Dynamical Formation of Black Hole Binaries in Star Clusters

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Based on the following papers

- Lee (2001) CQG, 18, 3977
Outline

• Globular Clusters

• Dynamical Formation of BH Binaries in Globular Clusters

• Properties of the Dynamical Binaries

• Summary
Short Description of Globular Clusters

• Nearly spherical objects
• Extremely dense systems
• Considered to be the oldest objects in the Galaxy
• Our Galaxy has ~150 clusters, and about 50 more could be hidden by Galactic extinction.
• Stars are formed at the same time
  • GCs have been thought to be ideal objects to study N-body dynamics and stellar evolution
Physical Parameters

- Typical masses: $10^4 - 10^6 \, M_{\text{Sun}}$.
- Metallicity is lower than the Sun.
- Current luminosity function is nearly log-normal: maybe a consequence of long dynamical evolution.

**Graph 1:**
- Cluster Metallicity [Fe/H]
- Number per Bin
- Harris 1999
- N=137

**Graph 2:**
- Milky Way GCLF
- $E(B-V) < 1$ (N=121)
- $M_{\nu}^0 = 7.4$
- $\sigma = 1.15$
Globular Clusters Host Many Compact Binaries

- Millisecond pulsars
  - ~80 times more frequent per stellar mass than the Galaxy at large
- X-ray binaries
  - Also over abundant in globular clusters by a large factor
Most Galaxies also Host Globular Clusters:

- Specific frequency $S_N$: number of GCs per luminosity

- $S_N \sim 1$ for spirals, and much higher for ellipticals

- Are extragalactic GCs similar to Galactic GC?

- What is the mean $S_N$?

Harris 1999
Dynamics of Globular Clusters

- Self-gravitating
- Composed of mostly point masses
- Weak field (i.e., Newtonian dynamics)
- General evolution is well understood, but we do not know the details mostly due to
  - Long range nature of the gravity
  - Poor understanding of the initial conditions
  - Complicated microphysics such as stellar evolution, effects of external fields, etc.
Time scales

• Dynamical time: time for a star to make a single trip across the entire system
  \[ t_{\text{dyn}} \approx \frac{v}{R} \]

• Relaxation time: time for a star to lose its initial memory of orbit (or energy)

• If the system is self-gravitating (i.e., in virial equilibrium)
  \[ t_{\text{rel}} \approx \frac{0.1N}{\ln N} t_{\text{dyn}} \]

\[ \rightarrow t_{\text{rel}} \gg t_{\text{dyn}} \text{ if } N \text{ becomes large (}> 1000) \]
Relaxation Times of Globular Clusters

- During $t \ll t_{\text{rel}}$, stellar orbit is determined by the smooth potential of the system: collisionless
- In the long run, the stellar orbit will deviate from the original one: collisional
- Dynamical evolution takes place in relaxation time scales
- Most of the globular clusters have $t_{\text{rel}} < \text{age of the universe}$
  - Dynamical evolution has to be taken into account

\[ t_{rh} = 0.138 \frac{N^{1/2} \rho_{h}^{3/2}}{m^{1/2} G^{1/2} \ln(\gamma N)} \]

Hyung Mok Lee

Globular Clusters in the Milky Way (N=146)
Course of Evolution

- Self-gravitating systems undergo core-collapse via two-body relaxation
- The central part becomes very dense
- The time scale for this is \( \sim 10 t_{rh} \)

Based on the integration of Fokker-Planck equation
Example of Cluster Evolution with Black Holes

Lee 1995

(a) Central Density
- dotted: 0.7 $M_\odot$ stars
- short-dashed: 10 $M_\odot$ BH
- long-dashed: 20 $M_\odot$ BH
- solid: Total
Dynamical Friction

- Black holes become the most massive component within $\sim 10^7$ years in globular clusters.
- In order to reach equipartition, massive component lose energy and settle into the central region (dynamical friction).
- Friction time scale is $\sim m^*/m_{BH} < 1/10$ times relaxation time.
- Central parts are completely dominated by BHs.

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Lee 2001

$m_* = 0.7M_\odot$, $m_{deg} = 10M_\odot$
Formation Processes of Black Hole Binaries

- In extremely dense systems purely dynamical processes can lead to the formation of compact binaries
- Three-body processes
- Two-body capture
Formation Rates of BH Binaries

- Gravitational Wave Capture in parabolic approximation
  \[ \Sigma_{\text{cap}} \approx 17 \frac{G^2 m^2}{c^{10/7} v_{\infty}^{18/7}} \]
  \[ \left( \frac{dn}{dt} \right)_{\text{cap}} = \frac{1}{2} < n^2 \Sigma_{\text{cap}} v_{\text{rel}} > \]

- Three-body processes (Goodman & Hut 1983)
  \[ \left( \frac{dn}{dt} \right)_{3B} \approx C \times n^3 \frac{(Gm)^5}{\sigma^9}, \quad C \sim 0.2 \]
Which is more efficient?

