Facing the challenge of testing GR with gravitational waves

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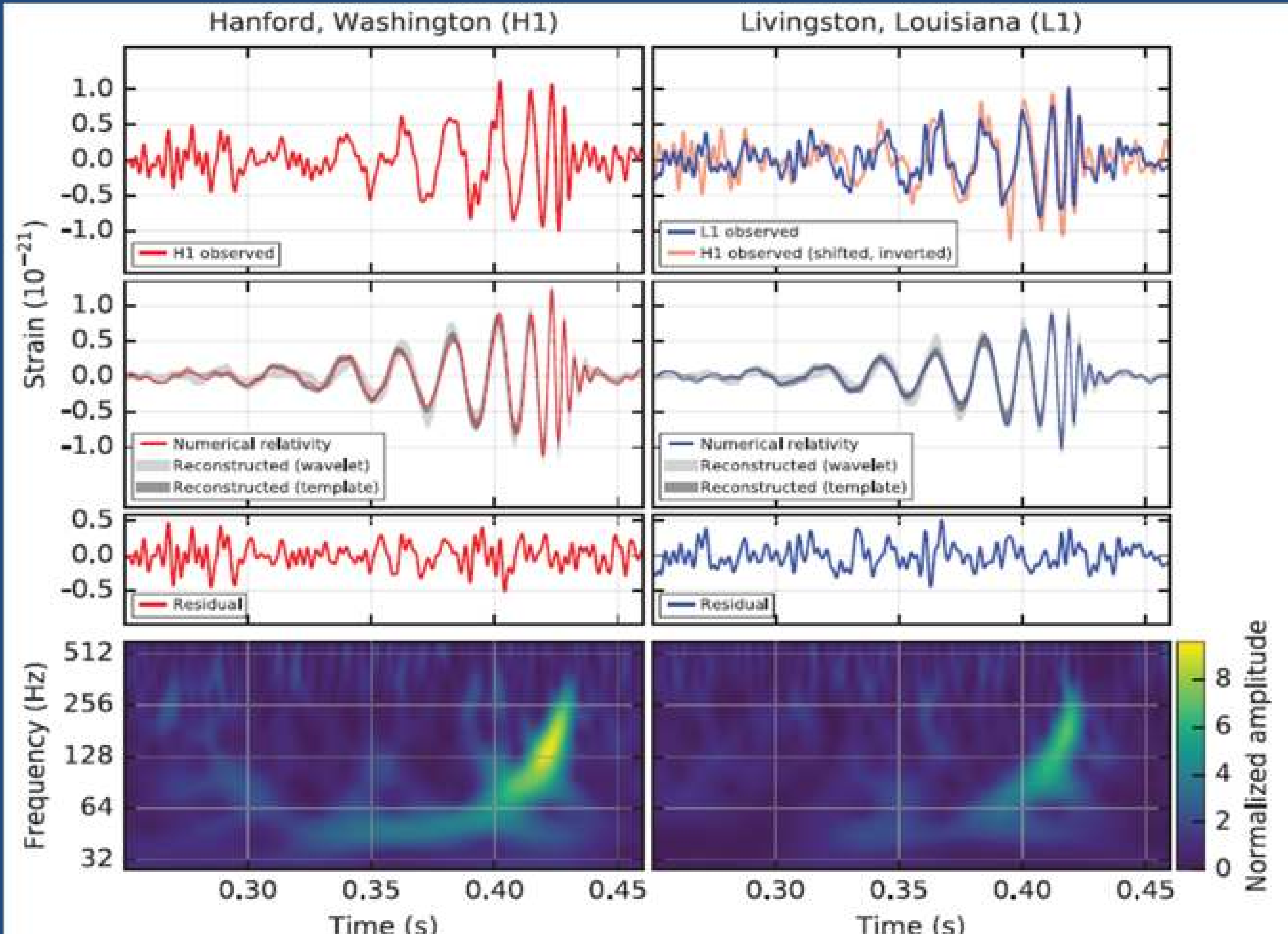
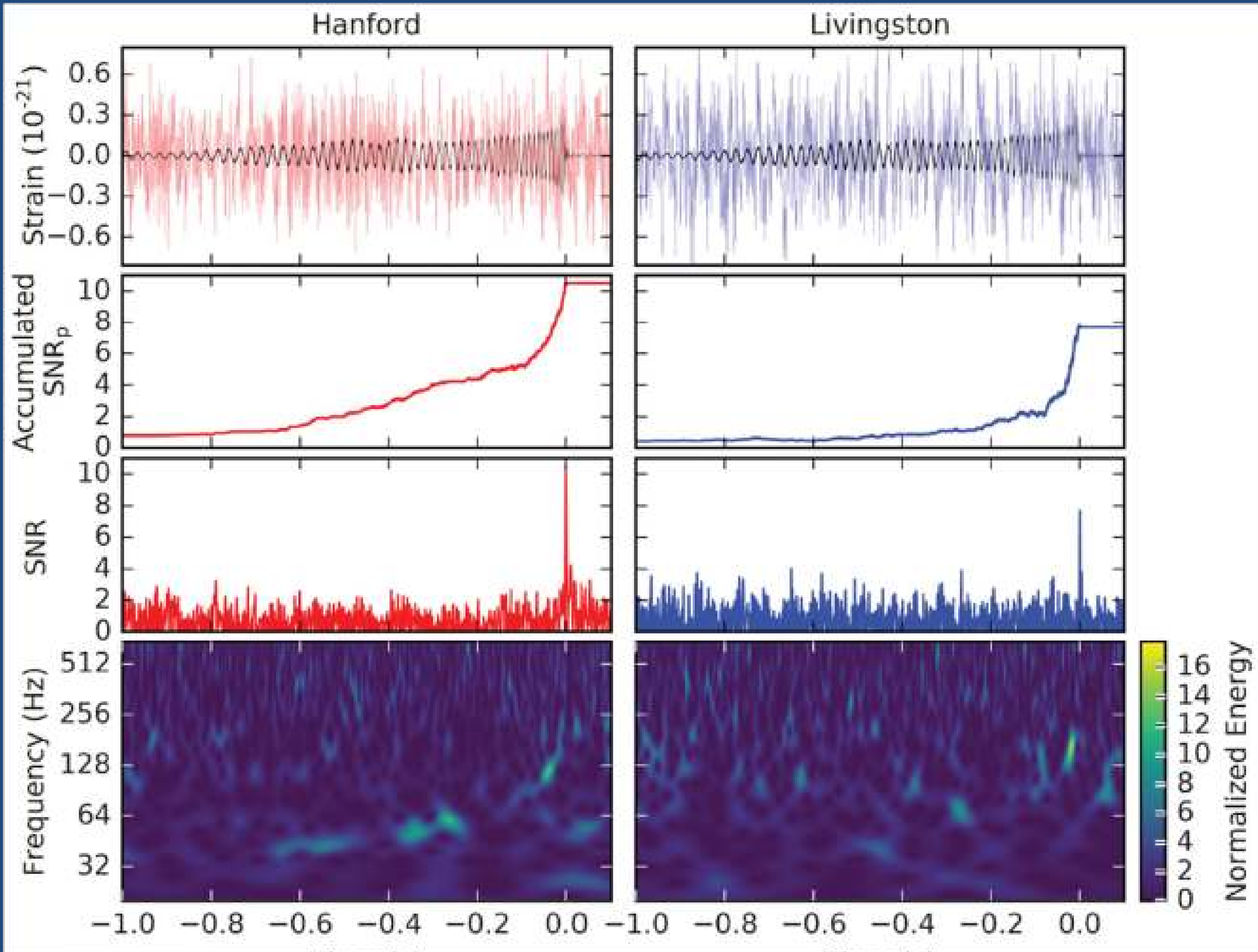


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
0.30
0.35
0.40
0.45
Time (s)
Normalized amplitude
0
2
4
6
8

GW151226


Hanford
Livingston
Strain (10^{-21})
0.6
0.3
0.0
−0.3
−0.6
Accumulated SNR_p
10
8
6
4
2
0
SNR
10
8
6
4
2
0
Frequency (Hz)
512
256
128
64
32
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
Normalized Energy
16
14
12
10
8
6
4
2
0

Implications & some framework

- GW exist & we can detect them [*and what we do & we don't detect tells us much!*]
- Signal is consistent with existence of black hole binaries as described in GR
- Much astrophysics that I won't get into

Nonlinear/highly dynamical GR meets data…

- *Is gravity described as in GR?*
- *How should one face testing extensions to GR? [deviations could be subtle]*

For this talk…

Two fronts:

- *Differences in well-defined theories*

- *How to deal with other ('loosely defined') extensions to GR?*

In both cases need predictions for digging signals out !

Gravity, beyond GR?

