

Confronting Braneworld Models of Dark Energy with Supernova and other Datasets

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Plan of Talk :

- ⇒ Introduction : Dark Energy
- ⇒ Braneworld Models of Dark Energy
- ⇒ Observations & Methodology
- ⇒ Current Results
 - ⇒ Results from SNe
 - ⇒ Results from complementary data
- ⇒ Conclusion

Observations of Dark Energy :

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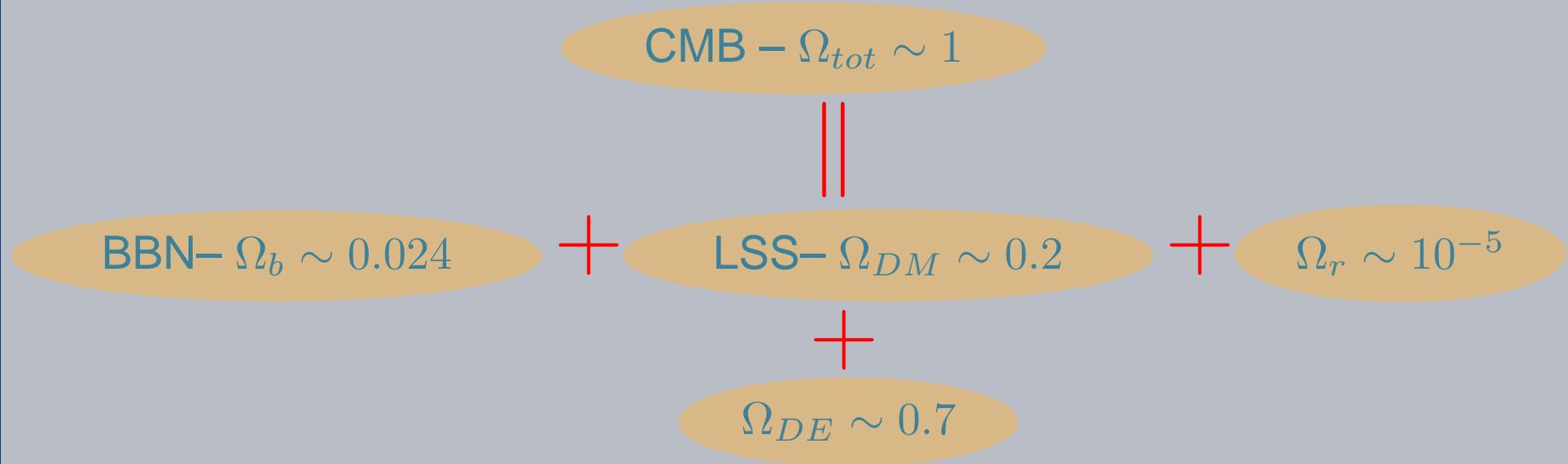
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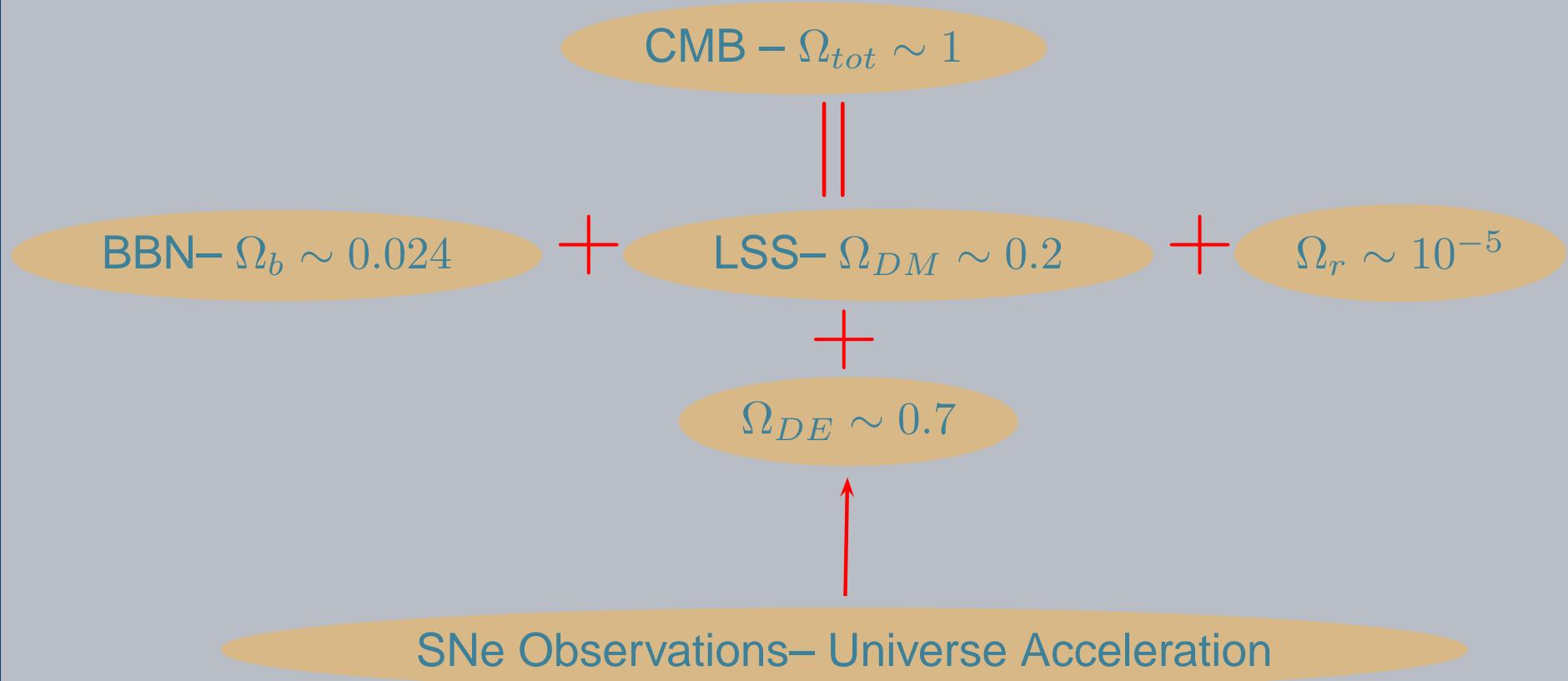
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$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p)$$

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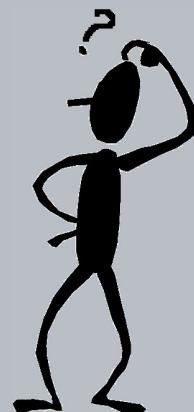
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Theoretical explanation–

Zero point vacuum fluctuation $\langle T_{\mu\nu} \rangle = \Lambda g_{\mu\nu}$
– Zeldovich (1968)

Problems :

⇒ Cosmological Constant Problem–

Divergence problem– $\Lambda/8\pi G = \langle T_{00} \rangle_{\text{vac}} \propto \int_0^\infty k^2 \sqrt{k^2 + m^2} dk$

Planck scale cut-off– $\langle T_{00} \rangle_{\text{vac}} \simeq 10^{76} \text{Gev}^4$

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④ Fine-tuning Problem–

DE density today– $\rho_\Lambda \simeq 10^{-47} \text{Gev}^4$

Slightly smaller density– $\rho_\Lambda \simeq 10^{-50} \text{Gev}^4$ – Recollapse

Slightly larger density– $\rho_\Lambda \simeq 10^{-43} \text{Gev}^4$ – Inhibits structure formation

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⇒ Braneworld models ⇒ Modifying the left-hand side of Einstein's equations !

Brane Models of Dark Energy :

$$S = M^3 \left[\int_{\text{bulk}} (R_5 - 2\Lambda_b) - 2 \int_{\text{brane}} K \right] + \int_{\text{brane}} [m^2 R_4 - 2\sigma + L(h_{\alpha\beta}, \phi)] .$$

(Sahni & Shtanov, 2003)

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$$H^2(a) = \frac{A}{a^3} + B + \frac{2}{l^2} \left[1 \mp \sqrt{1 + l^2 \left(\frac{A}{a^3} + B - \frac{\Lambda_b}{6} - \frac{C}{a^4} \right)} \right] .$$

$$A = \frac{\rho_0 a_0^3}{3m^2} , B = \frac{\sigma}{3m^2} .$$

Brane Models of Dark Energy :



$m = 0 \Rightarrow$ FRW generalization of Randall Sundrum :

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⇒ $\sigma = \Lambda_b = 0 \Rightarrow$ Dvali Gabagadze Porratti (DGP) model :

$$H^2 = \frac{A}{a^3} + \frac{2}{l^2} \mp \frac{2}{l} \sqrt{\frac{1}{l} + \frac{A}{a^3}} .$$

(+) sign leads to self-accelerating braneworld model.

