GRAVITATIONAL POLARIZATION AND THE PHENOMENOLOGY OF MOND

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Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
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Based on

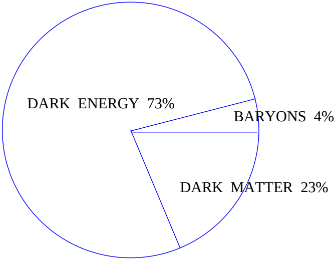
Gravitational polarization and the phenomenology of MOND (astro-ph/0605637)

Dipolar particles in general relativity (gr-qc/0609121)

Outline of the talk

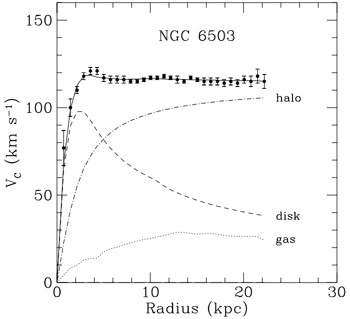
PHENOMENOLOGY OF DARK MATTER

Concordance model in cosmology



- Einstein field equations with a cosmological constant Λ are assumed to be correct and $\Omega_\Lambda = 73\%$ is measured from the Hubble diagram of supernovas
- Density of baryons $\Omega_\mathrm{B} = 4\%$ is known from Big Bang nucleosynthesis and from CMB anisotropies
- Dark matter is non-baryonic and accounts for the observed dynamical mass of bounded astrophysical systems [Oort 1932, Zwicky 1933]

Galaxies are dominated by non-baryonic dark matter



For a circular orbit $v(r) = \sqrt{\frac{G\,M(r)}{r}}$

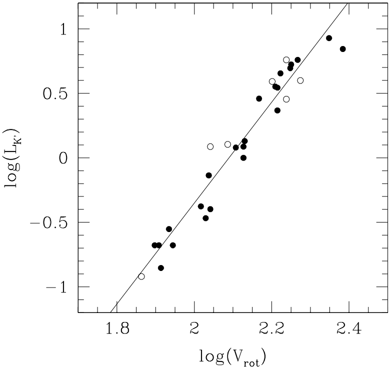
The fact that $v(r)$ is approximately constant implies that beyond the optical disk

$$M_{\text{halo}}(r) \approx r \qquad \rho_{\text{halo}}(r) \approx \frac{1}{r^2}$$

The dark matter paradigm [Bertone, Hooper & Silk 2004]

1. Dark matter is made of unknown non-baryonic particles, *e.g.*
   - neutrinos (but $\Omega_\nu \lesssim 7\%$)
   - neutralinos (predicted by super-symmetric extensions of the standard model)
   - light scalar [Boehm, Cassé & Fayet 2003]
   - axions
   - $\cdots$
2. It accounts for the observed mass discrepancy between the dynamical and luminous masses of bounded astrophysical systems at the scale of clusters of galaxies
3. It triggers the formation of large-scale structures by gravitational collapse and predicts the scale-dependence of density fluctuations
4. It suggests some universal dark matter density profile [Navarro, Frenk & White 1995]
5. It has difficulties at explaining naturally the flat rotation curves of galaxies and the Tully-Fisher relation

The Tully-Fisher empirical relation [Tully & Fisher 1977]



The relation between the asymptotic flat velocity and the luminosity of spirals is

$$v_{\text{flat}} \propto L^{1/4}$$

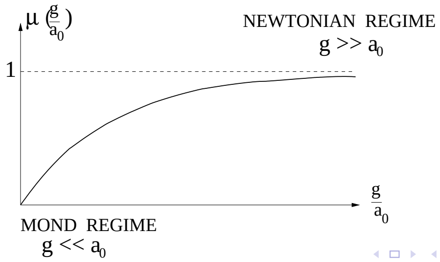
PHENOMENOLOGY OF MOND

Modified Newtonian dynamics (MOND) [Milgrom 1983]

1. MOND was proposed as an alternative to the dark matter paradigm
2. It states that there is no dark matter and we witness a violation of the fundamental law of gravity
3. MOND is designed to account for the basic phenomenology of the flat rotation curves of galaxies and the Tully-Fisher relation

