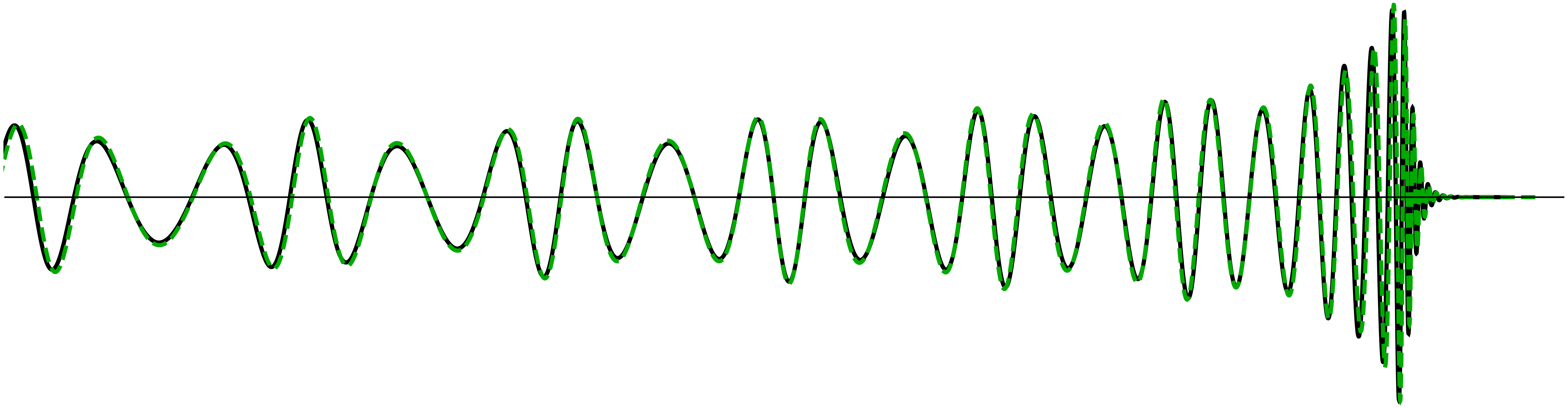

An eccentric binary black hole inspiral-merger-ringdown gravitational waveform model from post-Newtonian and numerical relativity

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Simulating
eXtreme
Spacetimes

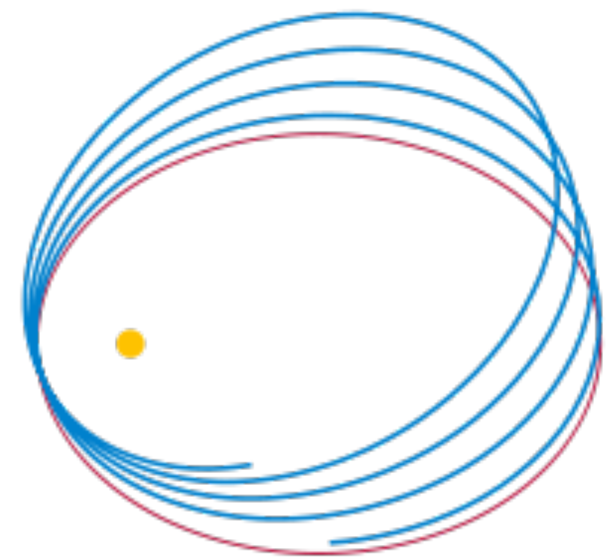
TEGrAW, Paris, June 2017

Introduction

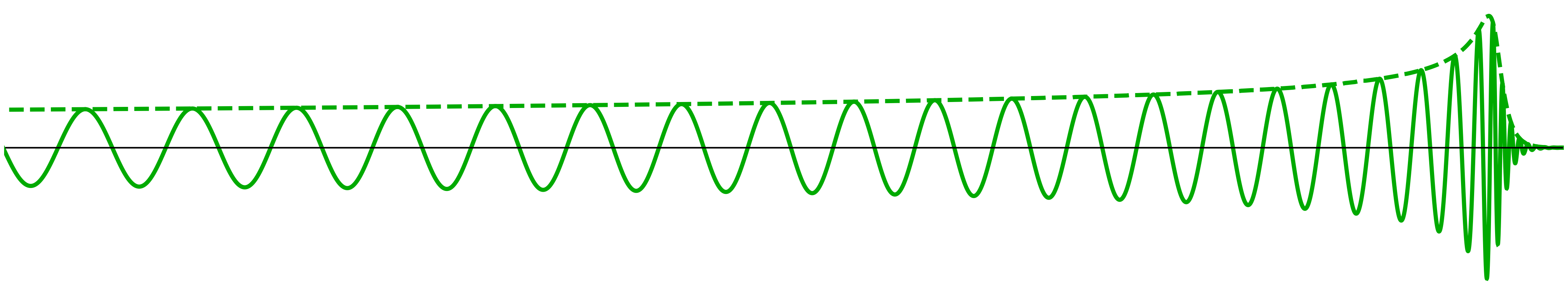
- Eccentric binary systems **circularise** as E and L are emitted (Peters 1964)
- Eccentricity of BBH **expected** to be 0 well before merger
- Eccentric binaries in LIGO band?
- Can we measure (bound) eccentricity of **GW events** such as GW150914?
- **Eccentric waveform model** could be compared with GW data to measure/constrain eccentricity)
- Construct and test such a model using **Post-Newtonian** approximation and **Numerical Relativity**
- Only need late inspiral+merger; e.g. **last 5 orbits** for GW150914

A selection of eccentric Numerical Relativity simulations

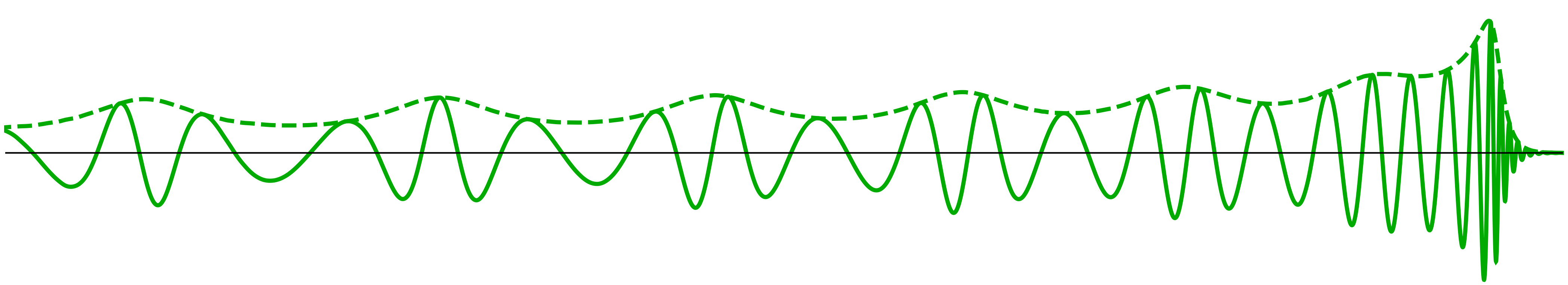
- 19 new accurate NR simulations, ~25 cycles, **SpEC** code
- Non-spinning
- Initial eccentricity **$e \leq 0.2$**
- $q = m_1/m_2 \leq 3$