Brane Models of Dark Energy :

$$H^2 = \frac{A}{a^3} + \Lambda_{\text{eff}}$$
$$\Lambda_{\text{eff}} = \underbrace{\left(B + \frac{2}{l^2} \right)}_{\Downarrow \Lambda} \mp \underbrace{\frac{2}{l^2} \sqrt{1 + l^2 \left(\frac{A}{a^3} + B - \frac{\Lambda_b}{6} \right)}}_{\Downarrow \text{Screening term}} .$$

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$$w_{\text{eff}}(a = a_0) = -1 \mp \frac{1}{\left(H_0^2 - \frac{A}{a_0^3} \right) \sqrt{1 + l^2 \left(\frac{A}{a_0^3} + B - \frac{\Lambda_b}{6} \right)}} .$$

Brane Models of Dark Energy :

Brane1 :

$$\frac{H^2(z)}{H_0^2} = \Omega_{0m}(1+z)^3 + \Omega_\sigma + 2\Omega_l \left(-2\sqrt{\Omega_l} \sqrt{\Omega_{0m}(1+z)^3 + \Omega_\sigma + \Omega_l + \Omega_{\Lambda_b}} \right) ,$$

$$\Omega_{0m} = \frac{\rho_0}{3m^2 H_0^2}, \quad \Omega_\sigma = \frac{\sigma}{3m^2 H_0^2}, \quad \Omega_l = \frac{1}{l^2 H_0^2}, \quad \Omega_{\Lambda_b} = \frac{\Lambda_b}{6H_0^2}$$

$$\text{Flat universe} \Rightarrow \Omega_\sigma = 1 - \Omega_{0m} + \sqrt{\Omega_l} \sqrt{1 + \Omega_{\Lambda_b}}$$

$$w_0 = -1 - \frac{\Omega_{0m}}{1 - \Omega_{0m}} \sqrt{\frac{\Omega_l}{\Omega_{0m} + \Omega_\sigma + \Omega_l + \Omega_{\Lambda_b}}} \leq -1 .$$

Brane Models of Dark Energy :

Brane2 :

$$\frac{H^2(z)}{H_0^2} = \Omega_{0m}(1+z)^3 + \Omega_\sigma + 2\Omega_l \circledplus 2\sqrt{\Omega_l} \sqrt{\Omega_{0m}(1+z)^3 + \Omega_\sigma + \Omega_l + \Omega_{\Lambda_b}} ,$$

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SNe Ia \Rightarrow Thermonuclear explosion in C+O white dwarf

Strong correlation between peak magnitude & light curve shape \rightarrow calibrated candles

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High- z SNe \rightarrow Accelerating Universe \rightarrow Dark Energy

1. Calan Tololo low z data + Supernova Cosmology Project (SCP) + High- z SNe Search Team (HZT) + Hubble Space Telescope (HST)–

Gold Sample

\Rightarrow 157 SNe between $z = 0 - 1.7$

2. Calan Tololo + SuperNova Legacy Survey 2 yr data

SNLS sample

\Rightarrow 115 SNe between $z = 0 - 1.0$

Complementary Datsets :

⇒ Baryon Acoustic Oscillations (BAO) :

For SDSS data at $z_{ob} = 0.35$

$$A = \frac{\sqrt{\Omega_{0m}}}{h(z_{ob})^{1/3}} \left[\frac{1}{z_{ob}} \int_0^{z_{ob}} \frac{dz}{h(z)} \right]^{2/3} = 0.469 \pm 0.017 ,$$

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⇒ Cosmic Microwave Background (CMB) :

For WMAP3 data with $\Omega_{0m} h^2 = 0.127_{-0.013}^{+0.007}$

$$R = \sqrt{\Omega_{0m}} \int_0^{z_{ls}} \frac{dz}{h(z)} = 1.70 \pm 0.03$$

Model Fitting :

Observations ⇒

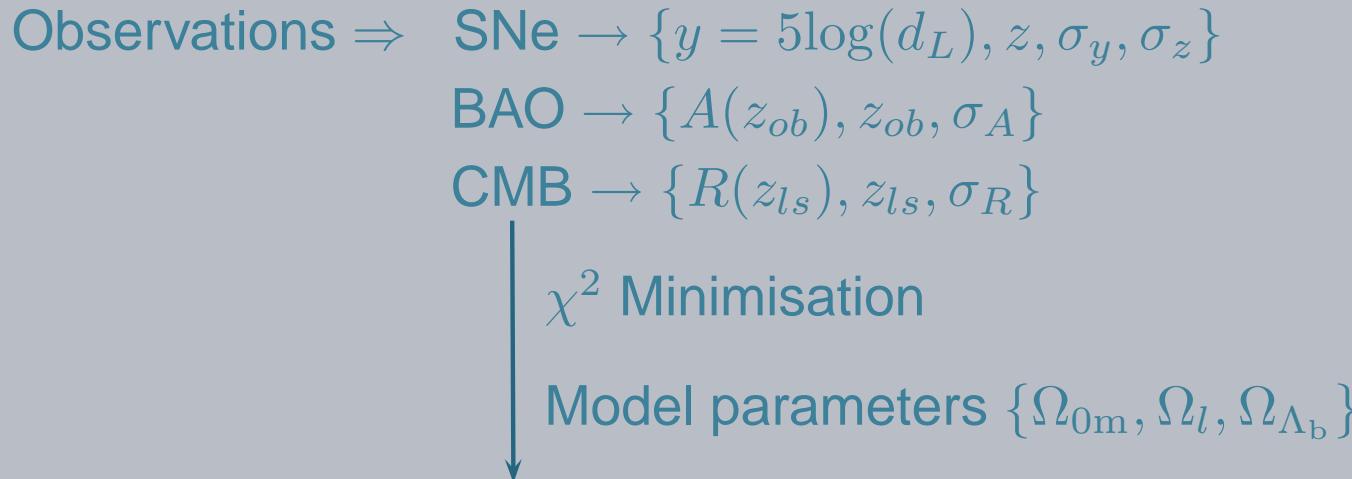
Model Fitting :

Observations \Rightarrow SNe $\rightarrow \{y = 5\log(d_L), z, \sigma_y, \sigma_z\}$

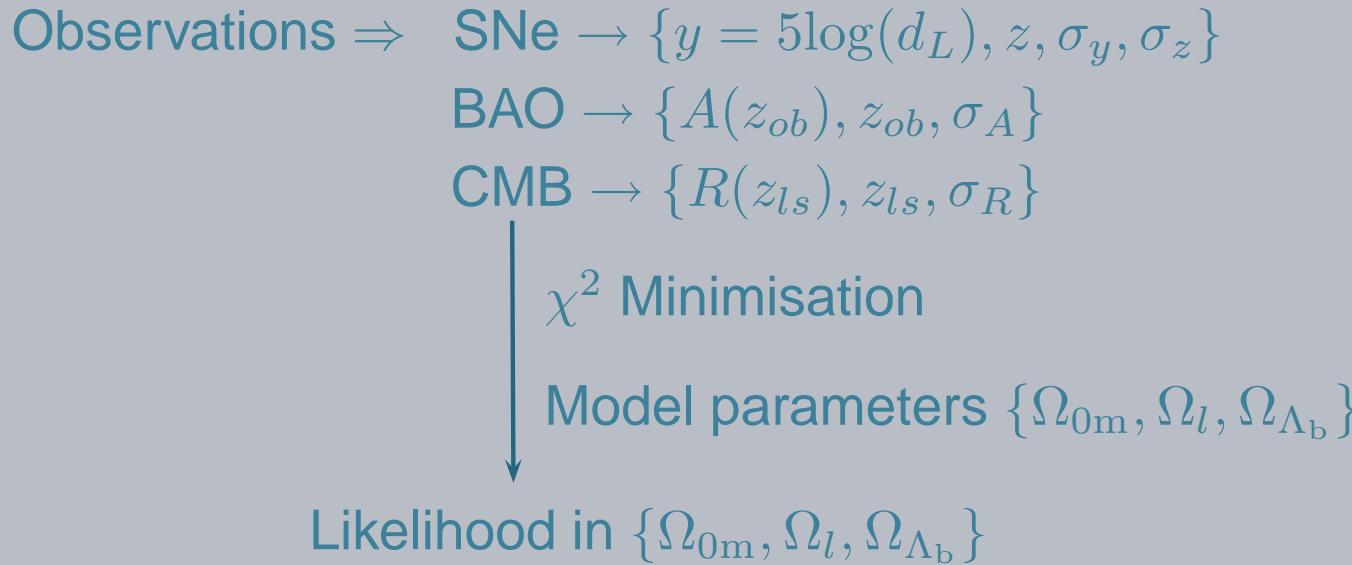
BAO $\rightarrow \{A(z_{ob}), z_{ob}, \sigma_A\}$

