Fourier-domain response and Bayesian parameter estimation for LISA

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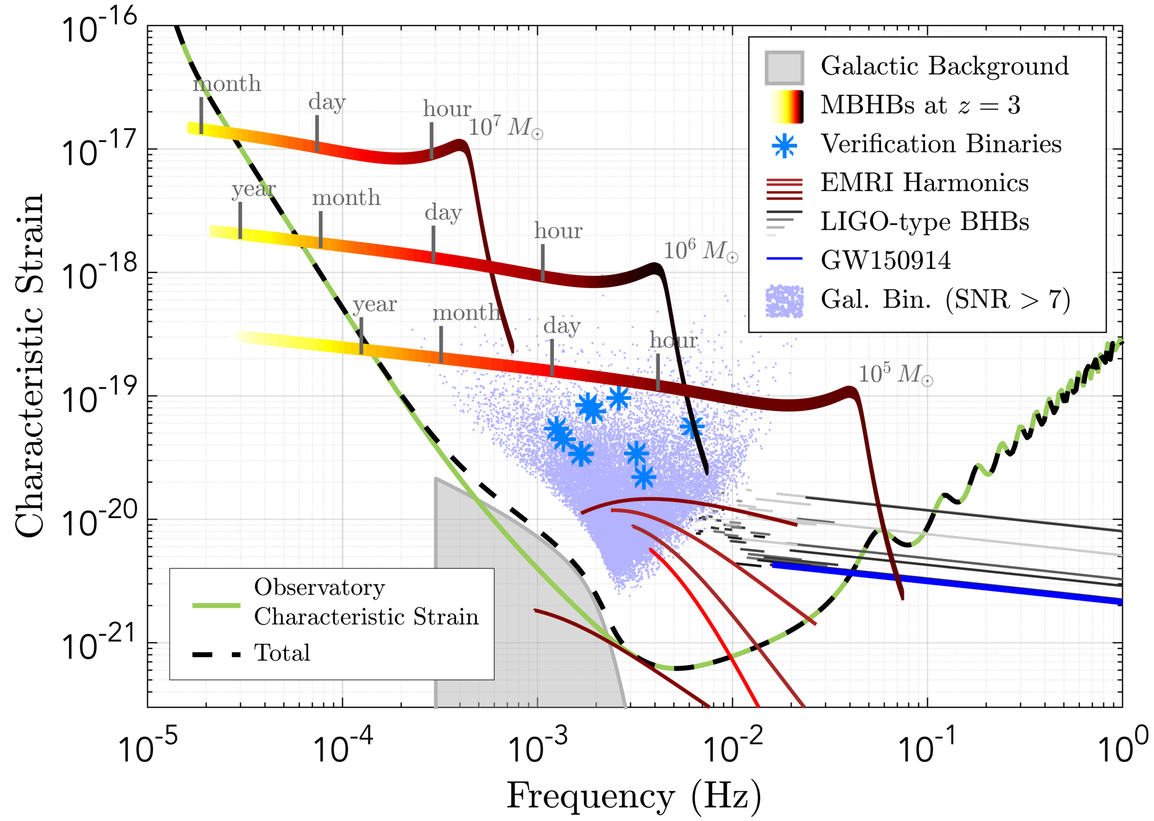
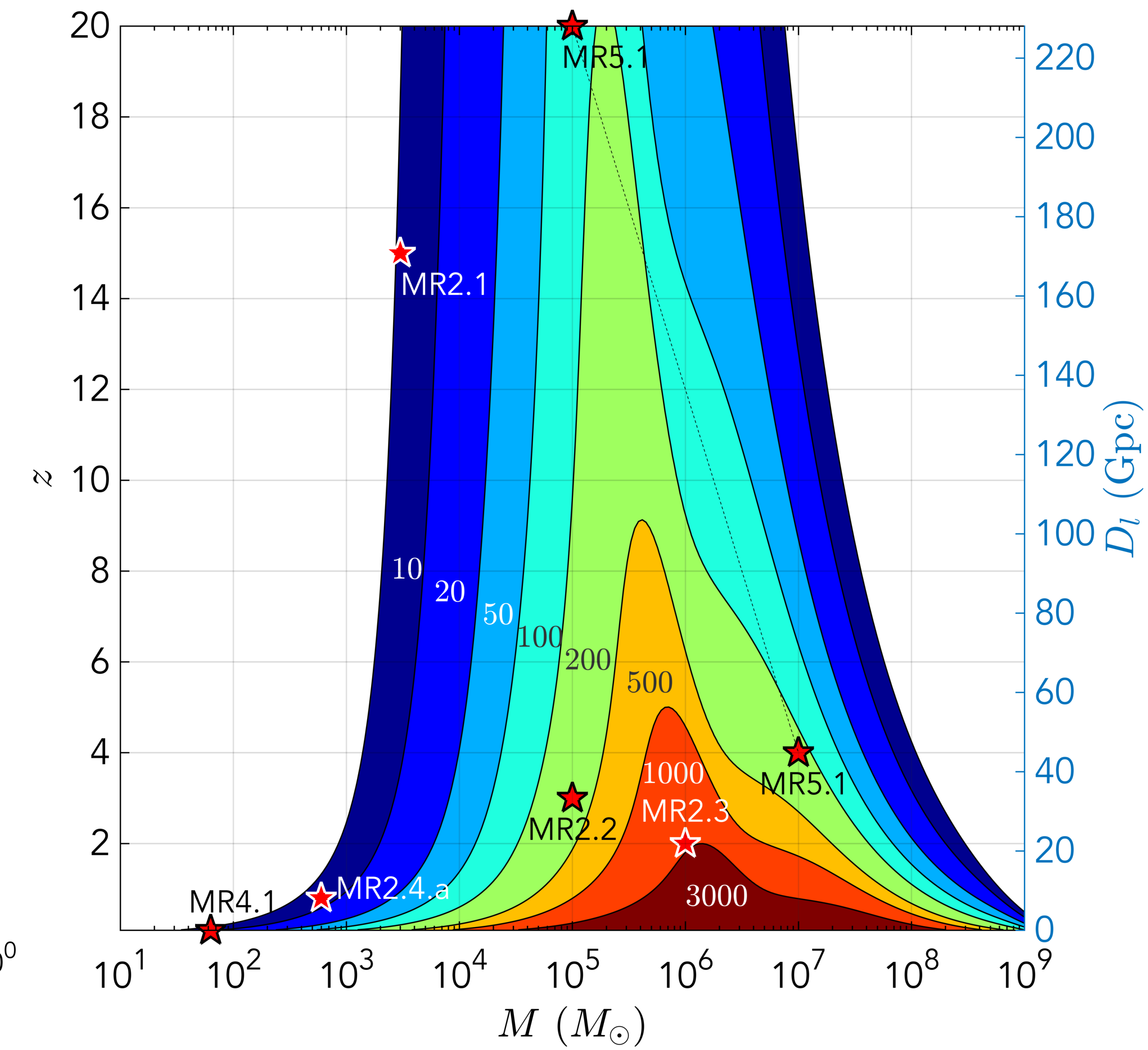
LISA BBH targets

[LISA L3 proposal]





MR1.1 Galactic Binaries
MR2.1 Light, seed black holes at high redshift
MR2.2 Blackhole growth over cosmic history
MR2.3a Mergers of Milky-way type galaxies
MR2.4a Detection of Intermediate Mass Black Holes
MR2.4b High mass ratio Intermediate Mass Black Holes
MR3.1 EMRIs around massive black holes
MR4.1 LIGO-type black holes
MR5.1 Tests of GR with high SNR ring-down signals
MR7.1 Astrophysical stochastic background
MR7.2 Cosmological stochastic background

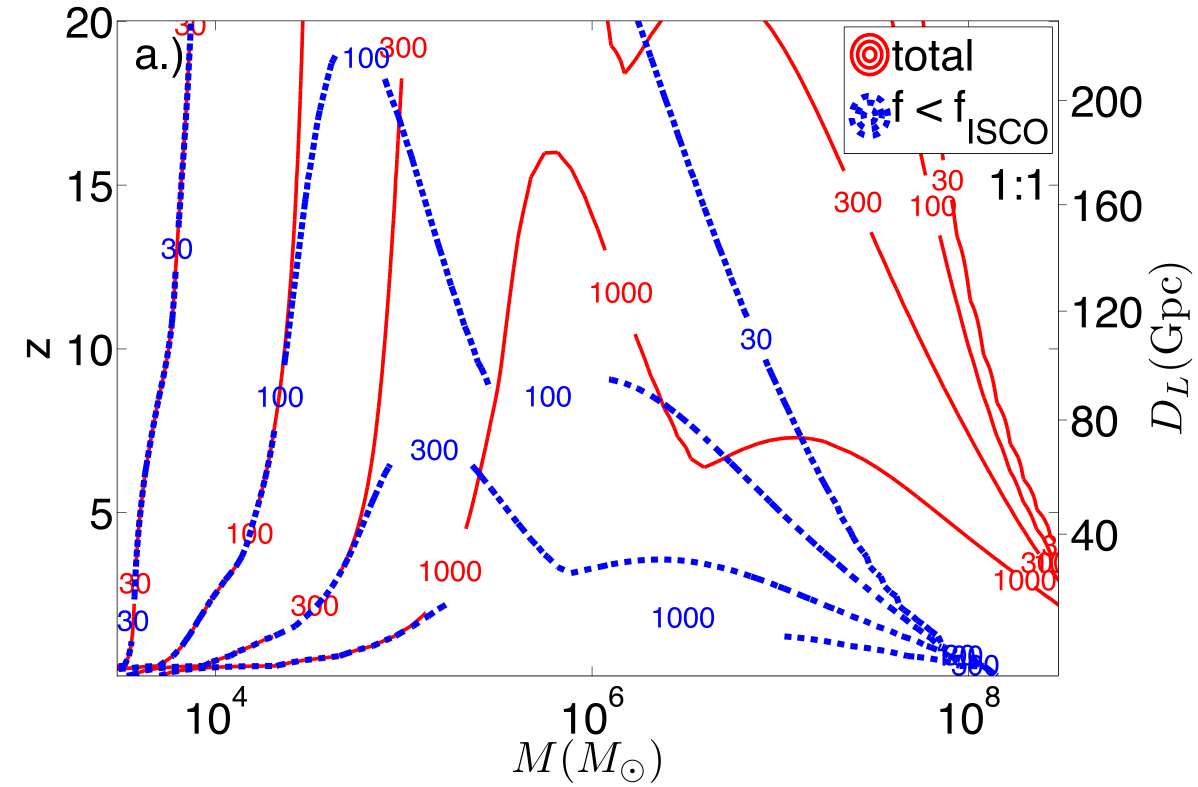
How accurately will LISA measure parameters of BBH coalescences across parameter space ?

LISA BBH parameter estimation

LISA prospective parameter estimation

PE	Fisher	Bayes
Inspiral	✓	✓ [MLDC]
IMR	✓ [effective MRD, extrinsic]	X [extrinsic]

Bayesian analyses are expensive: >10^6 likelihoods
Simplified low-f response used for inspiral signals



[McWilliams&al 2011]

Improvements in waveforms

- IMR waveforms with spins (SEOBNRv4, PhenomD)
- IMR waveforms with precession (SEOBNRv3, PhenomP)
- ROM acceleration for SEOB (spins aligned) and NR (full 7d surrogate q<2)
- Higher modes so far only non-spinning (EOBNRv2HM ROM - TF2 ext.)

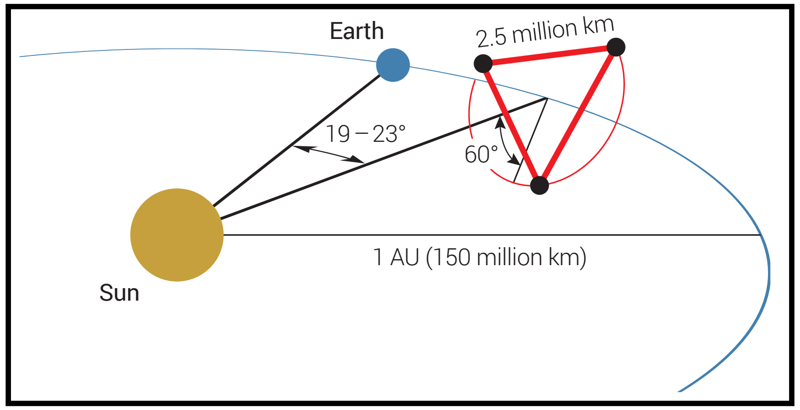
Objective: use fast IMR waveforms and fast FD LISA response to enable Bayesian analyses for prospective parameter estimation

LISA instrument response

Frequency observables: $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2}\frac{1}{1-\hat{k}\cdot n_l} n_l \cdot \Big(h(t-\hat{k}\cdot p_s) - h(t-\hat{k}\cdot p_r) \Big) \cdot n_l$$

TDI: combinations of delayed y_{slr}



- Orbital delay
- Change of orientation with time
- Armlength delays

Formal problem: modulated and delayed signal

$$\mathrm{FT}[F(t)h(t+d(t))] \leftrightarrow \tilde{h}(f),\ F(t),\ d(t)$$

Separation of timescales: 1/yr<<f

Low-f response: LIGO-like
Unsufficient for IMR and low-mass signals

Analogy with precessing signals

- Approximating I-frame hI as rotation of P-frame non-precessing waveform hP
- Used in SEOB (TD) and PhenomP (FD)

Extension through merger/ringdown given FD hP ?

$$h^{\mathrm{I}}_{\ell m} = \sum_{m'} D^{\ell\,*}_{m'm}(\alpha,\beta,\gamma)\ h^{\mathrm{P}}_{\ell m'}$$

FD modulations and delays: formalism

A general view

Input: $\tilde{h}(f) = A(f)e^{-i\Psi(f)}$

$$s(t) = F(t)h(t + d(t))$$
$$\tilde{s}(f) = \int df' \, \tilde{h}(f - f')\tilde{G}(f - f', f')$$
$$\tilde{G}(f, f') = \int dt \, e^{2i\pi f' t} e^{-2i\pi f d(t)} F(t)$$

Convolution with frequency-dependent kernel

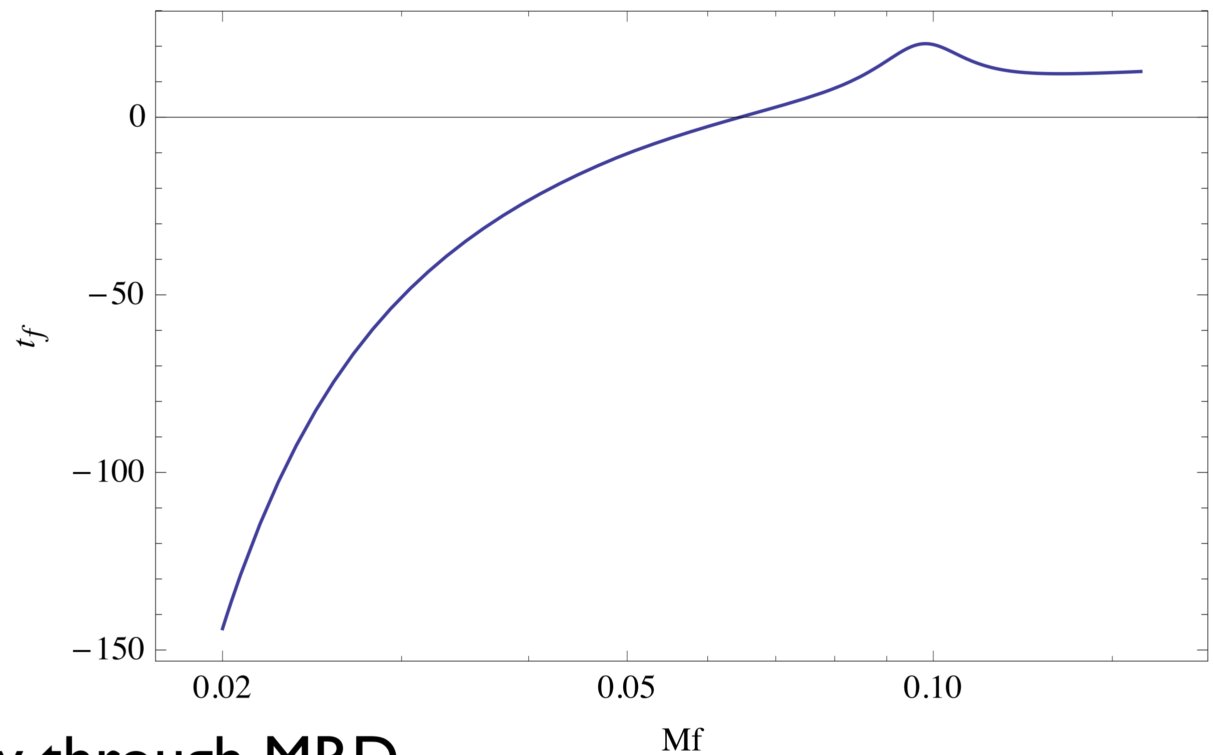
→ Separation of timescales: if F, d have only frequencies <<f, local convolution - expand h(f-f') in f'