- Signal is perfectly consistent with General Relativity. Did we expect anything else? *[parameterized deviations better by 2-3 orders of mag]*
  - Weinberg: massless spin 2 field + diff invariance *linearized* Einstein eqns [*quadrupolar leading mode, 2 polarizations, chirping behavior, ring-down to a Kerr-Newman BH*]
  - What if additional fields (or massive) / higher curvature corrections are involved? dipolar/scalar radiation? 6-modes of polarization. Is there a well-defined theory? [very few, e.g. the ‘family’ dubbed Scalar-Tensor. Interesting phenomena but the latter requires, eg non-vacuum scenarios to show differences]

- So far, evidence is that any departure will be subtle! Predictions & new ideas on how to go after them are important

Digging deeper in the data

- Binary black hole problem *``essentially solved'' just in GR*
  - Given physical parameters from the binary, at some given (large) separation, the full wavetrain is *uniquely* determined. That is, *the full content of the signal* is a priori known
  - Particularly clear stages of the dynamics can be identified.
  - Properties of such stages can be exploited in multiple detections for digging deeper
    - E.g. QNMs a la GR   No hair property

Example

- Let's concentrate on the 'after-merger' regime. GR predicts the signal is dominated by QNMs. For simplicity let's focus on the leading (2-2) and one of the sub-leading modes (3-3). The signal at the detector is:

$$s_j = n_j + h_{22,j} + h_{33,j}\,, \tag{1}$$

where the subscript j refers to the jth event, n_j is the corresponding detector noise, and $h_{\ell m,j}$ is a ringdown mode of the form (for $t > 0$)

$$h_{\ell m,j}(t) = A_{\ell m,j} e^{-\gamma_{\ell m,j} t} \sin(\omega_{\ell m,j} t - \phi_{\ell m,j})\,. \tag{2}$$

For each ringdown mode, $(\omega_{\ell m,j} + i\gamma_{\ell m,j})$ is its complex frequency, $A_{\ell m,j}$ its real amplitude, and $\phi_{\ell m,j}$ its constant phase offset.

A key observation is that *A, ω and ϕ are known from the model*

- Thus, we can consider *adding coherently* different signals targeting specific modes/behavior with N events:

1. Pick any given event and define $\omega_{33,1} =: \omega_{33}$ & $\phi_{33,1} =: \phi_{33}$
2. Define $a_j = \omega_{33,j} / \omega_{33}$ & $\Delta_j = (\phi_{33,j} - \phi_{33}) / \omega_{33,j}$
3. Shift/rescale each $s_j(t) = s_j(t/a_j + \Delta_j)$
4. Add them!, $s = \Sigma\, c_j\, s_j$

The resulting sum contains a single oscillating frequency ω_{33} and a collection of rescaled ω_{22}'s

The rest… are details of Bayes analysis and facing the fact that parameters have uncertainties.

[Yang,Yagi,Blackman,LL,Pascalidis,Pretorius,Yunes '17]

Hypothesis testing

- For simplicity work in freqn domain and consider N events
- Hypothesis:

$$H1 : y := s - h_{22} = n + A\, h_{33;} \quad H2 : y := s - h_{22} = n$$

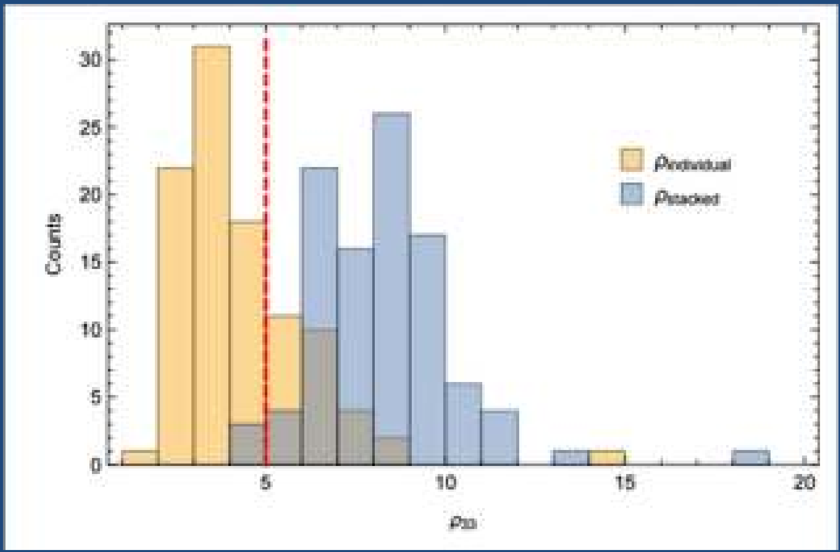
- $P_A \sim \exp(- \Pi_f \; 2\, |y - Ah_{33}| / S_n \; |^2)$ (Probability function for the 2nd mode to be present)

- With P_A perform a Generalized Likelihood Ratio Test ($<h_{33},y>/|h_{33}|> \gamma$), obtaining the requirement to favour H1 over H2 & $H_{33} \sim <h_{33}>$ (1+offsets)

$$\rho_{33} \equiv \frac{\sqrt{\langle \mathbf{H}_{33} | \mathbf{H}_{33} \rangle}}{\sqrt{1 + \sigma_{\mathrm{p}}^2}} \geq \rho_{\mathrm{crit}} \,,$$

- With ρ_{crit} related to the false alarm and detection rates and σ_p the variance of distribution. For 0.01 and 0.99 respectively $\rho_{crit} = 4.65$.

- Assume uniform merger rate (40 Gpc^{-3} yr^{-1})
- For simplicity no spins in individual BHs (in the binary) and masses in [10-50]M_O
- Adopt zero-detuned, high-power noise spectral density for aLIGO at design sensitivity.
- Distribute events up to z=1
- Use MC for sampling
- 40-65 events with $\rho_{22} > 8$



- Without 'stacking' 28% chance
- With 'staking' 97% chance of detecting 33 mode in 1yr of observation. [if rate is 13 Gpc^{-3} yr^{-1}, 12% and 50% instead]

- Of course, the idea is more general than this application
  - Dig main mode in low SNR/low mass events
  - Dig pre-merger modulations
  - Etc.

- Also, if *full waveforms are unknown,* but particular features are: ‘incoherent’ stacking [e.g. without known phase] can still be implemented (to be submitted) ; e.g
  - Post-merger oscillations in BNS
  - QNMs in extensions to GR
  - etc

Post-merger oscillations in BNS merger (in ET/CE!)

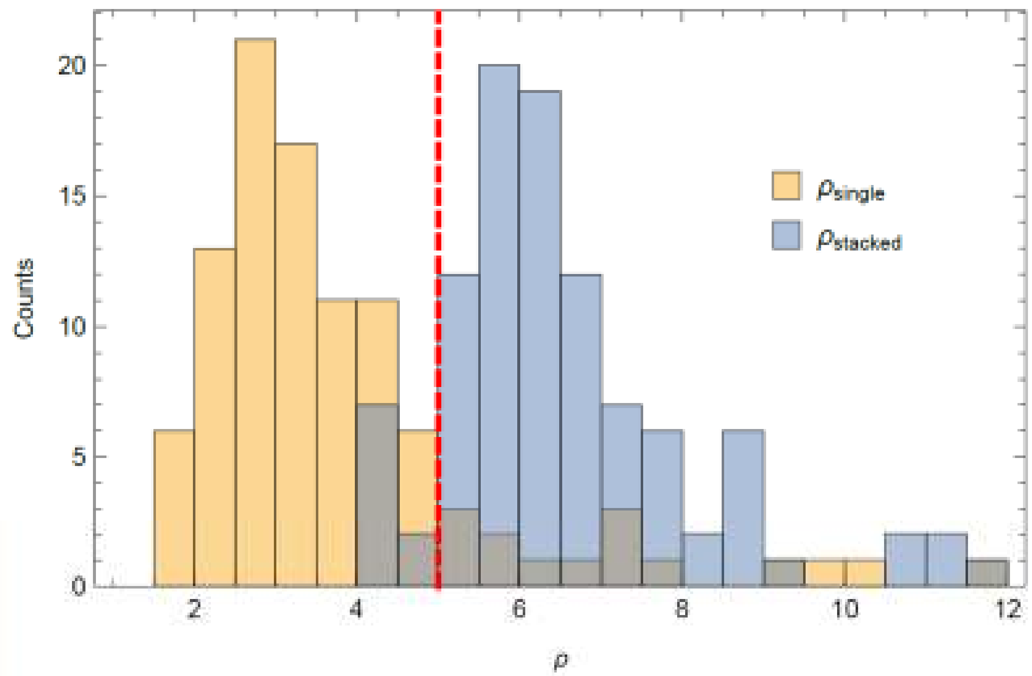


FIG. 1. Histogram for single event signal-to-noise ratio ρ (orange bins) and ρ for the stacked signal constructed from 15 loudest events in each MC realization (blue bins) with the TM1 EOS assuming one year observation with CE. The detection threshold is set to $\rho = 5$ (red, dashed line). Note that the detection chance is increased from $\sim 15\%$ to $\sim 90\%$ after stacking.

