Secular evolution of stellar cluster @GC

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The case of quasi-Keplerian systems

- Describe the secular evolution driven by finite-\(N\) effects for a quasi-Keplerian system
- inhomogeneous
- dynamically degenerate
- stable
- self-gravitating
- discrete

- How efficiently are BHs fed?

Some references:
- Rauch, Tremaine (1996): Resonant relaxation
- Meritt et al. (2011): Schwarzschild barrier
- Bar-Oz, Alexander (2014, 2016): \(\eta\)-formalism
- Sridhar, Touma (2016): Gilbert’s method for Landau
- Fouvrty, Pichon, Magorrian (2016): BBGKY approach

BBGKY Hierarchy truncation @ 3 pt function

\( \overline{r} = (J, \theta) \)

- BBGKY-\(n=1\) equation

\[ \frac{\partial F}{\partial t} + \left[ \bar{F}, \bar{\nabla} + \bar{\nabla}_T \right] \frac{1}{N} \int dR_1 |\bar{C}(R_1, R_2, T_{11})| dJ_{11} = 0. \]

- BBGKY-\(n=2\) equation

\[ \frac{1}{2} \frac{\partial}{\partial t} \left[ \bar{C}(R_1, R_2), \bar{C}(R_2, R_1) \right] + \bar{F}(R_1) \bar{F}(R_2), T_{11} \frac{1}{(2\pi)^4} \left( 2\pi^2 k^2 \right) \]

\[ \frac{3}{2} \bar{\nabla} \left[ \bar{C}(R_1, R_2), \bar{C}(R_1, R_1) \right] \bar{F}(R_1), T_{11} |dJ_{11}| + (1 \leftrightarrow 2) = 0. \] (50)

using averaging over fast angle

\[ \bar{F}(J, \theta) = \int \frac{d\theta}{(2\pi)^2} \bar{F}(J, \theta, \theta^0). \]

Bogoliubov’s synchronization hypothesis

Physical origin of Schwarzschild barrier

One PN and 1.5PN relativistic correction

\[ \Omega = \frac{\partial \bar{F}}{\partial J} \]

\( \Omega^{rel} \)

\( \Omega^{self} \)

\( \Omega^{eff} \)

\( \omega_J \)

Disc radius

Effect scales like the square density of wires

Stochastic diffusion

Resonant relaxation drives the disc to a configuration of lower angular momentum @ fixed semi major axis

\( J_L + L = \text{const} \)

Flux map in action space

\[ \Delta N \]

\( \Delta t \)

\( \bar{F}(J_1, J_2) \)

\( L_c (10^{10}) \)

Angular momentum

Increasing eccentricity

\( L_c (10^{10}) \)

Mass segregation near SMBH

Multi species?

Quasi-Keplerian systems

- BH dominates the dynamics: \( r=M_s/M_\ast < 1 \)
  \( \Rightarrow \) Keplerian orbits are closed.
- Dynamical degeneracy: \( \forall J, n, \Omega_{Kep}(J) = 0 \).
  \( \Rightarrow \) Delaunay variables

\[ J = (I, \Omega, \nu) \]

\( \Rightarrow \) Keplerian potentials, one has to deal with additional dynamical degeneracies

\( E = \bar{E}(J, \theta) \)

System phase-mixed w.r.t. the Keplerian phase

\[ F(J, \theta) \neq \bar{F}(\epsilon) \]

Keplerian wires precess in \( \theta^0 \)

\[ \Omega^0 = \frac{\partial \bar{F}}{\partial J} = \frac{\partial \bar{F}}{\partial J} + \frac{\partial \bar{F}}{\partial J} \]

Disc has mass

SMBH relativistic correction

The degenerate Balescu-Lenard equation

- The master equation of resonant relaxation

\[ \frac{\partial \bar{F}(J, \theta)}{\partial t} = - \frac{1}{N} \sum_{m_1 m_2} \int dJ_1 \frac{dJ_2}{2\pi^2} \left[ \frac{\partial}{\partial J_1} \frac{\partial}{\partial J_2} \right] \bar{F}(J_1, \theta) \bar{F}(J_2, \theta) \]

- Some properties:

\( \bar{F}(J, \theta) \) Orbital distortion.

\( \bar{F}(J, \theta) \) KH resonant relaxation.

\( \bar{F}(J, \theta) \) Adiabatic conservation.

\( \delta \) Resonance on precessions.

\( 1/\Delta_{\text{ext}} \) Self-gravity.

Individual stochastic diffusion

- Self-consistent diffusion of the system as a whole

\( \Rightarrow \) Anisotropic Balescu-Lenard equation

\[ \frac{\partial \bar{F}(J, \theta)}{\partial t} = \frac{\partial}{\partial J} A(J, \tau) \bar{F}(J, \theta) + D(J, \tau) \frac{\partial \bar{F}(J, \theta)}{\partial J} \]

- Individual dynamics of a wire at position \( J(\tau) \)

\( \Rightarrow \) Stochastic Langevin equation \( \bar{F}(\tau) \)

\[ \frac{\partial J}{\partial \tau} = h(J, \tau) + g(J, \tau) \bar{F}(\tau) \]

\( -\tau \) Process

- In the Langevin’s rewriting, \( F \) particles are dressed orbits.

\( \Rightarrow \) Huge gains in steps for integration.