CMB $\rightarrow \{R(z_{ls}), z_{ls}, \sigma_R\}$

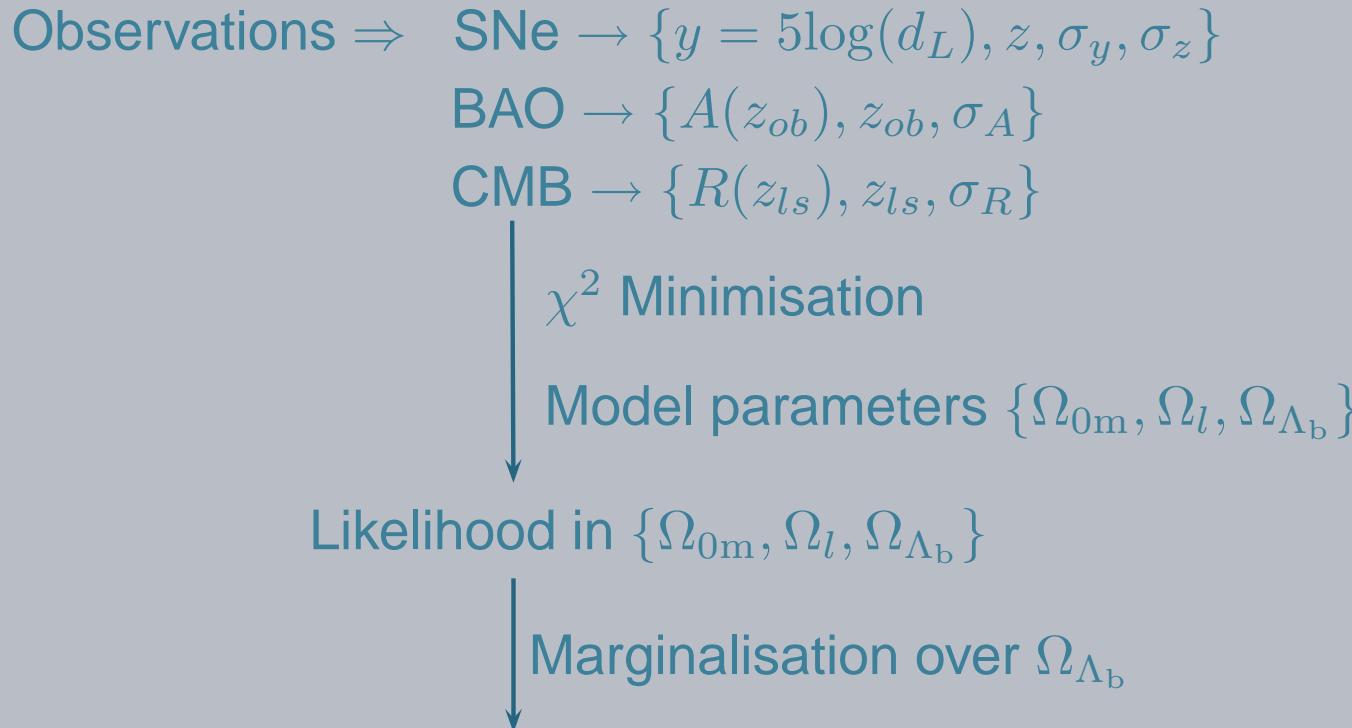
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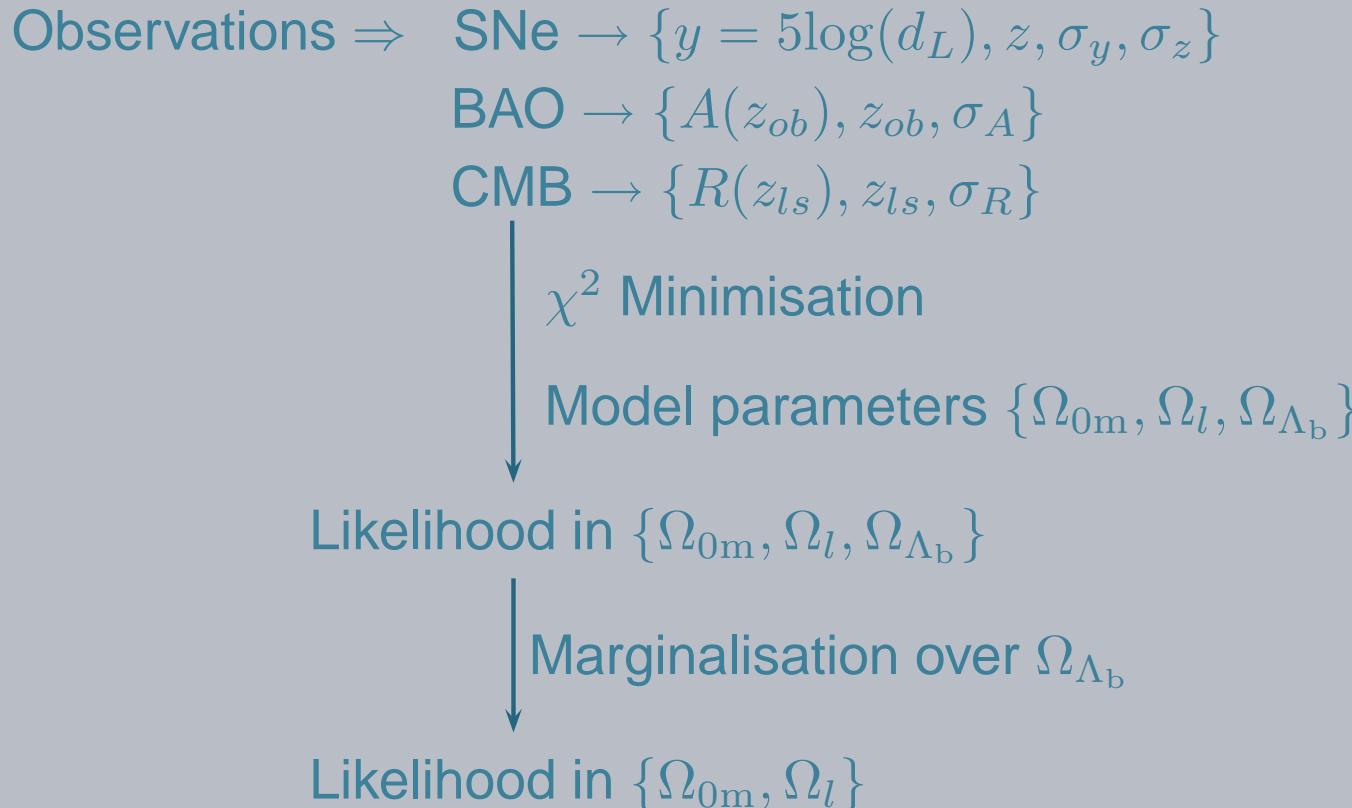
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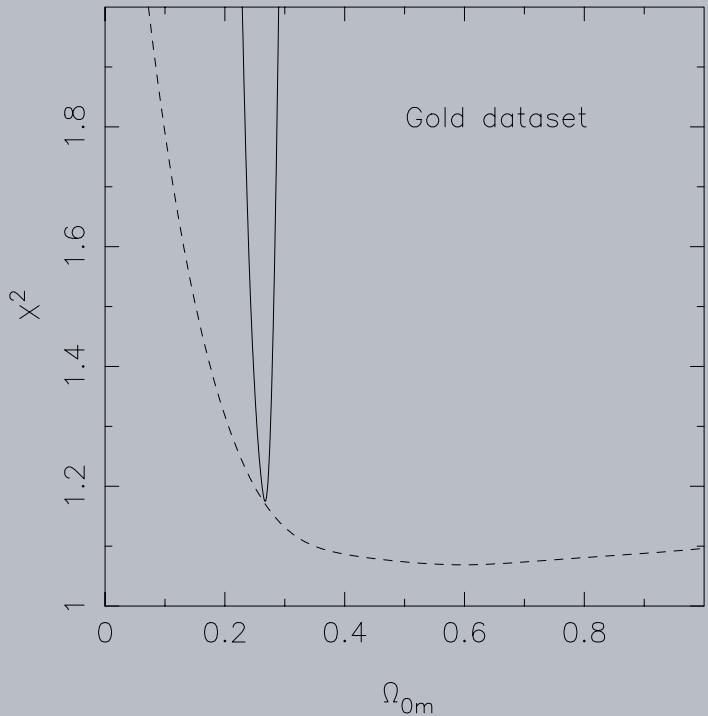
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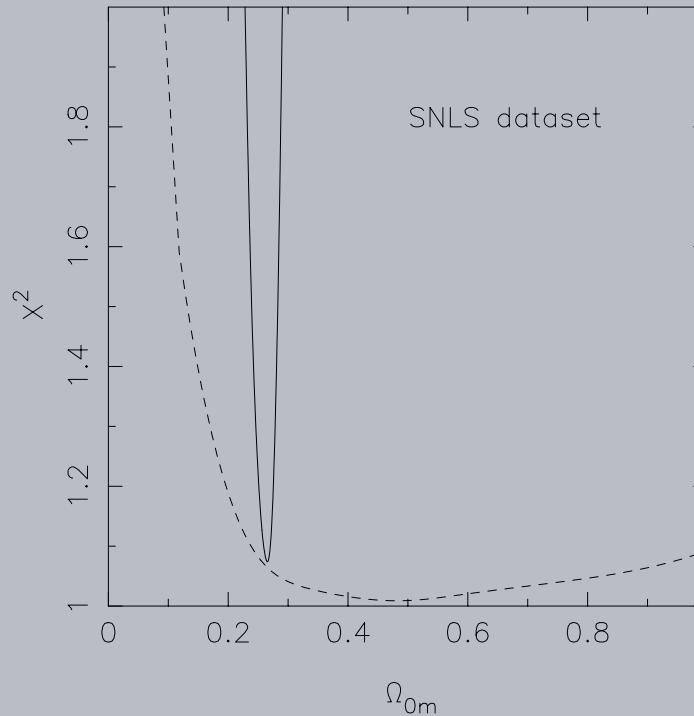
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Results for B1 :