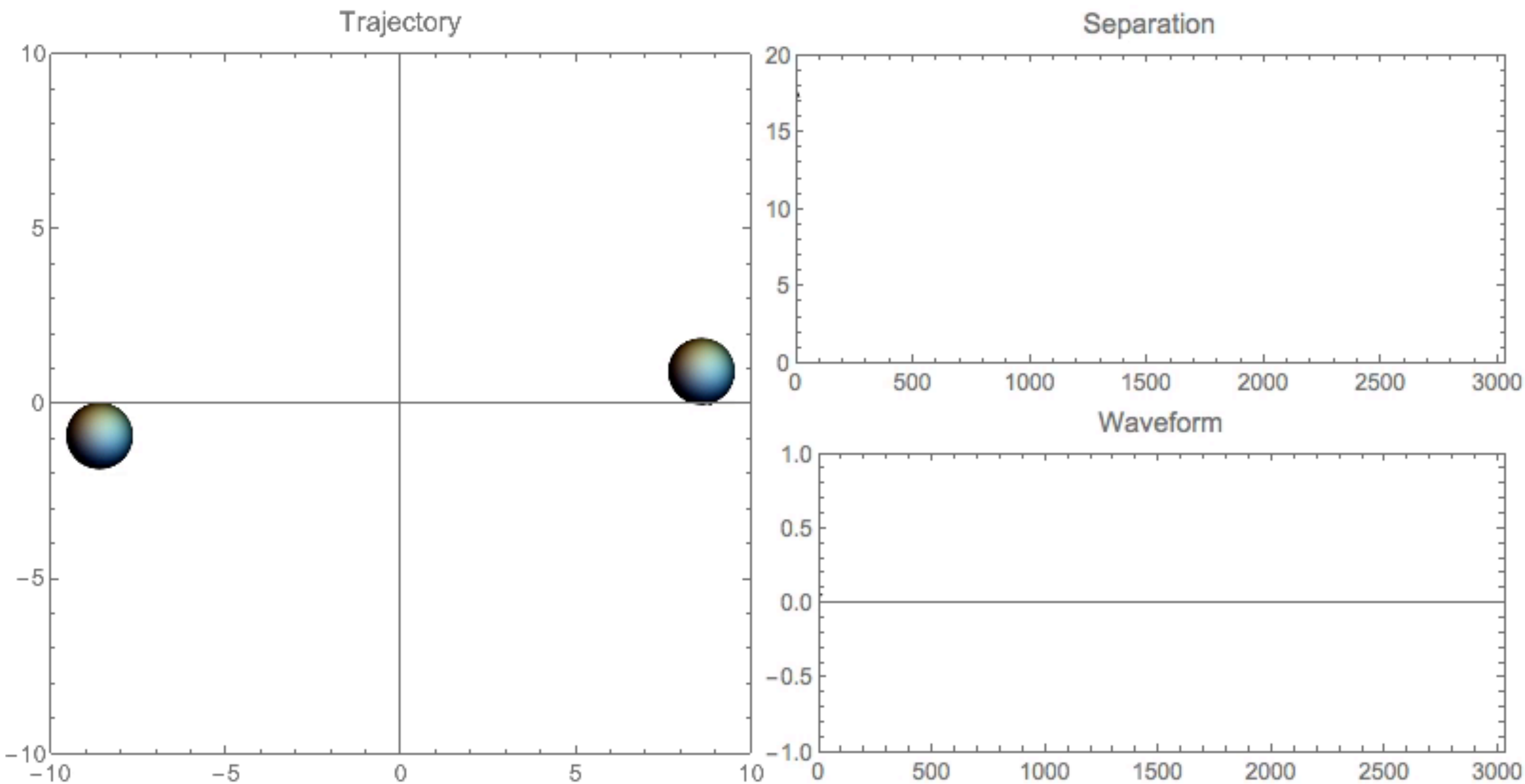
$e_0 = 0.00$



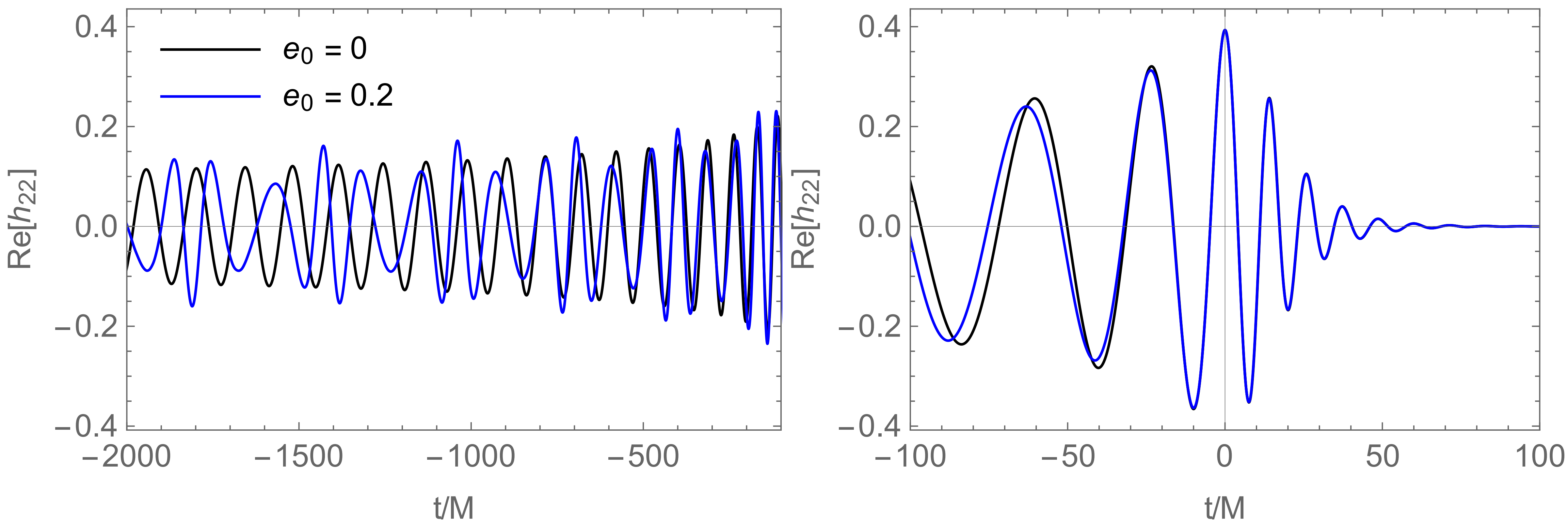
$e_0 = 0.20$



Eccentric simulation



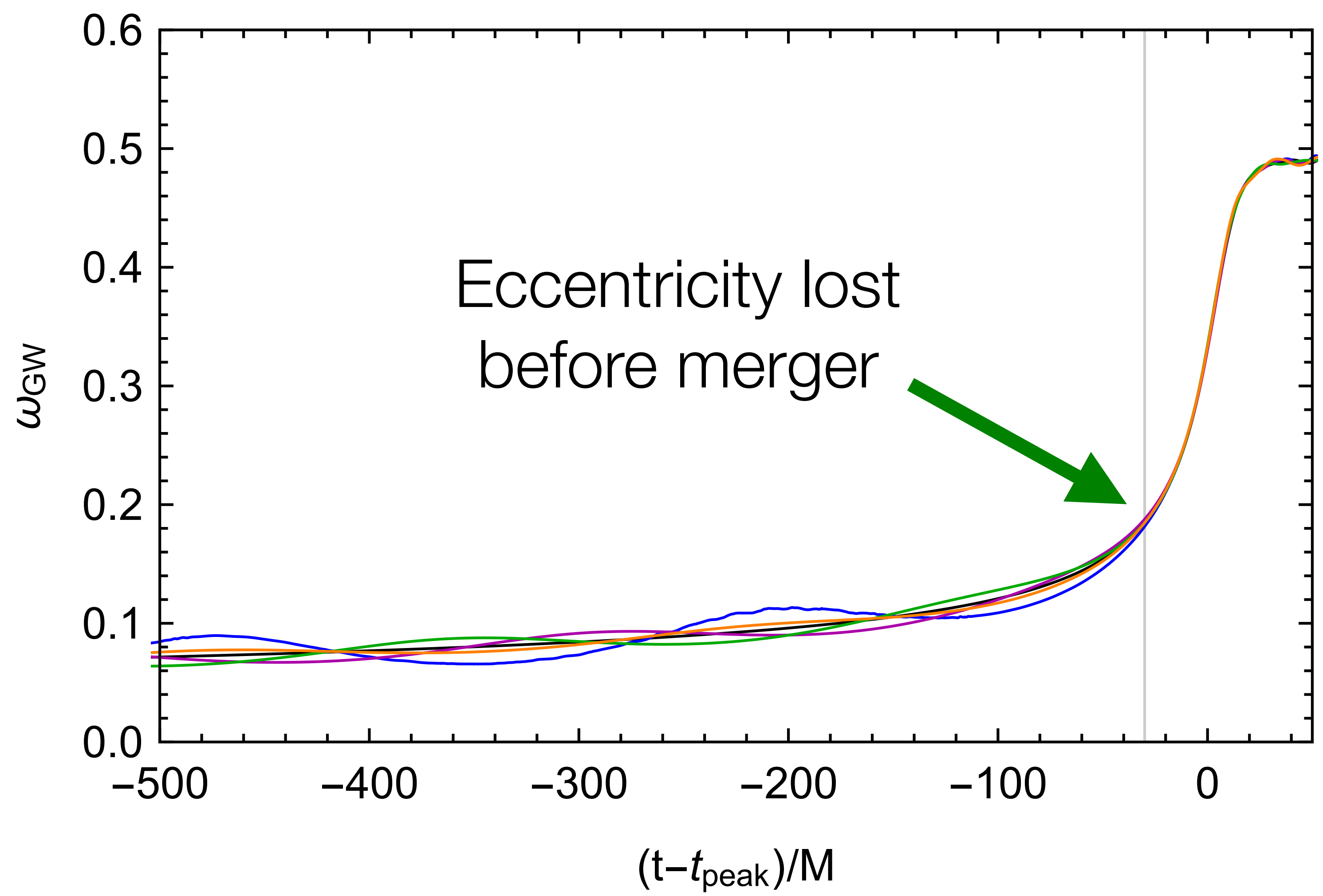
What does an eccentric BBH merger look like?



- Eccentric mergers are **circular**!

What does an eccentric BBH merger look like?

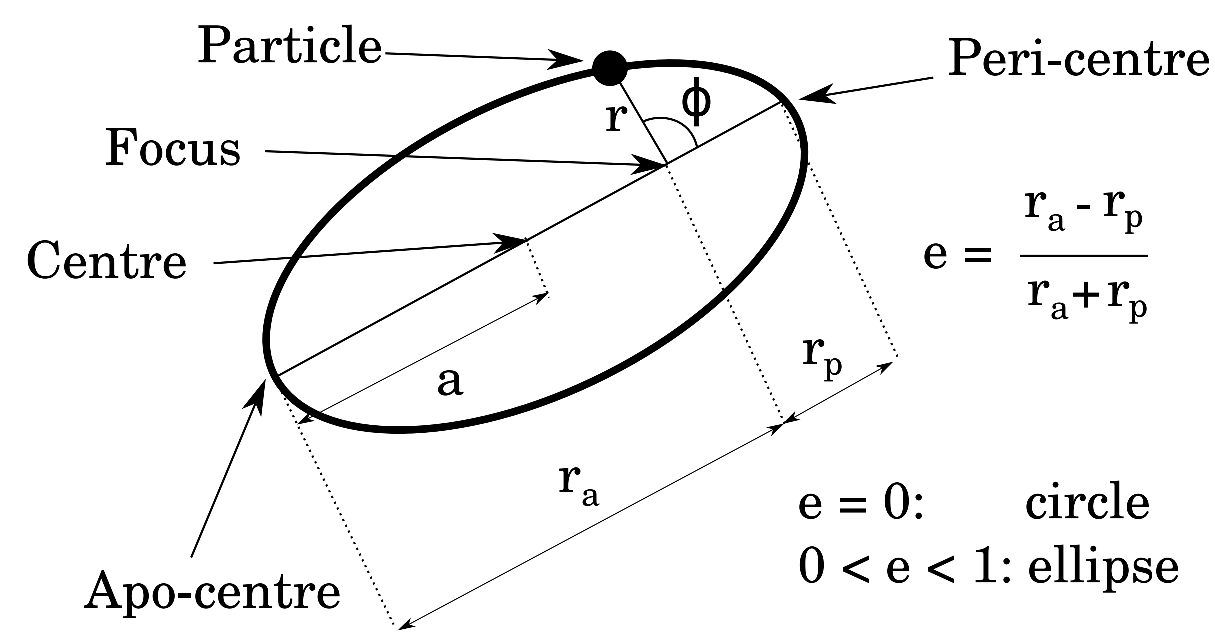
- **Circularisation** just before merger for q=1 [Hinder et al. 2008]
- Now extend to **q=3**
- For all eccentricities,
  - **Same waveform** for $t > t_{peak}$ - 30 M
  - Merger remnant has **same mass and spin**
- Can use **circular** merger model



GW instantaneous frequency (q=3) independent of e for $t > t_{peak}$ - 30 M (similar for amplitude)