Going after 'beyond' GR

- Make sure we know what to expect in GR
- 'Phenomenological' approaches [ppE,Tiger,EOB modified…]
  - (pro) rather generic, (difficulty): tie to physical principles, (warnings) are modifications monotonic in frequency?
- 'Specific' to particular theories
  - (pro) direct veto power to principles/theories, (difficulty) many options lead to ill-defined problems, (warnings) yet another axis on template parameter space

Beyond GR I?

- Restricting to theories known to allow for well-posed problems. I.e. those where one can show $|u(T)| \leq ae^{bt}|u(0)|$
- Few options known to be amenable to well defined initial (boundary) value problems. Examples: Scalar-Vector-Tensor theories.

Scalar-Tensor (ST) *[Damour-Esposito Farese]*

$$S = \int d^4x \frac{\sqrt{-g}}{2\kappa}\left[\phi R - \frac{\omega(\phi)}{\phi}\partial_\mu\phi\partial^\mu\phi\right] + S_M[g_{\mu\nu}, \psi]$$

Scalar-Vector-Tensor (EMD)

$$S = \int d^4x\sqrt{-\tilde{g}}\, e^{-2\phi}\left[R + \Lambda + 4(\nabla\phi)^2 - F^2 - \frac{H^2}{12}\right]$$

What's new here?

- Additional charge on compact object: 'α'
  [sensitive to compactness and asympt value of scalar]
- Renormalize G
- Dipole radiation
- Dynamical/induced scalarization non-monotonicity of scalar charges!

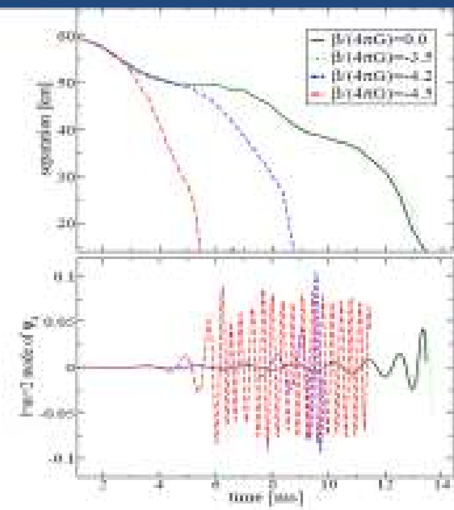


FIG. 1: The separation and the dominant mode of the ψ_4 scalar (encoding the effect of GWs) for a binary with gravitational masses $[1.58, 1.67] M_{\odot}$, and for different values of β.

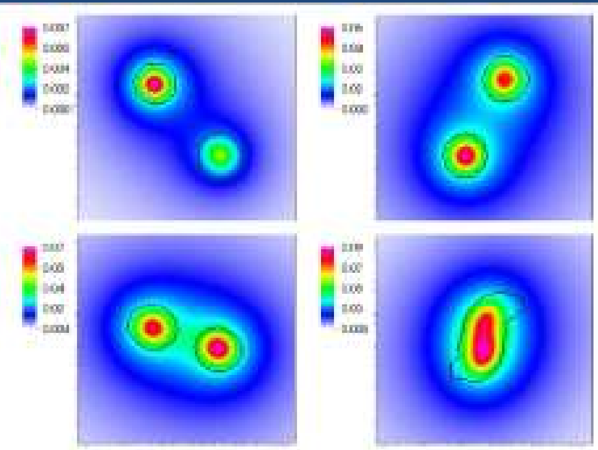
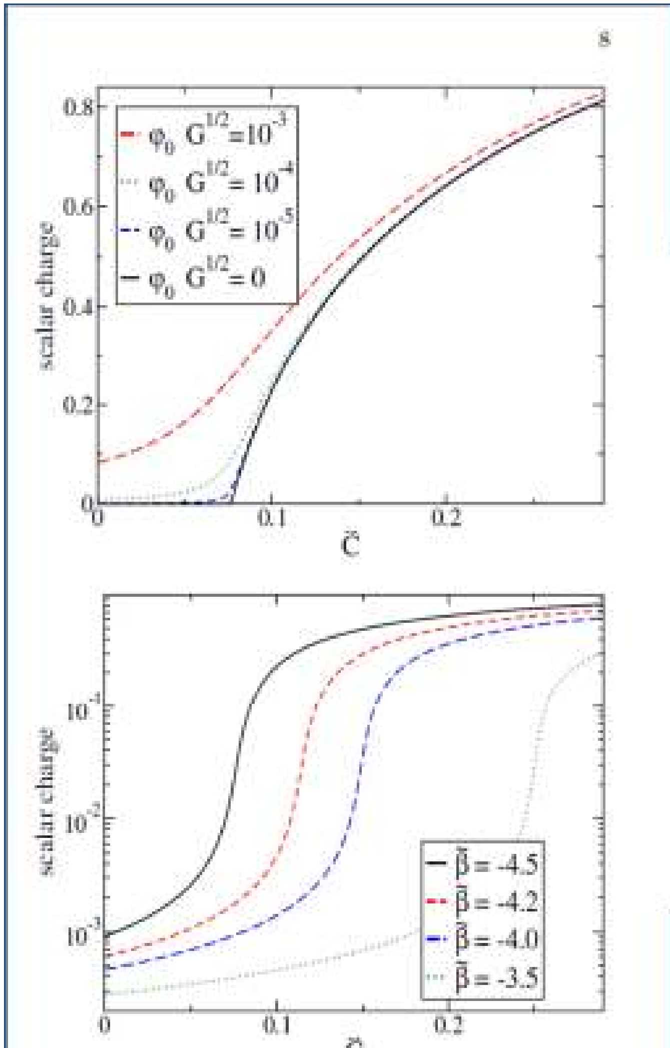


FIG. 2: The scalar field $\varphi G^{1/2}$ (color code) and the NS surfaces (solid black line) at $t = \{1.8, 3.1, 4.0, 5.3\}$ms for $\beta/(4\pi G) = -4.5$, and the binary of Fig. 1



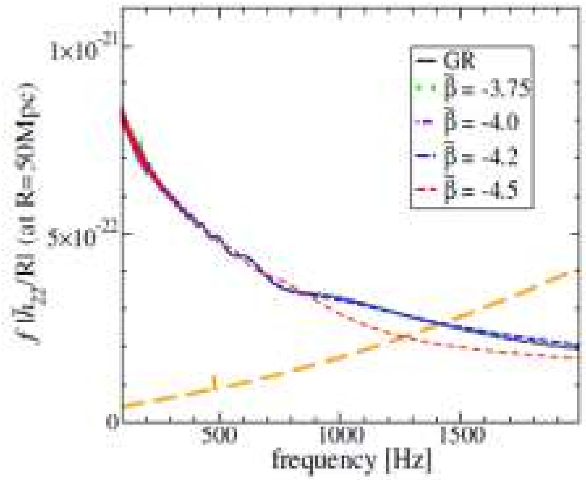


FIG. 8: Fourier spectrum of the $l = m = 2$ gravitational-wave mode for a binary with $m_1 = m_2 = 1.41 M_\odot$, for different values of $\bar{\beta}$. The dashed, monotonic line shows $\sqrt{f S_n(f)}$, where $S_n(f)$ is the noise power spectrum of Advanced LIGO. Departures from GR become visible at $f \simeq 800$ Hz for $\bar{\beta} = -4.5$.

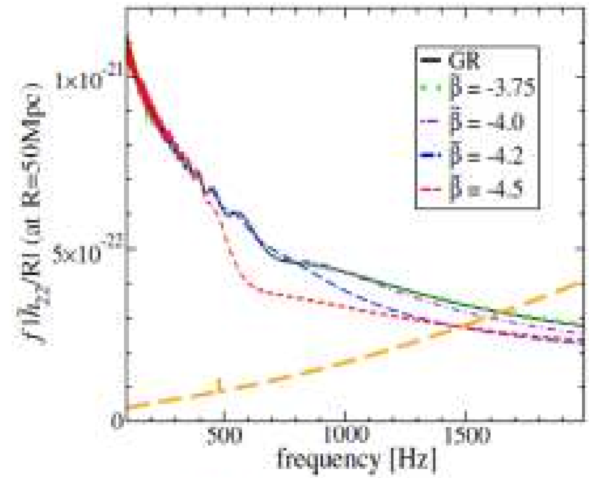


FIG. 10: Same as Fig. 8 but for a binary with $m_1 = m_2 = 1.51 M_\odot$. Deviations from the expected GR behavior arise as early as 500 Hz for $\bar{\beta} = -4.5$, and 800 Hz for $\bar{\beta} = -4.2$.