The leading order transfer function

Keeping linear term in the phase:

$$t_f \equiv -\frac{1}{2\pi}\frac{\mathrm{d}\Psi}{\mathrm{d}f}$$

$$\tilde{s}(f) = \mathcal{T}(f)\tilde{h}(f)$$
$$\mathcal{T}(f) = G(f, t_f)$$



Close to the SPA - but extends naturally through MRD

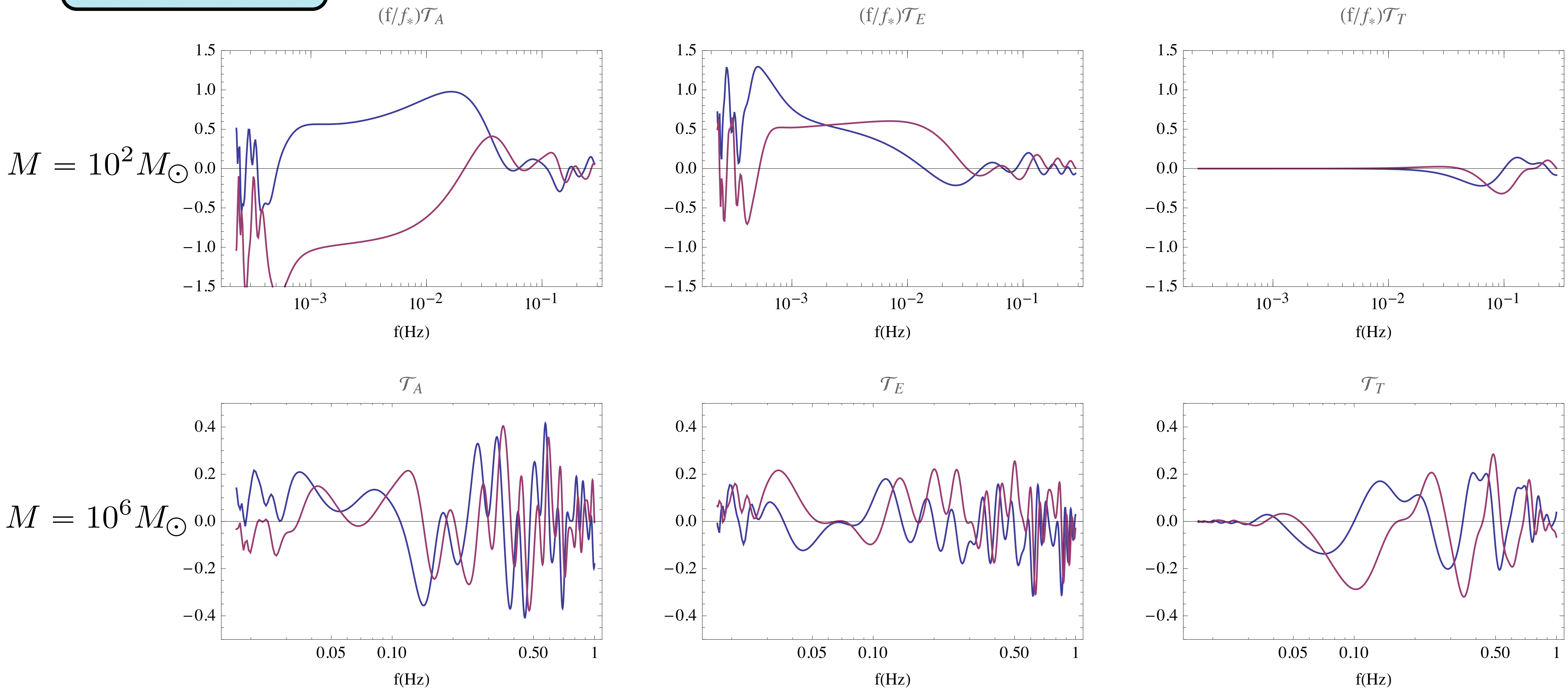
FD LISA response

One-arm transfer

$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \mathrm{sinc}\left[\pi f L \left(1 - \hat{k}\cdot n_l\right)\right] \exp\left[i\pi f \left(L + \hat{k}\cdot(p_1 + p_2)\right)\right] n_l \cdot P \cdot n_l$$

TDI transfer

[common f-dependence scaled out]



Compact spline representation: 300 pts for h, 800 pts for low-f and high-f response

FD response: figures of merit of approximation

Higher-order corrections

$$\tilde{s}(f) = \mathcal{T}(f)\tilde{h}(f)$$

Leading order: $\mathcal{T}(f) = G(f, t_f)$

Phase (quadratic term): $\mathcal{T}(f) = \sum \frac{1}{p!} \left(\frac{i}{8\pi^2} \frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2} \right) \partial_t^{2p} G(f, t_f) \rightarrow T_{\mathrm{RR}}^2 = -\frac{1}{4\pi^2}\frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2}$

rederivation of [Klein&al 2014]

Amplitude: $\mathcal{T}(f) = \sum \frac{1}{(2i\pi)^p p!} \frac{1}{A} \frac{\mathrm{d}^p A}{\mathrm{d}f^p} \partial_t^p G(f, t_f)$

f-dependence (delays): $\mathcal{T}(f) = \sum \frac{1}{(2i\pi)^p p!} \partial_t^p \partial_f^p G(f, t_f)$

Improved delays: $\mathcal{T}(f) \simeq F(t_f) \exp\left[-2i\pi f d(t_f)(1 - \dot{d}(t_f)) \right]$

Separation of timescales

(e)LISA:

$$\partial_t G \sim 2\pi f_0 G$$

$$f_0 = 1/\mathrm{yr} = 3.10^{-8}\mathrm{Hz} \ll f$$

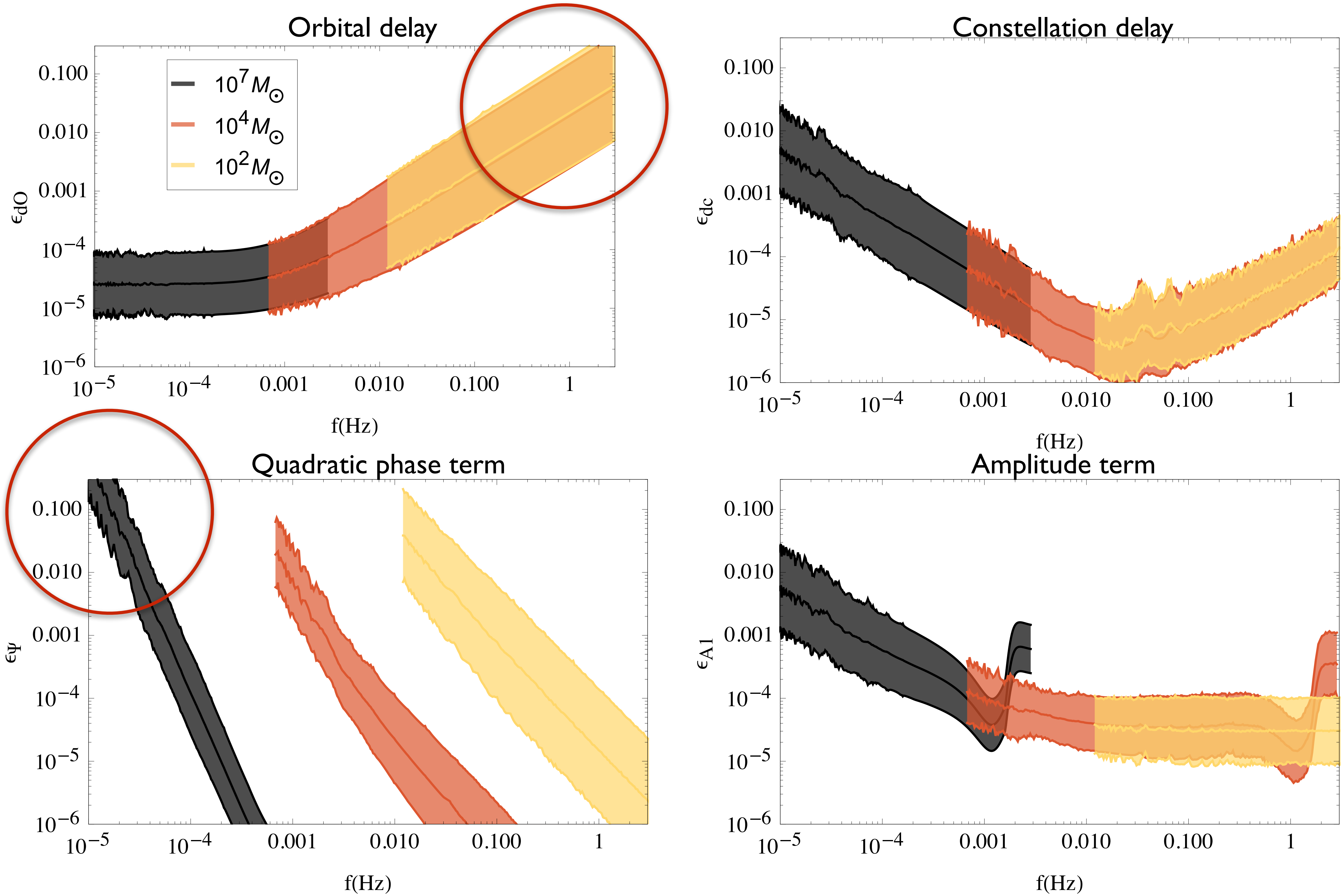
Precessing binaries: $G = F(t)$

Inspiral: $\partial_t^2 F \sim \Omega_{\mathrm{prec}}^2 \sim 2\mathrm{PN}$

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2} \sim T_{\mathrm{RR}}^2 \sim -2.5\mathrm{PN}$$

+ Merger-ringdown ?

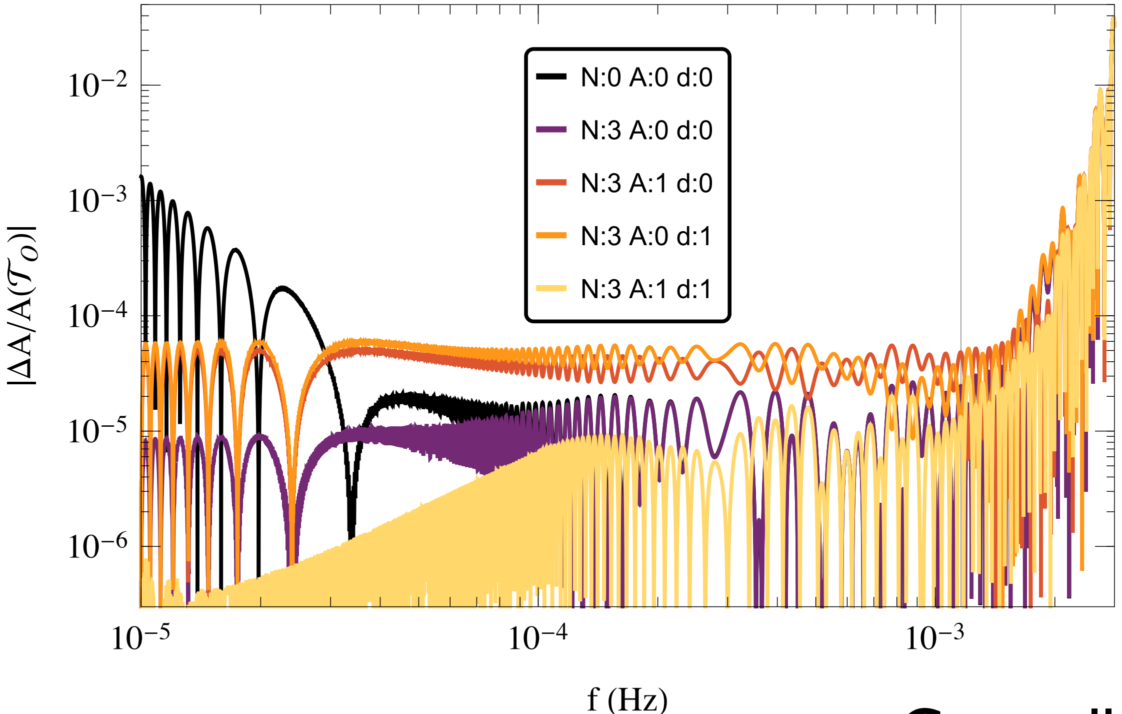
FD response: figures of merit of approximation