Modelling the inspiral: the building blocks

- **Post-Newtonian** model:
  - **Conservative** motion (without inspiral):
    - constant E and L
    - eccentricity **e**, semi-major axis **a**
    - r, ϕ in E and L (**3 PN**)
  - **Radiation reaction**:
    - Adiabatic constants E and L integrated from **2 PN fluxes**
  - **Waveforms** 0 PN (restricted approximation):
    - h_+, h_x in r, ϕ



- See [IH et al. 2010] for details
- Empirically found best agreement with NR for PN expansion variable x (TaylorT4 x when e → 0)

PN developments

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K. Arun, Ph.D. thesis, Jawaharlal Nehru University, New Delhi (2006).
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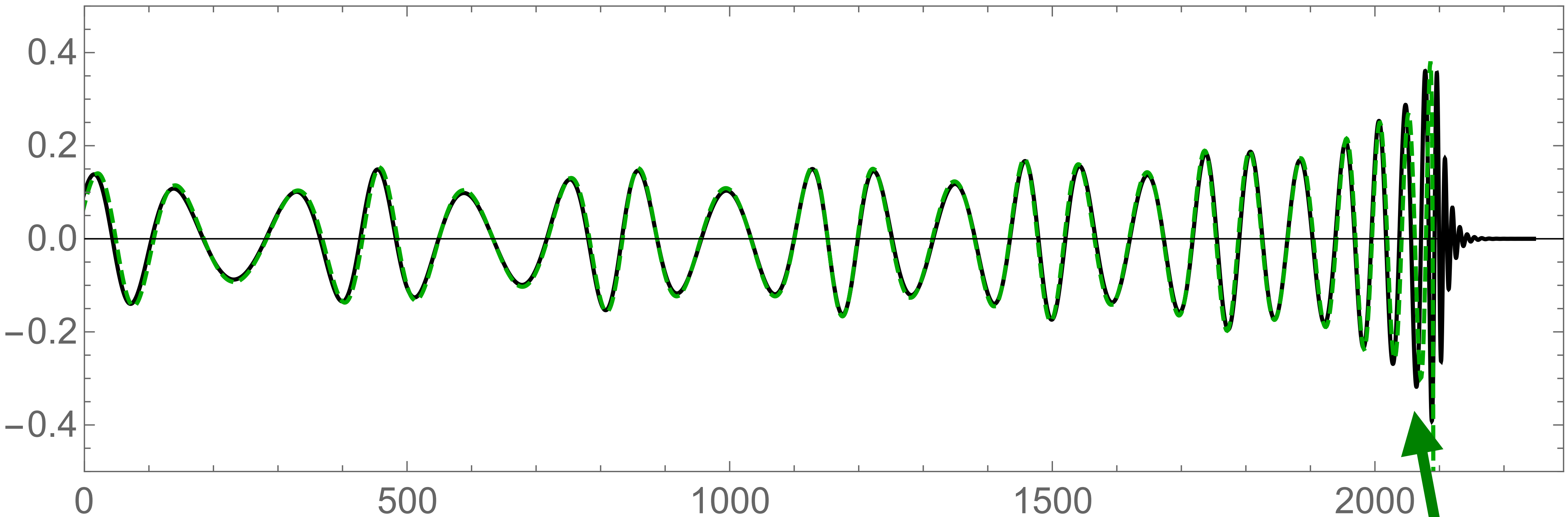
+ others

Validation of PN inspiral against Numerical Relativity

- NR and PN agree well in **inspiral** for last ~10 orbits



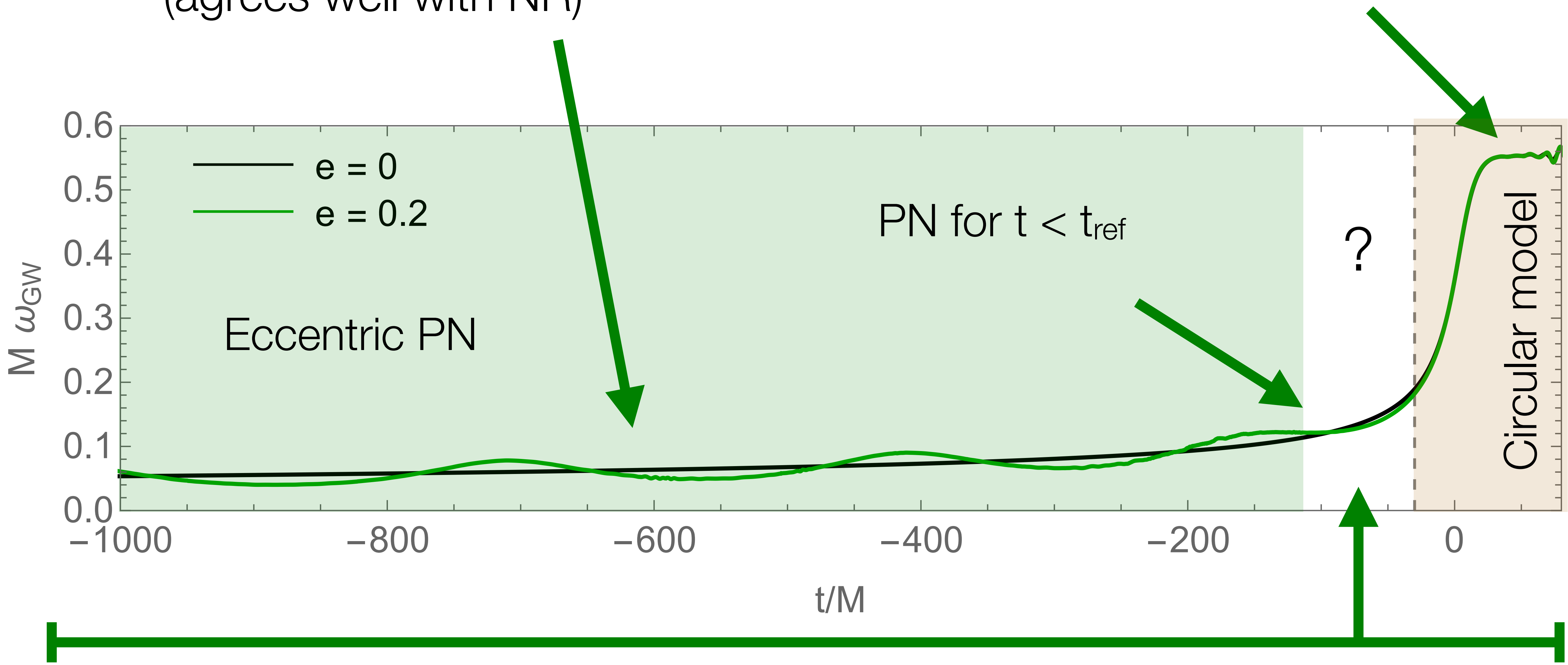
- **Fit** best PN (e,x,l,ϕ) over inspiral

PN breaks down near merger

Eccentric model construction

Use eccentric PN for inspiral (agrees well with NR)