- Differences can be significant, but could be ‘degenerate’ with equation of state variations
- EM counterparts can be significant aids

[Barausse,Palenzuela,Ponce,LL][Sampson etal]

Black holes (EMD)

- In the absence of matter, scalar charge ‘induced’ through coupling with vector field [or time dependent cosmological constant]
- Charge largely independent of asymptotic value of scalar field. Proportional to $\alpha_0(Q/M)^2$ or Q/M [for small/large coupling]

--behavior interpolates KN to Kerr!

- For small values only subtle differences in dynamics and radiation characteristics

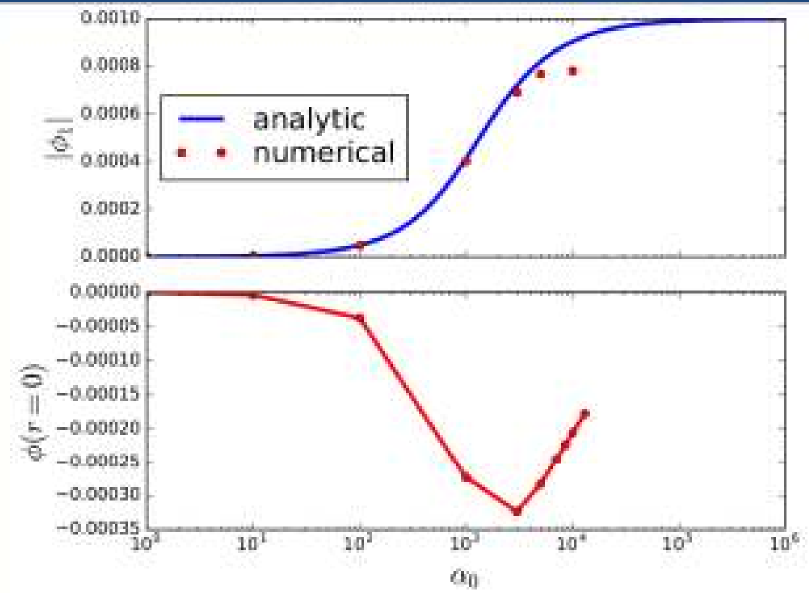
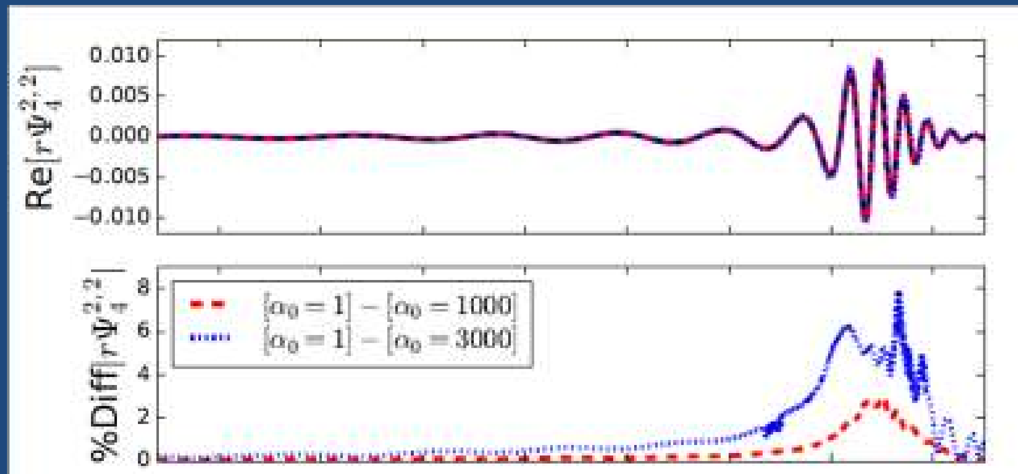
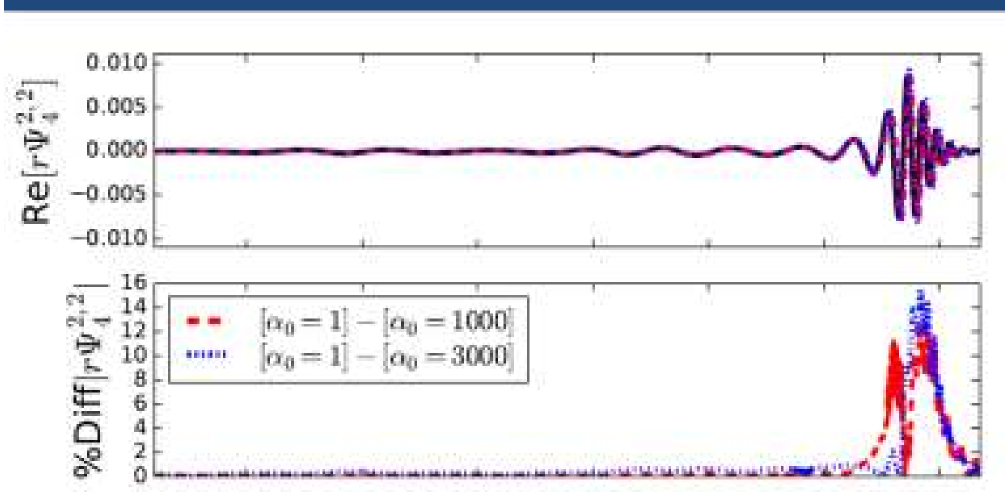


FIG. 3: Dilaton behavior for a single, electrically charged, non-spinning black hole with $q_e = 0.001$, $M = 1$, and $\phi_0 = -10^{-10}$. These quantities have been extracted at late times when the solution settled down to a roughly stationary solution. (Top) The dilaton scalar charge along with the analytic value, expression (38), for $q_e = 0.001$ and $M = 1$. (Bottom) The central value of the scalar field at late times.

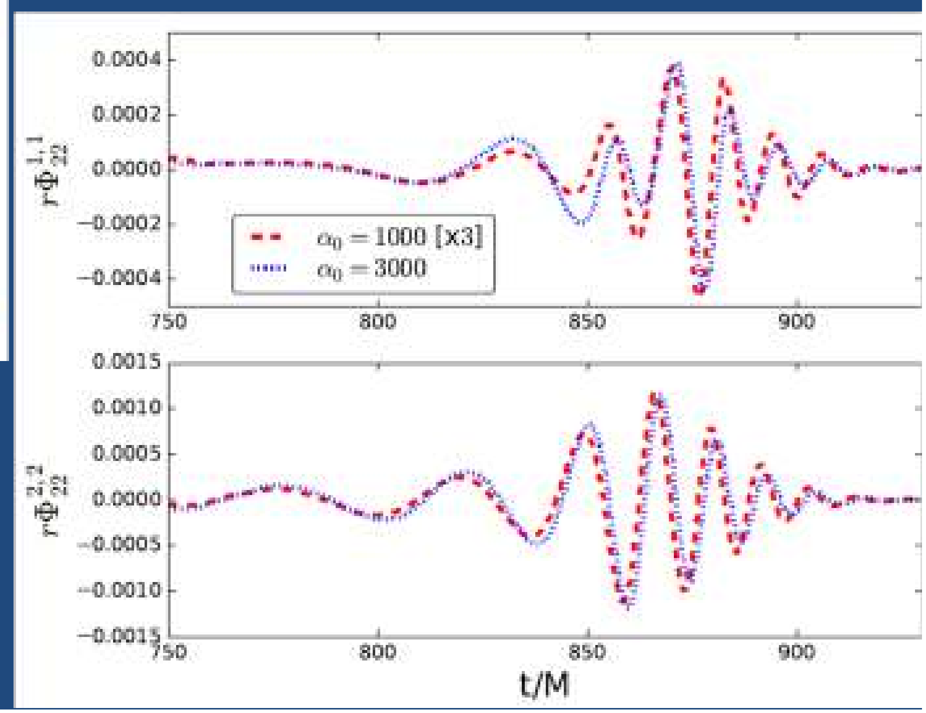
[Hirschman,LL,Liebling,Palenzuela, 1706.09875; Jai-akson,Chatrabhuti,Evnin,LL 1706.06519; also, Glampedakis,Pappas,Silva,Berti 1706.07658]



[equal mass case]



[unequal mass case q=2/3]



Beyond GR II?

