# Orbiting a naked singularity in large-$\omega$ Brans-Dicke gravity

## Expected incidence on Gravitational Radiation in the EMRI case

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Acronyms:

- ABD = Asymptotic Brans-Dicke
- BD = Brans-Dicke
- GR = General Relativity
- JNW = Janis-Neuman-Winicour
- LSCO = Last Stable Circular Orbit
- ST = Scalar-tensor

Brans's Class I (vacuum BD) solution [1] can be written [2]

$$\Phi = \left(\frac{r-k}{r+k}\right)^{s} \qquad \& \qquad ds^2 = -\left(\frac{r-k}{r+k}\right)^{-s+2\lambda} dt^2 + \left(\frac{r+k}{r}\right)^{4} \left(\frac{r-k}{r+k}\right)^{2-s-2\lambda} \left(dr^2 + r^2 d\Omega^2\right)$$

where $|s| \leq \frac{2}{\sqrt{3+2\omega}}$ $\qquad 2\lambda = \sqrt{4-(3+2\omega)s^2}$ $\qquad m = (2\lambda - s)k$ (mass)

$s = 0$ (→ $\lambda = 1$ for any **finite** $\omega$) → vacuum GR (Schwarzschild) …

… ie standard **non rotating GR black hole** …

… otherwise the solution describes a **naked singularity** or a wormhole spacetime [3]

To what extent is a (naked) **singularity** a problem ?

- classical gravity point of view: singularity = breakdown of physical predictivity
- quantum gravity point of view: **the presence of a singularity is just a mark that one enters a spacetime region where the non quantum description of spacetime can no longer be relevant**

→ in the naked case, the spacetime region where quantum gravity processes are at work is no longer hidden behind an horizon

Experimental constraints on ST gravity [4]: $\omega > 4.10^4 \rightarrow s \cong 0$

approximate Brans's Class I by a Janis-Newman-Winicour (JNW) metric [5] …

$$ds^2 = -\left(\frac{r-k}{r+k}\right)^{2\lambda} dt^2 + \left(\frac{r+k}{r}\right)^4 \left(\frac{r-k}{r+k}\right)^{2-2\lambda} \left(dr^2 + r^2 d\Omega^2\right) \quad \text{with} \quad \lambda \in [0,1] \qquad (m = 2\lambda k)$$

… that solves GR filled by a massless scalar (ie not GR vacuum)

$$R_{ab} = \partial_a \varphi \, \partial_b \varphi \quad \text{with (spherical Brans's Class I)} \quad \varphi = \sqrt{2\left(1-\lambda^2\right)} \ln \frac{r-k}{r+k}$$

**In general**, for large $\omega$, $T_{ab}$ filled BD gravity (and to some extent for a large class of BD like ST theories) is asymptotically equivalent to GR, but filled by $T_{ab}$ + a massless scalar [6]

$$R_{ab} = 8\pi\left(T_{ab} - \frac{1}{2} T g_{ab}\right) + \partial_a \varphi \, \partial_b \varphi \quad \text{with} \quad Dalemb(\varphi) = 0$$

Asymptotic ($T_{ab}$ filled) Brans-Dicke (ABD)

$\lambda = 1 \rightarrow$ Schwarzschild (GR) $\longrightarrow$ circular orbits: - exist iff $R > 3m$