The Newtonian gravitational field $\vec{g}_{\mathrm{N}}$ is modified in an *ad hoc* way

$$\mu\left(\frac{g}{a_0}\right)\vec{g} = \vec{g}_{\mathrm{N}}$$



The deep MOND regime [Milgrom 1983]

$$\mu\left(\frac{g}{a_0}\right) \approx \frac{g}{a_0} \quad \text{in the MOND regime (when } g \ll a_0\text{)}$$

- For a spherical mass $g_{\mathrm{N}} = \frac{G\,M}{r^2}$ hence

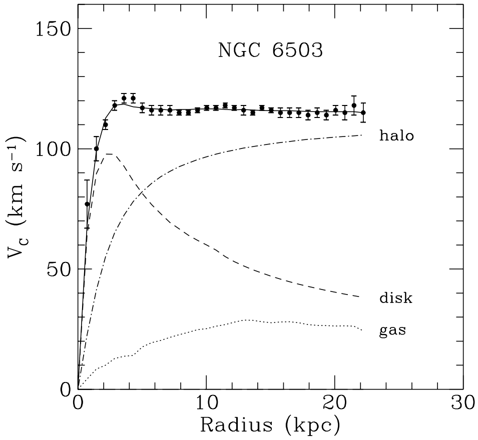
$$g \approx \frac{\sqrt{G\,M\,a_0}}{r} \quad \text{and} \quad U \approx -\sqrt{G\,M\,a_0}\,\ln r$$

- For circular motion $\frac{v^2}{r} = g$ thus

$$v_{\mathrm{flat}} \approx (G\,M\,a_0)^{1/4}$$

- Assuming that $L/M \approx$ const we recover the Tully-Fisher relation
- The numerical value of the critical acceleration is $a_0 \approx 10^{-10}\,\mathrm{m/s^2}$

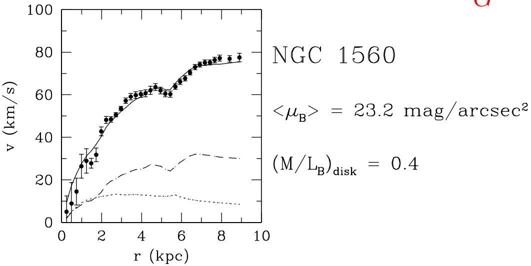
The MOND fit of galactic rotation curves



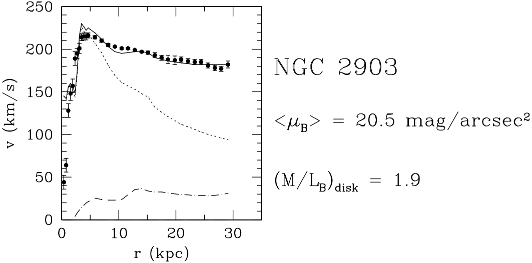
$\Longleftarrow$ the solid line is the MOND fit