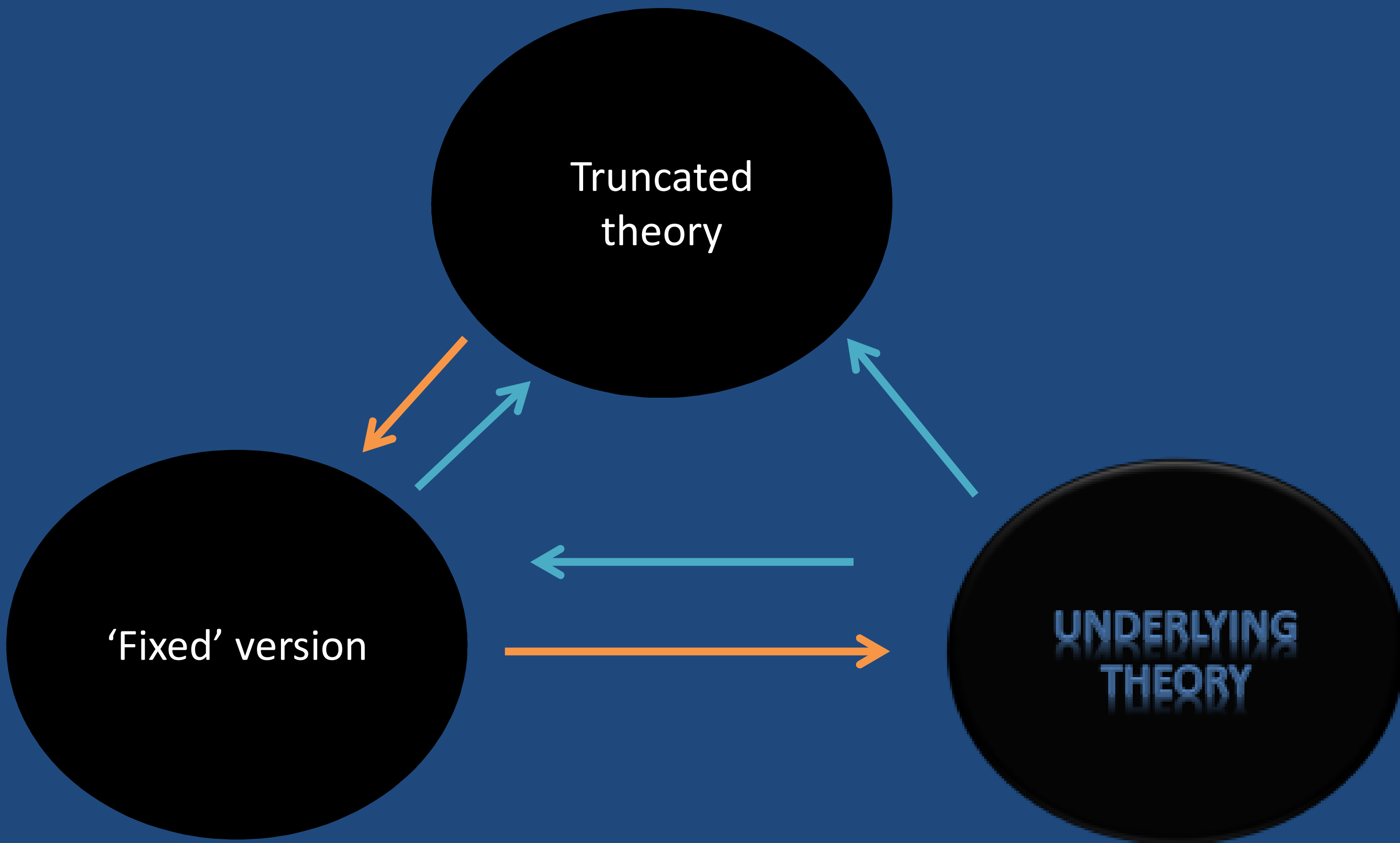
- Many extensions contemplate: *R + k (higher order curv)*
  - Nicely/loftily motivated by QG, Λ/DM considerations, EFT, etc
  - Some degree of testing in weak-field scenarios over *very specialized* backgrounds
  - Problems: higher derivatives, very dubious character (or unknown character) of resulting equations of motion, possible runaway of energy to UV, etc. [ill-posednes… Hadamard would even say throw it away!]
  - How to ask nonlinear qns from potentially (arguably/bettably/swearably) sick theories? Need to understand non-linear regime and what to expect, or hope!, will happen with higher order curvature corrections

'clashing' philosophy/expectations

- What to do in non-linear regimes?
- Deviations from GR 'should stay small', if they don't one is outside the domain of applicability.
  - But, generically, ill-posed systems will have a runaway to the UV
  - At the linear level, a freqn cut-off can be introduced
  - At the non-linear level: lower-order terms & background solution generate, in particular, higher frequencies even if ID is well within the cut-off.

  - So, for progress, something *somehow justifiable* must be done at the *'practical' level*

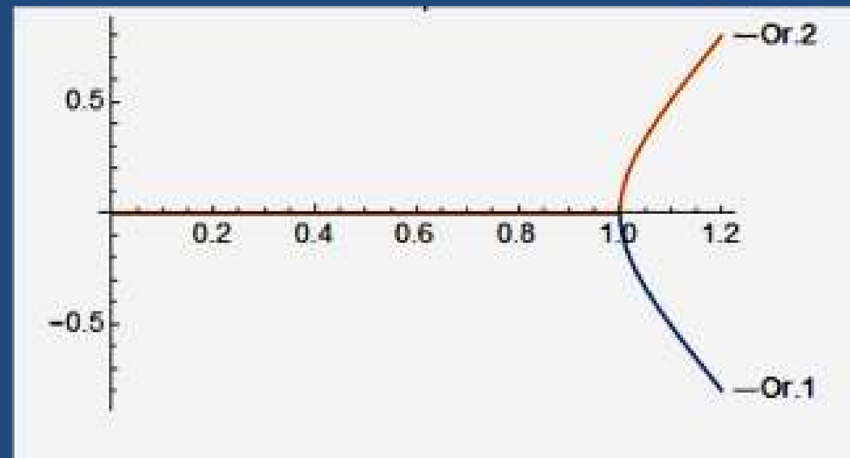
Ie, something that could provide realistic answers to the underlying theory we would like to study.



‘Practical level’: example

- Take:

$$\Box\phi = \lambda\partial_x^4\phi,$$



- One option: consider $\phi = \psi + \varphi$
- Assume φ is $O(\lambda)$
- Study Box(ψ) $= 0$; Box(φ) $= \lambda\ \psi_{,xxxx}$
- Consistent solution if $|\lambda\ \varphi_{,xxxx}|$ stays small
- Why would this be the case?

example redux

- Define $\Pi = \phi_{,xx}$

$$\begin{aligned} \Box\phi &= \lambda\partial_x^2\Pi, \\ \sigma\Box\Pi &= \partial_x^2\phi - \tau\partial_t\Pi - \Pi \end{aligned}$$

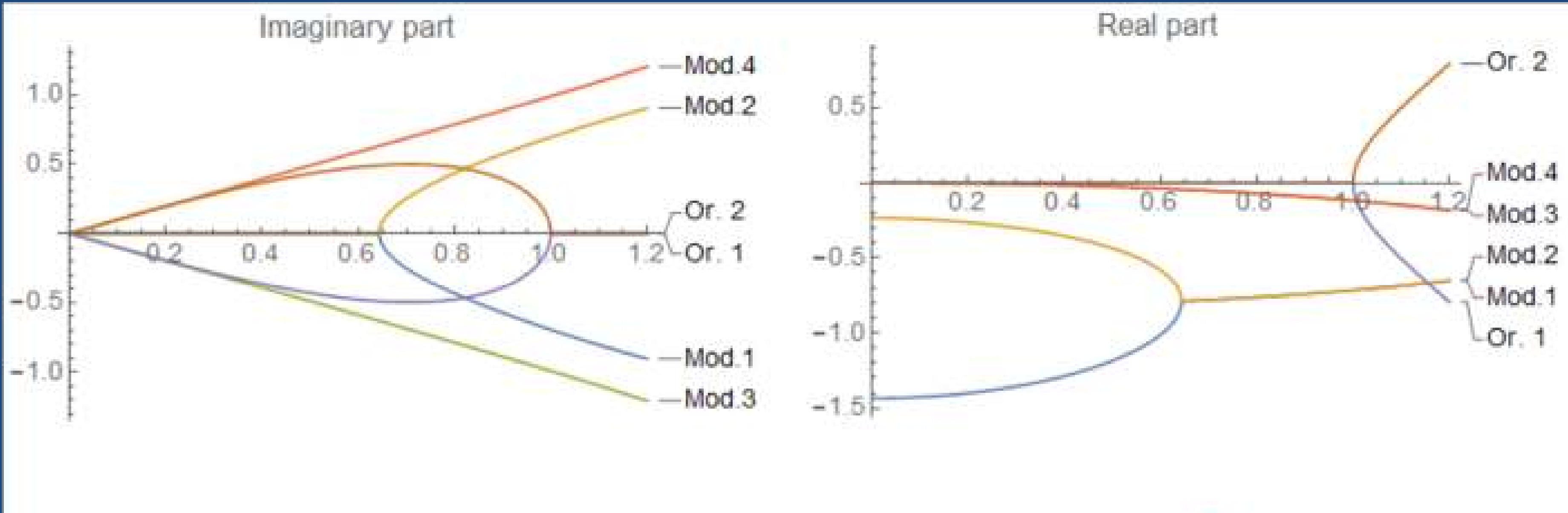
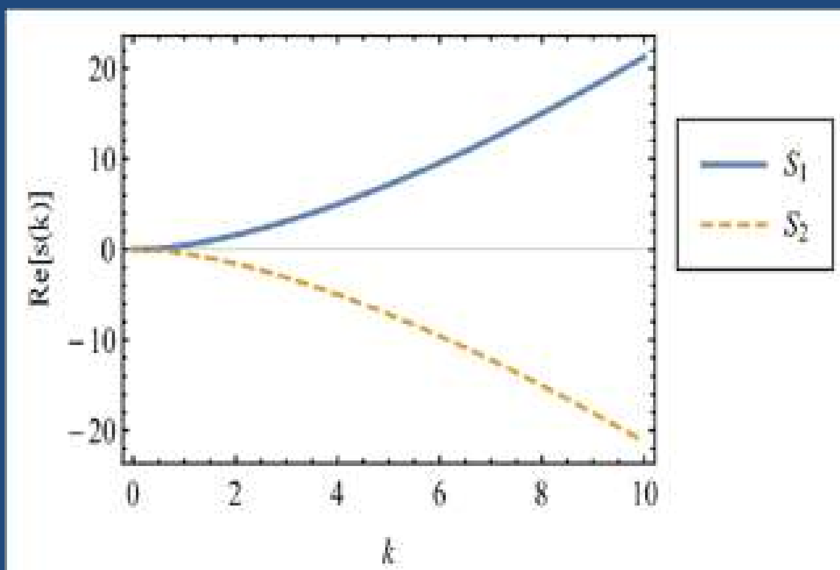


