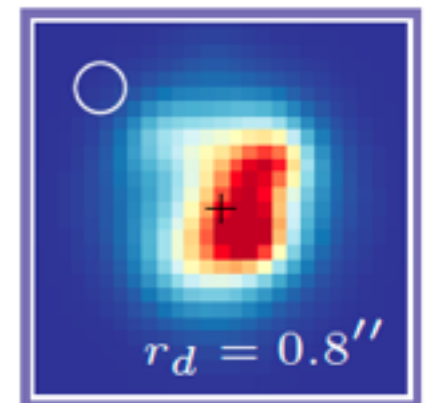
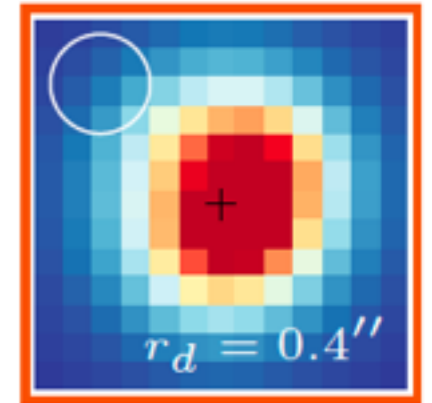
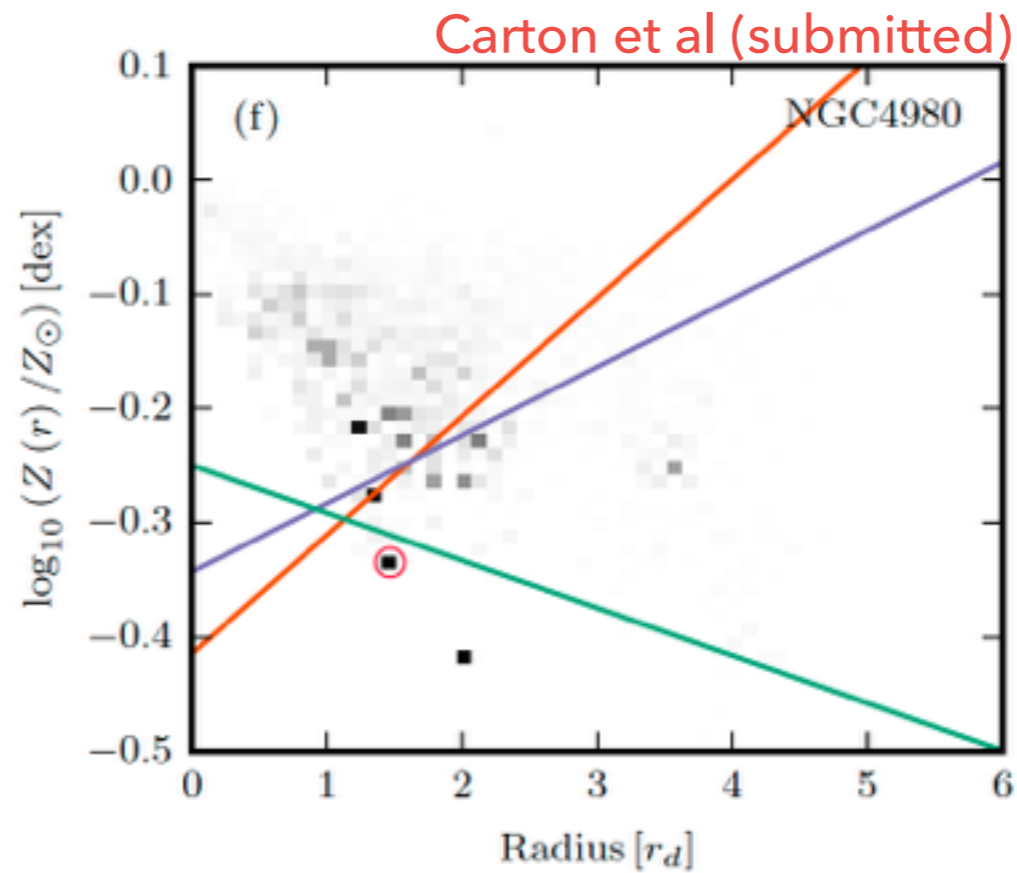
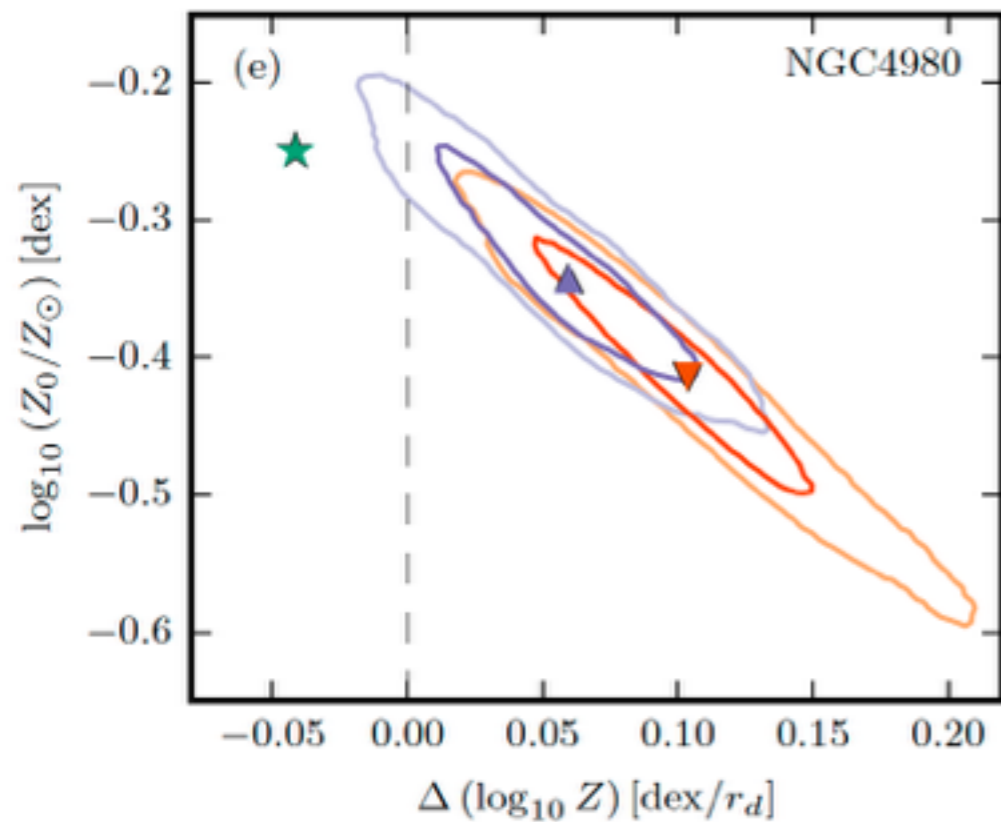
Way forwards: Forward modelling

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Carton et al (submitted)



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# METALLICITY PROFILES