Use quasi-circular for merger

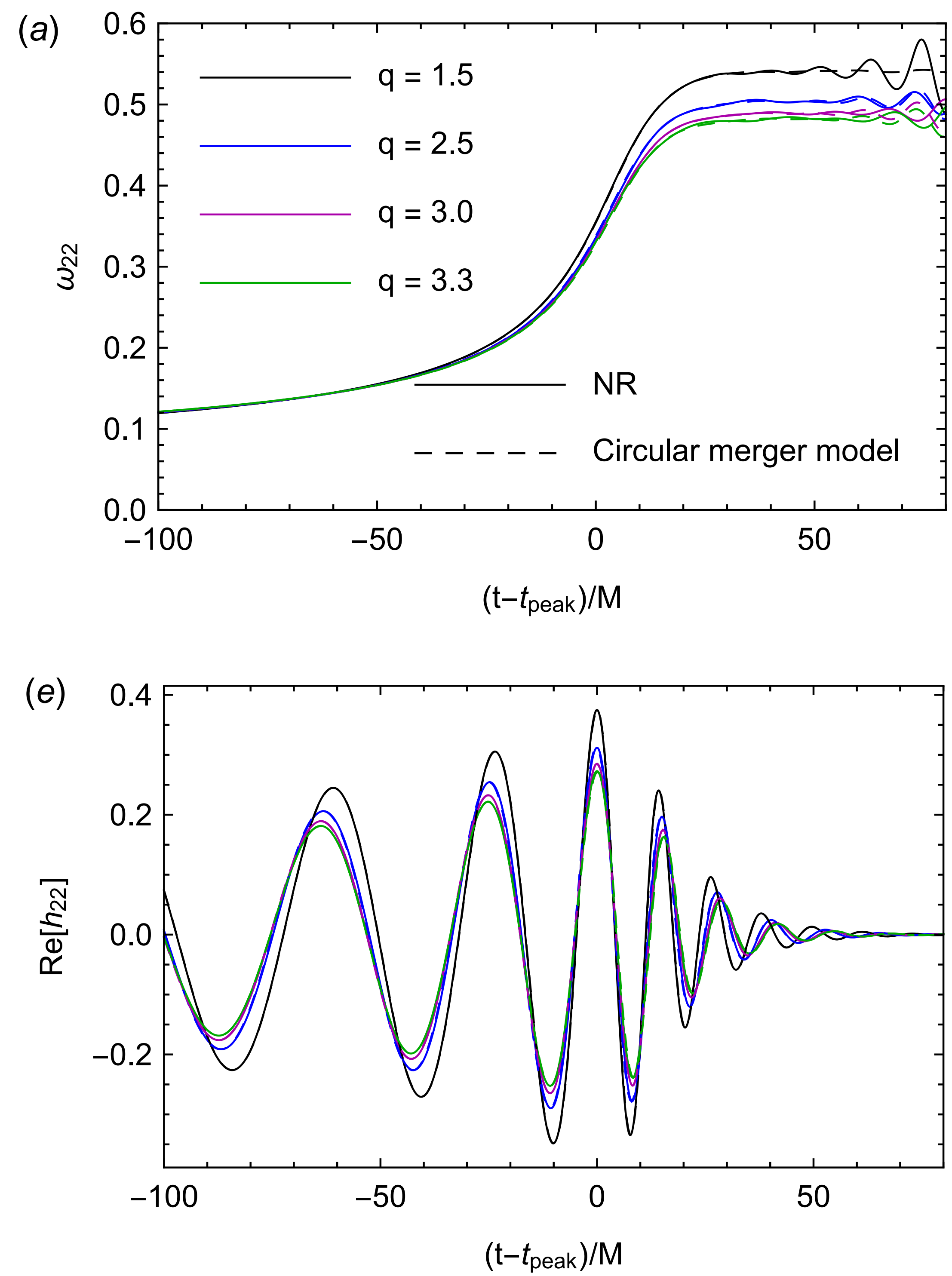


- Use a fitted ansatz for **Δt** (time to peak)
- Blend in **frequency** and **amplitude** of waveform between eccentric PN and circular

Eccentric model construction: Merger

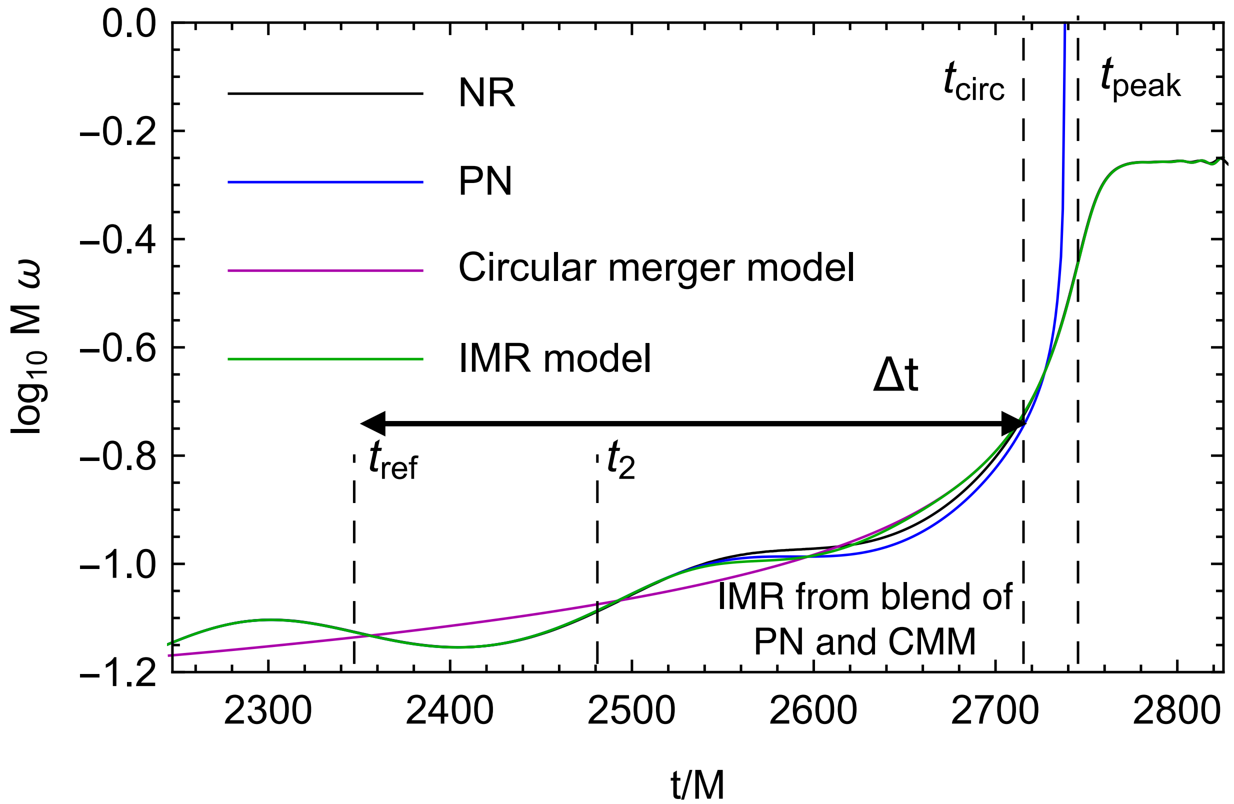
- Circular Merger Model (CMM):
  - **One-parameter** (q) family of e=0 waveforms
  - **Interpolate** ω(t) and A(t) for q ∈ {1, 2, 4} from SXS public catalogue
  - **Test** against 4 additional e=0 waveforms
- Modelling error **negligible**