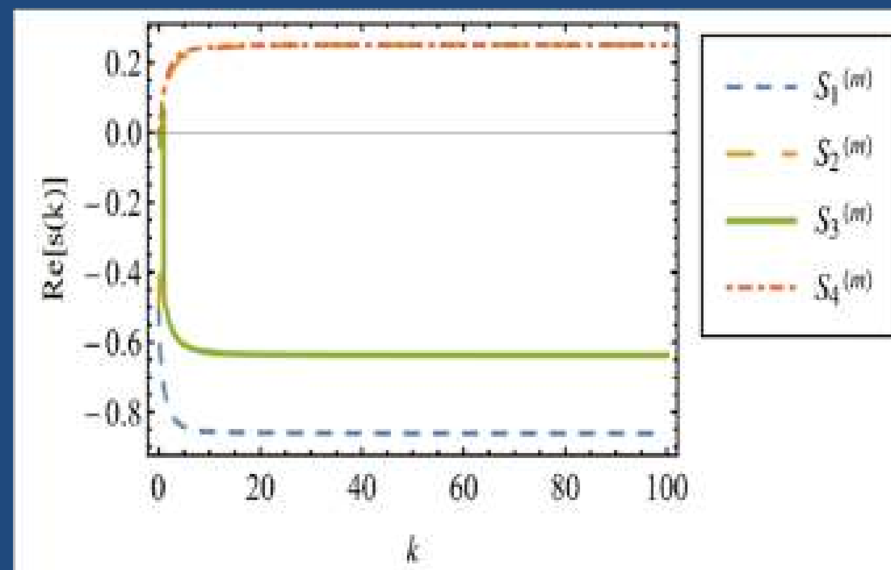
FIG. 2: Real and imaginary parts of $s(k)$ for the modified model Eq. (7), with $\tau = 5$, $\sigma = 3$.

example 2

- $\blacksquare\phi = \lambda\partial^3\phi$

$$
\begin{aligned}
\Box\phi &= \lambda\partial_x\Pi, \\
\alpha\Box\Pi + \sigma\partial_t\partial_x\Pi &= \partial_x^2\phi - \tau\partial_t\Pi - \Pi,
\end{aligned}
$$





- What to do? What to trust? Let’s take a detour…

Relativistic hydrodynamics

- $T_{ab} = (PF)_{ab} + (shear/bulk)_{ab} + Grad(shear/bulk..)_{ab}$ …
- Conservation of PF   well posed system of eqns
- Conservation of T_{ab} acausal/ill-posed, etc…

- Why? Higher derivatives and nonlinearities *can* drive runaway behavior of energy towards high frequency
  - But! The theory above was written within a gradient expansion, once it runs away one goes beyond the regime of applicability. Should not use/trust it in general unless there is a good reason for it. *[And here one typically ends up in a circular discussion…]*

Fixing relativistic hydro: Israel-Stewart

- I-S formulation: enforce staying within the validity of a gradient expansion.
- Define $\Pi =$ (shear/bulk)$_{ab}$ + Grad(shear/bulk..)$_{ab}$ as new and independent variables
- Force an eqn on Π such that $\Pi \sim$ (shear/bulk)$_{ab}$ to leading order always

- $\Pi_{,t}$ = - $\Pi +$ (shear/bulk)$_{ab}$ …. [Geroch, details shouldn't matter]

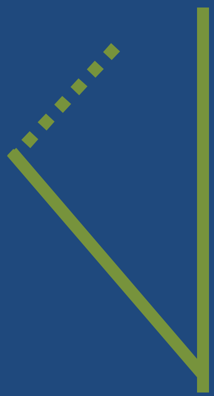
- So, mathematically all now correct. How about physically?

For Navier-Stokes, nonlinearities induce

- Energy cascade (direct d>3, *inverse/direct d=2*)
- $E(k) \sim k^{-p}$ (5/3 and 3 for 2+1)
- Correlations: $< v(r)^3 > \sim r$ (but $\{-r, r^3\}$ in 2+1) [in the relativistic case [Fouxon-Oz] [Westernacher-Schneider,Oz,LL '15], [Westernacher-Schneider,LL arXiv:1706.07480]
- Inverse cascade behavior induced by enstrophy 'quasi-conservation'
- For relativistic hydrodynamics, the analogous quantity exists, is conserved for perfect fluids and also induce an inverse cascade [Carrasco,LL,Myers,Reula,Singh; Westernacher-Schneider,Oz,LL]
- *Can we expect anything related in gravity?*

- AdS/CFT gravity/fluid correspondence
  [Bhattacharya,Hubeny,Minwalla,Rangamani; VanRaamsdonk; Baier,Romatschke,Son,Starinets,Stephanov]
- Take EEs but cast perturbation in a gradient expansion
  – $g = g\big(M(x), a(x)\big)\ s.t.\ \partial^n F < \partial^{n-1} F \ \ (F = \{M, a\})$

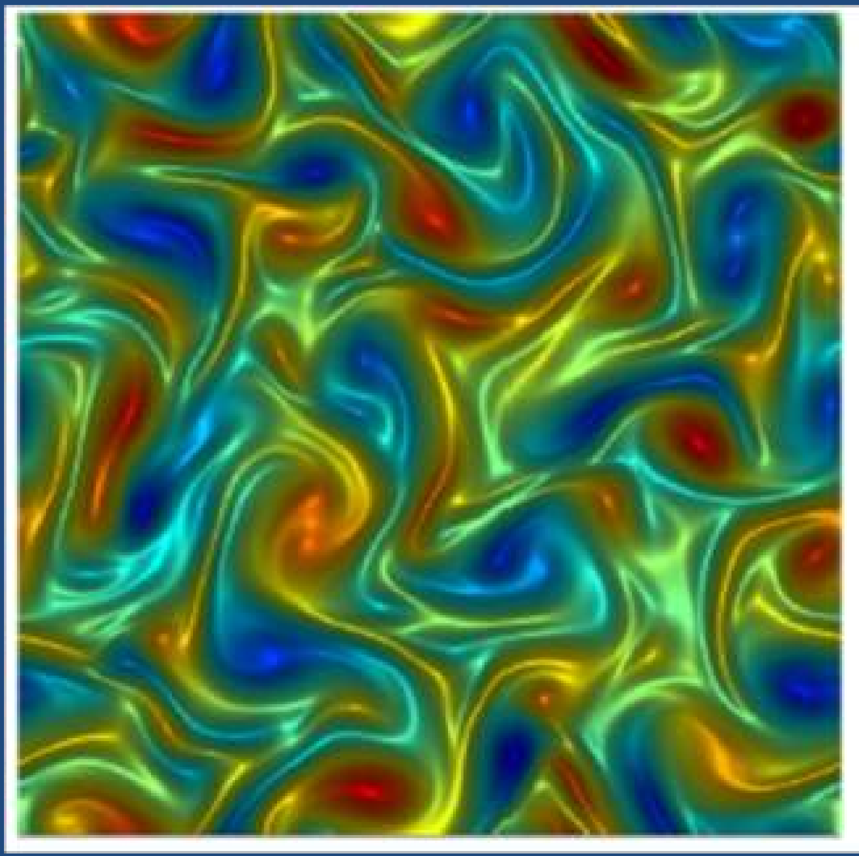
  Hierarchy of eqns:
  – at the AdS bdry: $\nabla_a (S)^{ab} = 0, S^a_a = 0$
  – Off the AdS bdry (into de bulk) simple 'radial' eqns