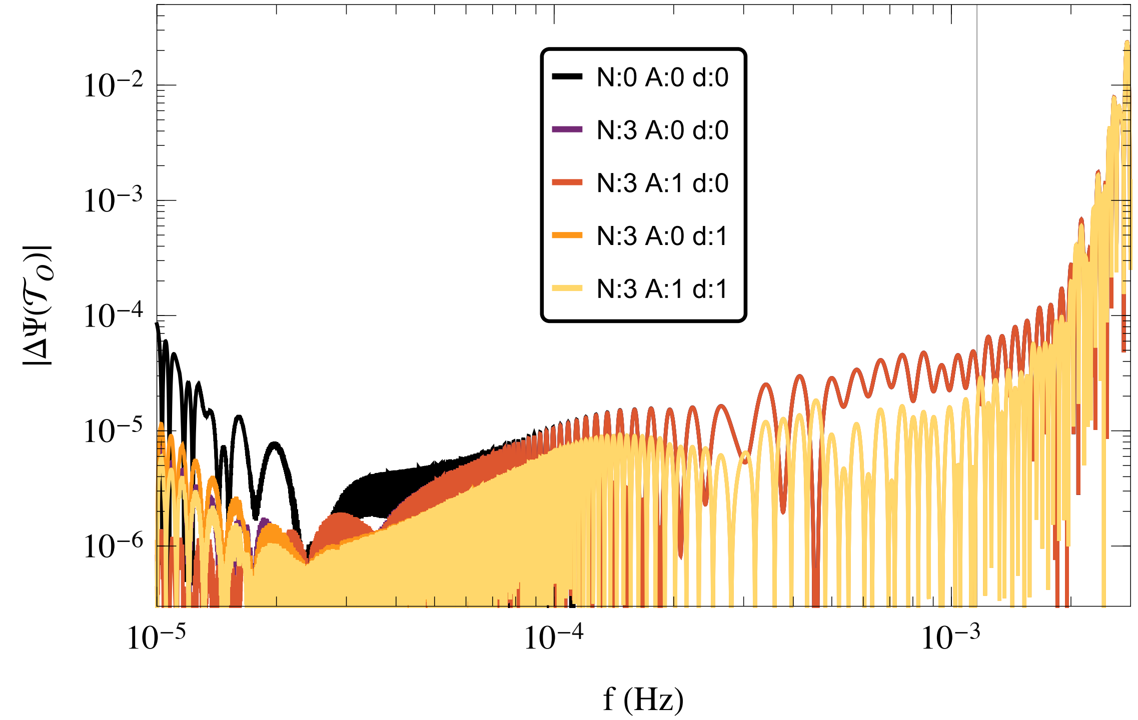


FD response: reconstruction errors

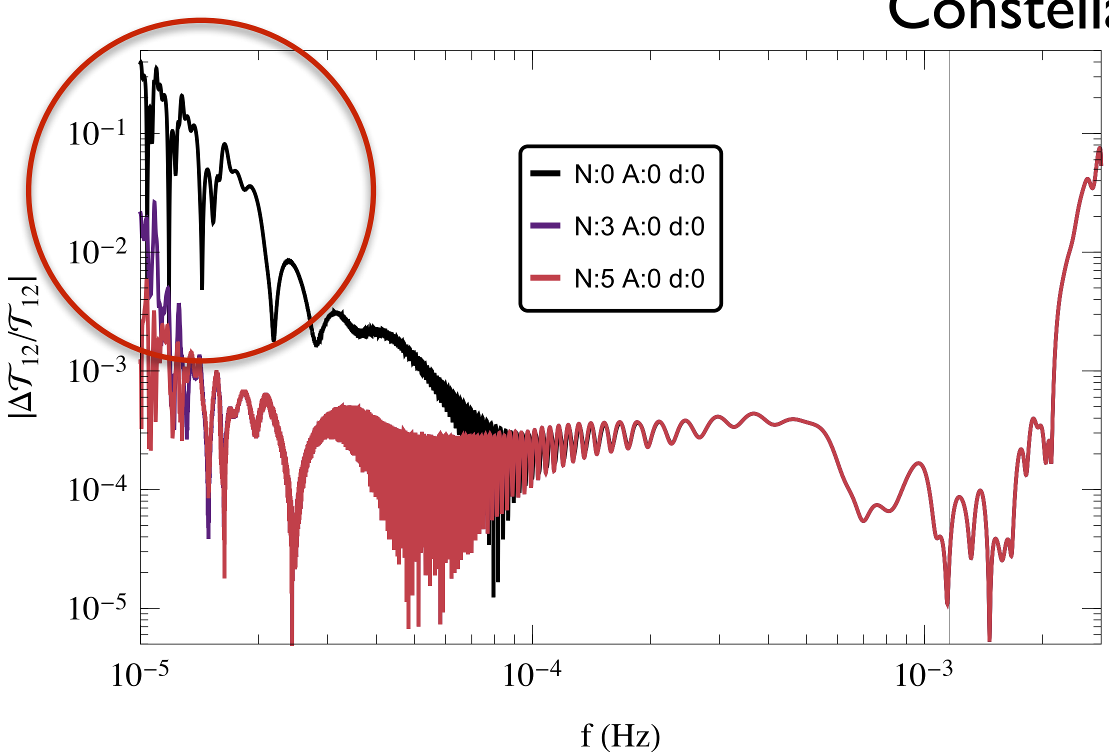
$M = 10^7 M_\odot$

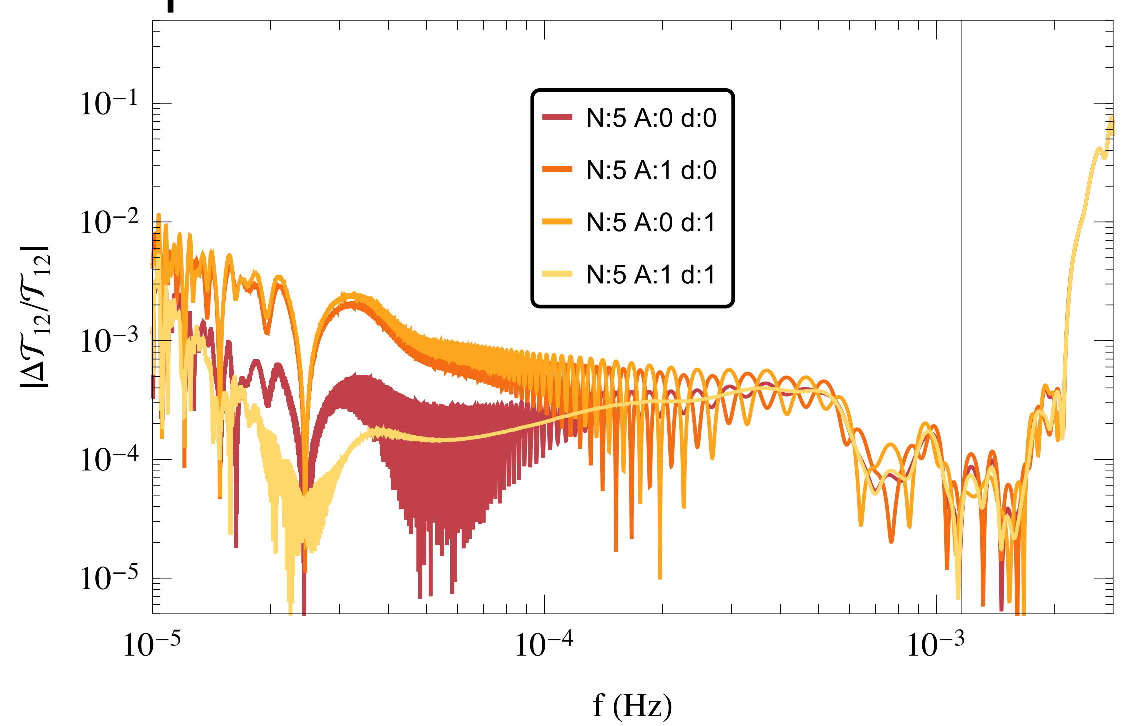
Orbital delay





Constellation response

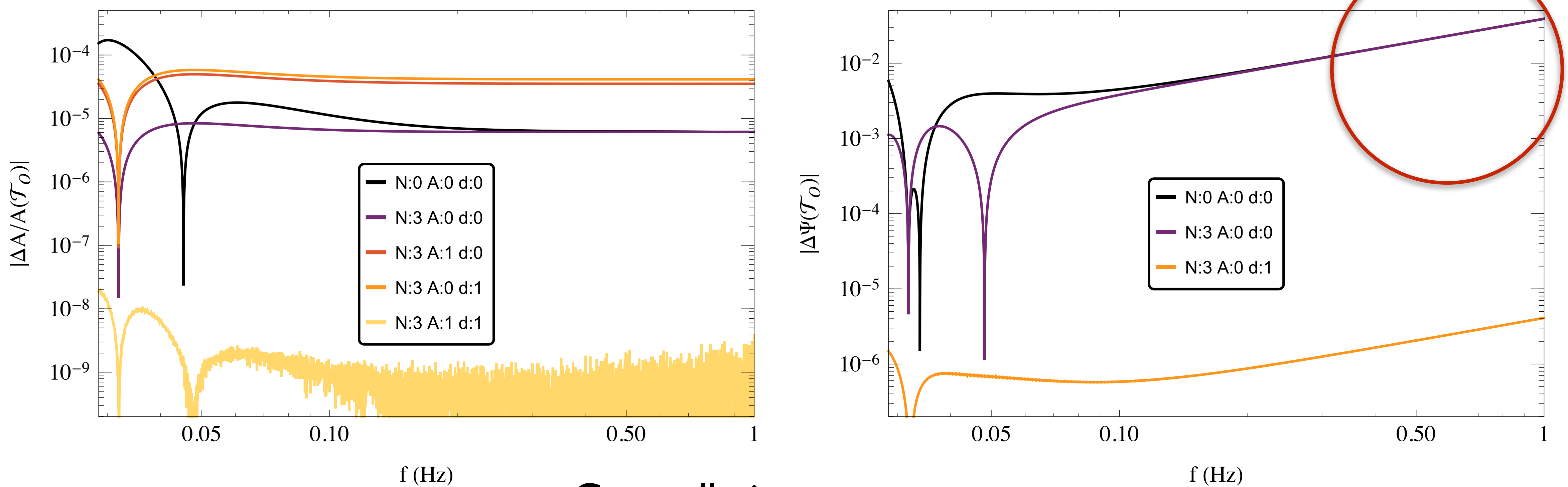




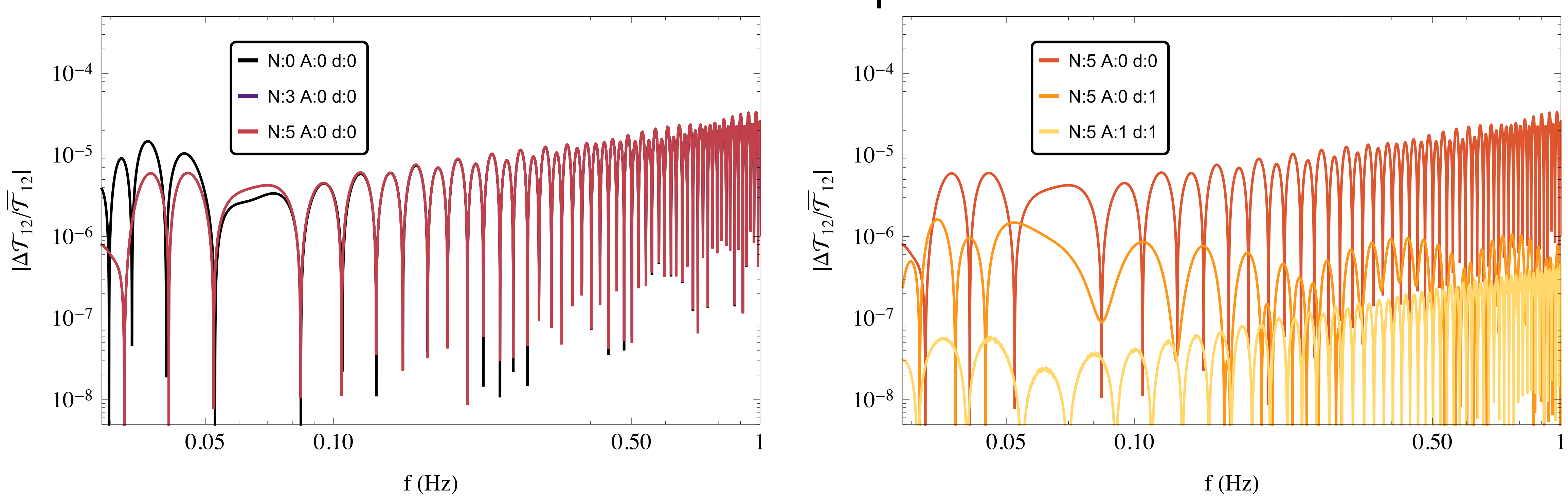
FD response: reconstruction errors

$M = 10^2 M_\odot$

Orbital delay



Constellation response

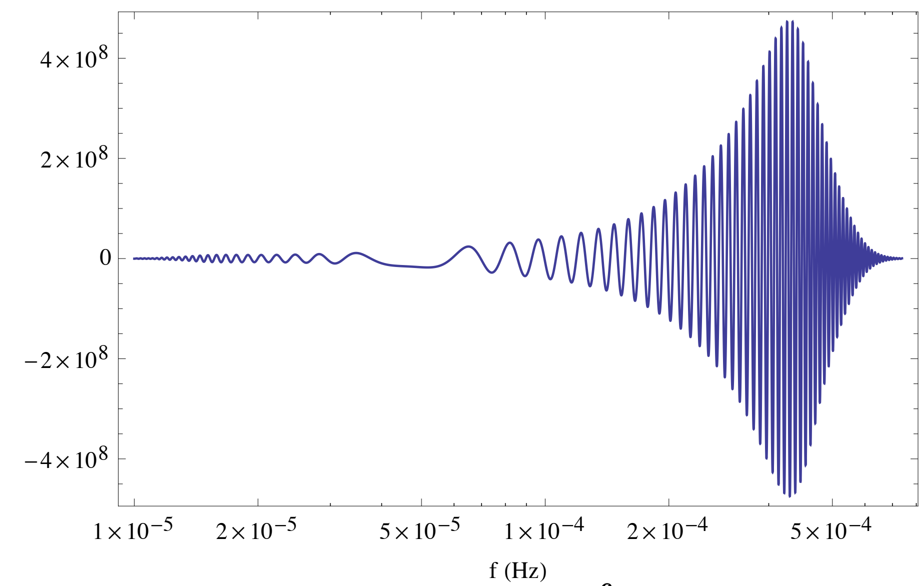


Bayesian inference implementation

Accelerated no-noise overlaps

- **Sparse grid:** Amplitude/phase and response
- 1D Spline representation 300-800 pts
- Cost increases when including HM

Overlaps: oscillatory integrands



$$(h_1|h_2) = 4\mathrm{Re}\int df\, \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} \longrightarrow \int_{f_i}^{f_{i+1}} P(f)e^{i[af+bf^2]} \longrightarrow \int_{f_i}^{f_{i+1}} e^{i[af+bf^2]}$$

Implementation

- EOBNRv2HM waveforms (ROM) (non-spinning, 22,21,33,44,55 modes)
- Accelerated overlaps for amplitude/phase
- Sampler: MultiNEST, PTMCMC
- **0-noise, single signal**

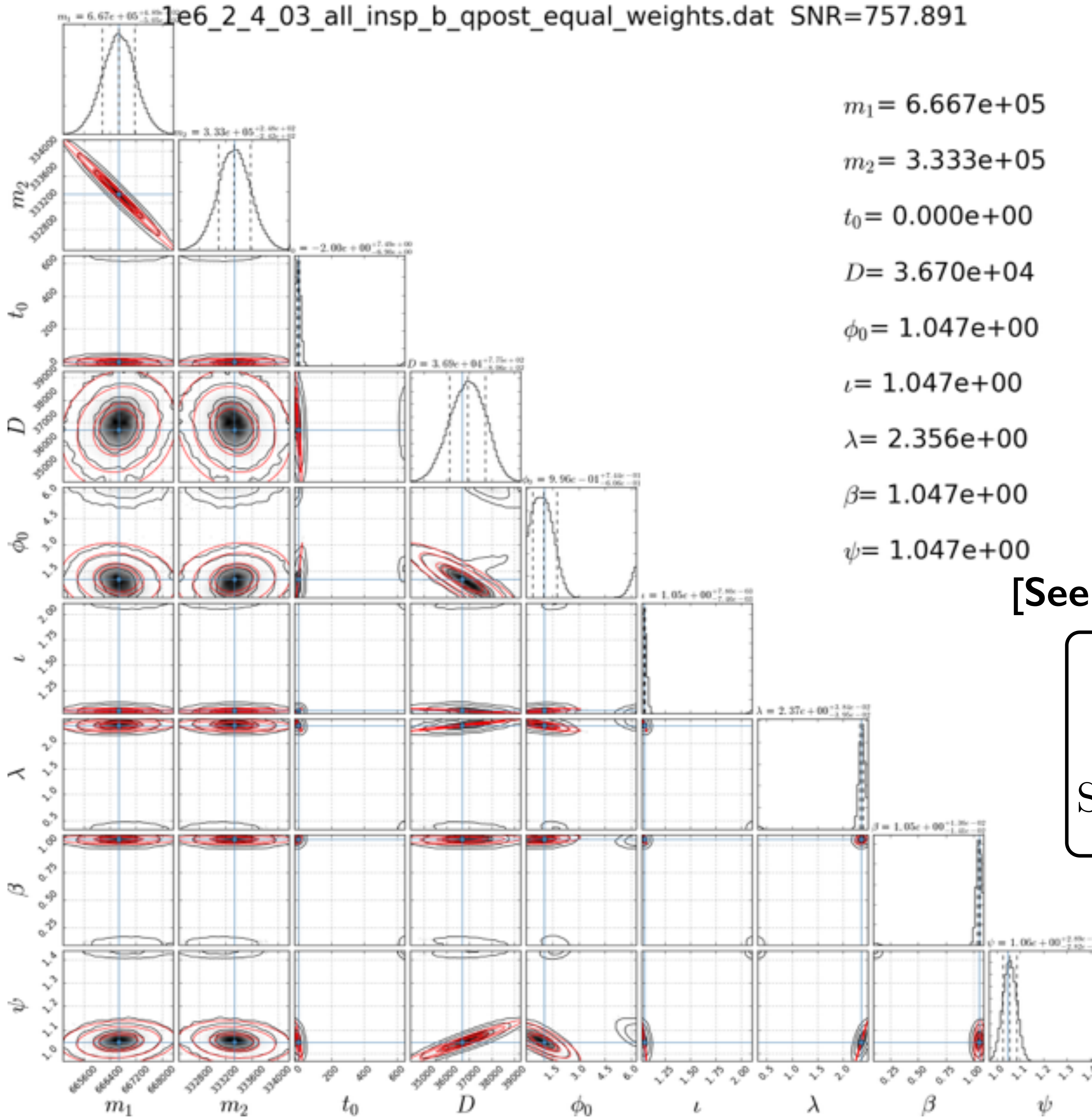
Likelihood cost
Single mode h22: 2-10ms
5 modes hlm: 30-100ms

Number of samples

$$M = 10^2 :\ 15 - 20.10^6$$

$$M = 10^6 :\ 40.10^6\sqrt{\mathrm{SNR}/200}$$

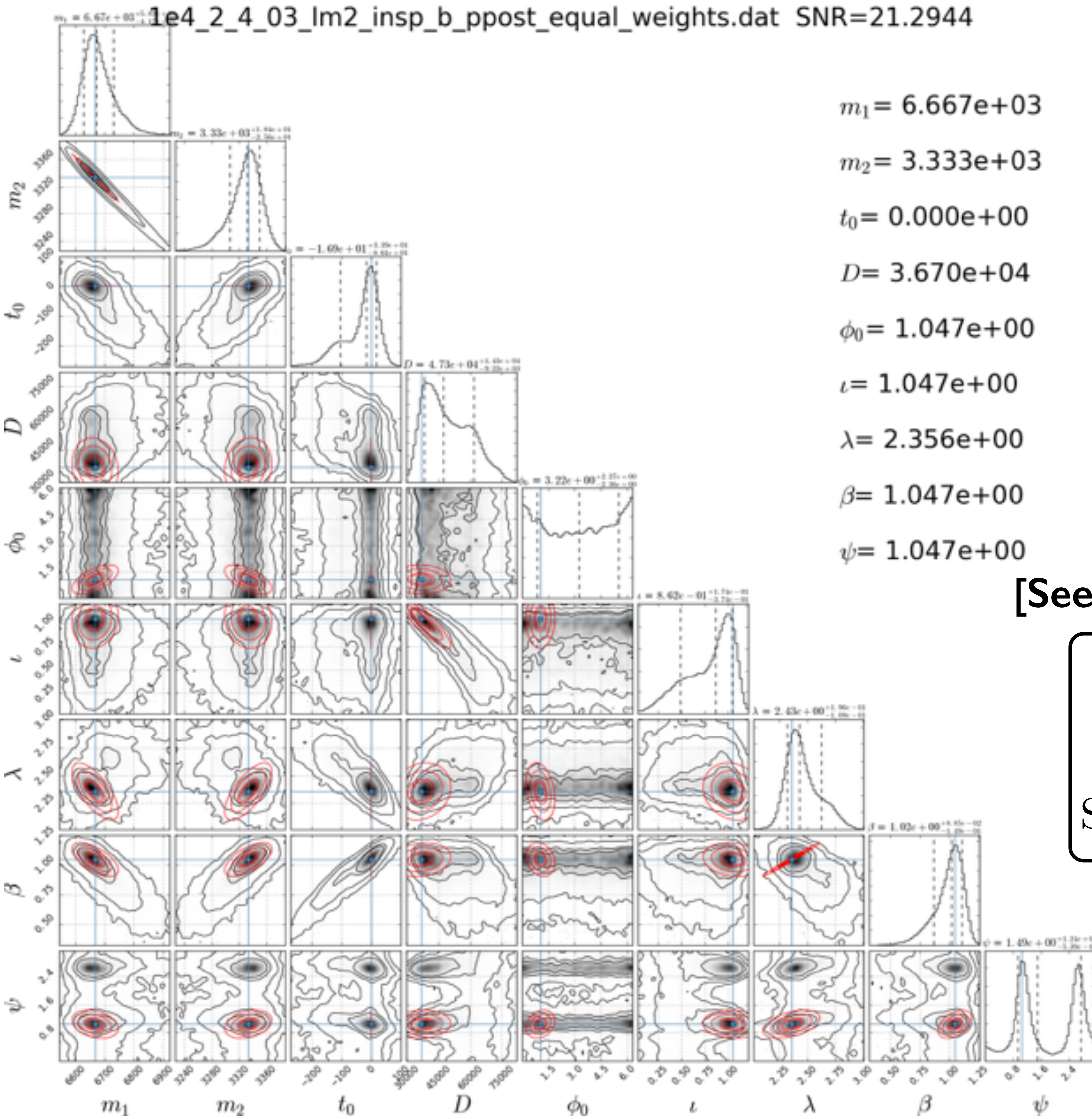
LISA Bayesian inference example: high-mass



[See J. Baker's poster]

$$M = 10^6 M_\odot$$
$$z = 4$$
$$\text{SNR} = 760$$

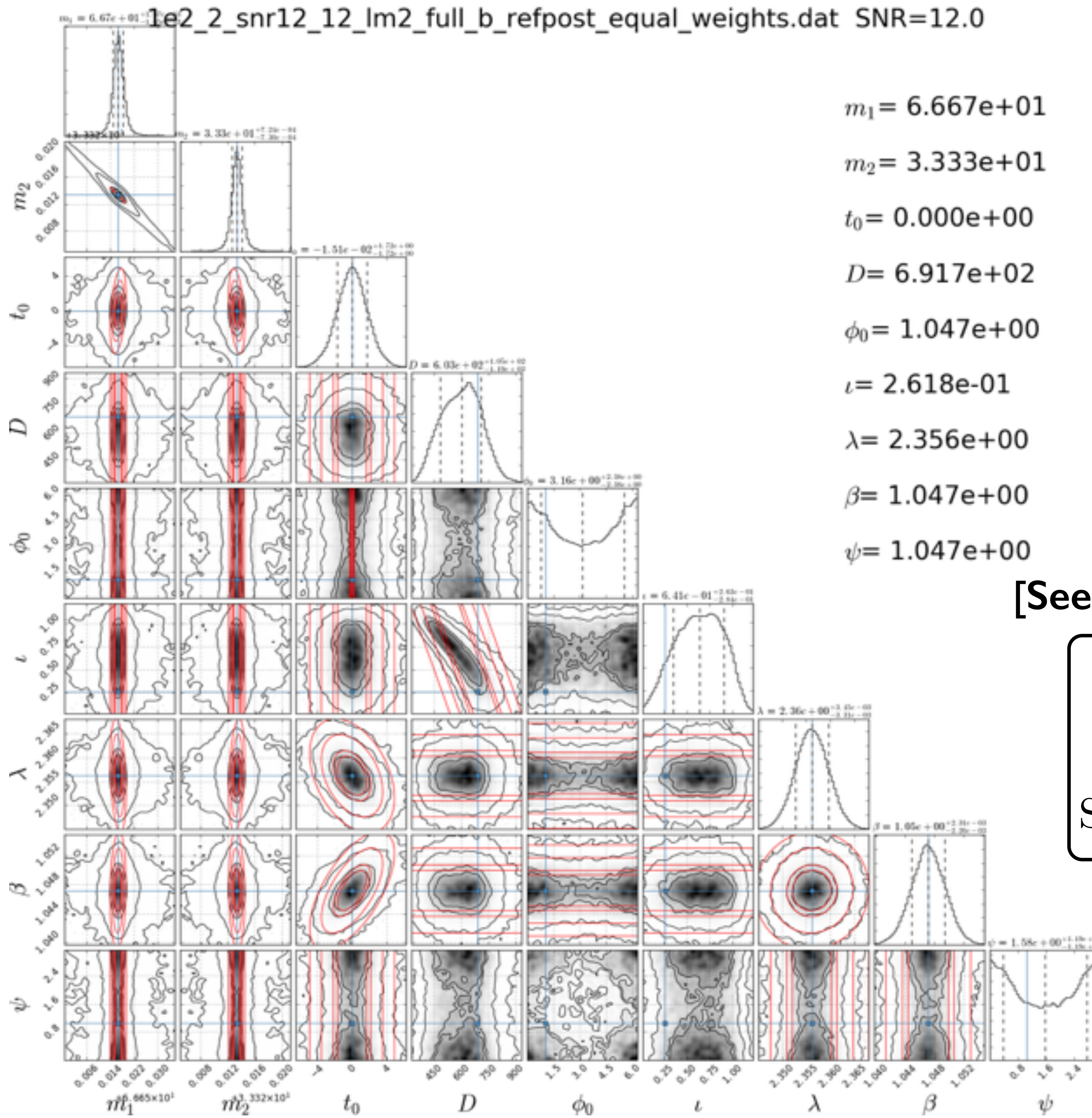
LISA Bayesian inference example: intermediate-mass