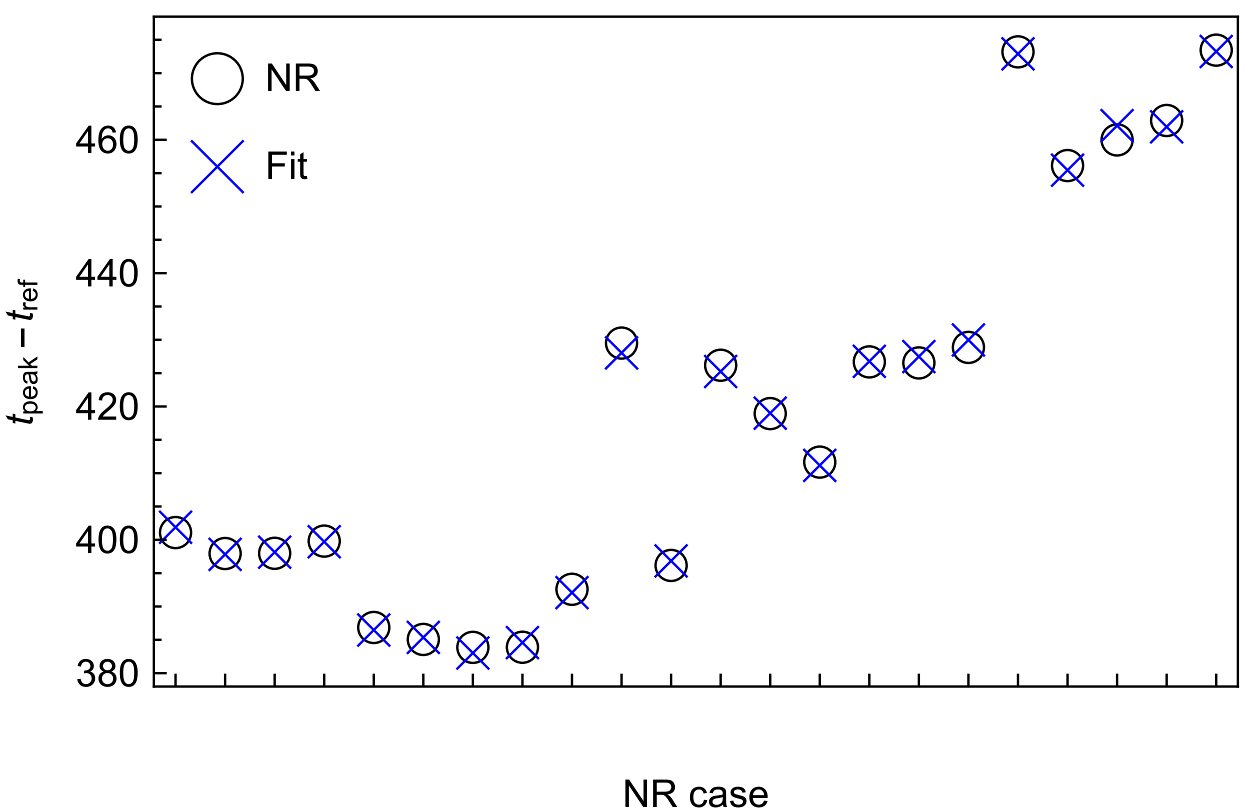
Comparison between NR and CMM for 4 quasi-circular waveforms not used to construct the model



Eccentric model construction: Transition

- Smoothly blend **PN inspiral** and **circular merger**





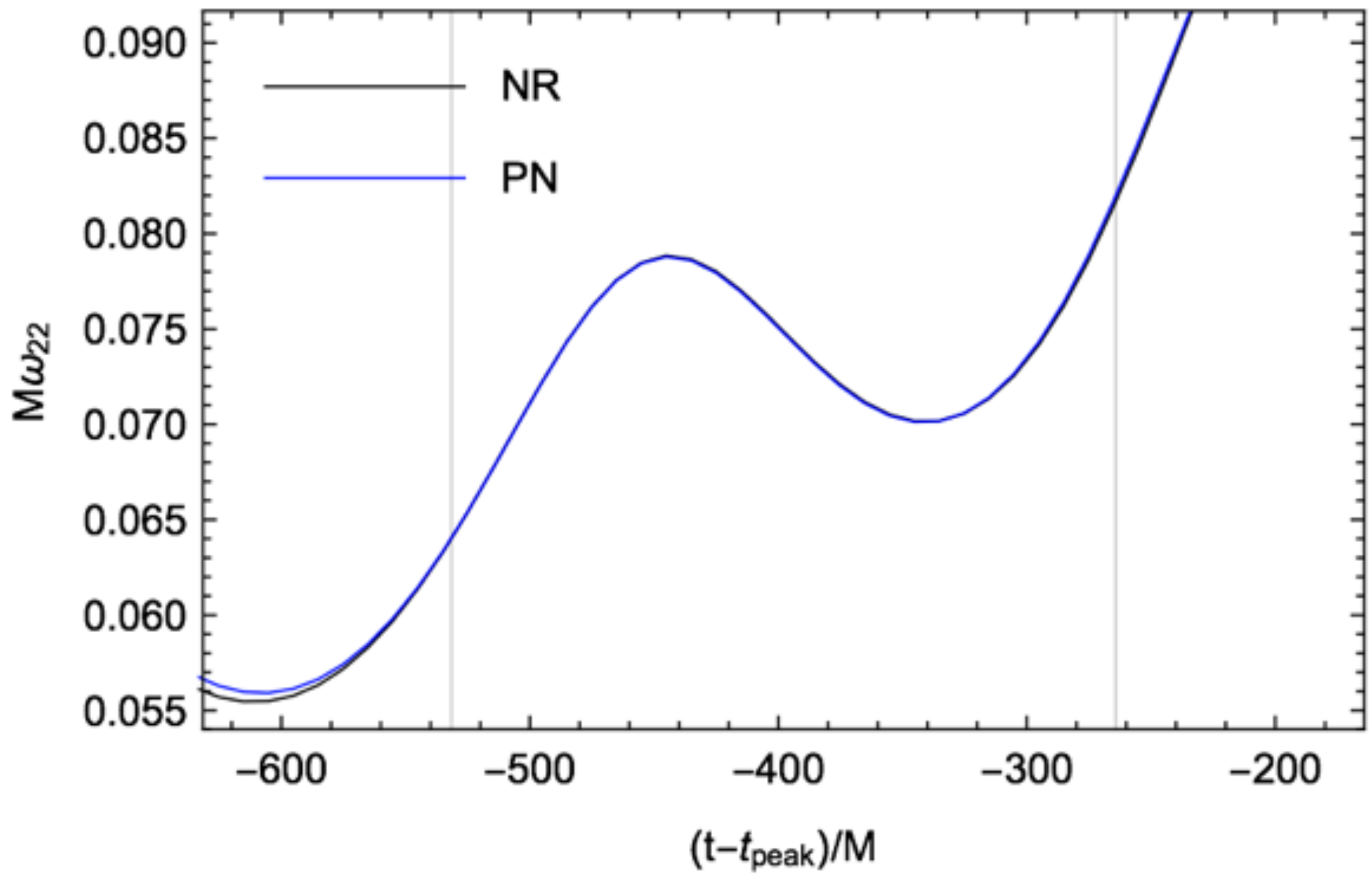
- **Blending parameters** from NR simulations
- Most important:
  - **Δt**: where is the **peak** of the merger waveform? **Fit** from NR. Fit error ±1 M.