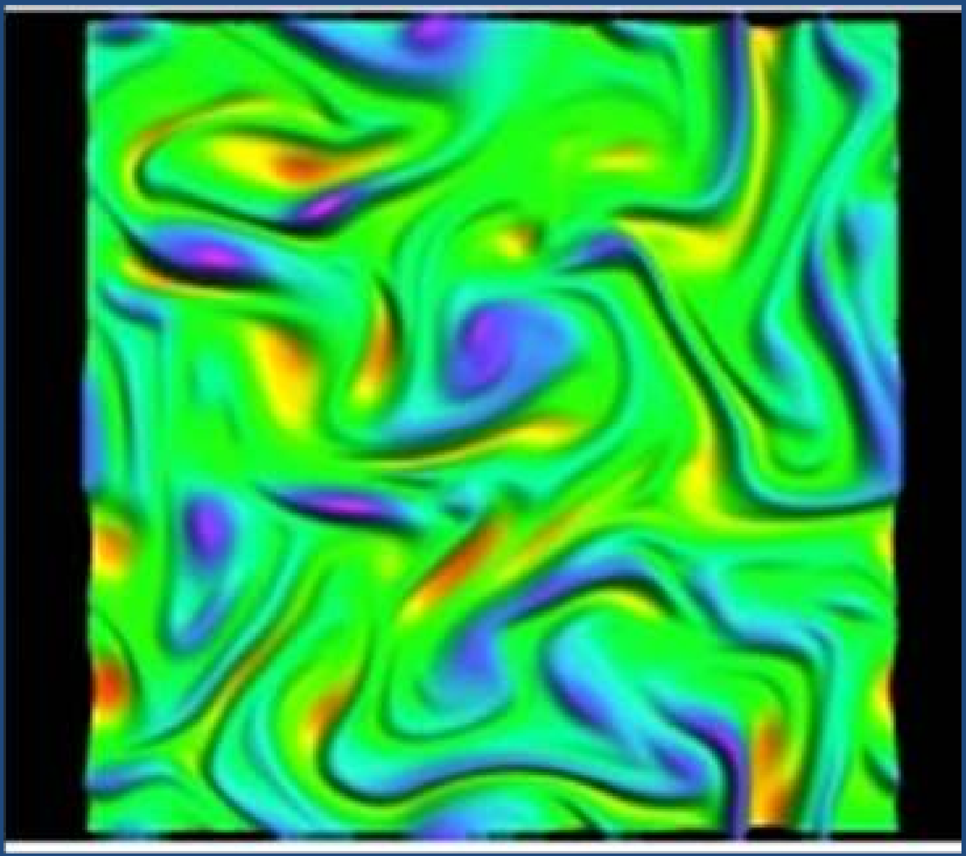
- $\mathrm{T}_{ab} = T_{ab} = \frac{\rho}{d-1}(d u_a u_b + \eta_{ab}) + \Pi_{ab}$
- Subject to :
  – $u_a u^a = -1$; $T^a{}_a = 0$; $\Pi_{ab} = -2\eta\sigma_{ab} + \cdots$
  – $\nabla_a T^{ab} = 0.$

[Carrasco,LL,Myers,Reula,Singh '13]

Bulk & holographic calculation



[Adams,Chesler,Liu. PRL 2014]



[Green,Carrasco,LL, PRX 2013]

On to the ‘real world’

- Ultimately what allowed for turbulence?
  - AdS ‘trapping energy’ slowly decaying QNMs & turbulence
  - Or slowly decaying QNMs time for non-linearities to ``do something’’?

- In AF spacetimes, membrane paradigm! **However* this is delicate.* Let’s try something else, taking though a page from what we learnt from fluids.

- First, recall the behavior of parametric oscillators:
  - $q_{,tt} + \omega^2 (1 + f(t)) q + \gamma q_{,t} = 0$
  - Soln is generically bounded in time *except* when f(t) oscillates approximately with $\omega' \sim 2\omega$. [e.g. $f(t) = f_o \cos(\omega' t)$] . If so, an unbounded solution is triggered behaving as $e^{\alpha t}$ with $\alpha = (f_o^2 \omega^2/16 - (\omega'-\omega)^2)^{1/2} - \gamma$
  - (referred to as *parametric instability* in classical mechanics and optics)

[Yang-Zimmerman,LL PRL ‘14]

Take a Kerr BH

- Let's consider now a BH with a mode that perturbs it with *(l,m)*
- Now, to linear order $g_{full} = g_{kerr} + h_1$ ($h_1 \to h_o(t) = \varepsilon\, e^{i\omega t}\, Y_{lm}$)
- QNMs → $\omega_{lmn} = {}^{m}/_{2} - {}^{\delta\sqrt{\kappa}}/_{\sqrt{2}} - i\,(n + {}^{1}/_{2})\, {}^{\sqrt{\kappa}}/_{\sqrt{2}}$
- with $\kappa = [1\text{-}a/m]^{1/2}$: thus,if sufficiently highly spinning, QNMs decay → 0.
- Consider the next order as determined by this –time dependent– background → parametric oscillator analogue!

- As a simplification, consider a single mode for h_1 and we'll take only a scalar perturbation (the general case is similar). One obtains:

$$[\ \mathrm{Box}_{\mathrm{kerr}} + O(h_1)\ ]\ \Phi = 0.$$

- With the solution having the form: $e^{t(\alpha - \omega_I)}$ with

$$\alpha = \pm\sqrt{|Hh_0(t)/Qm'|^2 - (\omega'_R - \omega_R/2)^2}\,,$$

- So exponentially growing solution if:

$$h_0(t)/(m'\omega'_I) - |Q/H|\sqrt{(\omega'_R - \omega_R/2)^2/\omega'^2_I + 1} > 0.$$

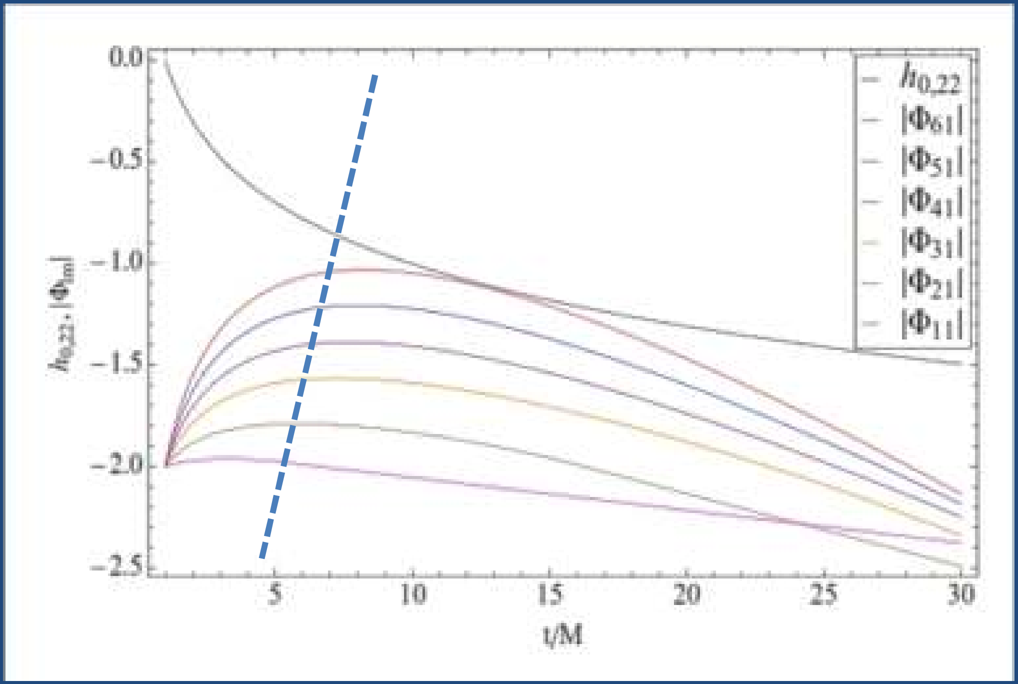
- if Φ has l, m/2 a parametric instability can turn on; i.e. inverse cascade.
- Further, one can find ‘critical values’ for growth onset.
- And can define a max value as:

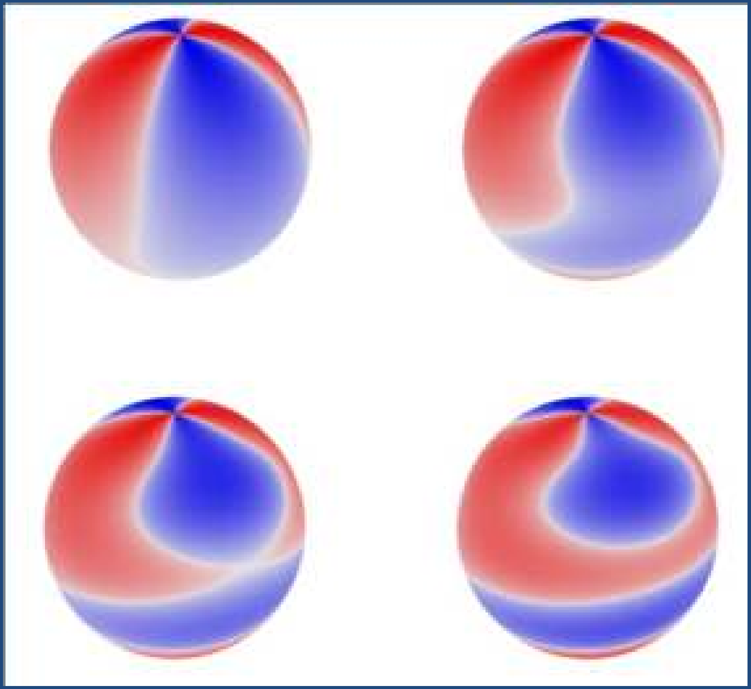
$$Re_g = h_o/(m\,\omega_\nu)$$

- identify λ<–> 1/m ; v <-> h_o ; η/ρ <-> ω_ν

$$Re_g = Re$$

Critical ``Reynolds'' number & instability





a = 0.998, perturbation ~ 0.02%, initial mode l=2,m2

Could 'potentially' have observational consequences (especially if 'gargantua' exists beyond Hollywood). Signal is different from that expected at the linear level.

So…

- Nonlinearities in GR appear to induce cascade to longer wavelengths [under long wavelength perturbs]
  - Fluid/gravity correspondence; specific calculations in AF; Bianchi-identities at future null infinity (NP formalism); gravitational waves detected by LIGO!
  - A geometric analog to enstrophy should be 'somewhere' identifiable [Green, ongoing]

  - Within this, any theory which to leading order is GR should have it as well. And, *in 3+1*, would cascade inversely [in the right regimes]

- What to do in non-linear regimes?
  - Option 1: ``reduction of order'': $L(\emptyset) = S(\emptyset)$. Treat *S* 'iteratively' keeping *L* a well defined hyperbolic operator. *Depending on the scheme* can render the problem well posed, but is it physically doing the right job?

  - *Option 2: 'modify' the equations in analogy with Israel-Stewart's formulation of relativistic hydro:*
    - *Get's the job done at the practical level*
    - *Doest not involve iterations*
    - *'captures' possibly exponential growing modes without need to resum…*

(some) examples

- Dynamical Chern-Simons & non-commutative gravity

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R - 32\pi G\alpha_0 \left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = 8\pi G T^{\mu\nu}_{\text{matter}}$$

$$\downarrow$$

$$\Box h_{ij} = \lambda \Box\Box h_{ij}\,.$$

$$\downarrow$$

$$\begin{aligned} \Box h_{ij} &= \lambda \Box \Pi_{ij}\,, \\ \Pi_{ij} &= \Box h_{ij} - \tau\partial_t \Pi_{ij} - \sigma\Box\Pi_{ij}\,. \end{aligned}$$

Dynamical Chern-Simmons [e.g. Okounkova etal '17]

$$G_{ab} + 2\,\alpha\kappa\, C_{ab} = \kappa^2 T_{ab}\,,$$

where G_{ab} is the Einstein tensor, and the traceless 'C-tensor' is defined as

$$C^{ab} = (\nabla_c\vartheta)\ \epsilon^{cde(a}\nabla_e R^{b)}{}_d + (\nabla_c\nabla_d\vartheta)\ {}^*R^{d(ab)c}\,.$$

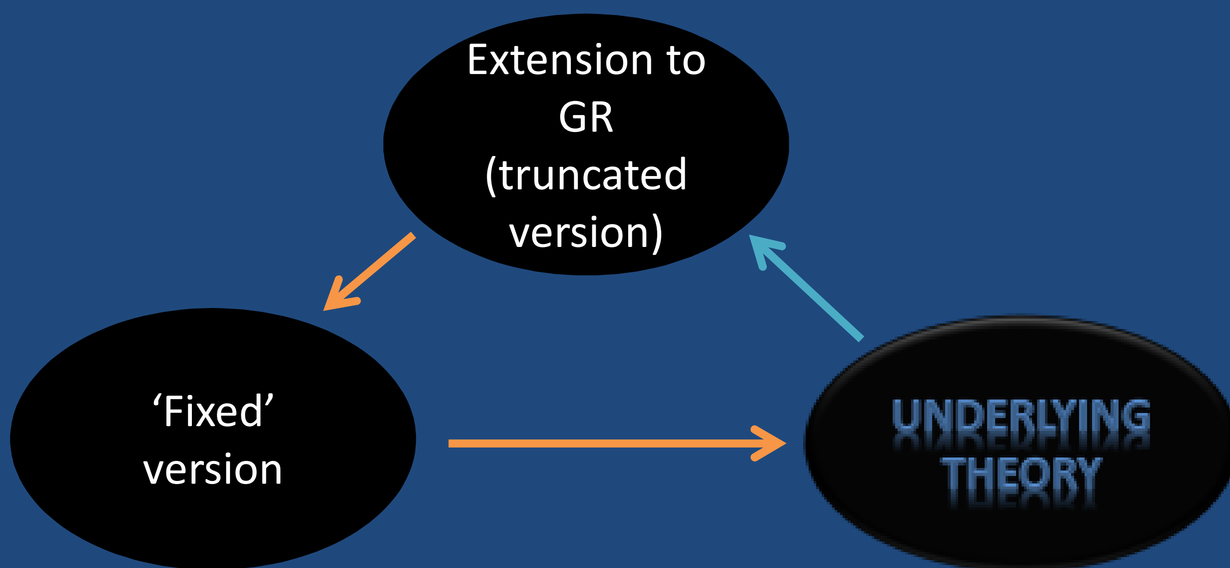
$$\downarrow$$

$$\Box\Box\varphi + a\partial_x^2\Box\varphi + b\partial_x^4\varphi + \lambda\partial_x\Box\Box\varphi = 0,$$

$$\downarrow$$

$$\begin{aligned}\Box\Box\varphi + a\partial_x^2\Box\varphi + b\partial_x^4\varphi + \lambda\partial_x\Pi &= 0,\\ \tau\partial_t\Pi + \Pi - \Box\Box\varphi + \sigma\Box\Pi &= 0.\end{aligned}$$

- *Can be justified in 3+1 dimensions!*
  - In extensions to GR, which have GR as a low curvature limit and scenarios with long-wavelength perturbations 3+1 dimensions should capture the right physics but not in higher dimensions.
- There seems to be a way to avoid ‘not going to non-linear-land’ with (many) GR alternatives and face LIGO’s data

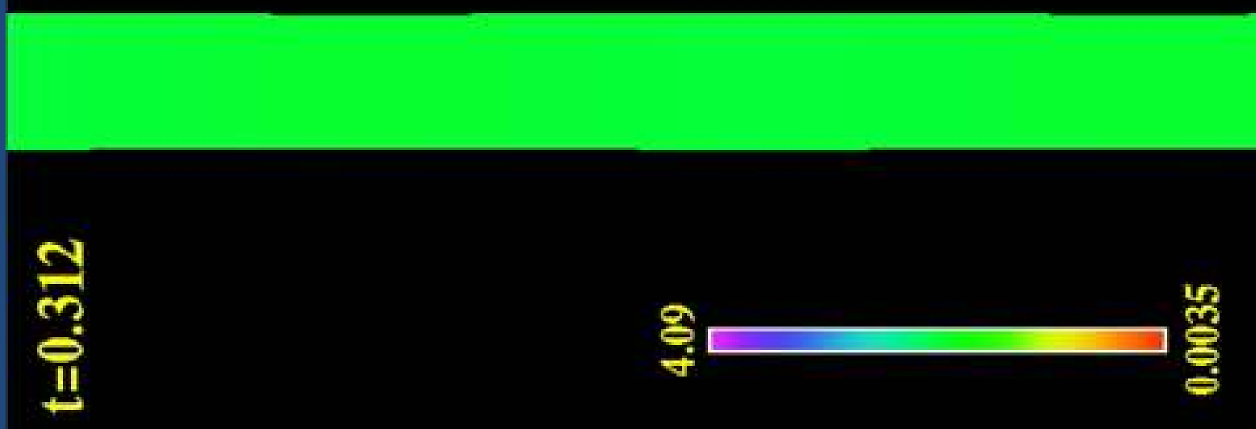


[Cayuso,Ortiz,LL arXiv:1706.07421]

Final thoughts

- We are in a new era. Still to be decided if we have
  - Beyond a solid new tool for astrophysics, a way to obtain guidance for what replaces GR. Ripe time to think new ideas and explore new prospects

  - Can dig deeper in the data [with the right model/knowledge]
  - Pheno modifications can be non-monotonic
  - Extensions to GR are dangerous, but if desired, can be fixed and the fix is possibly justified



Capillary Breakup of human Saliva

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