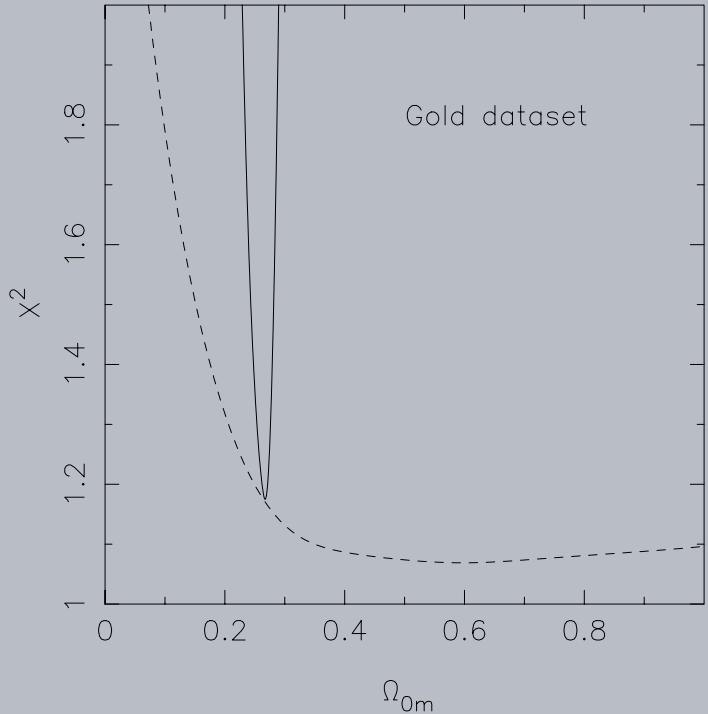


Gold



SNLS

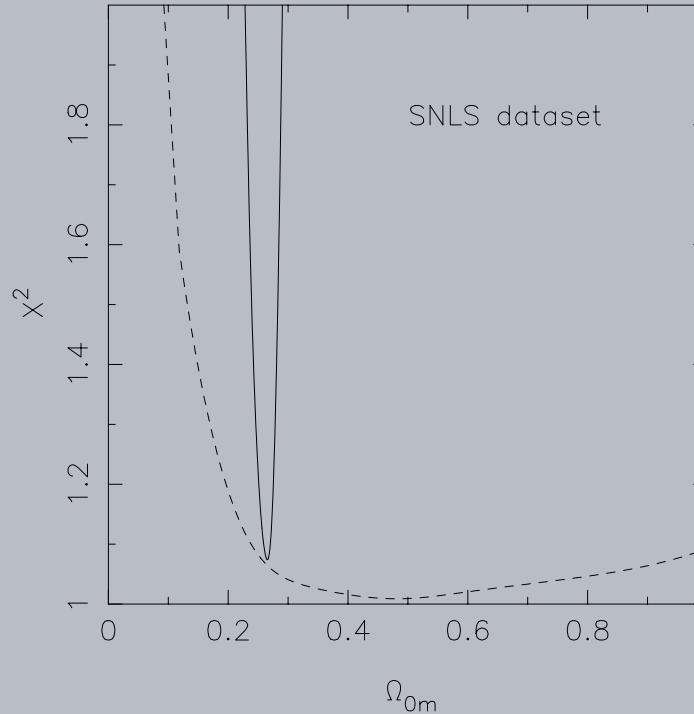
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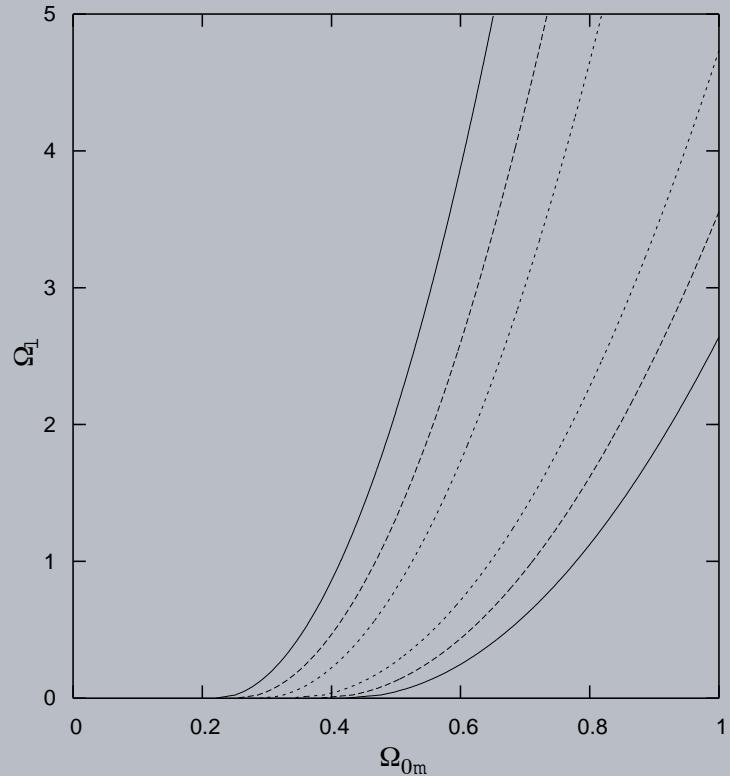
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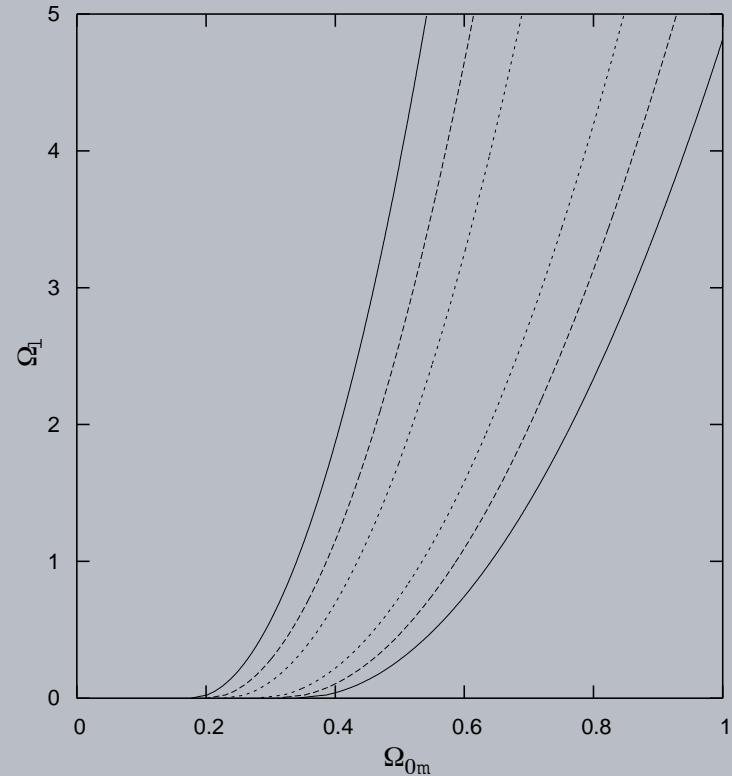