[See J. Baker's poster]

$$M = 10^4 M_\odot$$
$$z = 4$$
$$\text{SNR} = 21$$

LISA Bayesian inference example: low-mass



[See J. Baker's poster]

$$M = 10^2 M_\odot$$
$$z = 0.15$$
$$\mathrm{SNR} = 12$$

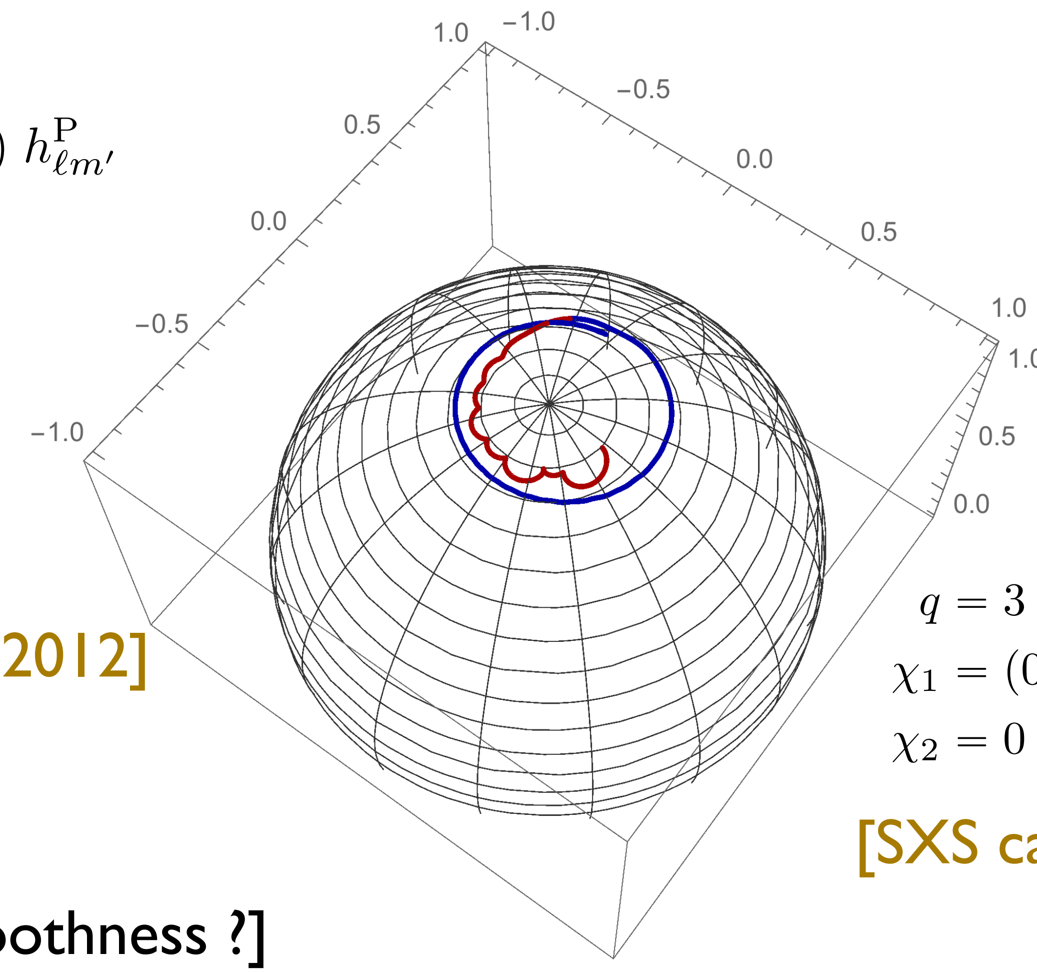
Precessing modulations in Fourier domain

Frame trajectory

$$h^{\mathrm{I}}_{\ell m} = \sum_{m'} D^{\ell\,*}_{m'm}(\alpha,\beta,\gamma)\; h^{\mathrm{P}}_{\ell m'}$$

- PN dynamics: $Z_{\mathrm{frame}} = \hat{L}$
- Extracting frame from the waveform (IMR) [O'Shaughnessy&al 2011]
- Approximate behaviour post-merger:

$\Omega_{\mathrm{frame}} \sim \omega^{\mathrm{QNM}}_{220} - \omega^{\mathrm{QNM}}_{210}$ [O'Shaughnessy&al 2012]



$q = 3$

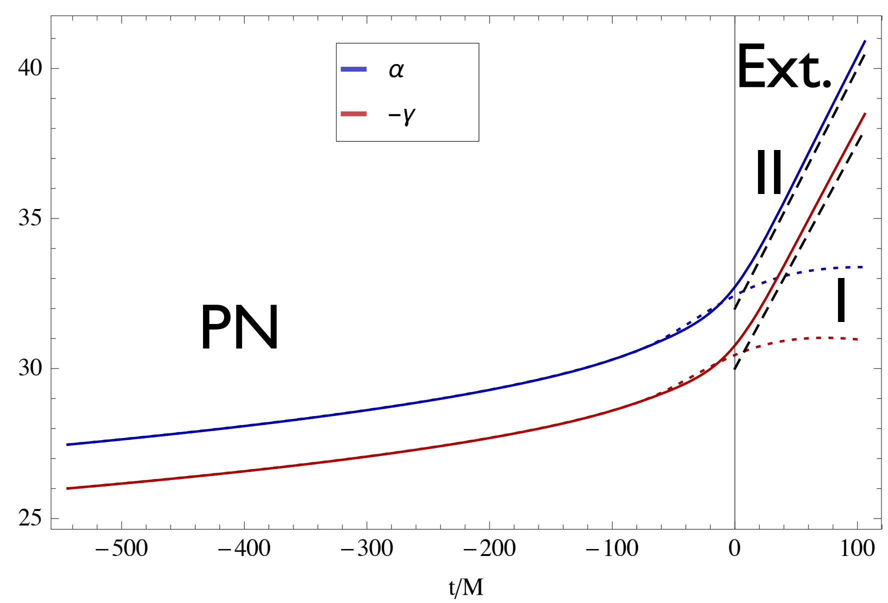
$\chi_1 = (0.5, 0, 0)$

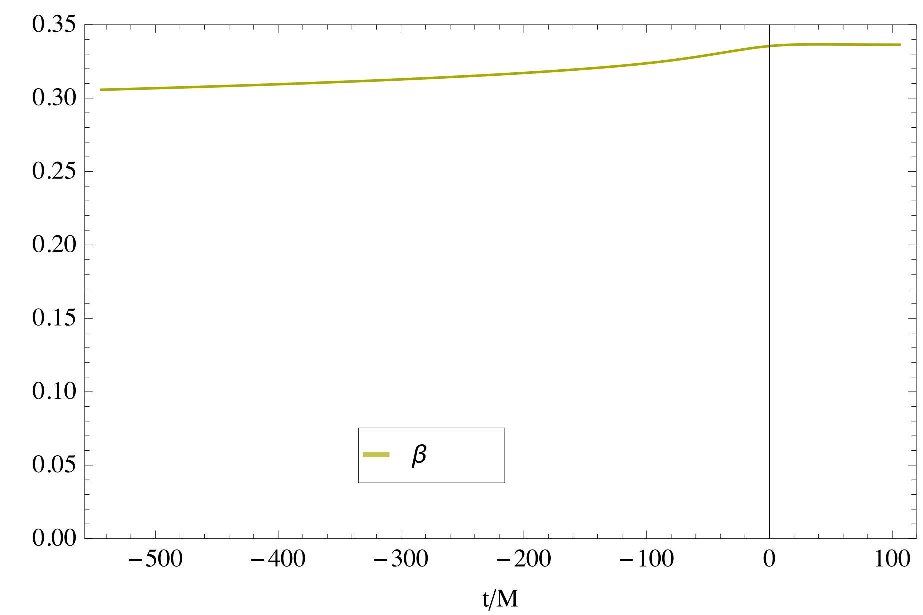
$\chi_2 = 0$

[SXS catalog]

Pre- and post-merger frame

[Smoothness ?]





Relation to previous work

Previous works:

- Leading order (different MR) [SpinTaylorF2, PhenomP]
- Quadratic phase (SUA) [Klein&al 2014]

SPA/SUA	Fourier domain approach
$t_f \; : \; \omega(t_f) = \pi f$ (SPA)	$t_f = -\frac{1}{2\pi}\frac{\mathrm{d}\Psi}{\mathrm{d}f}$ (IMR)
$T_f = \frac{1}{\sqrt{2\dot{\omega}(t_f)}}$ Rad. Reac. (SUA)	$T_f^2 = \frac{1}{4\pi^2}\left\lvert\frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2}\right\rvert$ (IMR)
$\tilde{s}(f) = \tilde{h}(f)\sum\frac{(-i)^p}{2^p p!}T_f^{2p}\partial_t^{2p}F$ (SUA)	$\tilde{s}(f) = \tilde{h}(f)\sum\frac{(-i)^p}{2^p p!}T_f^{2p}\partial_t^{2p}F$ Taylor FD Quad. phase
$\tilde{s}(f) = \tilde{h}(f)\sum a_k F(t_f \pm kT_f)$ (Resum.)	$\tilde{s}(f) = \tilde{h}(f)\int dt\, \exp\left[-\frac{i}{2}\left(\frac{t-t_f}{T_f}\right)^2\right]F(t)$

New corrections:

- Higher-order amplitude corrections $\mathrm{d}^p A/\mathrm{d}f^p$
- Local convolution approach for post-merger

Summary

LISA prospective parameter estimation

- Bayesian parameter estimation using full IMR signals for the full mass range
- Full Fourier-domain response of the instrument using t(f) correspondence
- Higher-order corrections in the response available
- Including (non-spinning) merger and higher modes: EOBNRv2HM ROM waveforms
- Implementation using accelerated no-noise overlaps: few 10s of ms/likelihood
- Analogy of formalism with FD precession
- Still preliminary **- See J. Baker's poster for more results**

Outlook

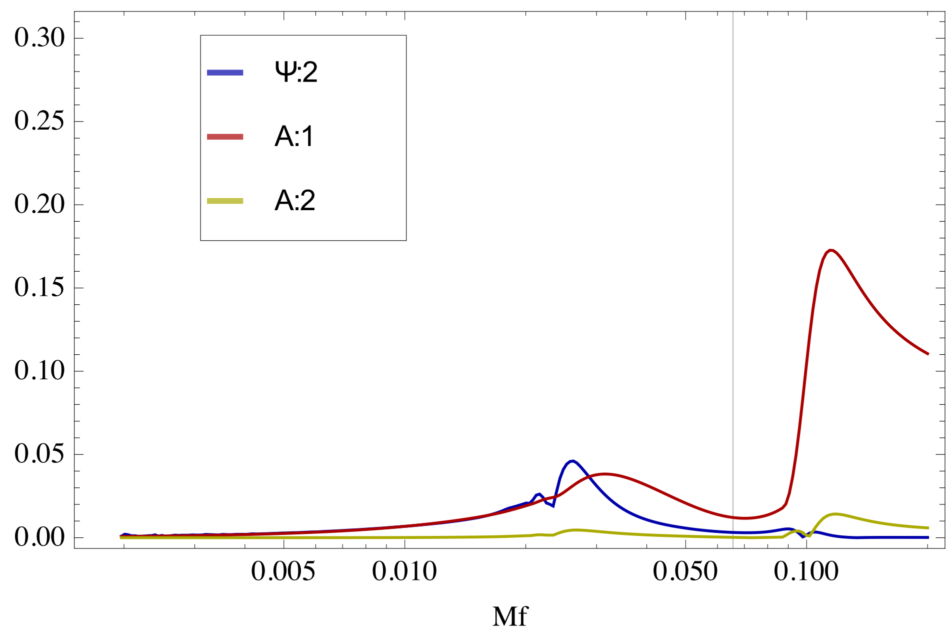
- Including spins (SEOBNRv4ROM, PhenomD/P)
- Including eccentricity
- Joint LIGO/LISA parameter estimation
- Parameter estimation as a function of time: accumulation of the signal
- Cosmology with LISA: standard sirens (EM or pure GW)
- Investigate superposition of signals
- Testing GR at high SNR / with multiband GW observations

Precession: magnitude of corrections

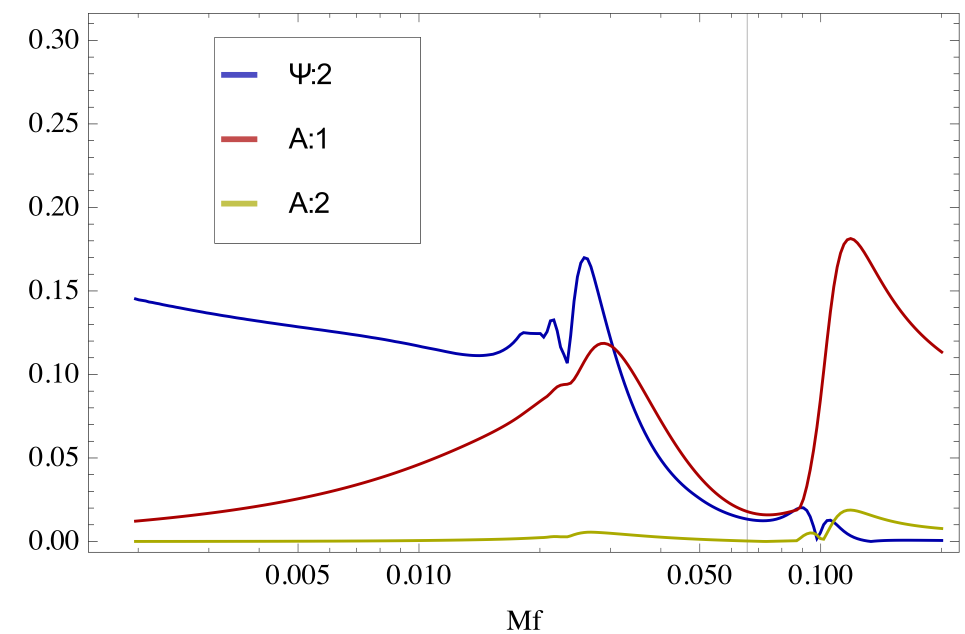
Example: $q = 3,\ \chi_1 = (-0.3, 0.5, 0.7),\ \chi_2 = (0.3, -0.2, -0.5)$