- stable iff $R > 6m$

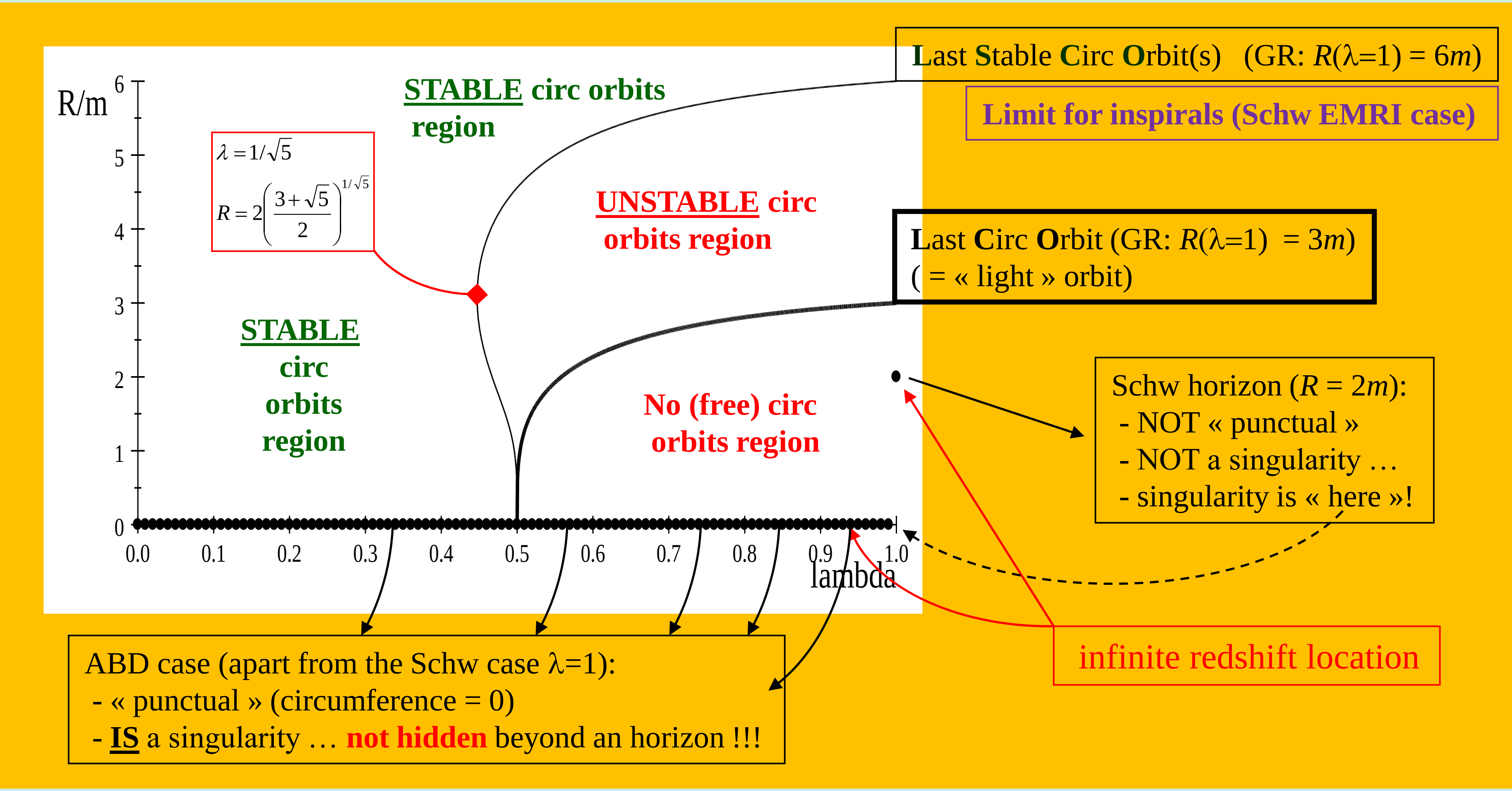
$R$ = areal radius

$\lambda < 1 \rightarrow$ naked singularity (no longer wormholes (ABD) ...)

→ circular orbits → ???

$$R_{ABD}(r) = r\left(1+\frac{k}{r}\right)^2 \left(\frac{r-k}{r+k}\right)^{1-\lambda}$$

## Circular orbits properties in JNW metric [2][7]



Orbital frequency measured by a **far away** observer

combines
- « local » orbital frequency
- gravitational doppler

$$\lambda = 1 \longrightarrow \nu_\infty < \nu_\infty(R = 6m) = \left(12\pi\sqrt{6}m\right)^{-1} \approx (2.2\ \text{kHz}) \times \left(\frac{m}{m_{Sun}}\right)^{-1}$$

$$\frac{1}{2} < \lambda < 1 \longrightarrow \nu_\infty < \nu_\infty(R = R_{LSCO}) = \dots$$

$$\frac{1}{\sqrt{5}} < \lambda \longrightarrow$$

- stability for all (areal) radius,
  until bumping into the naked singularity,
  with increasing « local » frequency
- time « freezing », naked singularity
  being and infinite redshift location

**???**

$$\frac{1}{\sqrt{5}} < \lambda < \frac{1}{2} \longrightarrow$$ inside recircularisation ???

Orbital frequency measured by a **far away** observer
on a circular orbit of Brans's Class I ABD metric:

$$\nu_\infty(\lambda,r)=\frac{(2\lambda)^{3/2}}{2\pi m}\left(\frac{2\lambda r}{m}\right)^{3/2}\frac{\left(\frac{2\lambda r}{m}-1\right)^{2\lambda-1}\left(\frac{2\lambda r}{m}+1\right)^{-2\lambda-1}}{\sqrt{\left(\frac{2\lambda r}{m}\right)^2-2\lambda\frac{2\lambda r}{m}+1}}$$

When approaching the singularity (decreasing radius ← gravitational radiation) :

$$\nu_\infty\left(\lambda, r\to\frac{m}{2\lambda}\right)\sim\frac{1}{2\pi m}\frac{(2\lambda)^{3/2}}{2^{2\lambda+1}\sqrt{2-2\lambda}}\left(\frac{2\lambda r}{m}-1\right)^{-(1-2\lambda)}$$

$R\to 0$

$\to\infty\quad$ for $\quad\lambda<1/2$

$\leq\frac{1}{2}$ (for circular orbits to exist when $R \to 0$)

**Conclusions on the gravitational radiation emited by an object inspiraling a (hypothetic …) BD-like naked singularity**

If (1) gravity is of ST (BD-like) nature, and (2) BD-like naked singularities do exist, the (classically evaluated) gravitational frequency (twice the orbital one) emitted by and inspiraling object is expected to be **not bound**, unlike what happens in GR, for some values of the parameter $\lambda$.

This possibility **does not depend how high $\omega$ is**, ie how close to GR is ST gravity in the solar system, for instance. Indeed, the divergence of ABD from GR is of finite amplitude (both $\varphi$ and 1-$\lambda$ are finite quantities in the displayed ABD Brans's solution).

In the ST framework, naked singularities could be primordial objects (as primordial black holes could exist in both GR and ST frameworks).
(Possibility of creating naked singularity by gravitational ST collapse?)

➔ Naked singularities as hypothetic alternative sources of gravitational radiation, with very specific properties. More precise properties of the radiation in the Extreme Mass Ratio case should be reachable by perturbative calculations in the ABD framework (in order to both simplify the equations to solve and use what is known from to date experiments). Work in progress …

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