SNLS

SNe Results for B1 :

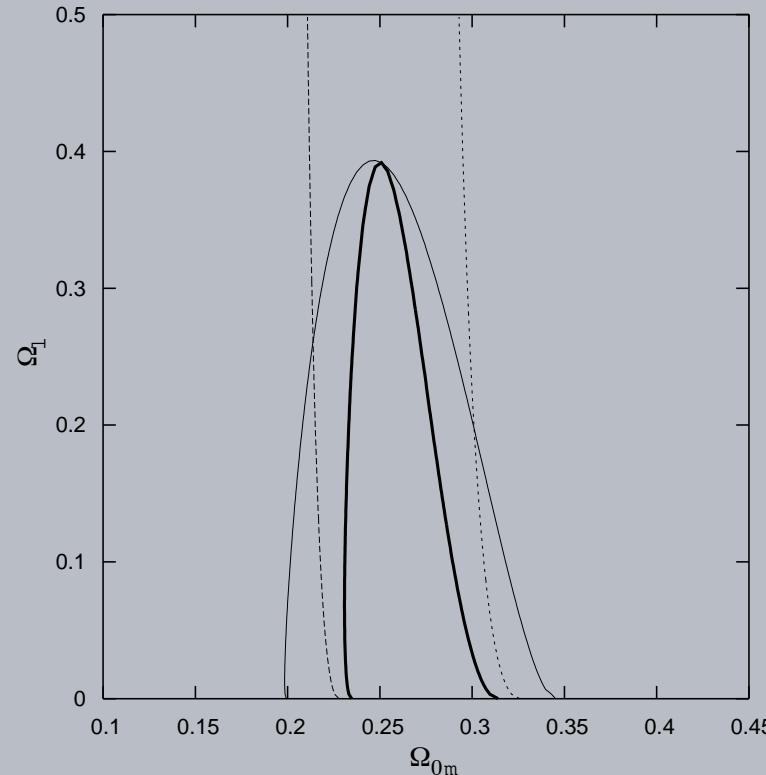


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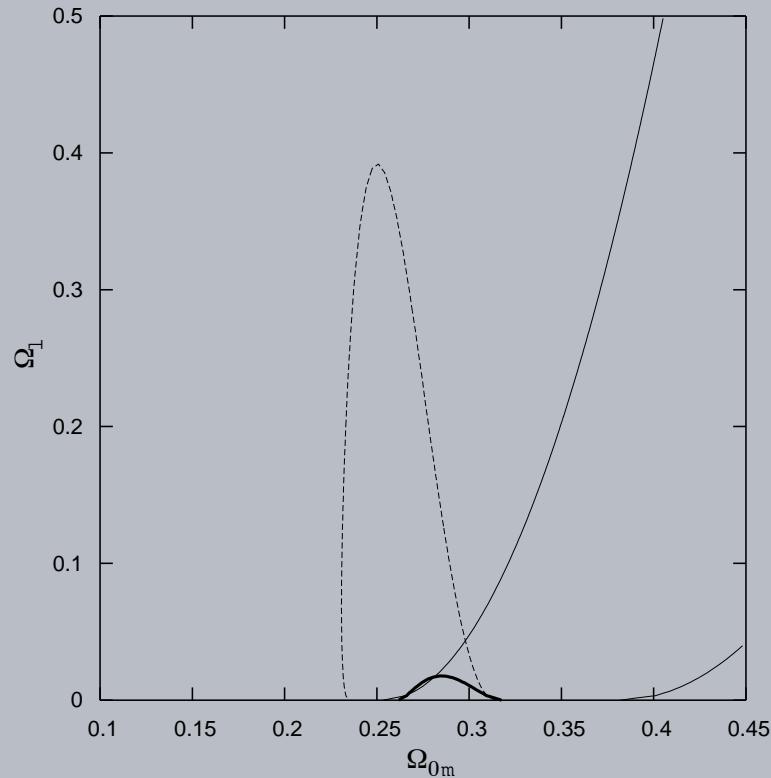


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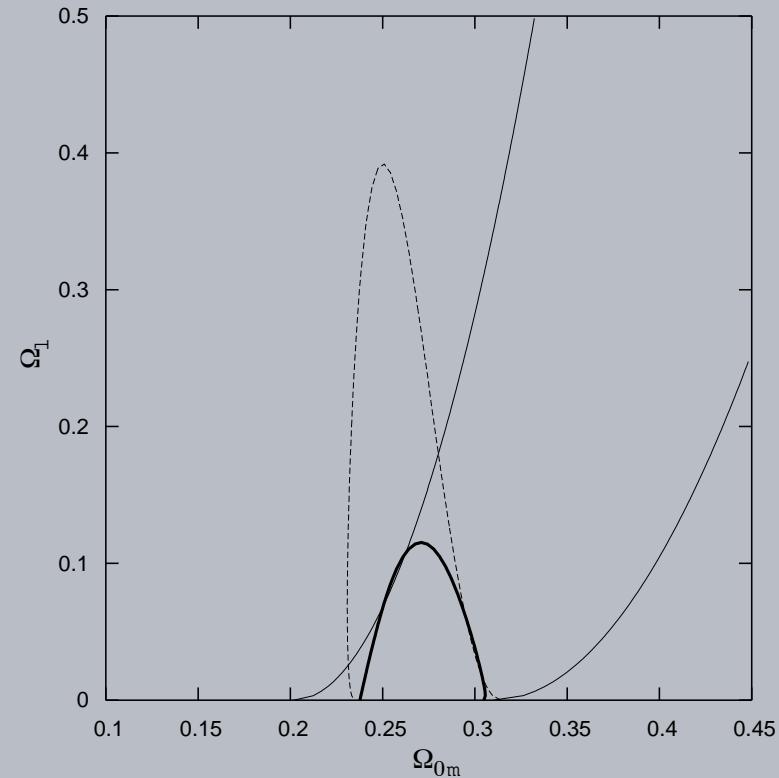
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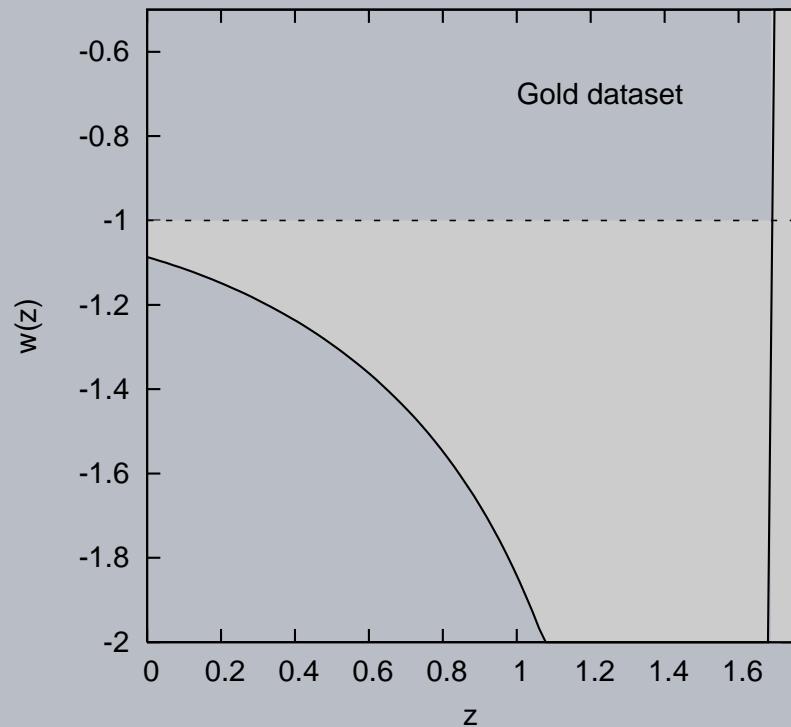