Case I

$h^{\rm P}_{22} \to h^{\rm I}_{22}$

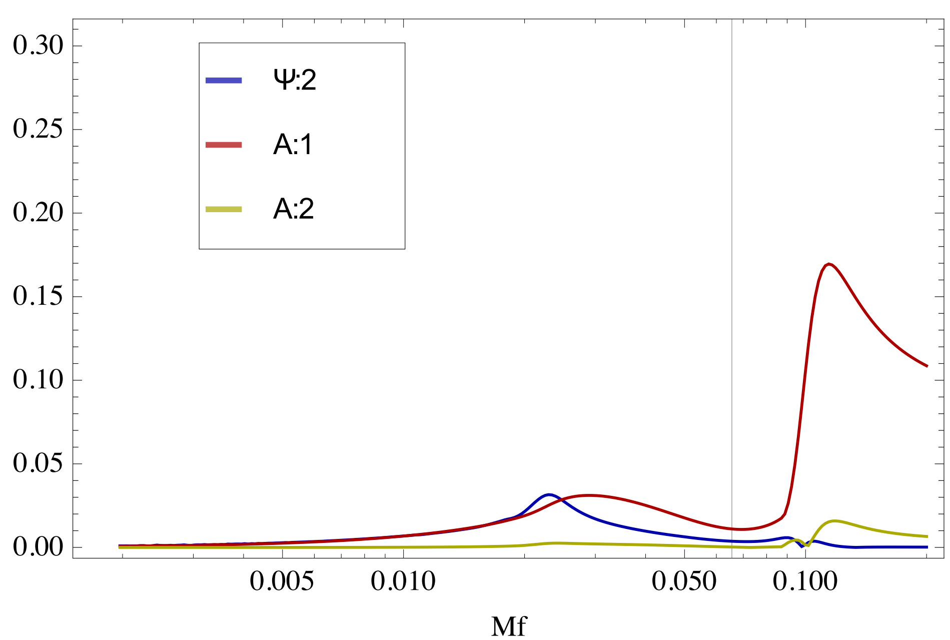


$h^{\rm P}_{22} \to h^{\rm I}_{21}$

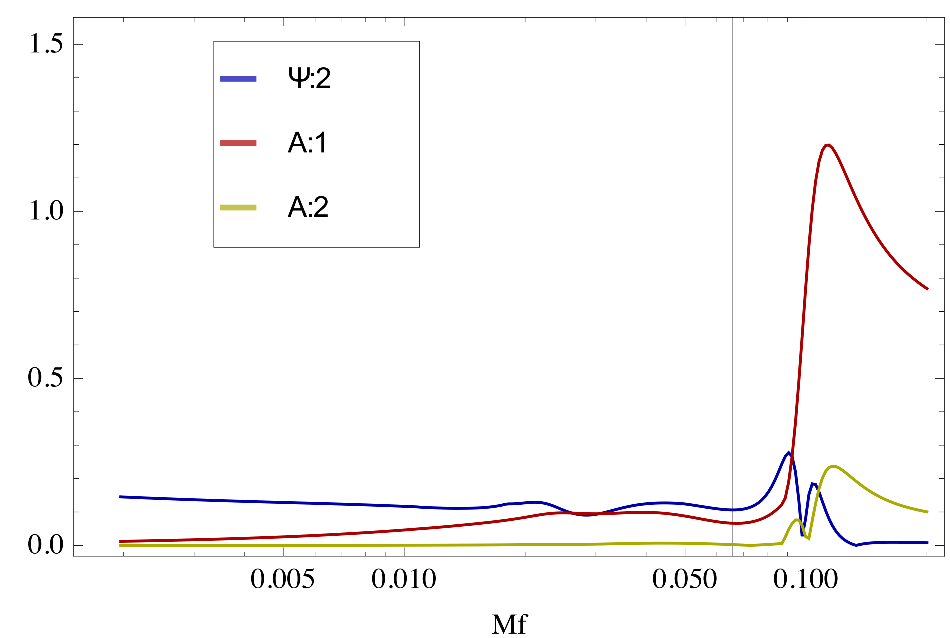


Case II

$h^{\rm P}_{22} \to h^{\rm I}_{22}$



$h^{\rm P}_{22} \to h^{\rm I}_{21}$

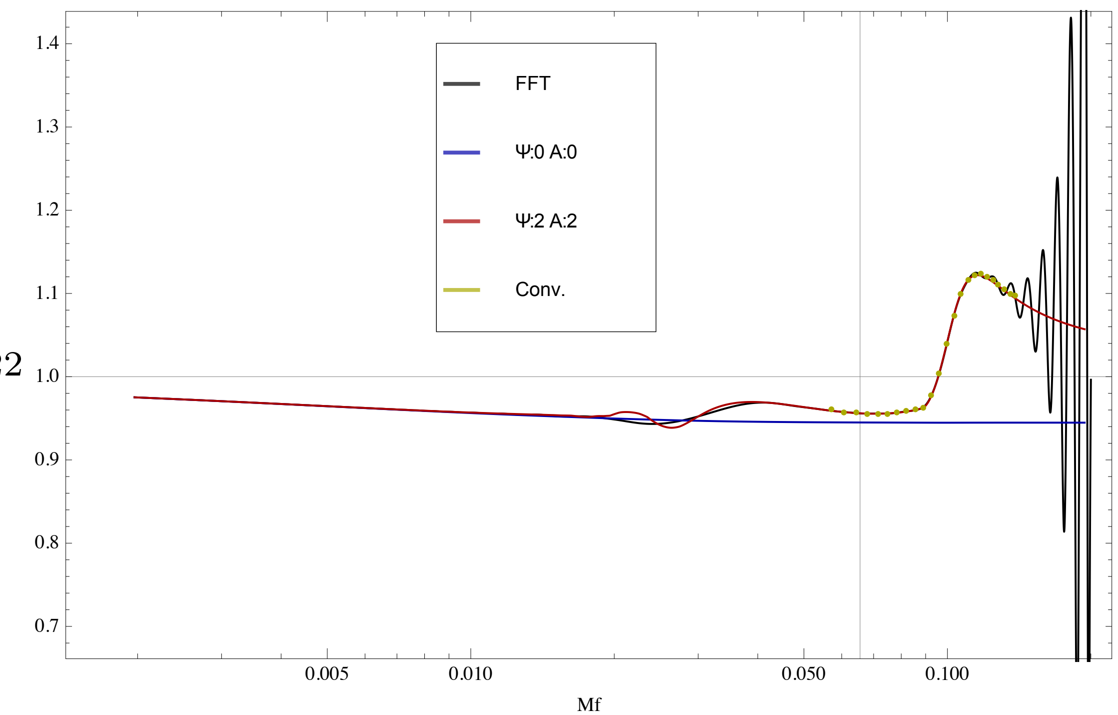


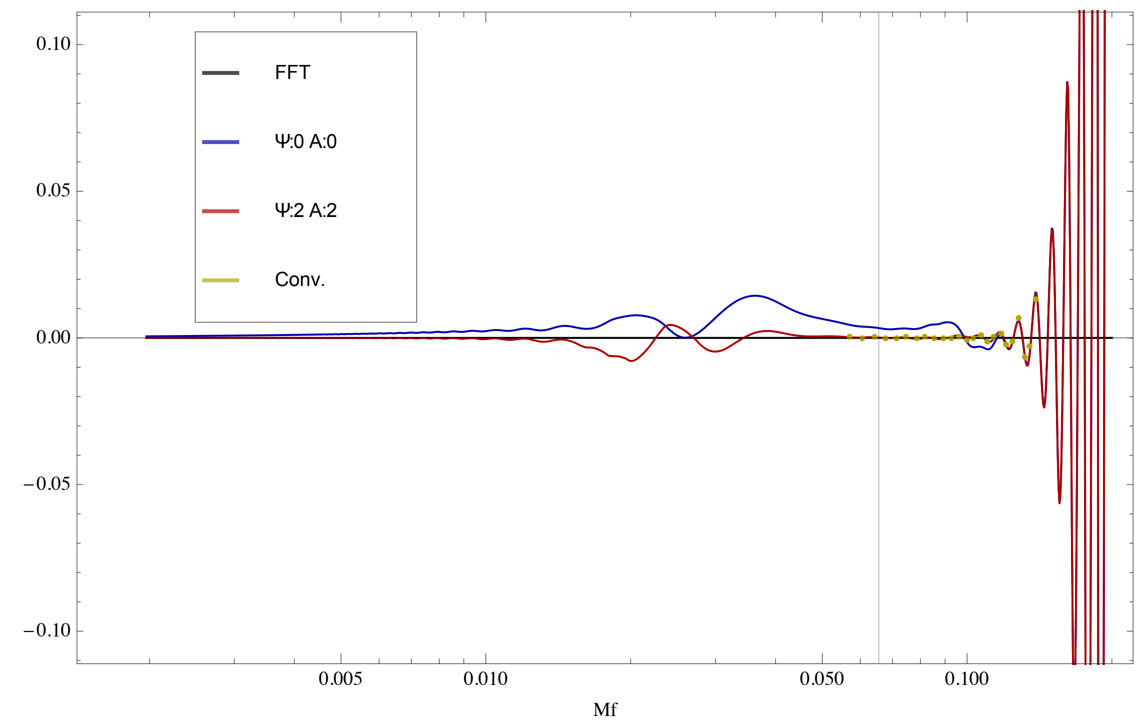
Precession: errors - case I

Amplitude relative to h22P

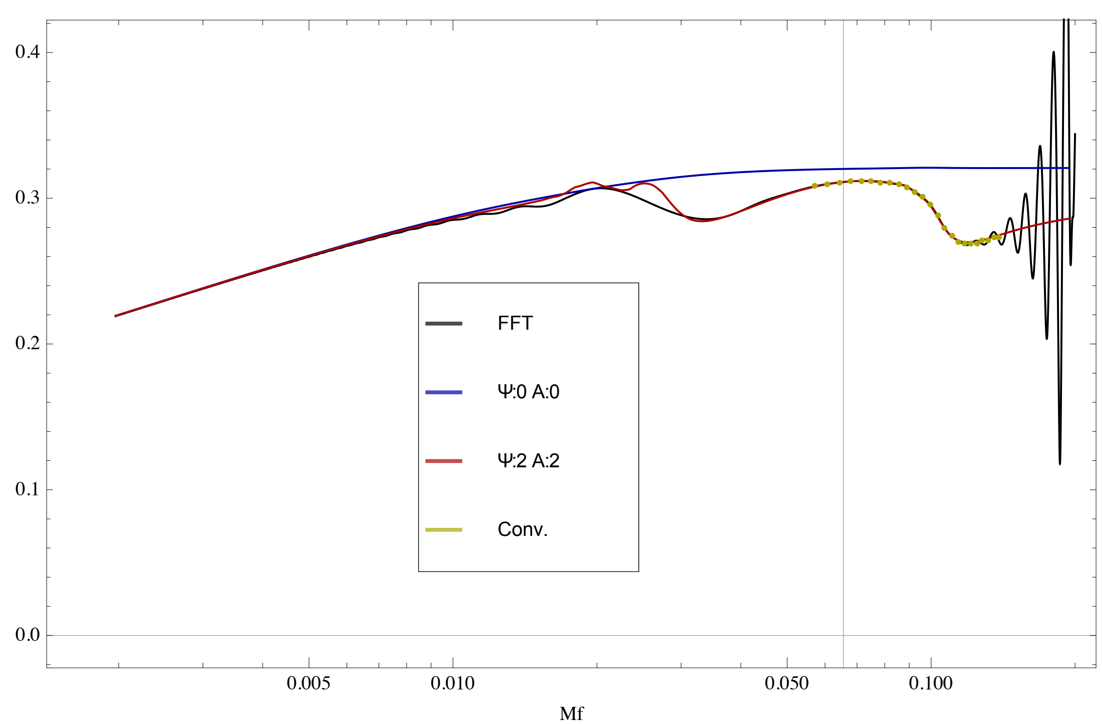
Phase difference

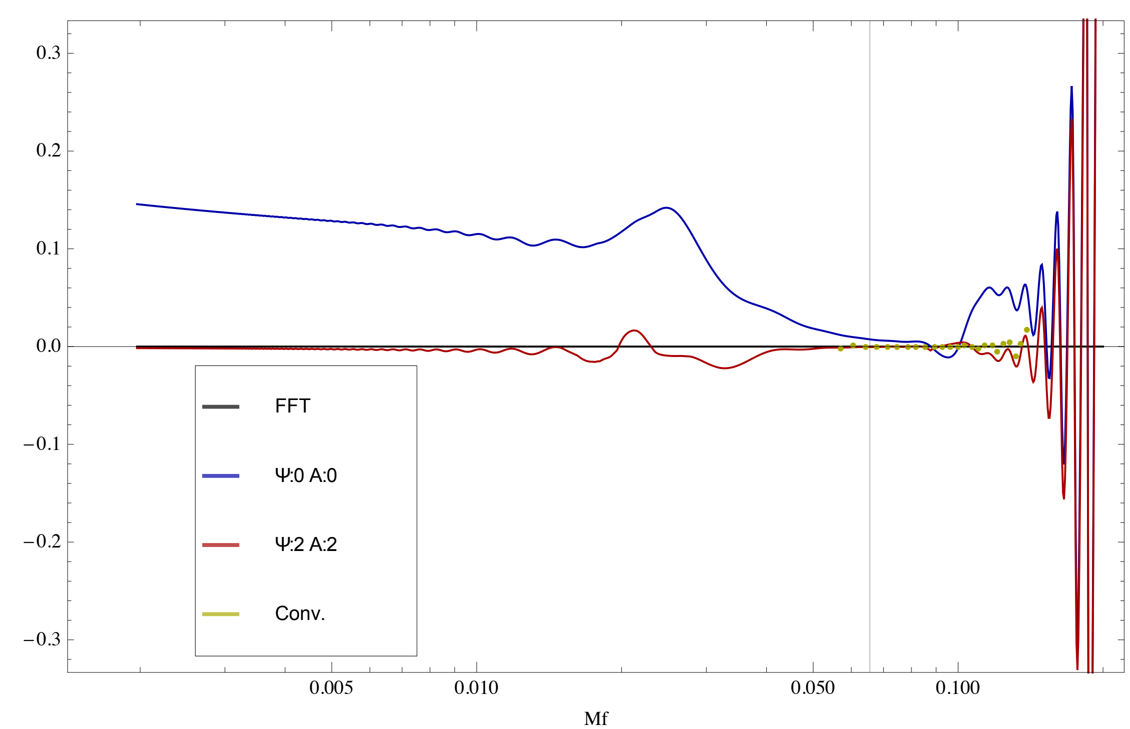
$h_{22}^{\rm P} \to h_{22}^{\rm I}$





$h_{22}^{\rm P} \to h_{21}^{\rm I}$



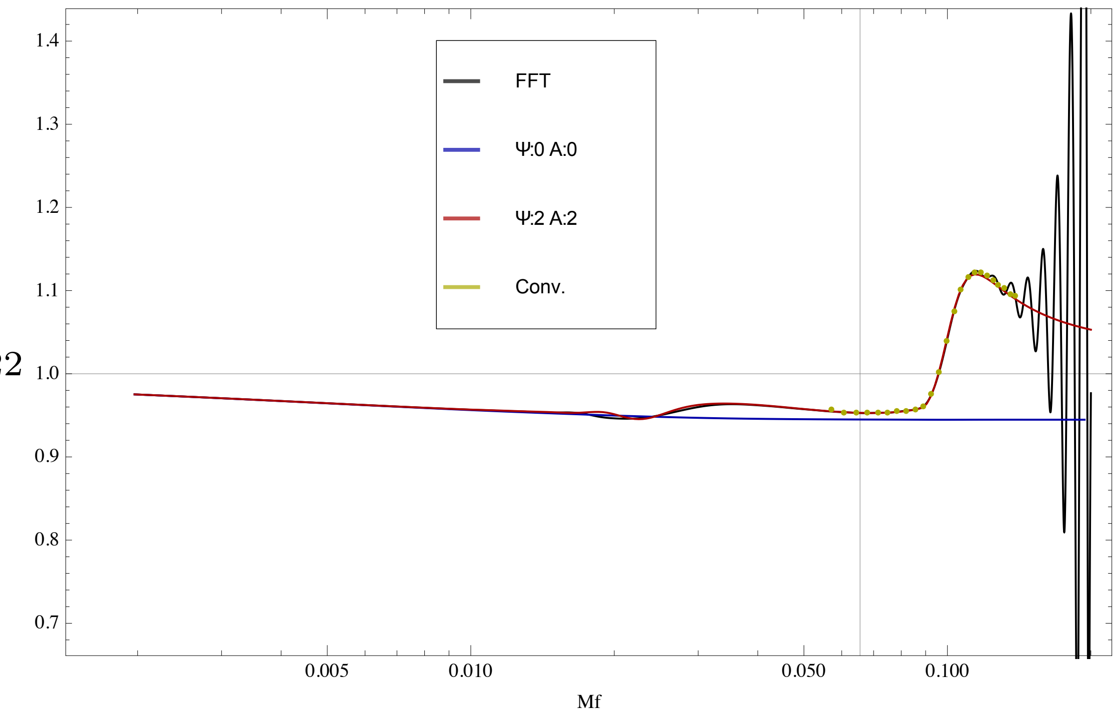


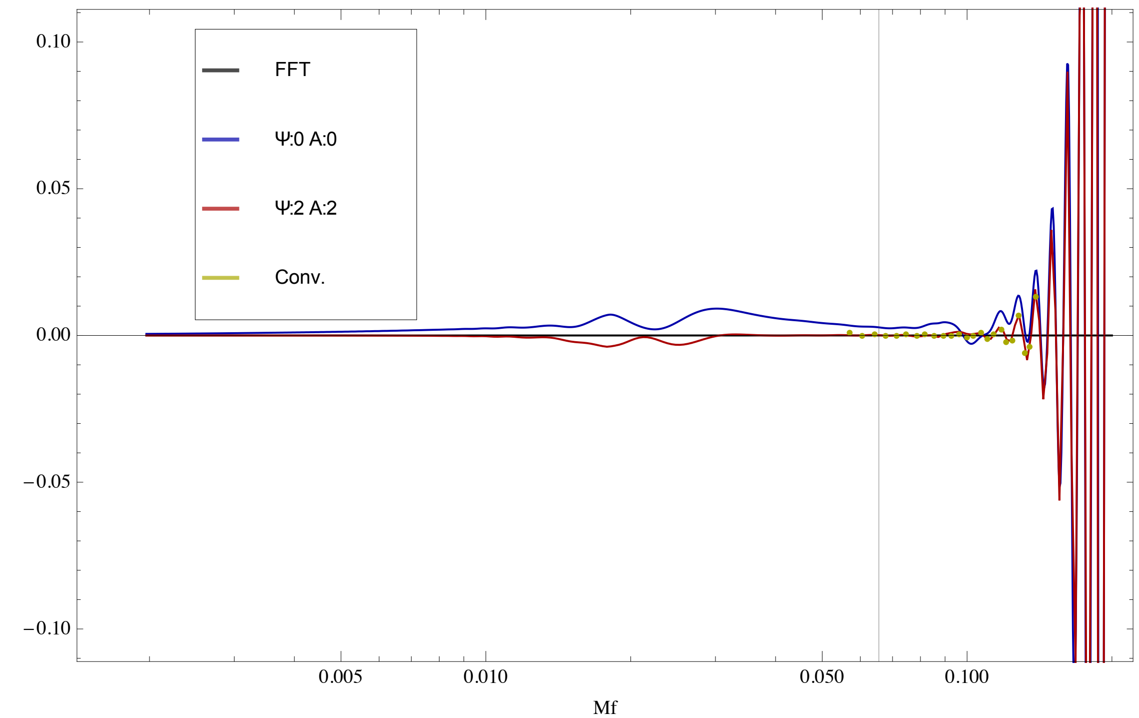
Precession: errors - case II

Amplitude relative to h22P

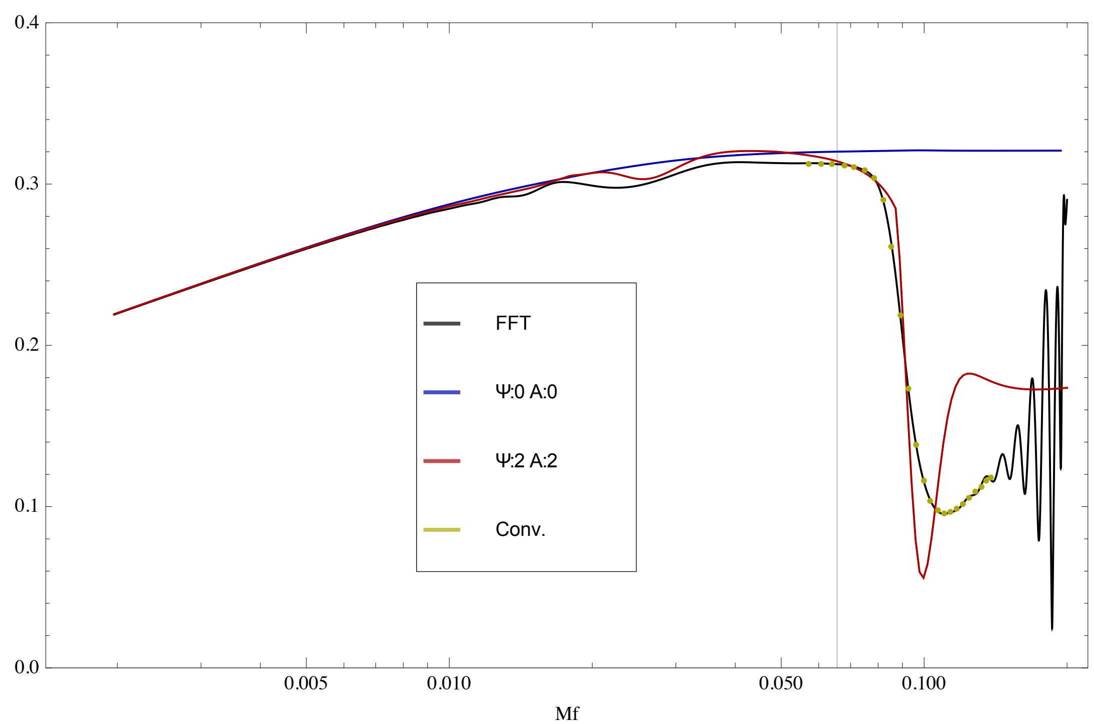
Phase difference

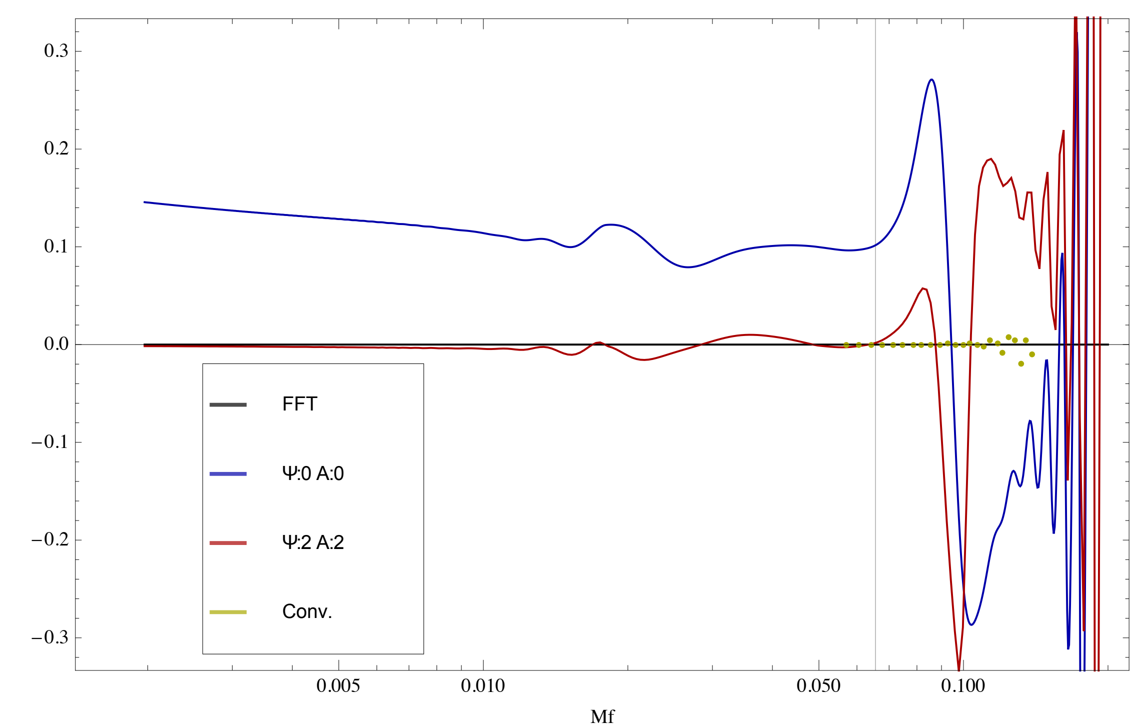
$h_{22}^{\mathrm{P}} \to h_{22}^{\mathrm{I}}$