Low and High surface brightness galaxies

$$\Sigma_\mathrm{m} = \frac{a_0}{G}$$

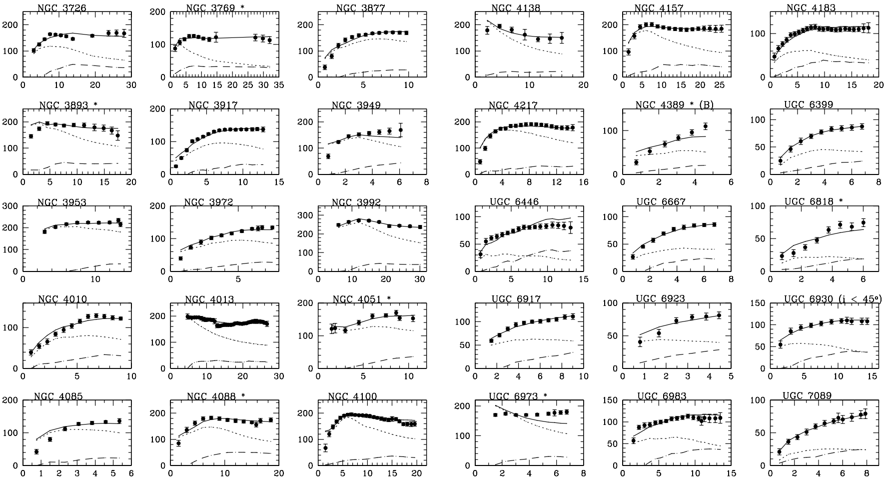


LSB $(\Sigma \lesssim \Sigma_\mathrm{m})$



HSB $(\Sigma \gtrsim \Sigma_\mathrm{m})$

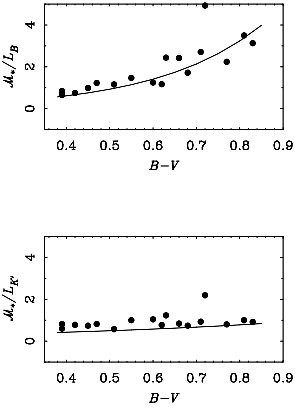
Many galaxies are fitted with MOND



Fit of the mass-to-luminosity ratio

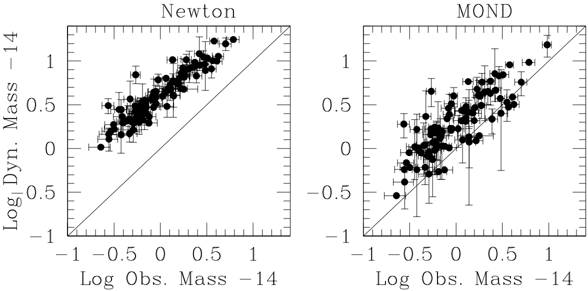
The fit of rotation curves is actually a one-parameter fit. The mass-to-luminosity ratio $\mathcal{M}/L$ of each galaxy is adjusted.

$\mathcal{M}/L$ shows the same trend with colour as is implied by models of population synthesis [Sanders & Verheijen 1998]



SOME PROBLEMS WITH MOND

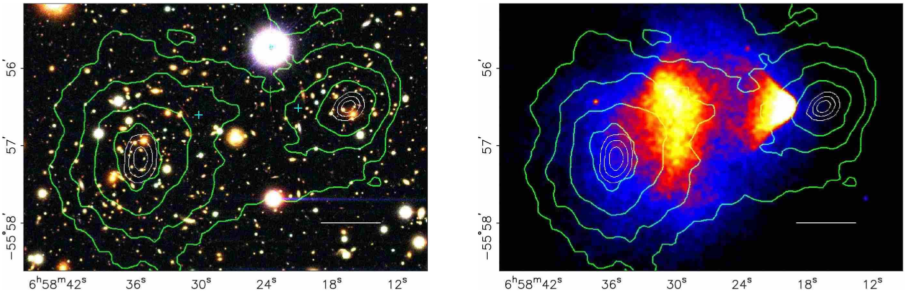
Problem with the galaxy clusters [Gerbal, Durret *et al* 1992, Sanders 1999]



The mass discrepancy is $\approx 4 - 5$ with Newton and ≈ 2 with MOND

- We may reject MOND
- Or say that the mass budget of galaxy clusters is not complete (the mass discrepancy was larger before the discovery that most of the mass is in hot X-ray emitting gas)
- And/or say that the agreement of MOND with the galactic rotation curves is impressive enough to look for an explanation

Collision of two clusters of galaxies [Clowe *et al* 2006]



This is a serious argument against MOND-like modifications of gravity

Dark matter versus MOND

The alternative seems to be

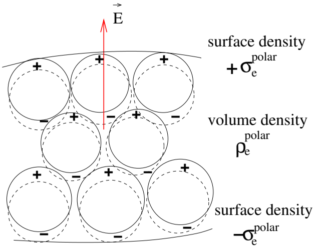
1. Either accept the existence of cold dark matter but
   - made of unknown non-baryonic particles yet to be discovered
   - which fails to reproduce in a natural way the flat rotation curves of galaxies
2. Or postulate a modification of the fundamental law of gravity (MOND and its relativistic extensions [Bekenstein & Sanders 2004]) but
   - on a completely *ad hoc* and physically unjustified basis
   - which has a problem at the scale of clusters of galaxies

Here we adopt a different approach

1. Keep the standard law of gravity namely general relativity and its Newtonian limit
2. Use the phenomenology of MOND to guess what could be the (probably unorthodox) nature of dark matter
3. Leave aside the problem of clusters of galaxies