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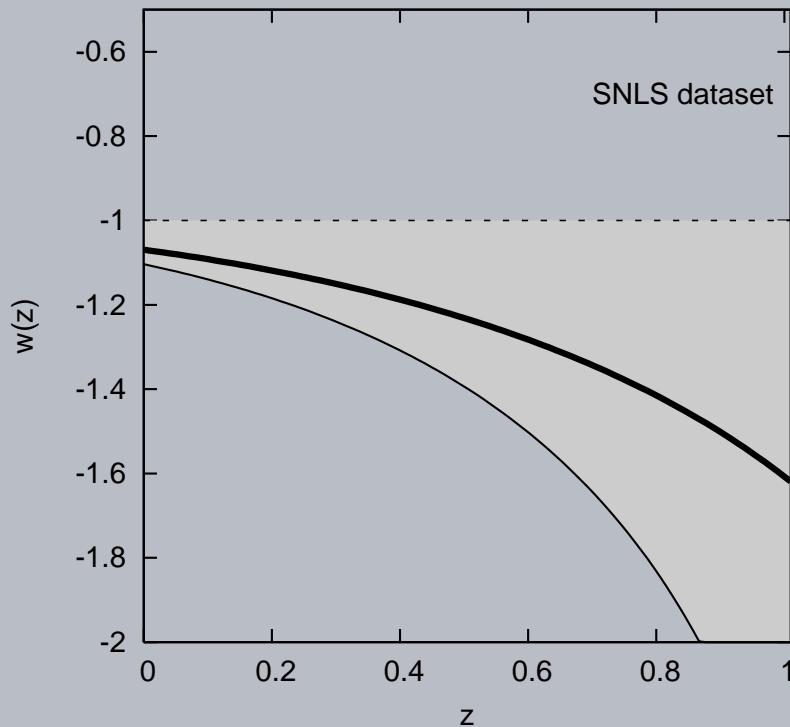


SNLS

w_{eff} for B1 :

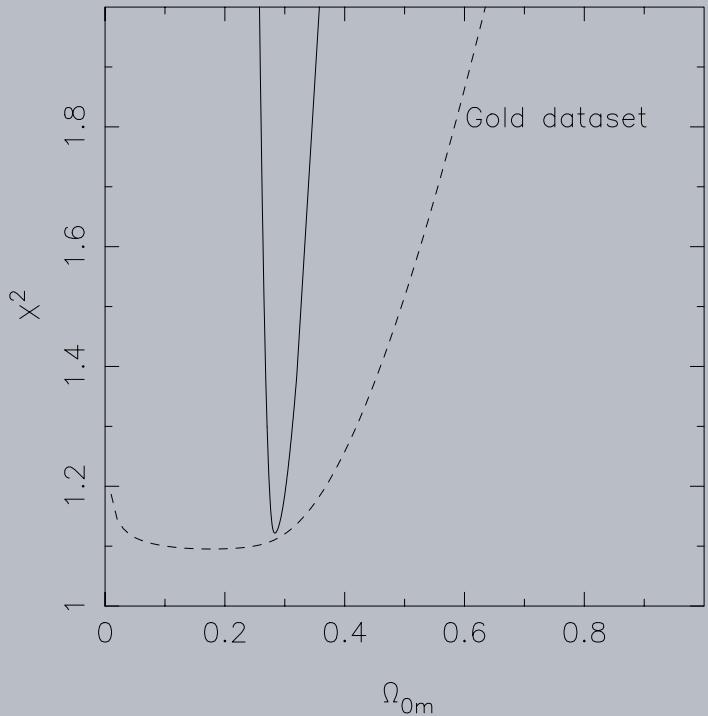


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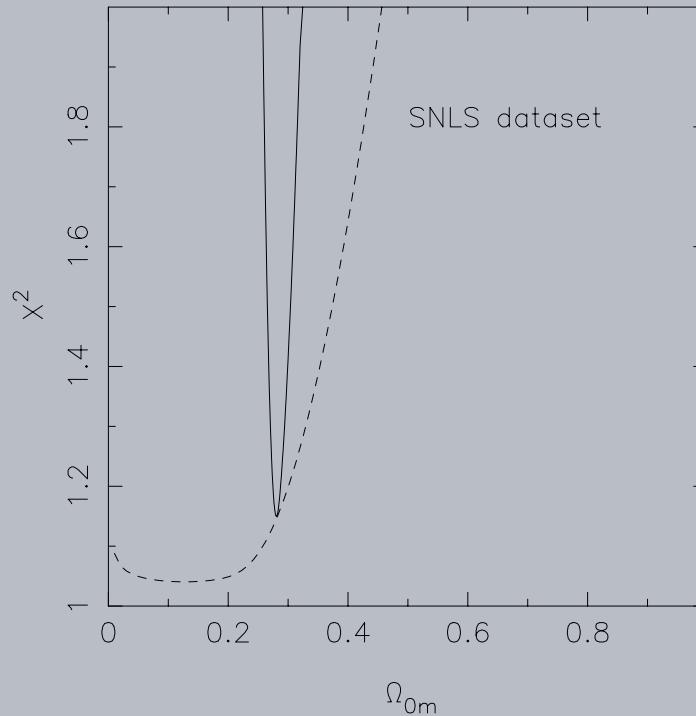


SNLS

Results for B2 :