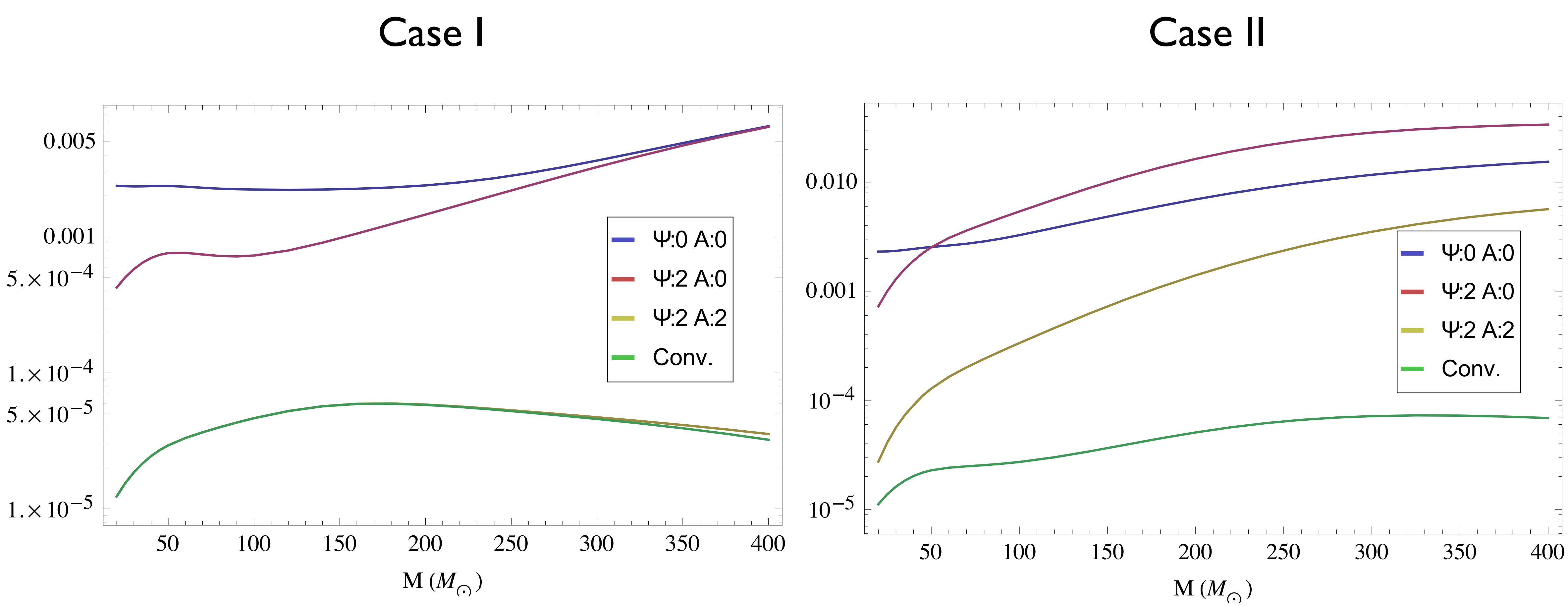


$h_{22}^{\mathrm{P}} \to h_{21}^{\mathrm{I}}$





Precession: mismatches

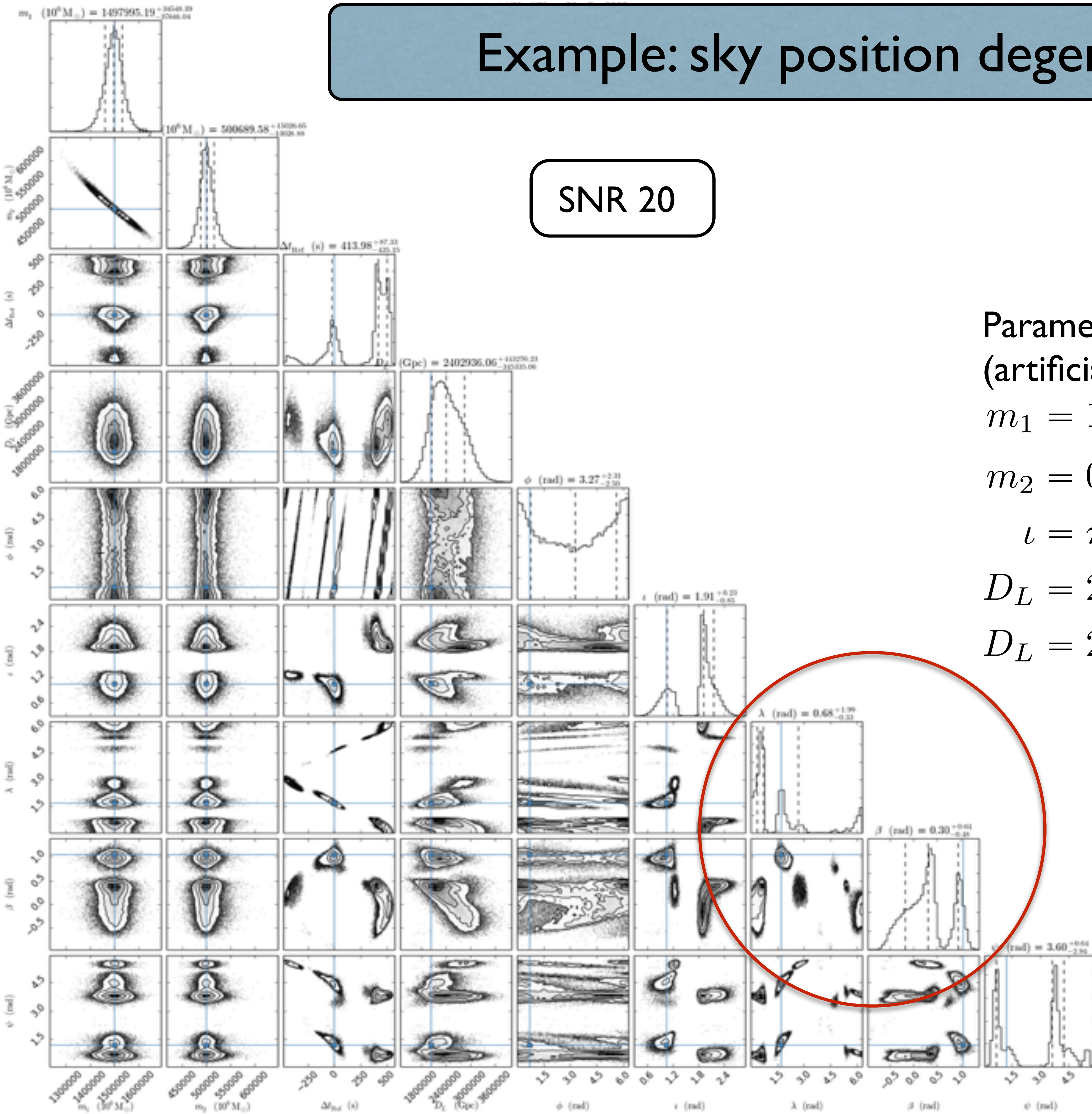


$$\iota = \pi/2$$

- possible building block for models of precessing signals
- robustness across parameter space ?

Example: sky position degeneracies

SNR 20



Parameters:
(artificially distant)

$$m_1 = 1.5 \times 10^6 M_\odot$$

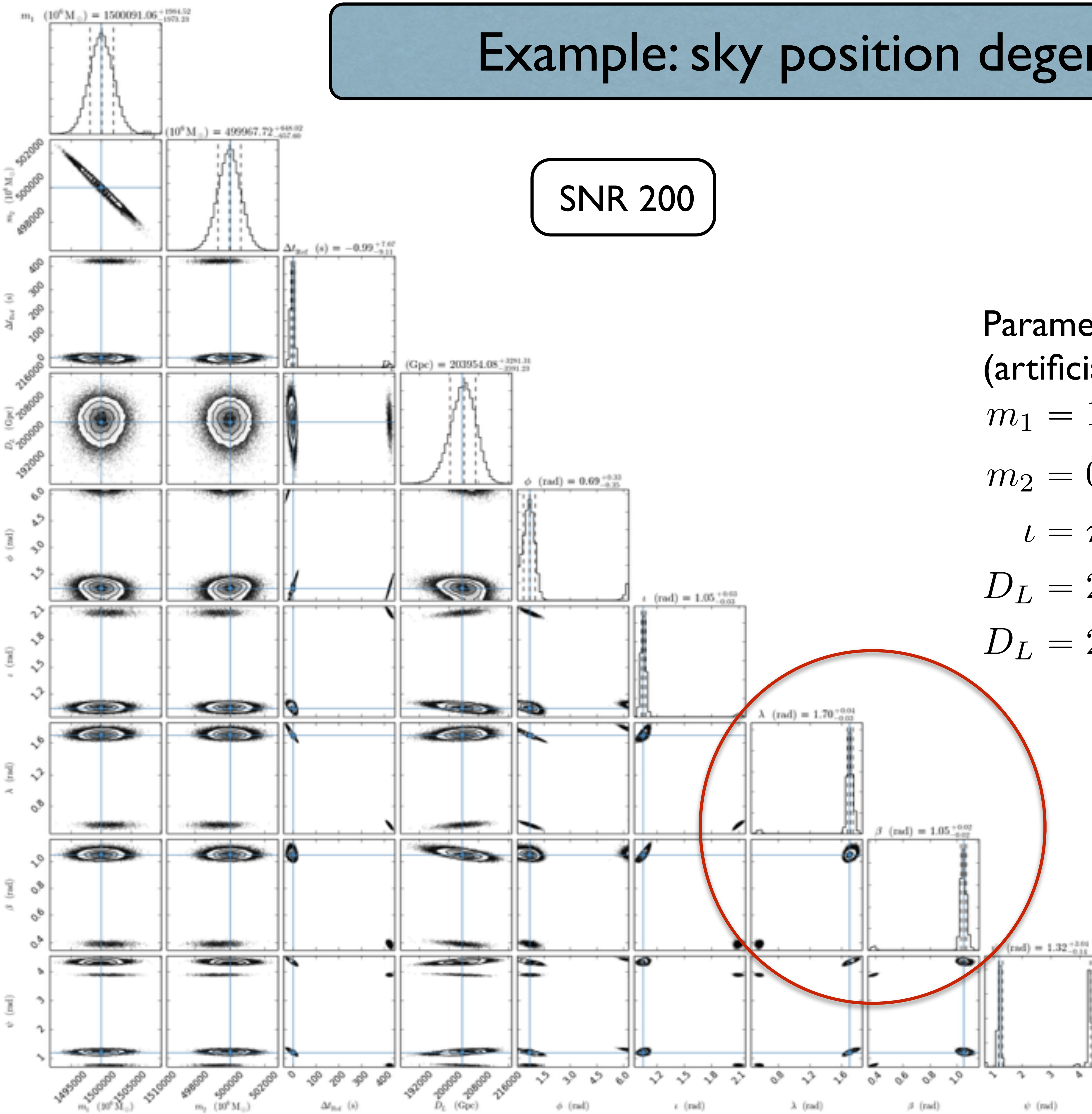
$$m_2 = 0.5 \times 10^6 M_\odot$$

$$\iota = \pi/3$$

$$D_L = 2036\text{Gpc (SNR 20)}$$

$$D_L = 203.6\text{Gpc (SNR 200)}$$

Example: sky position degeneracies



SNR 200

Parameters:
(artificially distant)