$$\Delta t(q,e,l) = \Delta t_0 + a_1 e + a_2 e^2 + b_1 q + b_2 q^2 + c_1 e \cos(l + c_2)$$

Results: Waveform comparison

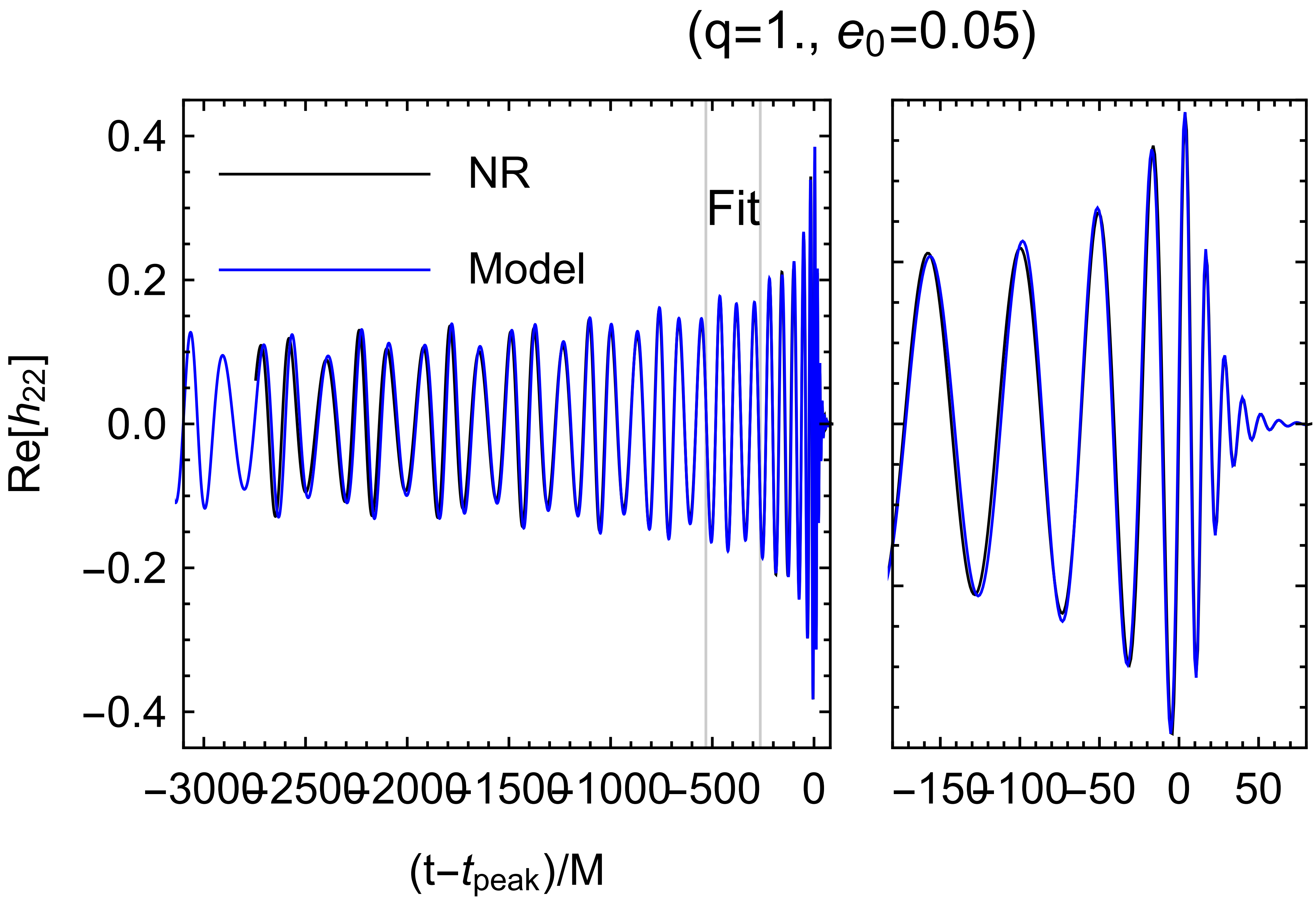
- Determine **PN parameters** of NR waveform via **1 orbit fit** ~7 cycles before peak