NEWTONIAN MODEL OF GRAVITATIONAL POLARIZATION

The electric field in a dielectric medium



The atoms in a dielectric are modelled by electric dipole moments

$$\pi_e^i = q\, d^i$$

The polarization vector is

$$\Pi_e^i = n\, \pi_e^i$$

Density of polarization charges $\rho_e^{\text{polar}} = -\partial_i \Pi_e^i$

$$\partial_i E^i = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0} \quad \Longleftrightarrow \quad \partial_i \left(E^i + \frac{\Pi_e^i}{\varepsilon_0} \right) = \frac{\rho_e}{\varepsilon_0}$$

The dipoles and polarization vector are aligned with the electric field

$$\frac{\Pi_e^i}{\varepsilon_0} = \underbrace{\chi_e(E)}_{\text{electric susceptibility}} \quad E^i$$

Interpretation of the MOND equation

The MOND equation derivable from a Lagrangian [Bekenstein & Milgrom 1984] reads

$$\partial_i \Big[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} g^i \Big] = -4\pi \, G \, \rho$$

It is formally analogous to the equation of electrostatics inside a dielectric

$$\mu = 1 + \underbrace{\chi(g)}_{\text{gravitational susceptibility}} \quad \text{and} \quad \underbrace{\Pi^i}_{\text{gravitational polarization}} = -\frac{\chi}{4\pi \, G} \, g^i$$

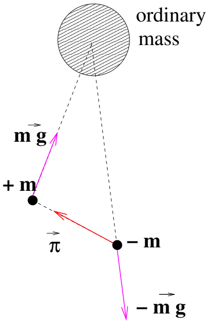
The MOND equation is equivalent to

$$\Delta U = -4\pi \, G \, (\rho + \rho_{\text{polar}})$$

In this interpretation the Newtonian law of gravity is not violated but we are postulating a new form of dark matter consisting of polarization masses with density

$$\rho_{\text{polar}} = -\partial_i \Pi^i$$

Sign of the gravitational susceptibility



The digravitational medium is modelled by individual dipole moments π^i

$$\pi^i = m\, d^i$$
$$\Pi^i = n\, \pi^i$$

We suppose that the dipoles consist of particles doublets with

- opposite gravitational masses $m_{\rm g} = \pm m$
- positive inertial masses $m_{\rm i} = m$

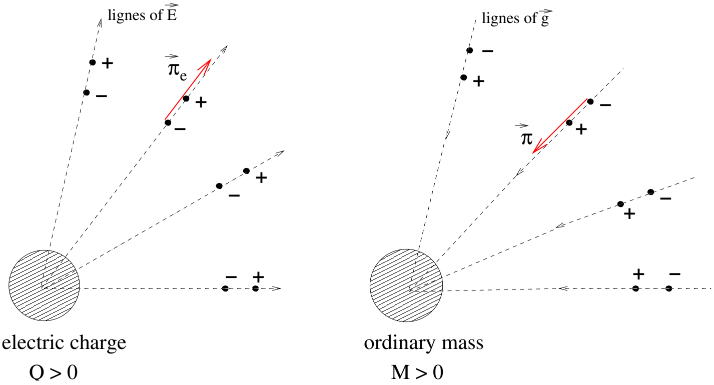
1. The gravitational force is governed by a negative Coulomb law [Bondi 1957]
2. The dipoles tend to align in the same direction as the gravitational field thus

$$\boxed{\chi < 0}$$

which is nicely compatible with MOND since $0 < \mu < 1 \Longrightarrow -1 < \chi < 0$

3. The dipole cannot be stable in a gravitational field and we must invoke some non-gravitational force

Electric screening versus gravitational anti-screening



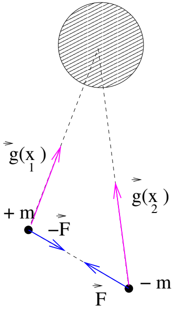
Screening by polarization charges

$\boxed{\chi_e > 0}$

Anti-screening by polarization masses

$\boxed{\chi < 0}$

Equations of motion of dipolar particles



$$m\,\frac{d^2x_1^i}{dt^2} = m\,g^i(x_1) - F^i(x_1 - x_2)$$

$$m\,\frac{d^2x_2^i}{dt^2} = -m\,g^i(x_2) + F^i(x_1 - x_2)$$

F^i is an attractive non-gravitational force

$$x^i = \frac{x_1^i + x_2^i}{2} \qquad \pi^i = m\,(x_1^i - x_2^i)$$

The equations of motion to first order in $d = |x_1 - x_2|$ are

$$2m\,\frac{d^2x^i}{dt^2} = \pi^j\,\partial_{ij}U + \mathcal{O}(d^2)$$

$$\frac{d^2\pi^i}{dt^2} = 2m\,g^i - 2F^i + \mathcal{O}(d^2)$$

The dipolar particle violates the equivalence principle since $M_\mathrm{i} = 2m$ while $M_\mathrm{g} = 0$. It is accelerated by the tidal gravitational field $\partial_{ij}U$