- Capture versus 3-body processes

\[
\frac{(dn/dt)_{Cap}}{(dn/dt)_{3B}} \approx 10^7 \left( \frac{10^5 \text{pc}^{-3}}{n_B} \right) \left( \frac{\sigma}{100 \text{km/s}} \right)^{52/7}
\]

- Globular clusters: \( \sigma \sim 10 \text{ km/s} \), and black holes would have even smaller \( \sigma \)
  - Three-body processes are more efficient
- Galactic Nuclei: \( \sigma \sim 100 \text{ km/s} \)
  - Direct capture is more efficient
Dynamical Evolution and Binary Formation

- Globular clusters are not static objects, but evolve in time
  - Core collapse followed by dynamical friction.
  - Black holes become the most massive component in short time (<few times $10^7$ years).
- Large fraction of the black holes form binaries through three-body processes
Orbits of Dynamical Binaries

- **Captured Binaries**:
  - **Very eccentric**: \(1 - e \sim 10 \left( \frac{\sigma}{c} \right)^{10/7}\)
  - **Semi-major axis**: \(a \sim 0.05 \frac{Gm}{\sigma^2}\)
  - **Merge very quickly**

- **Three-body binaries**
  - **Moderately eccentric** \(f(e) \, de \sim e \, de\)
  - **Semi-major axis**: \(a \sim 0.2 \frac{Gm}{\sigma^2}\)
  - **Merging times are very long**

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Hong & Lee 2015
Captured binaries
Binaries can interact with other stars frequently because the cross sections are large.

Hard binaries become harder (i.e., heat source) through resonant interactions (i.e., interactions with $p \sim a$).

$1/3$ goes to KE of binaries for equal mass case.

$E \sim 0.4 |E_{\text{bin}}|$.
Fate of BH Binaries

- As the orbits become tighter, the “collision time” with surrounding stars increases while merger time scale decreases.
- The recoil energy due to binary-single encounters also increases

\[ K_B \sim \frac{2}{3} \times 0.4E_B = \frac{2}{15} \frac{Gm^2}{a} \]

- If \( K_B > \frac{1}{2} m \nu_{esc}^2 \), the binaries get ejected.
- At some point recoil energy exceeds the escape energy from the cluster

\[ a \lesssim a_{crit} = \frac{Gm}{15 \nu_{esc}^2} \]

- Depending on \( \nu_{esc} \), ejection or merger take place first.
Hardness of Binaries

• It is useful to define hardness parameter $x$, as a binding energy of binaries normalized by the average kinetic energy of the background.

\[ x \equiv \frac{Gm_1 m_2 / 2a}{3m_* \sigma^2}, \quad m_*: \text{stellar mass} \]

• The condition for ejection corresponds to

\[ x_{\text{crit}} \sim 5 \left( \frac{m}{m_*} \right) \left( \frac{v_{\text{esc}}^2}{\sigma^2} \right) \left( \frac{v_{\text{esc}}^2}{\sigma^2} \right) \sim 12 \]

• Therefore \[ x_{\text{crit}} \sim 60 \left( \frac{m}{m_*} \right) \]
Schematic Evolution of Binary Parameters in $e$-$x$ space

Critical hardness for ‘merger’ before next interaction (depends on velocity dispersion)

Critical hardness for ‘ejection’

Evolution of binary in $(e,x)$ space
Numerical Simulation of BH Binary Formation and Evolution

- Direct N-body simulations
- Spherical, non-rotating models in static tidal field
- Composed of ordinary and degenerate stars
  - Ordinary stars: $0.7 \, M_\odot$ (actual average mass could be lower)
  - $1.4 \, M_\odot$, representing NS
  - $10 \sim 40 \, M_\odot$, representing BH
    - Single component
    - Two component
    - Continuous mass spectrum
- $N=5,000 \sim 50,000$
- No Primordial binaries
Some Concerns of the Simulations

- $N \ll$ actual number of stars in GC
  - We try to look for properties that are independent of $N$
- No stellar evolution
  - Stellar evolution is important only in the very early phase of evolution ($\sim 10^7 - 10^8$ years)
- No initial BH binaries
  - Soft binaries easily disrupted
  - Hard binaries behave same way as dynamical binaries, and therefore our results provide conservative lower limits
- No mass function for normal stars
- No direct capture is included
  - Ratio between three-body processes and direct capture depends strongly on $N$
  - Direct capture will be investigated separately with Fokker-Planck code
Example of N-body Run

- **BH** mass = 10 Msun
- **NS** mass = 1.4 Msun
- **Normal Stars** = 0.7 Msun
Multiple interactions lead to very tight binaries and binaries get ejected. Here, we show that the binary was ejected when $\frac{1}{t_r(0)} = 4.29$.