- Optimise PN $\omega_{GW}(x_0, e_0, l_0)$ to get best agreement with NR



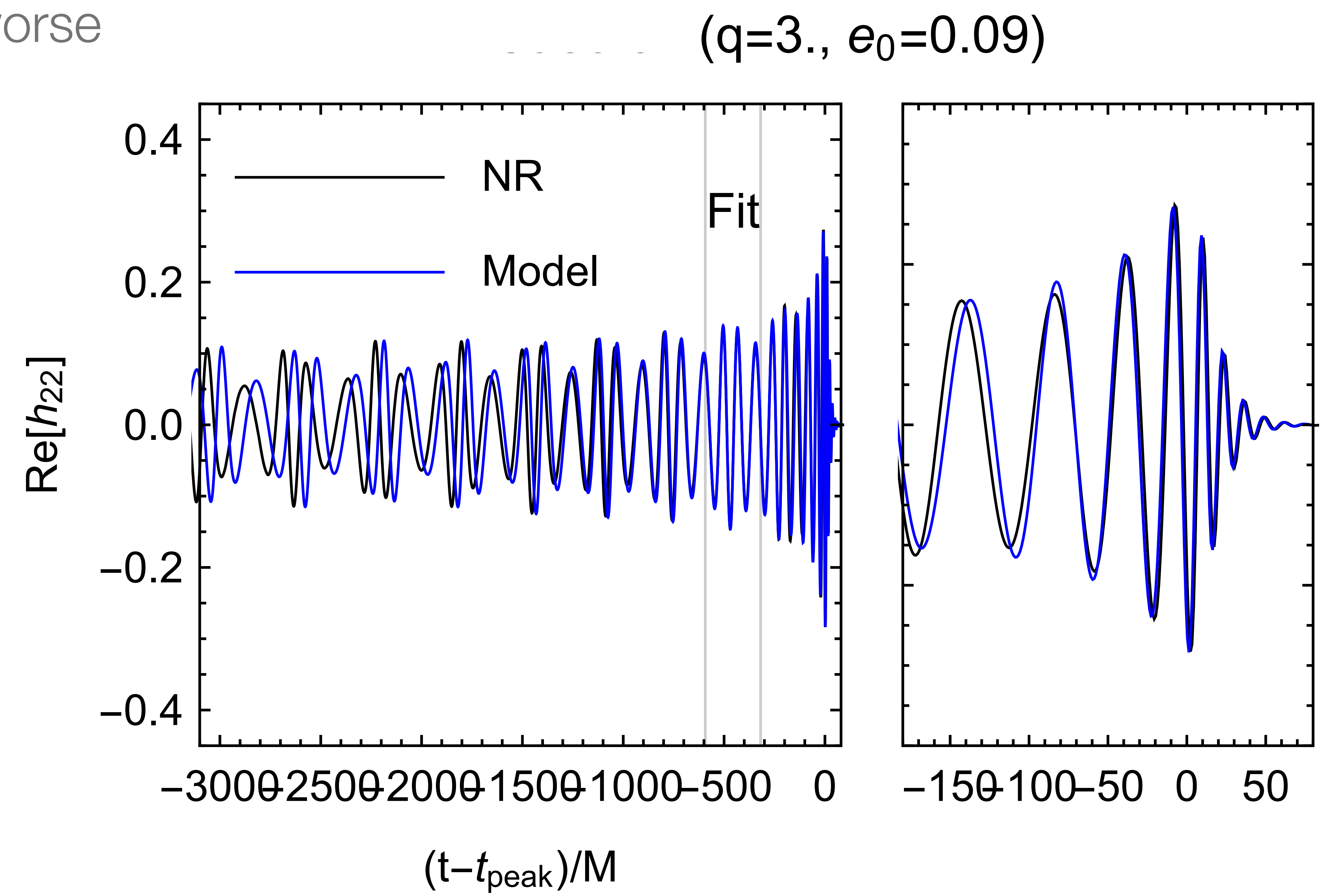
Results: Waveform comparison

- **Typical** case: **good agreement** over the ~25 cycles of the NR waveform



Results: Waveform comparison

- **Worst** case: **dephasing** of both Φ(t) (orbital oscillations) and l(t) (eccentric oscillations)
- Agreement is worse for **higher e** and **higher q**



Results: Faithfulness

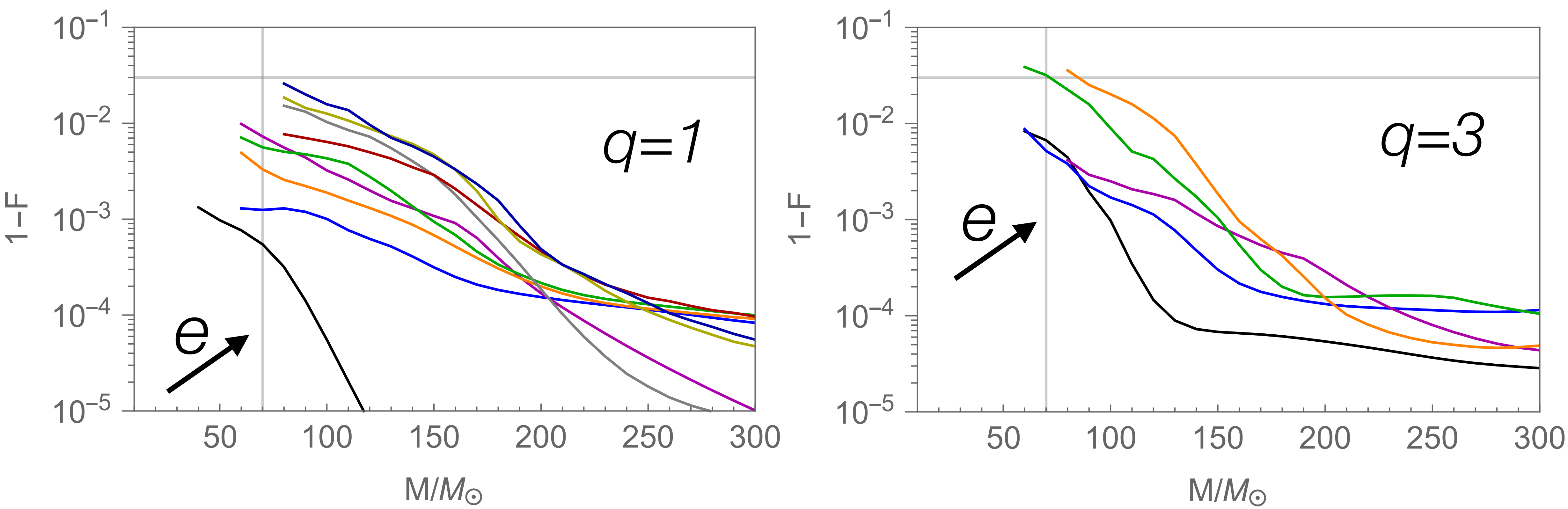
- Target **GW150914**:
  - O1 Advanced LIGO noise curve with **f_{min} = 30 Hz**
  - Short NR waveforms sufficient
- Label with **e_{ref}** from fit to PN ~**7 cycles before the merger**
- Overlap:

$$(h_1|h_2) \equiv 4\,\mathrm{Re}\int_{f_{\min}}^{f_{\max}} \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}\mathrm{d}f$$

- Faithfulness:

$$F = \max_{\phi_c, t_c} \frac{(h_1(\phi_c, t_c) \mid h_2)}{\sqrt{(h_1|h_1)(h_2|h_2)}}$$

Results: Faithfulness



- Eccentric model **faithful (97%)** with NR for $q \leq 3$:
    - For $e_{ref} < 0.05$, $M > 70\ M_\odot$
    - For $e_{ref} < 0.08$, $M > 93\ M_\odot$
- Limits on M from: (i) **length** of NR, (ii) accumulated **PN errors** (from RR)

Conclusions and outlook

- Eccentric **inspiral-merger-ringdown** BBH waveform model
  - See **[Huerta et al. 2016]** for a similar model, *not* calibrated to NR simulations
- Non-spinning, $q \lesssim 3$, $e_{ref} < 0.1$
- **Numerical Relativity** for calibration and testing
- < **3% unfaithfulness** to NR for GW150914-like events
- NR simulations and Mathematica code for model will be **public**
- Future:
  - Implications for **measurement of e** with GW detectors
  - **Longer** NR waveforms
  - **Spin**