The internal force law

At equilibrium $\pi^i = \text{const}$ hence

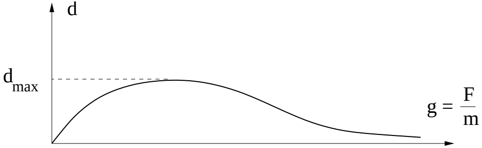
$$F^i = m\, g^i$$

Since $F^i \propto \pi^i$ the dipole is aligned with g^i, thus $F(d) = m\, g \Longrightarrow d = d(g)$. Using $\Pi = n\, m\, d = -\frac{\chi}{4\pi G} g$ we get the MOND function $\mu = 1 + \chi$ at equilibrium as $\mu\left(\frac{g}{a_0}\right) = 1 - k\, m \frac{d(g)}{g}$ where $k = 4\pi G n$

In the MOND regime $\mu = \frac{g}{a_0} + \mathcal{O}(g^2)$ hence

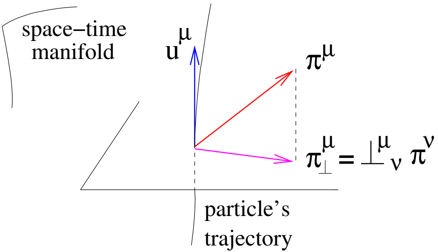
$$d = \frac{g}{k\, m}\left[1 - \frac{g}{a_0} + \mathcal{O}(g^2)\right]$$

$$F(d) = k\, m^2 d \left[1 + \frac{k\, m}{a_0} d + \mathcal{O}(d^2)\right]$$



DIPOLAR PARTICLES IN GENERAL RELATIVITY

Action principle



The particle's velocity is

$$u^\mu = \frac{dx^\mu}{d\tau}$$

The covariant time derivative of π^μ is

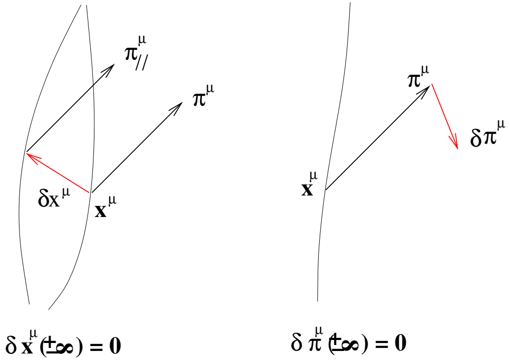
$$\dot{\pi}^\mu = \frac{D\pi^\mu}{d\tau} = u^\nu \nabla_\nu \pi^\mu$$

The action is

$$S = \int d\tau \left[-c\sqrt{g_{\mu\nu}\left(2m\, u^\mu - \dot{\pi}^\mu\right)\left(2m\, u^\nu - \dot{\pi}^\nu\right)} + \frac{\dot{\pi}_\mu\, \dot{\pi}^\mu}{4m} - V\left(\frac{\pi_\perp}{m}\right)\right]$$

- The first term is inspired by the action of spinning particles in general relativity [Bailey & Israel 1980]
- The second term is a kinetic-like term and will tell how the evolution of the dipole moment differs from parallel transport
- The third term is a potential term supposed to describe a non-gravitational force F^μ internal to the dipole

Equations of motion and evolution



$$\dot{\Omega}^\mu = -\frac{1}{2m} R^\mu{}_{\rho\nu\sigma} u^\rho \pi^\nu P^\sigma \qquad \dot{P}^\mu = -2F^\mu$$

where Ω^μ and P^μ are two basic linear momenta and F^μ is the internal force

$$F^\mu = \frac{\pi_\perp^\mu}{\pi_\perp} V'\left(\frac{\pi_\perp}{m}\right)$$

Expression of the linear momenta

The linear momenta read

$$\begin{aligned}\Omega^\mu &= \omega^\mu - p^\mu \\ P^\mu &= p^\mu + \dot{\pi}^\mu\end{aligned}$$