$$m_1 = 1.5 \times 10^6 M_\odot$$

$$m_2 = 0.5 \times 10^6 M_\odot$$

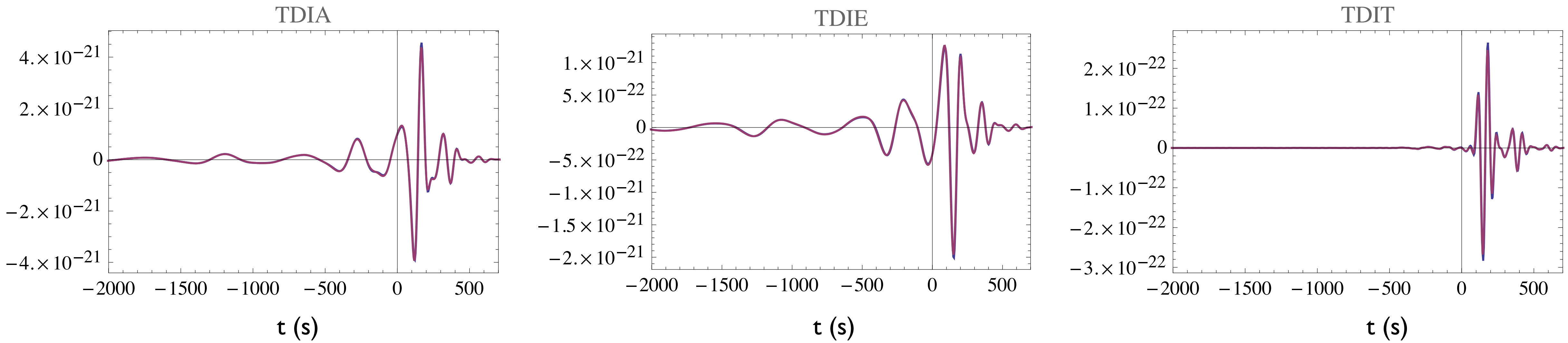
$$\iota = \pi/3$$

$$D_L = 2036\text{Gpc (SNR 20)}$$

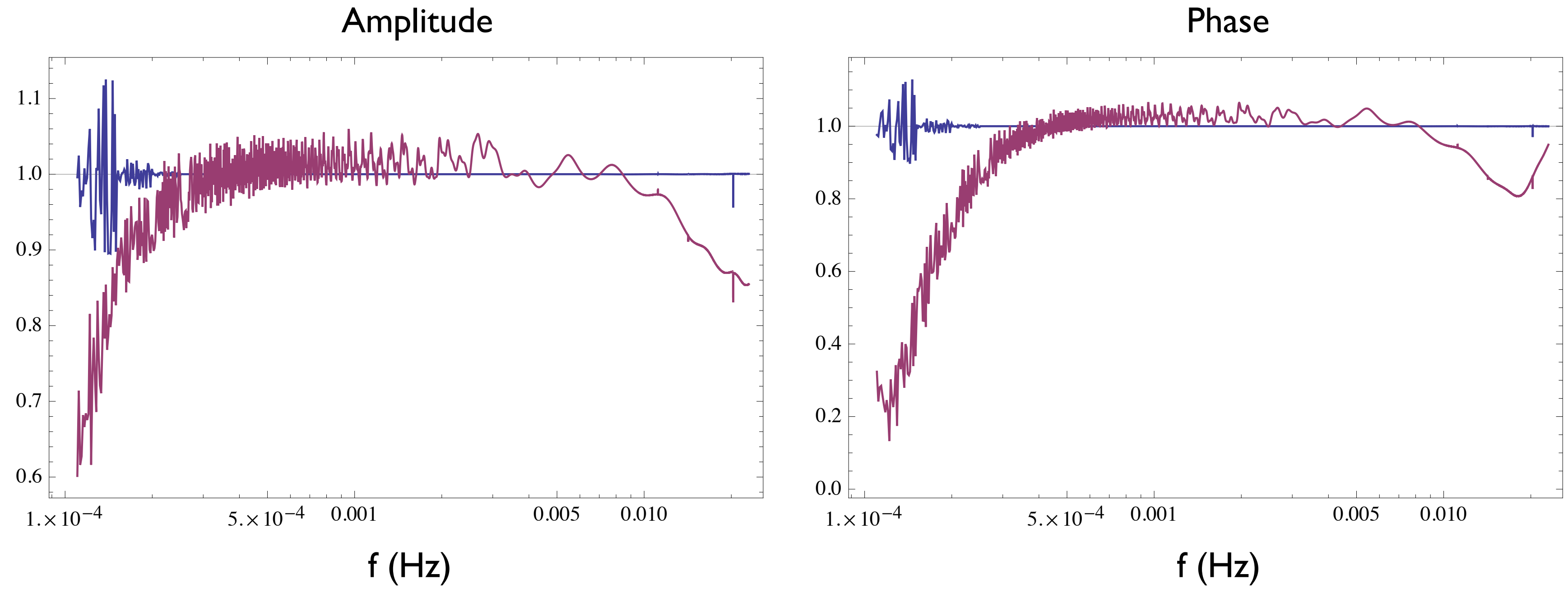
$$D_L = 203.6\text{Gpc (SNR 200)}$$

Example: sky position degeneracies

TD comparison to 2nd peak

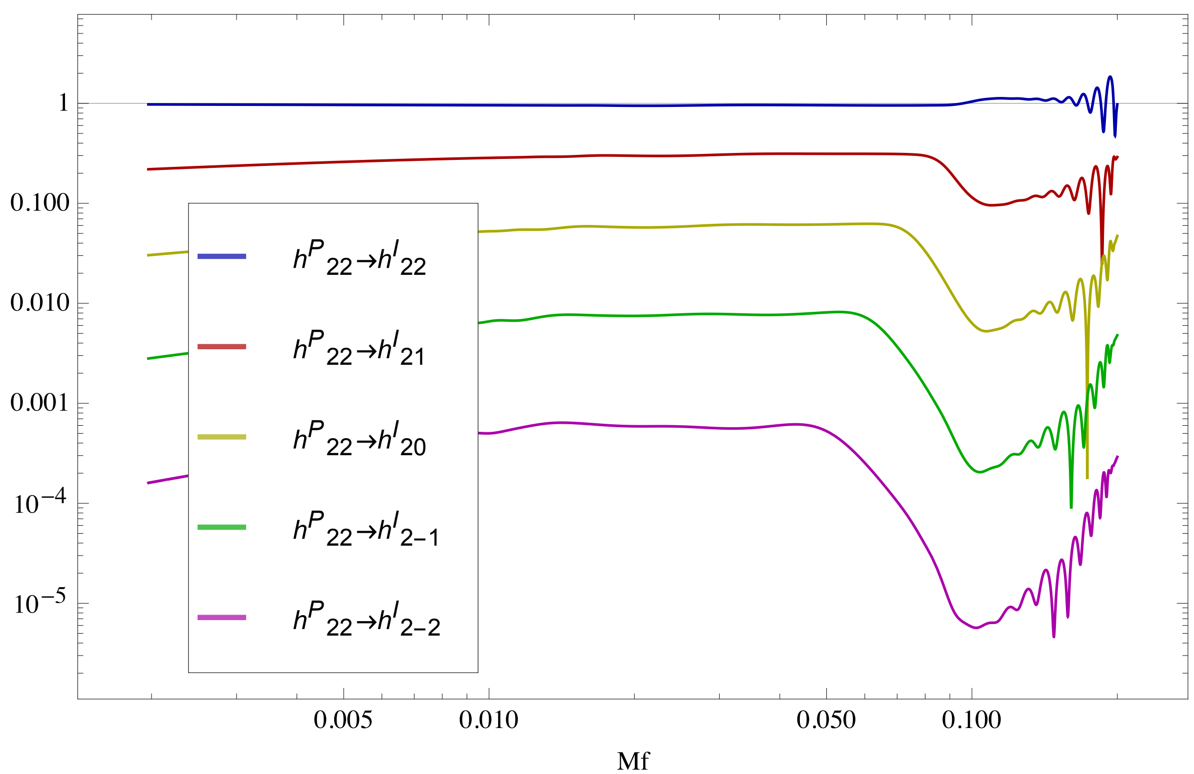


FD comparison to 2nd peak

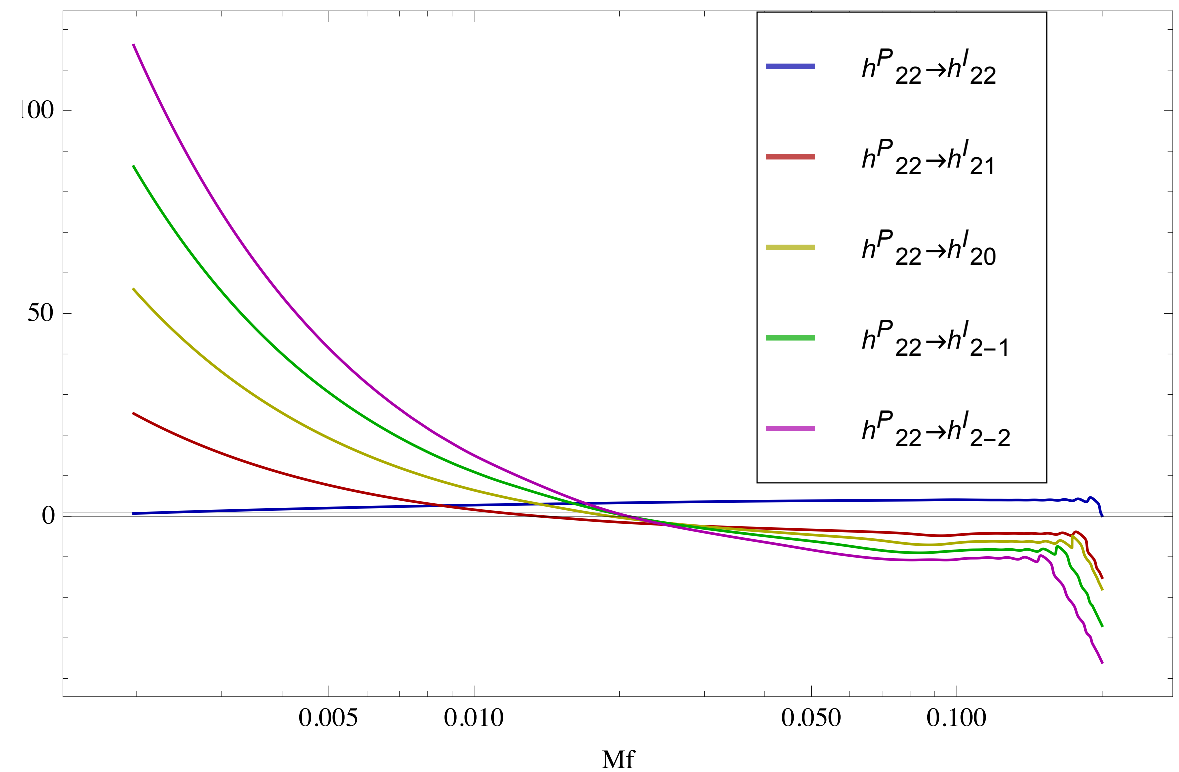


FD transfer functions for different modes

Normalized amplitude

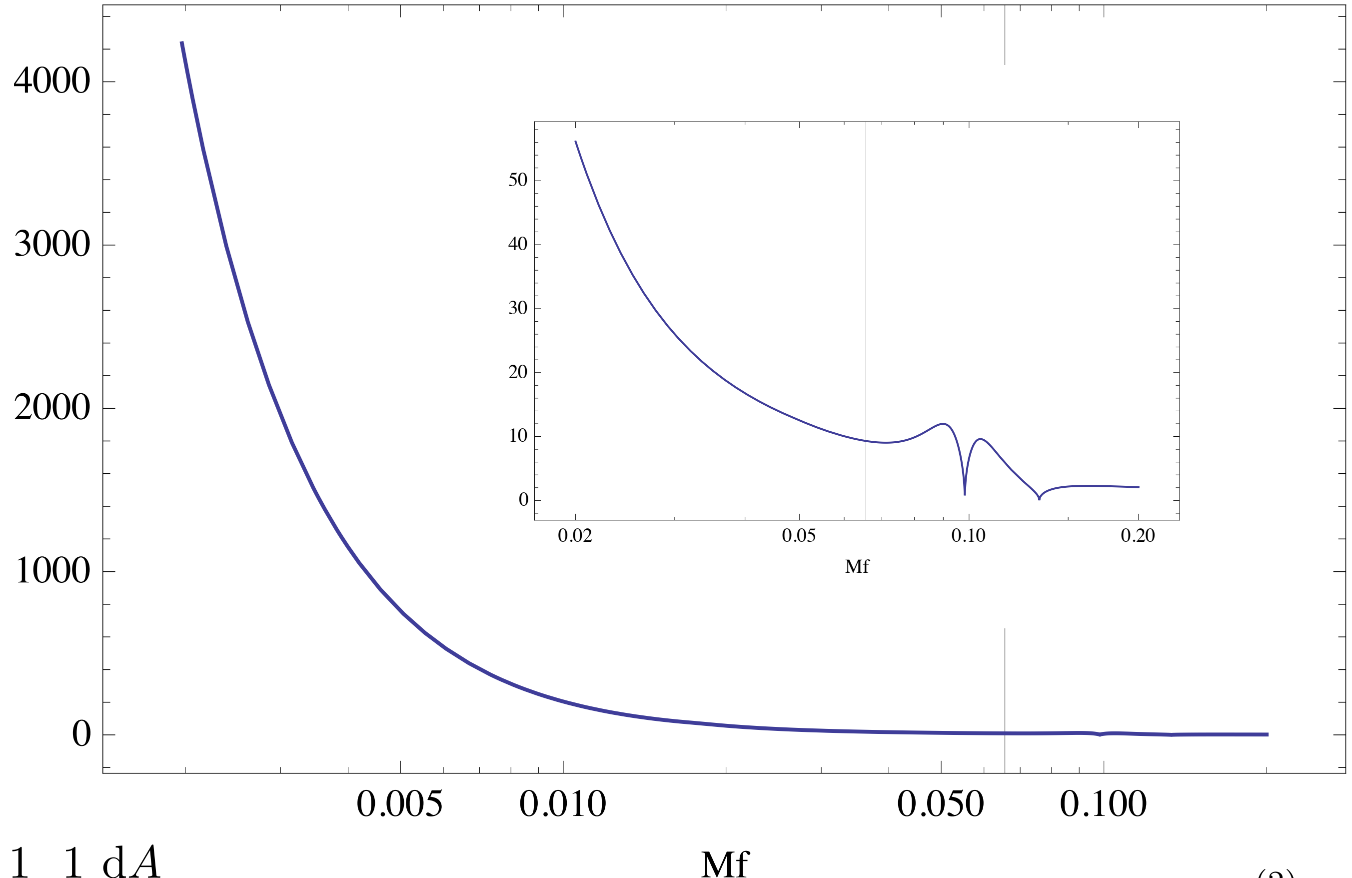


Phase

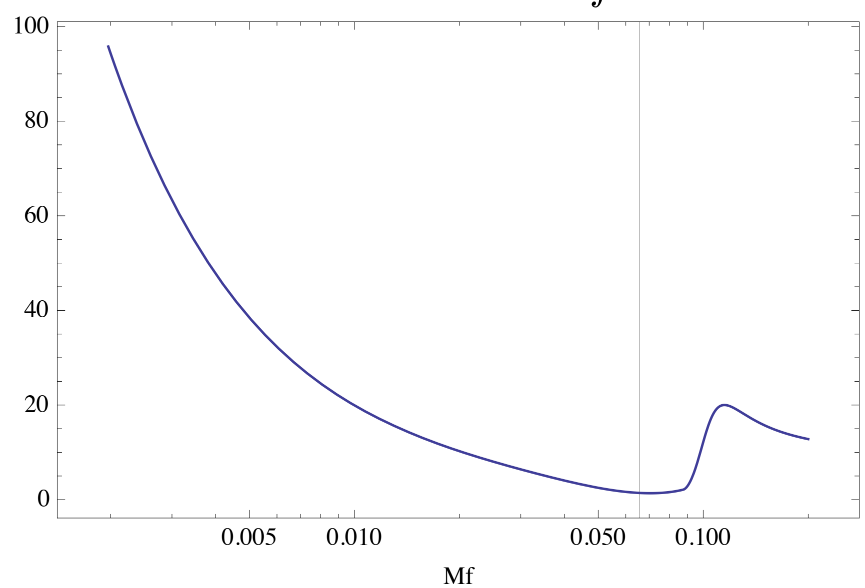


FD timescales

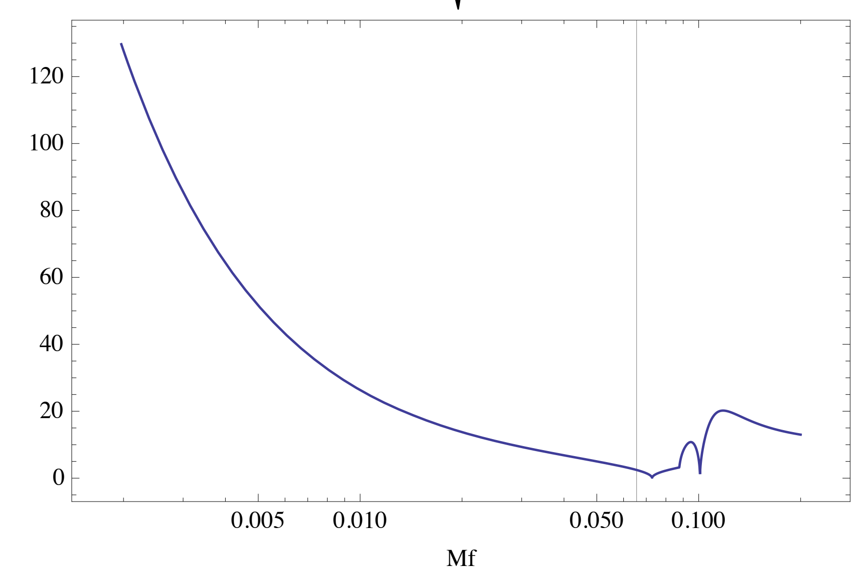
$$T_f = \sqrt{\frac{1}{4\pi^2} \left| \frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2} \right|}$$



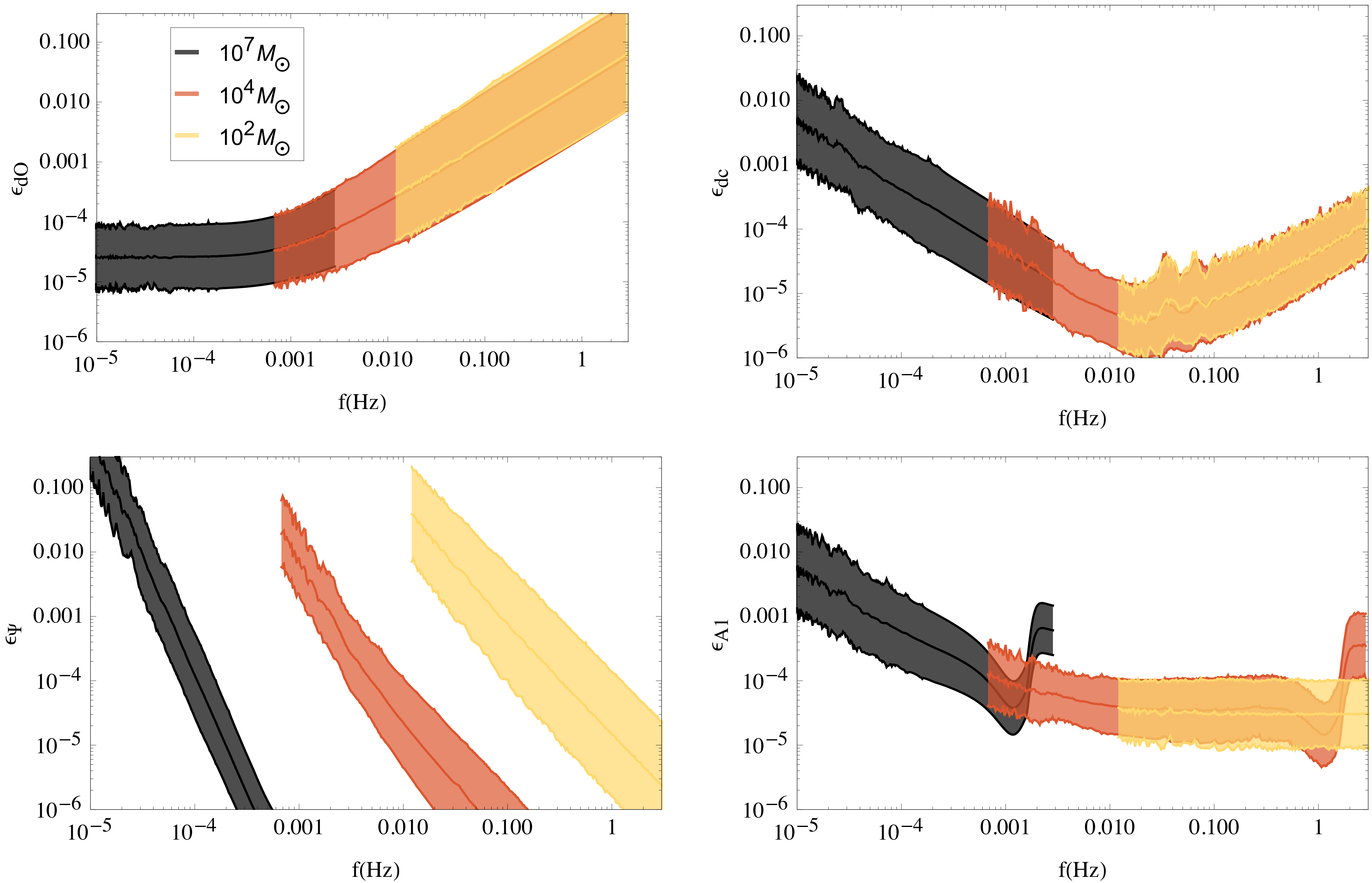
$$T_A^{(1)} = \frac{1}{2\pi}\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}f}$$



$$T_A^{(2)} = \sqrt{\frac{1}{4\pi^2}\frac{1}{A}\frac{\mathrm{d}^2A}{\mathrm{d}f^2}}$$