The x-axis represents the eccentricity, and the y-axis shows $\log_{10}(x)$. The equation for $x$ is:

$$x = \frac{Gm_1m_2}{2a} \frac{2}{3m*\sigma^2/2}$$
Properties of ejected binaries

- Ejection occurs as the binaries become very hard, consistent with the predictions
- But actual distribution of $x$ is broad
- The relative velocity to the cluster is $> v_e$. 

\[
x \equiv \frac{G m_1 m_2}{3a} \left( \frac{m_2}{a} \right) > \frac{\sigma^2}{2}
\]
How can we use these results?

- The distribution of orbital separation depends only on the velocity dispersion.

- The eccentricity distribution is almost invariant of any parameters (i.e., thermal distribution).

- Consequently ‘merging fraction’ depends on the central velocity dispersion.

Bae, Kim & Lee 2014
Models with two-mass BH: 10 & 20 M\(\odot\)

- Higher mass BHs form binaries first and the lower mass ones follow
- Time gap between formation and ejection is a few \(t_{\text{rh}}\), but it could be smaller for realistic systems

Park et al., (2017)
Time gap between formation and ejection

- As $N$ grows, $\Delta t$ becomes smaller.
- There is a long tail of large $\Delta t$
- Those with large $\Delta t$ would merge in the cluster, rather than get ejected.
Long lived binaries are very hard ones
Orbital Evolution of the Long-Lived Binary

- Experiences many weak encounters
- Composed of lower mass BHs

$t/t_{rh}(0) : 13.78$
Formed Late
Efficiency of Binary Formation with Mass

- Higher mass BHs form binaries more efficiently
- ‘Mass function’ obtained by GW observation would be skewed toward higher mass.

Park et al, (2017)
Continuous BH mass function

- BH mass function depends on metallicity
- Biased formation of higher mass BH binaries also seen here

Park et al., (2017)

BH mass function by Belczynski et al. (2016)
In two-component case, binaries with mixed masses (i.e., $10 - 20 \, M_\odot$) are very rare.

Similar trend is seen in models with continuous BH mass function.

Binaries with $m_2/m_1 > 2$ would be extremely rare.

Conflict with GW151226?

Park et al., (2017)
Galactic Globular Clusters

Median $V_{esc} = 20$ km/sec

Fraction of merger binaries within Hubble time

$f(e) = 2e$
Estimation of merger rate

- Assume 0.45 % NS (but 10% of this remains) and 0.18% BH by number in GC
- 15% of these objects escape in the form of compact binaries
- We used 142 clusters in the catalogue by Harris (2010) with mass and velocity dispersion [link](http://physwww.mcmaster.ca/~harris/Databases.html)
  - We computed the number of binaries whose merging time is shorter than Hubble time for each cluster and add them up
- Number density of globular clusters $n_{GC} = 8.7 \, h^3 \, \text{Mpc}^{-3}$ and $h=0.72$

Results

- $0.1 - 1 \, \text{yr}^{-1}$ within advanced detector range
- $\sim 10 \, \text{yr}^{-1} \, \text{Gpc}^{-3}$ for BH-BH merger
- Very small number of NS-BH binaries
Uncertainties

- Initial Mass function could have been top heavy for in low metallicity environments
- Clusters with relatively large $\sigma$ could have produced direct capture binaries
- Many GCs have been already disrupted due to
  - Galactic tidal field
  - Tidal Shocks
  - Dynamical friction
Planned Improvements

• More careful assessment of dependence on N

• Effects of the mass function of the ordinary stars
  • We assumed 0.7 Msun, but more realistic mean mass is much lower

• The rotation of the clusters could accelerate the dynamical

• Consideration of captured binaries.
Summary

• Black hole binaries can be formed by dynamical processes in globular clusters efficiently

• Most of them are formed by three-body processes.

• Some of them will merge in Hubble time after getting ejected
  • Mass ratios are likely to be less than 2
  • Massive BHs have higher chances in forming binaries
  • Merger rate is estimated to be $\sim 10 \text{ yr}^{-1} \text{ Gpc}^{-3}$
  • Some binaries could have been merged within the cluster (<10%). They are mostly composed of lower mass BHs.

• Substantial uncertainties in the estimated rate exist, but actual rate could be higher