- where p^μ is a space-like vector ($p^2 = +4m^2c^2$)

$$p^\mu = \frac{2m\,u^\mu - \dot{\pi}^\mu}{\Lambda}$$

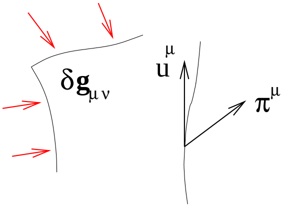
$$\text{where} \quad \Lambda = \sqrt{-1 - \frac{u_\nu \dot{\pi}^\nu}{mc^2} + \frac{\dot{\pi}_\nu \dot{\pi}^\nu}{4m^2c^2}}$$

(the first term in the action is given by the norm of p^μ)

- and ω^μ is given by

$$\omega^\mu = \frac{u^\mu}{c^2}\left(\frac{\dot{\pi}_\nu\,\dot{\pi}^\nu}{4m} + V\right) - \frac{u_\nu \pi^\nu}{mc^2} F^\mu$$

Derivation of the stress-energy tensor



$$\delta g_{\mu\nu} = 0 \quad \text{when} \quad |x^\lambda| \longrightarrow \infty$$

$$T^{\mu\nu} = n\,\Omega^{(\mu}u^{\nu)} - \frac{1}{2m}\nabla_\rho\Big(n\Big[\pi^\rho P^{(\mu} - P^\rho \pi^{(\mu}\Big]u^{\nu)}\Big)$$

where n is the number density of the particles

$$\nabla_\nu(n\,u^\nu) = 0$$

We verify that as a consequence of the equations of motion

$$\nabla_\nu T^{\mu\nu} = 0$$

A particular class of solutions

Two constraints:

- $u_\mu \dot{\Omega}^\mu = 0$ is identically satisfied
- $u_\mu \dot{P}^\mu = 0$ can be solved by choosing $\boxed{\Lambda = 1}$

Under this condition we find

$$\begin{aligned} P^\mu &= 2m\, u^\mu \\ \Omega^\mu &= \frac{V}{c^2}\, u^\mu + \perp^\mu_\nu\, \dot{\pi}^\nu_\perp \end{aligned}$$

P^μ is the linear momentum associated with the particle's inertial mass or equivalently passive gravitational mass

$$M_\mathrm{i} = 2m = M_\mathrm{p}$$

Ω^μ enters the monopole part of $T^{\mu\nu}$ and can be viewed as the active linear momentum and we may define the particle's active gravitational mass

$$M_\mathrm{a} = -\frac{1}{c^2} u_\mu \Omega^\mu = \frac{V}{c^2}$$

Equations of motion and stress-energy tensor

The final equations of motion are

$$\begin{aligned} 2m\, a^\mu &= -2F^\mu \\ \dot{\Omega}^\mu &\equiv \frac{D}{d\tau}\left[\frac{V}{c^2}\, u^\mu + \perp^\mu_\nu\, \dot{\pi}^\nu_\perp\right] = -\underbrace{\pi^\nu_\perp\, R^\mu{}_{\rho\nu\sigma} u^\rho u^\sigma}_{\text{coupling to curvature}} \end{aligned}$$

The stress-energy tensor reads

$$T^{\mu\nu} = n\,\Omega^{(\mu} u^{\nu)} - \nabla_\rho \underbrace{\left(n\left[\pi^\rho_\perp u^{(\mu} - u^\rho \pi^{(\mu}_\perp\right] u^{\nu)}\right)}_{\text{polarization tensor}}$$

We observe that the physical components of the dipole moment which enter the final equations are those of the orthogonal projection

$$\boxed{\pi^\mu_\perp = \perp^\mu_\nu\, \pi^\nu}$$

which is space-like. One can show that the component longitudinal to the velocity $u_\mu \pi^\mu$ (which is unobservable) is actually complex