Transfer functions figures of merit



Transfer functions and residuals

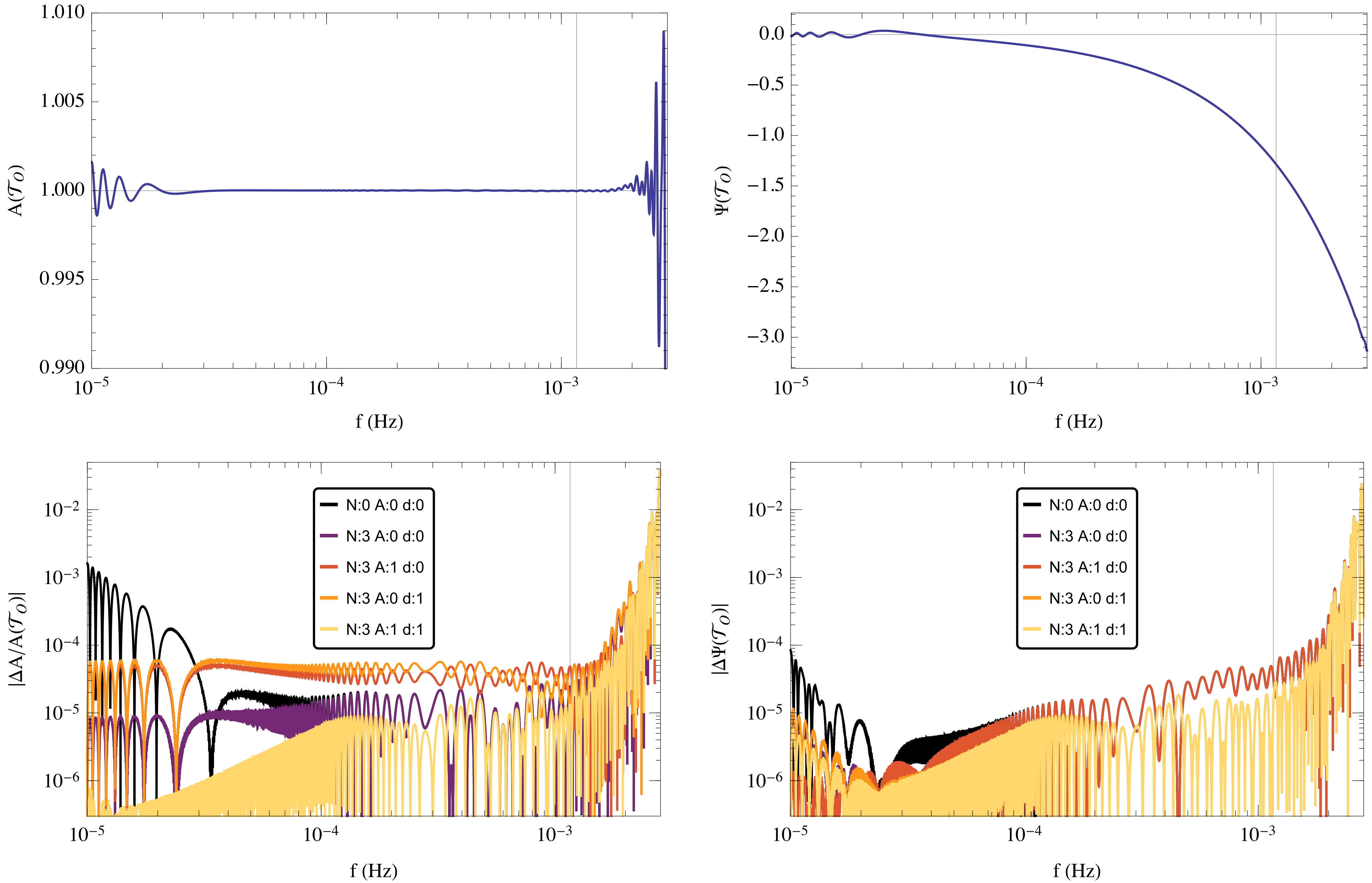


FIG. 5: Error in the transfer function for the orbital delay d_O, for $M = 10^7 M_\odot$.

Transfer functions and residuals

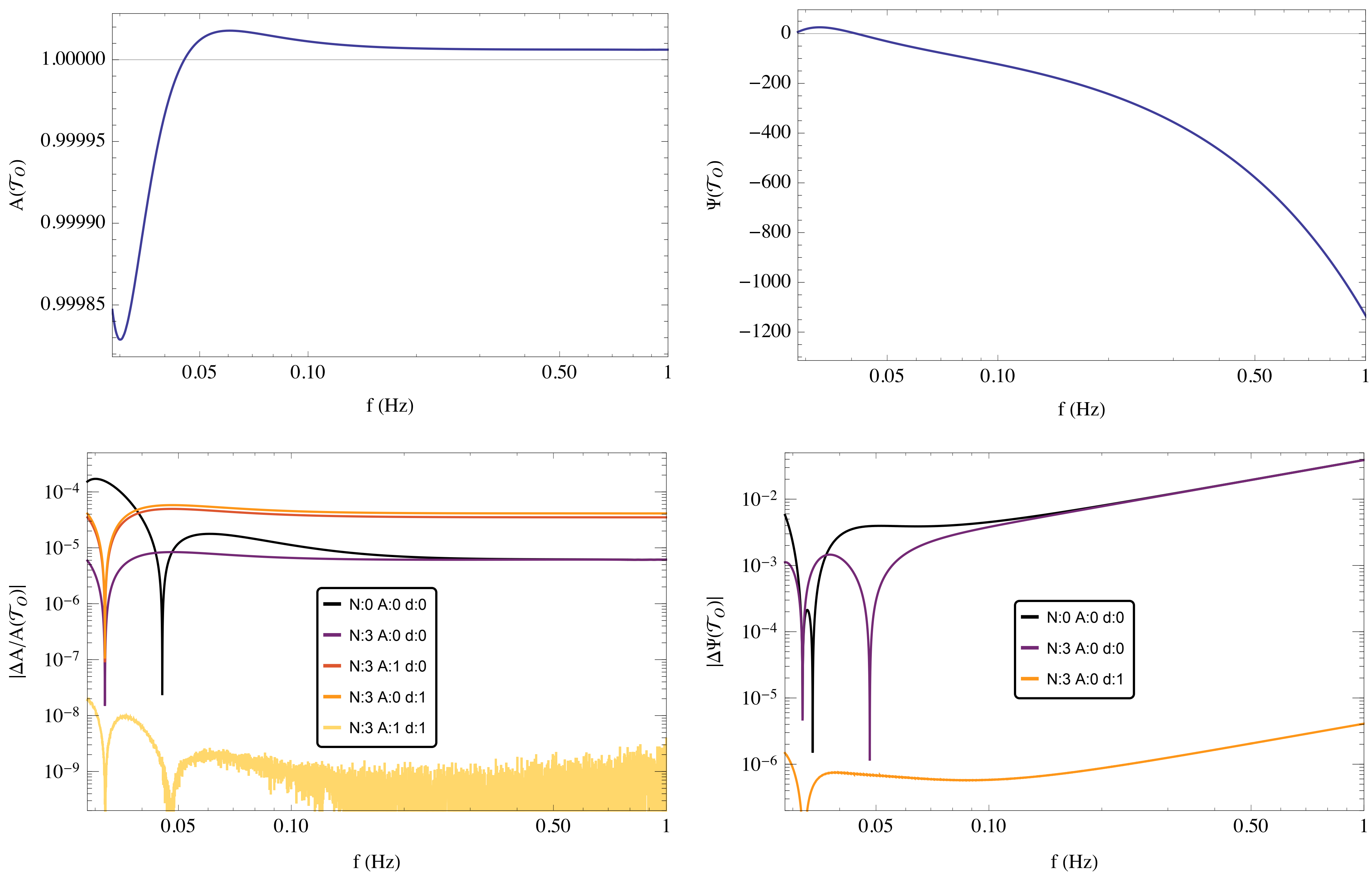


FIG. 7: Transfer function and reconstruction error for the orbital delay d_O, for $M = 10^2 M_\odot$.

Transfer functions and residuals

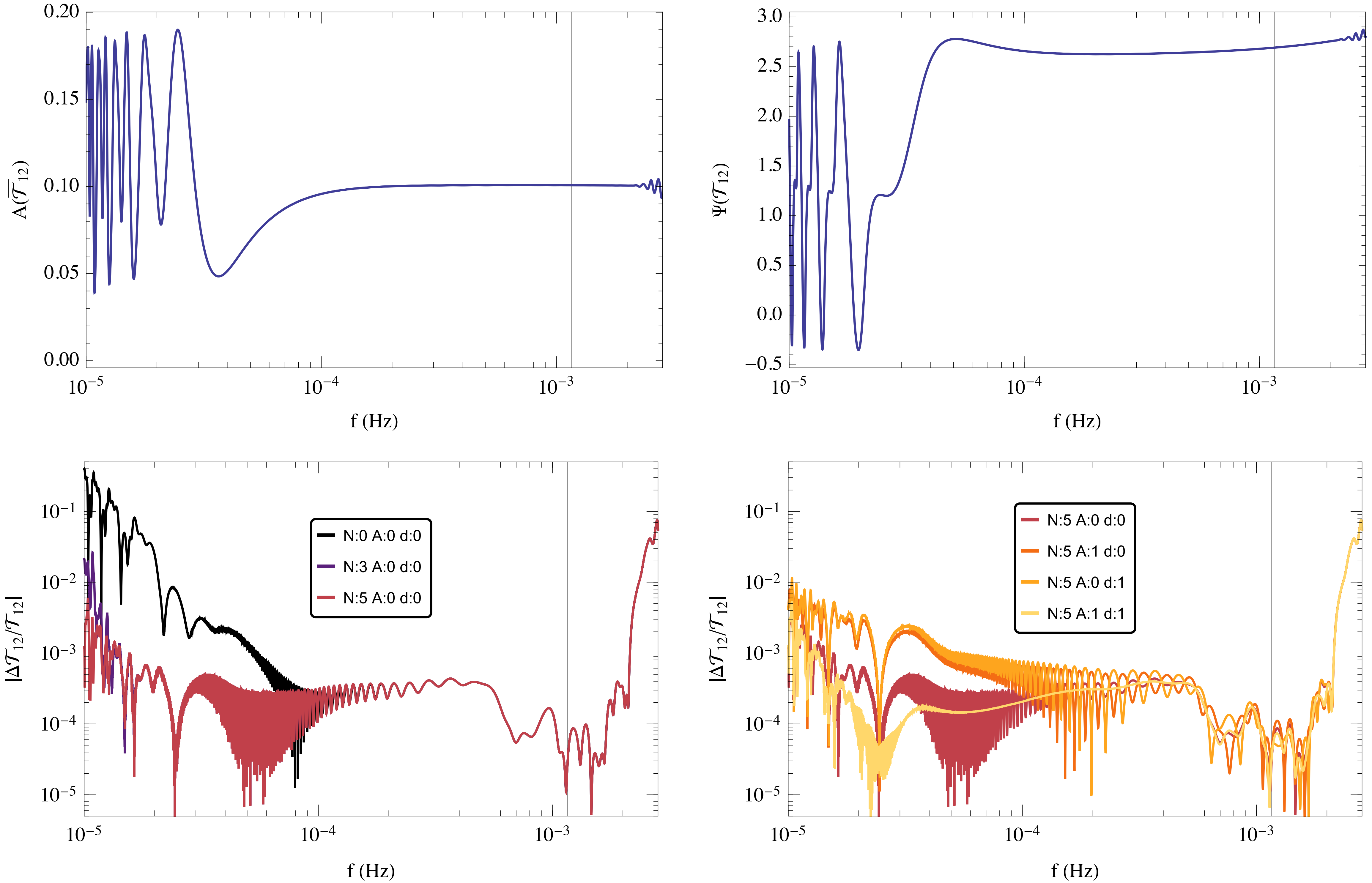


FIG. 6: Error in the transfer function for the basic observable y_{12}, for $M = 10^7 M_{\odot}$.

Transfer functions and residuals

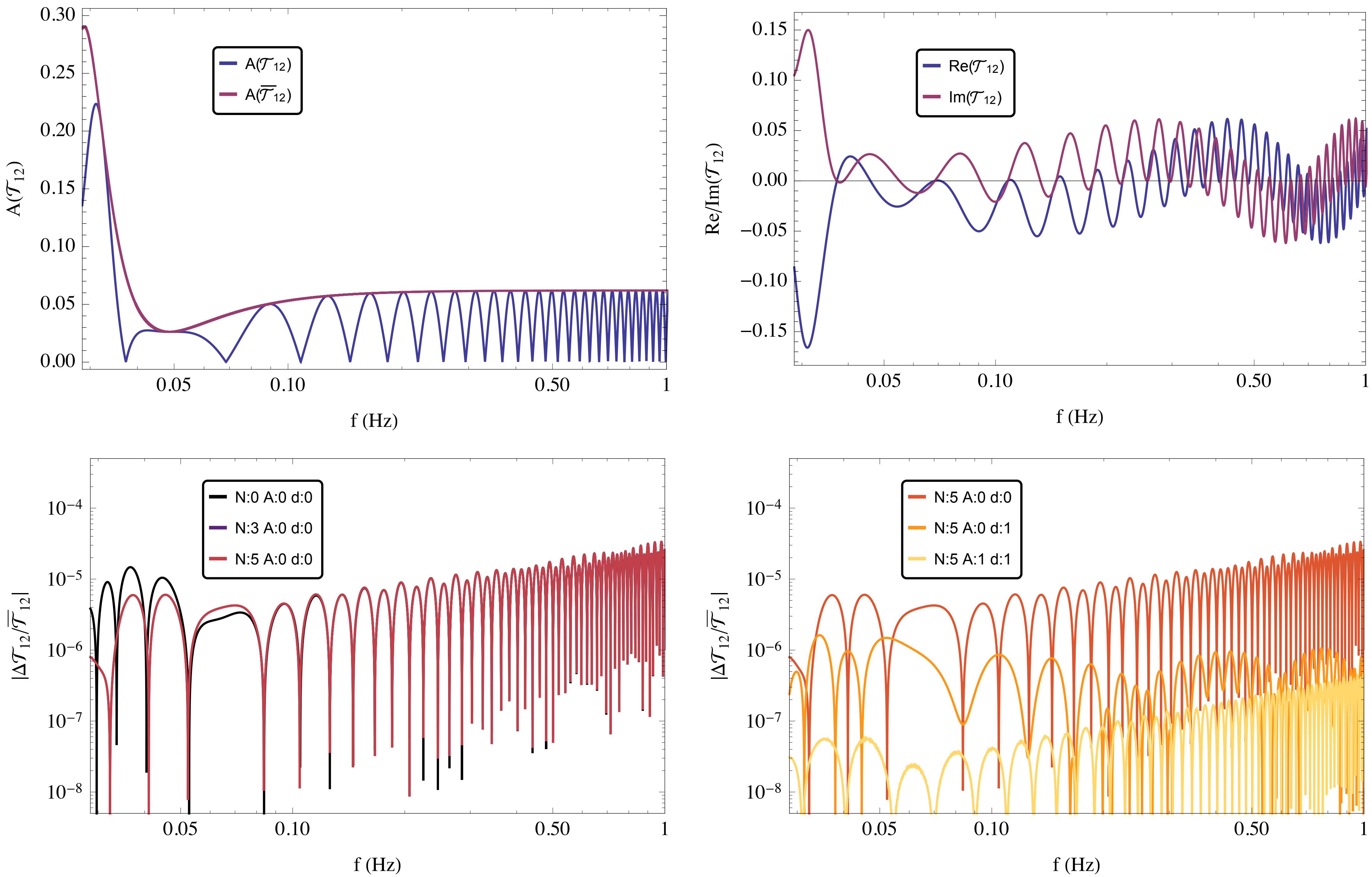
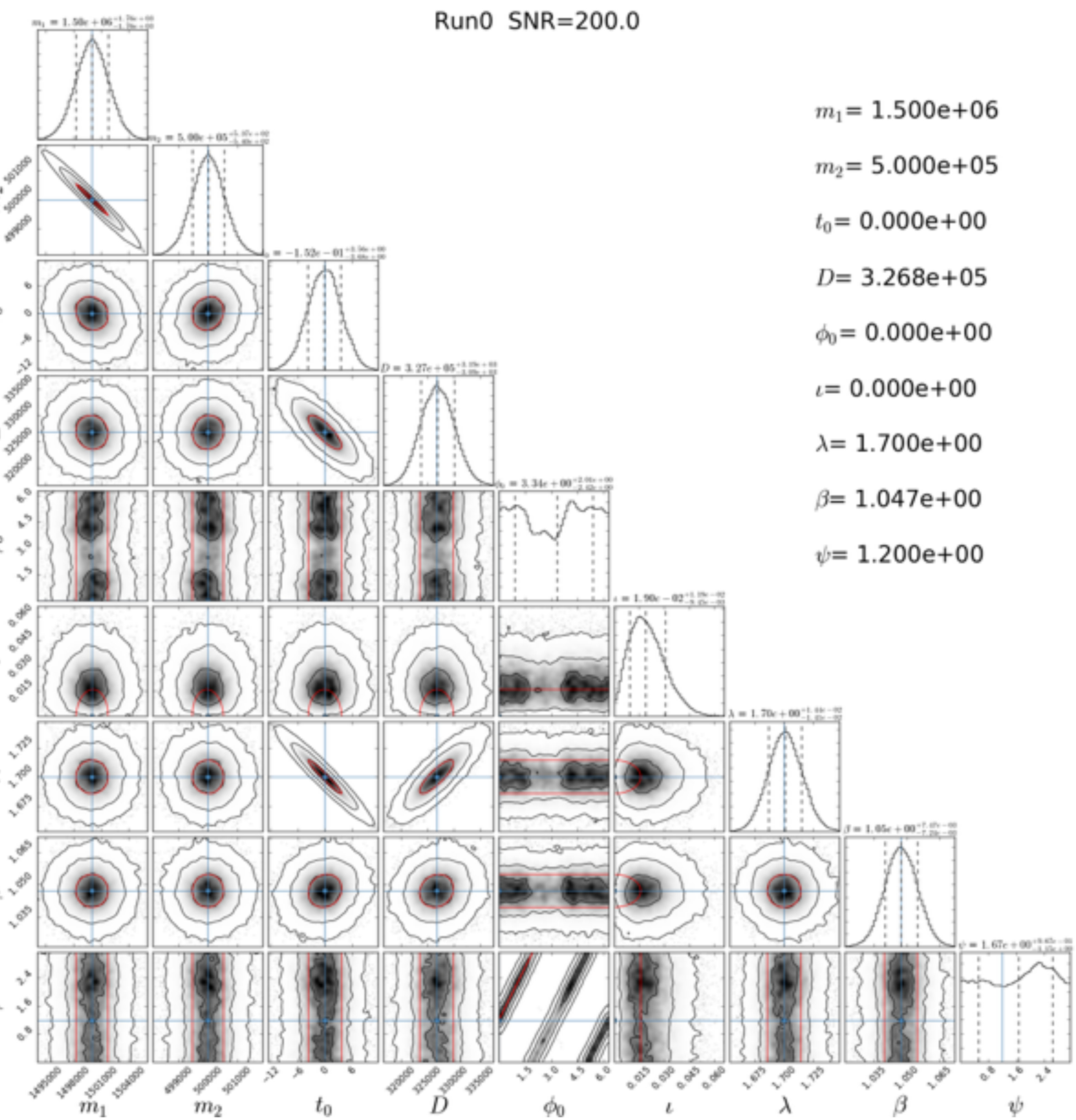


FIG. 8: Transfer function and reconstruction error for the basic observable y_{12}, for $M = 10^2 M_\odot$.

Example of Bayesian inference I



Example of Bayesian inference II