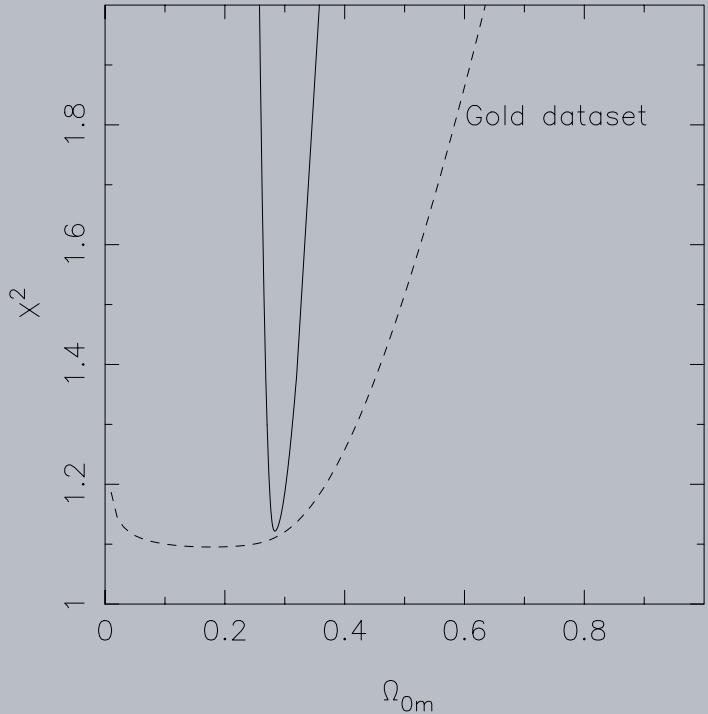


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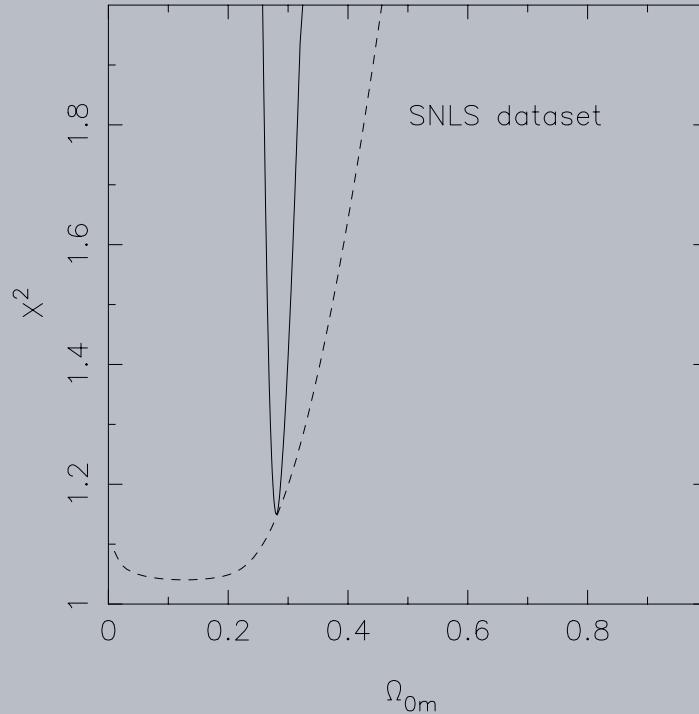
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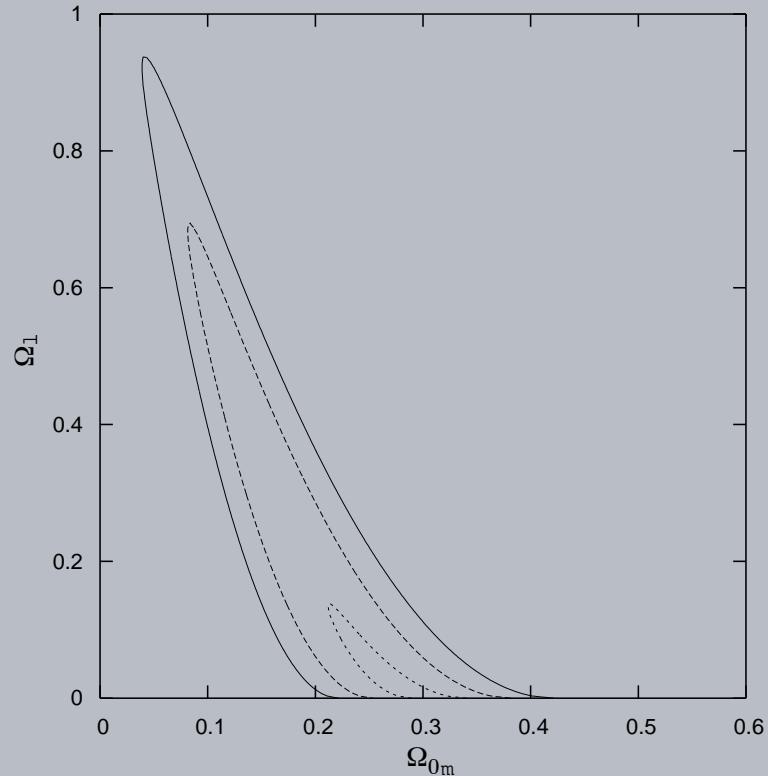
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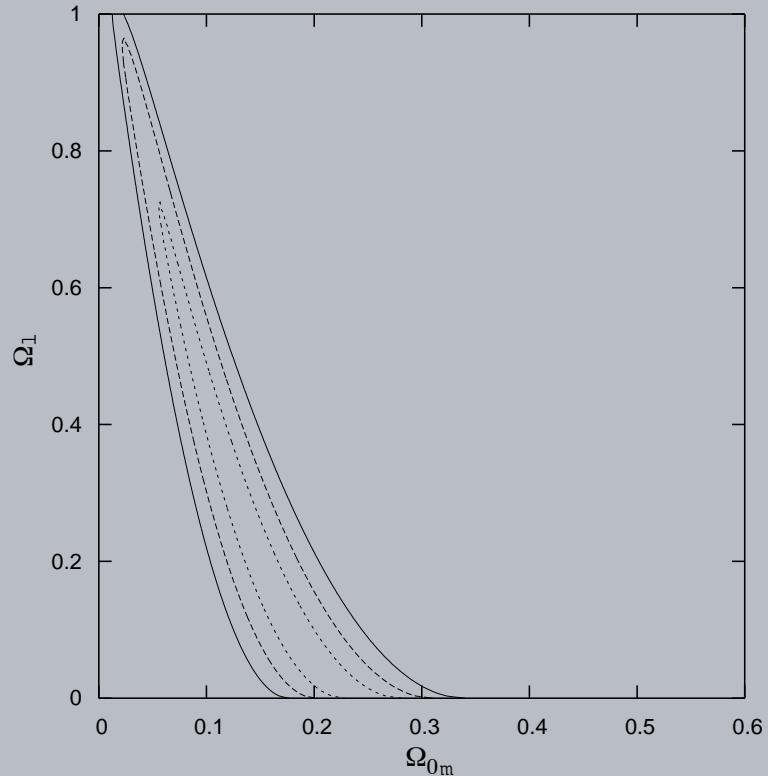


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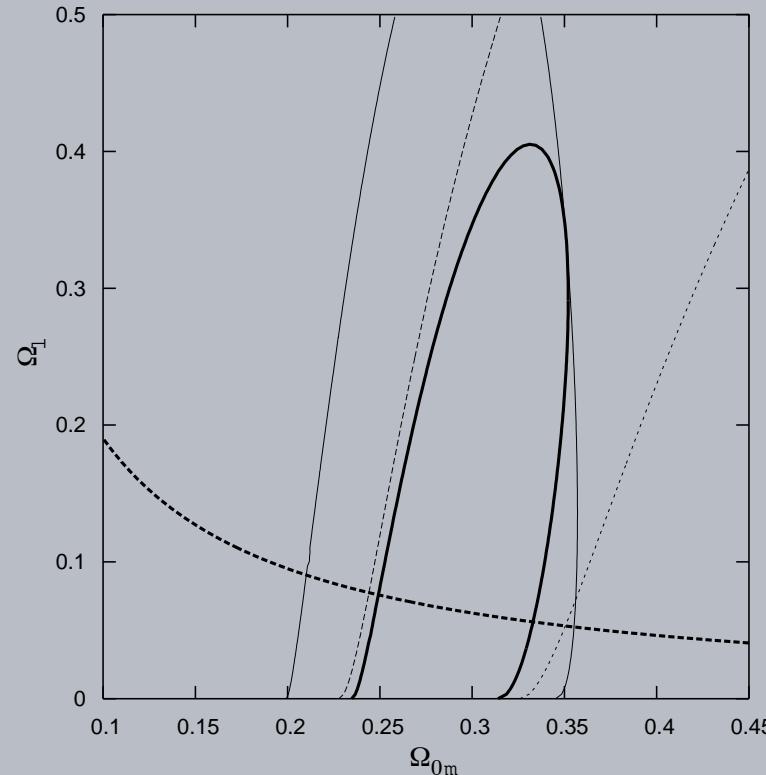


