The particle-without-particle approach to the self-force problem

Marius Oltean$^{1,2,3,4,5}$, Carlos F. Sopuerta$^1$ and Alessandro D.A.M. Spallicci$^{3,4,5}$

$^1$Institut de Ciències de l’Espai (IEEC-CSIC), Campus Universitat Autònoma de Barcelona, Spain
$^2$Departament de Física, Facultat de Ciències, Universitat Autònoma de Barcelona, Spain
$^3$Laboratoire de Physique et Chimie de l’Environnement et de l’Espace, CNRS, Orléans, France
$^4$Observatoire des Sciences de l’Univers en région Centre, Université d’Orléans, France
$^5$Pôle de Physique, Collegium Sciences et Techniques, Université d’Orléans, France

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Introduction
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The theoretical formalism to compute the self-force has been largely established, e.g. [S. Gralla and R. Wald, CQG 25, 205009 (2008)], but its mathematical implementation is still under development; we use the Particle-without-Particle (PwP) method.
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- A helpful testbed for the gravitational self-force is the scalar self-force—we tackle this using the PwP method in the frequency domain.
Scalar self-force: a simplified EMRI model
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- **Setup:** $m_*$ is a *charged scalar particle* (with charge $q$ associated to a scalar field $\Phi$) orbiting a *non-rotating black hole* (of fixed—Schwarzschild—geometry, $(M, g, \nabla)$) along a geodesic $\gamma$ with worldline $z(\tau)$ and 4-velocity $u = \dot{z}$. The EOMs are:

\[ t \quad (M, g, \nabla) \quad \gamma \quad u \quad z \quad x^i \quad m_*, q \]
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\nabla^2 \Phi = -4\pi q \int_{\gamma} d\tau \, \delta(x - z(\tau)),
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u \cdot \nabla (m_\ast u) = \mathcal{F} = q \left( \nabla \Phi \right)_{|_{\gamma}}.
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| Regge-Wheeler potential | proportional to $q$, dependent on $z$ | particle’s radial location | redefined (“tortoise”) radial coordinate |

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- proportional to \( q \), dependent on \( z \)
- particle’s radial location
- redefined (“tortoise”) radial coordinate

- Once the field is solved for, its singular part must be subtracted (via “mode-sum regularization” [L. Barack and A. Ori, PRD 61, 061502 (2000)]).
Scalar self-force: the PwP method
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- Split the computational domain into two disjoint regions. For any $Q(t, r)$:

$$Q = Q_0 \Theta_p^- + Q_1 \Theta_p^+,$$

$$\Theta_p^\pm = \Theta(\pm (r - r_p)),$$

$$[Q]_p = \lim_{r \to r_p(t)} (Q_1 - Q_0).$$
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- Split the computational domain into two disjoint regions. For any $Q(t, r)$:

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Q = Q_- \Theta^-_p + Q_+ \Theta^+_p, \quad [Q]_p = \lim_{r \to r_p(t)} (Q_+ - Q_-) .
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(\Box - V_\ell(r)) \psi_-^{\ell m} = 0 \quad [\partial_r \psi_-^{\ell m}]_p = 0 \quad [\psi_-^{\ell m}]_p = 0 \quad (\Box - V_\ell(r)) \psi_+^{\ell m} = 0
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\[
R_- \quad r_{\text{peri}}^* \quad r_p^*(t) \quad R_+ \quad r_{\text{apo}}^*
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*The particle-without-particle approach to the self-force problem*
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- We have shown [MO, CFS and ADAMS, forthcoming] that a general PwP method can be used for solving any arbitrary $m$-th order linear PDE with one-dimensional delta function (derivative) sources at $M$ particles.

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\mathcal{L}\psi(x, y) = \sum_{i=1}^{M} \sum_{j=0}^{m-1} f_{ij}(x, y) \delta^{(j)}(x - x_{p_i}(y)).
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The basic idea is to decompose $\psi = \sum_{i=0}^{M} \psi^i \Theta^i$ (with $\Theta^i$ suitably defined), and prove that one can match the LHS/RHS in Heaviside derivatives.
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- Example: Heat equation with constant source supported at a sinusoidally moving particle (using a Chebyshev-Lobatto grid in space and a standard finite-difference scheme in time).

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- The PwP has already been successfully used to compute the scalar self-force in the time domain [P. Cañizares and CFS, PRD 79, 084020 (2009); CQG 28, 134011 (2011)].
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- Move to the frequency domain! We have bound orbits, and thus discreet series:

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\psi_{\pm}^{\ell m}(t, r) = e^{-i m \omega \varphi^t} \sum_{n=-\infty}^{+\infty} e^{-i n \omega t} R_{\ell m n}^\pm(r).
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$$\psi_{\pm}^{lm}(t, r) = e^{-im \omega \varphi t} \sum_{n=-\infty}^{+\infty} e^{-in \omega_r t} R_{\pm}^{lmn}(r).$$

The wave-like PDEs for $\psi_{\pm}^{lm}$ become Schrödinger-like ODEs for $R_{\pm}^{lmn}$.
- We use a pseudospectral collocation method to find the homogeneous numerical solutions $\hat{R}^-_{lmn}$ and $\hat{R}^+_{lmn}$ for arbitrary BCs, then use the jump conditions to get the true solution,

$$R_{lmn} = C^-_{lmn} \hat{R}^-_{lmn} \Theta^-_p + C^+_{lmn} \hat{R}^+_{lmn} \Theta^+_p$$

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- For future work, another objective is to also extend this method to *rotating black holes* (Kerr).
Thanks for your attention!