Stress-energy tensor in the non relativistic limit

The active gravitational mass which parametrizes the monopolar part of the stress-energy tensor is negligible in the non relativistic (NR) limit

$$M_{\mathsf{a}} = \mathcal{O}\left(c^{-2}\right)$$

The stress-energy tensor is purely dipolar in the NR limit

$$\begin{aligned} T^{00} &= -c^2\,\partial_i\left(n\,\pi^i_\perp\right) + \mathcal{O}(c^0)\,, \\ T^{0i} &= \mathcal{O}(c)\,, \\ T^{ij} &= \mathcal{O}(c^0) \end{aligned}$$

The effect of the dipolar matter is to add to the Newtonian density of the matter fields the Newtonian density of polarization

$$\rho_{\mathsf{polar}} = -\partial_i \Pi^i \quad \text{where} \quad \Pi^i = n\,\pi^i_\perp$$

Dynamics of dipolar particles in the NR limit

1. Equation of motion of the dipolar particle

$$m\,\frac{d^2x^i}{dt^2} = m\,g^i - F^i + \mathcal{O}\left(c^{-2}\right)$$

2. Evolution equation of the dipole moment

$$\frac{d^2\pi^i_\perp}{dt^2} = \pi^j_\perp\,\partial_{ij}U + \mathcal{O}\left(c^{-2}\right)$$

3. Conservation of the number of particles

$$\partial_t\,n + \partial_i\big(n\,v^i\big) = \mathcal{O}\left(c^{-2}\right)$$

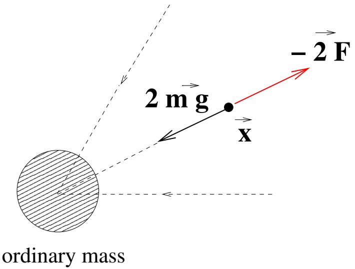
4. Poisson equation for the gravitational potential

$$\Delta U = -4\pi G\big(\rho + \rho_{\text{polar}}\big) + \mathcal{O}\left(c^{-2}\right)$$

In addition we have the standard equations of motion for ordinary matter (stars) and relativistic particles (photons). The theory predicts the standard general relativistic deviation of photons in the gravitational potential U

Recovering MOND

The dipolar particle is accelerated like a rocket by the internal force F^μ



Neglecting the tidal gravitational field $\partial_{ij}U$ we consider the solution for which the particle is at rest in the gravitational field

$$x^i \approx \text{const}$$

For this solution the internal force is equal to the weight

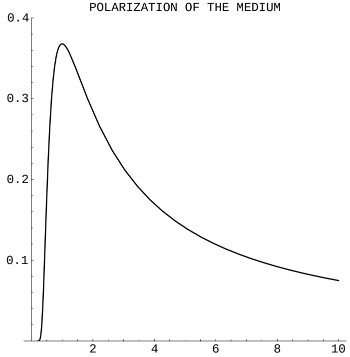
$$\boxed{F^i \approx m\, g^i}$$

so we recover MOND exactly like in the first (quasi Newtonian) model

Polarization of the dipolar medium around a point mass

Polarization $\Pi = n\,\pi_\perp$ of the gravitational field produced by a point mass M with susceptibility coefficient (equivalent to a choice for V)

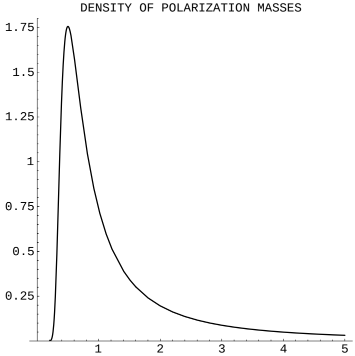
$$\chi = -e^{-g/a_0} \qquad \text{(hence } \mu = 1 - e^{-g/a_0}\text{)}$$



The unit of Π is $a_0/4\pi G$ and the unit of r is the distance at which the transition between the MOND and Newtonian regimes occurs, r_0 such that $g(r_0) = a_0$

Density of dipolar dark matter around a point mass

Density of polarization masses $\rho_{\text{polar}} = -\partial_i \Pi^i$ (in unit of $a_0/4\pi G r_0$)



When $r \to \infty$ we find that ρ_{polar} behaves like

$$\rho_{\text{polar}}(r) \sim \frac{1}{4\pi\, r^2}\sqrt{\frac{M\, a_0}{G}} \quad \Longrightarrow \quad M_{\text{polar}}(r) \sim \sqrt{\frac{M\, a_0}{G}}\, r$$

which accounts for the flat rotation curves of galaxies

Number density and velocity of dipole moments