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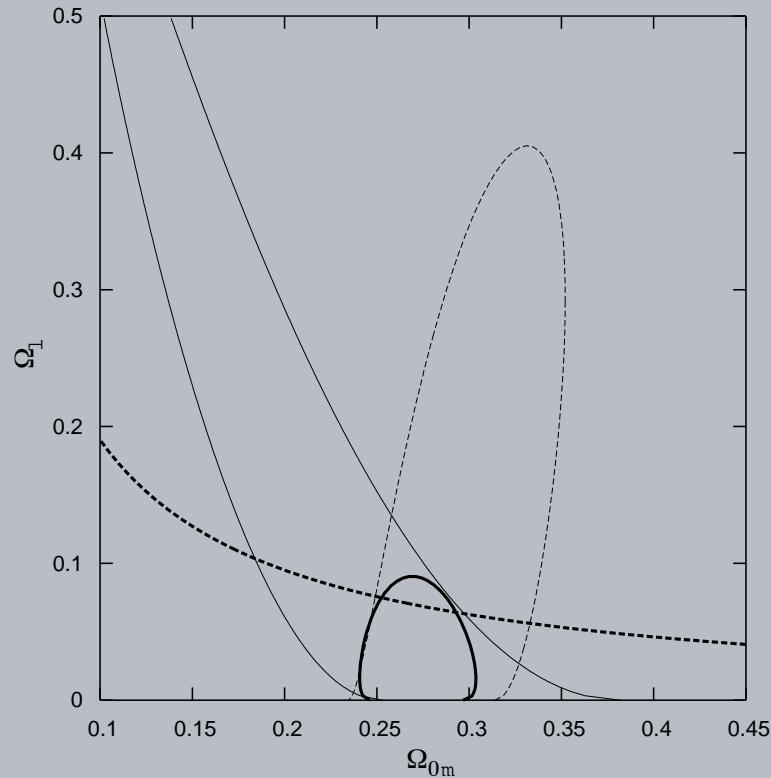


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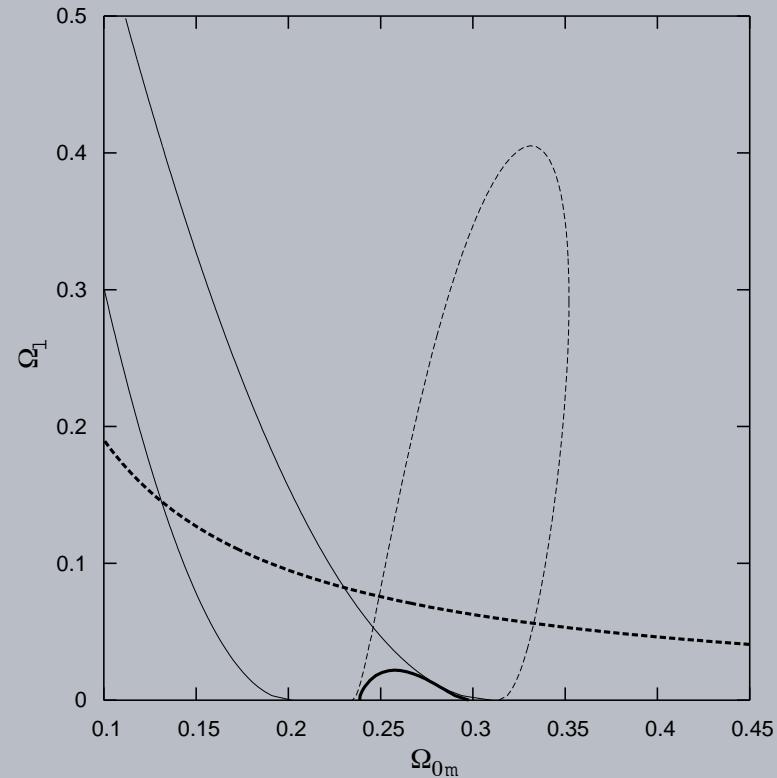
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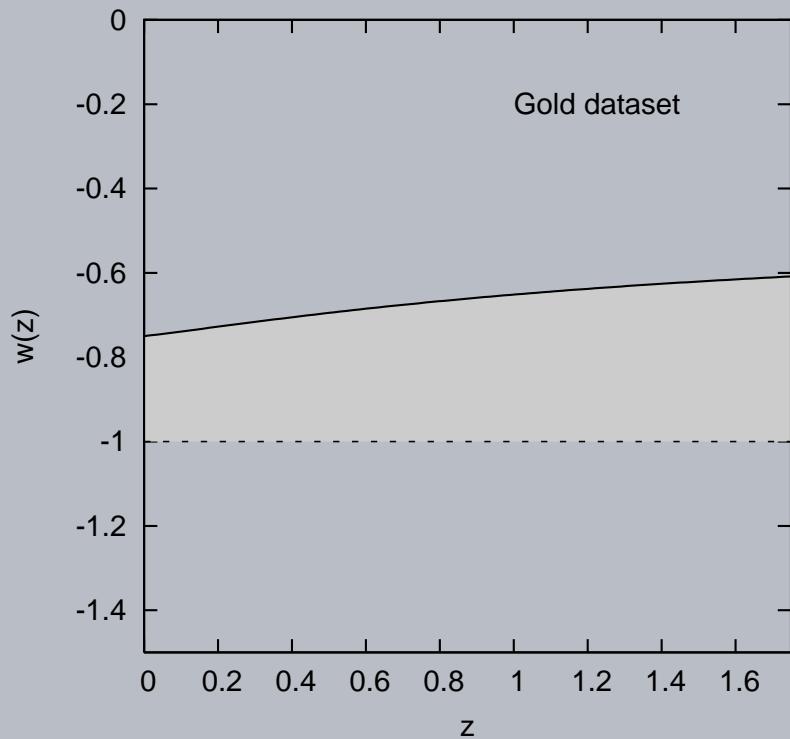


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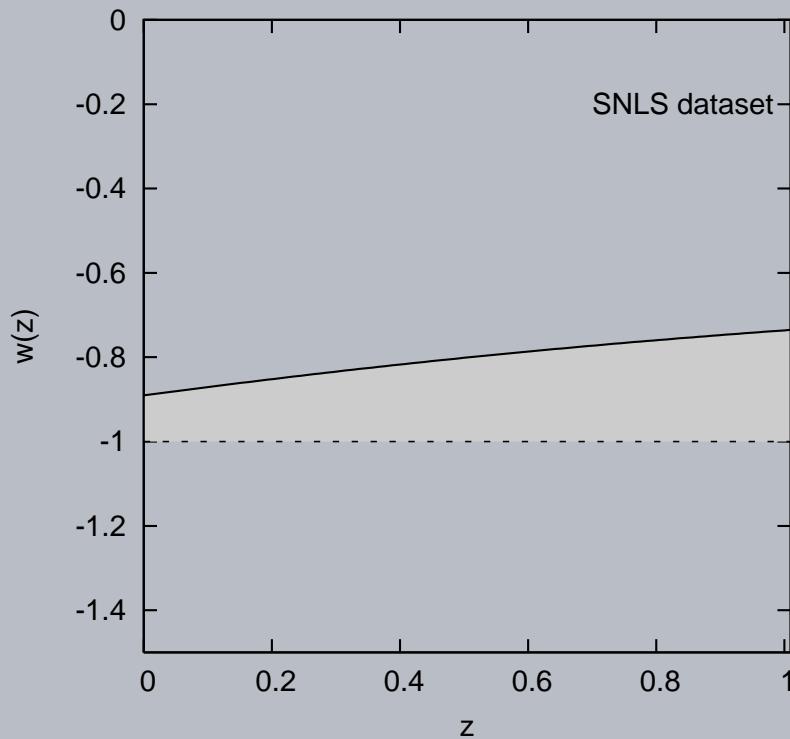


SNLS

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SNLS

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- ⇒ Joint analysis SNe Ia + CMB + BAO—
 - ⌚ No dataset by itself rules out braneworld models, but when taken together, they place strong constraints on the parameter space.

Conclusions :

- ⇒ We examine a general class of braneworld models in the light of current SNe Ia + CMB + BAO data.
- ⇒ The CMB & BAO data affect on the matter density of the model strongly.
- ⇒ SNe data— Slight discrepancy between results from two datasets—
 - ⌚ Gold sample prefers brane models with $w \geq -1$ (B2)
 - ⌚ SNLS sample prefers brane models with $w \leq -1$ (B1).
- ⇒ Joint analysis SNe Ia + CMB + BAO—
 - ⌚ No dataset by itself rules out braneworld models, but when taken together, they place strong constraints on the parameter space.
 - ⌚ DGP model is ruled out at 2σ by the joint analysis using SNLS data, but is allowed at 2σ by the Gold data.

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 - ⇒ Strong model independent constraints on the matter density.
 - ⇒ Larger quantity of SNe data to remove discrepancy between datasets.
 - ⇒ Complete analysis of CMB *etc.* datasets for braneworld models, taking into account the relevant perturbation theory.