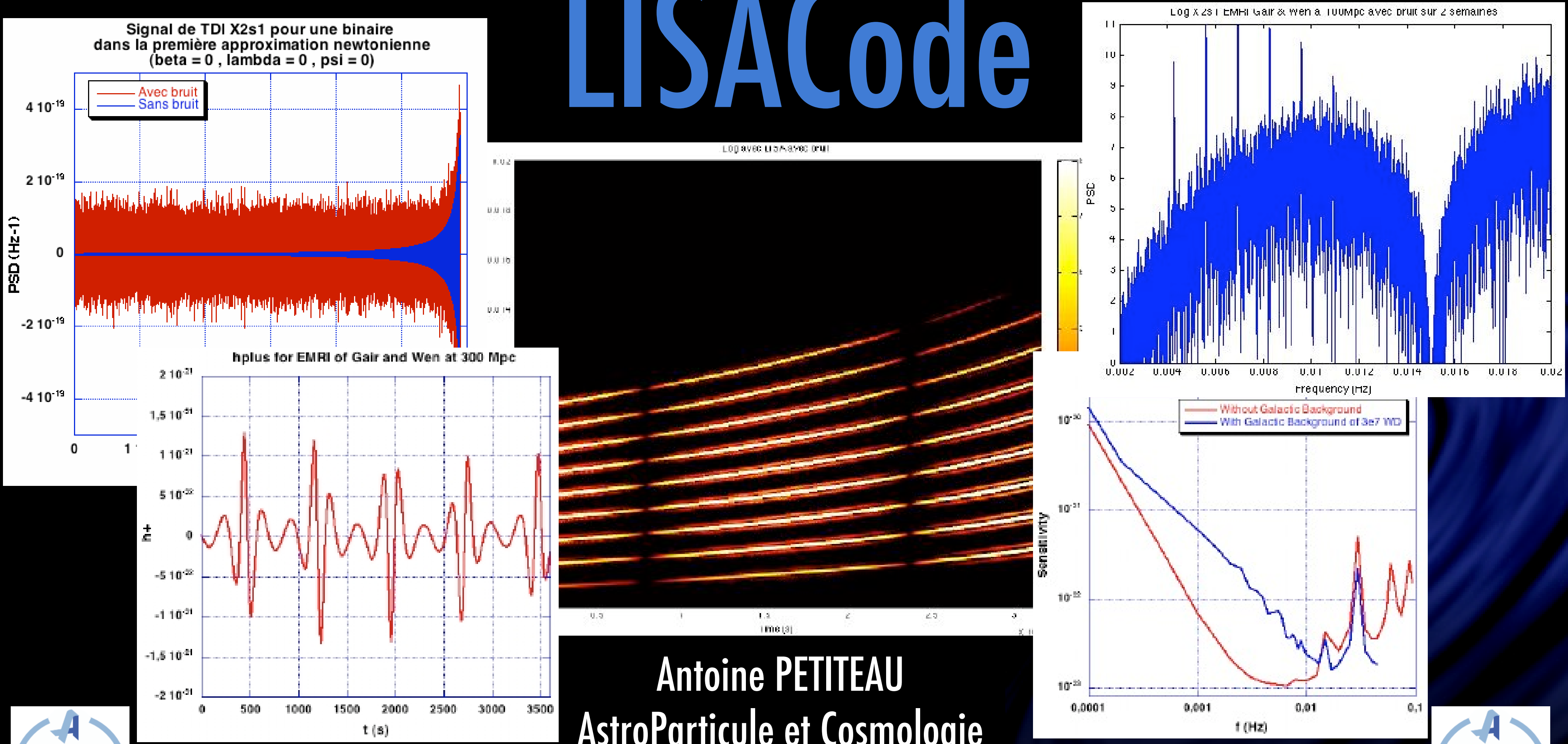
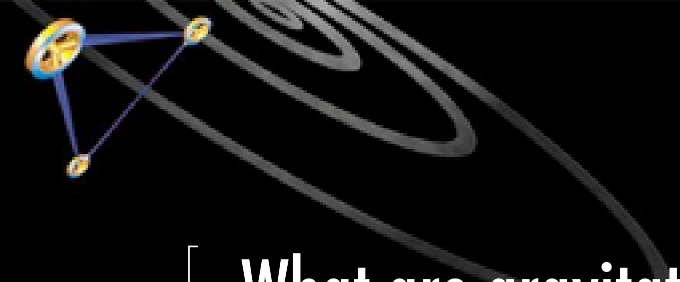
From gravitational wave sources to LISA data analysis :

LISACode

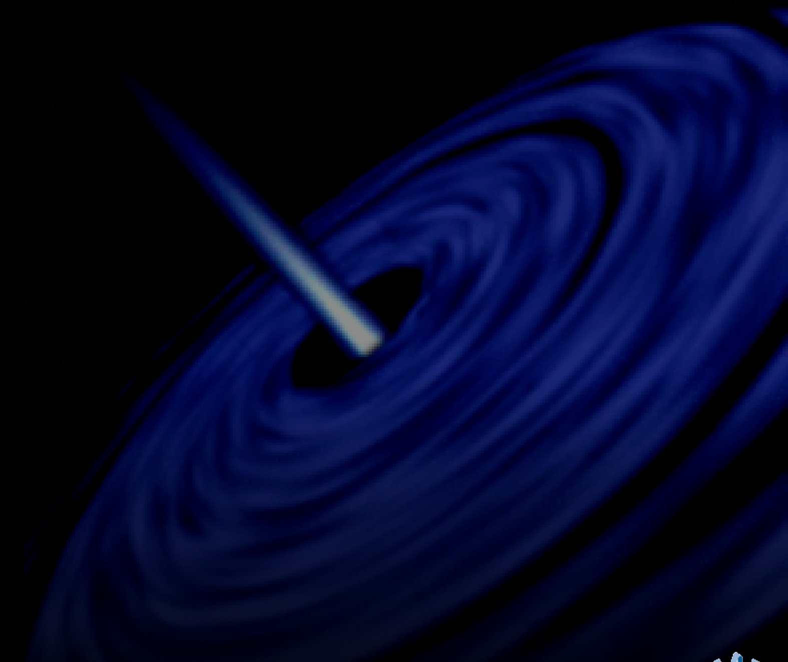


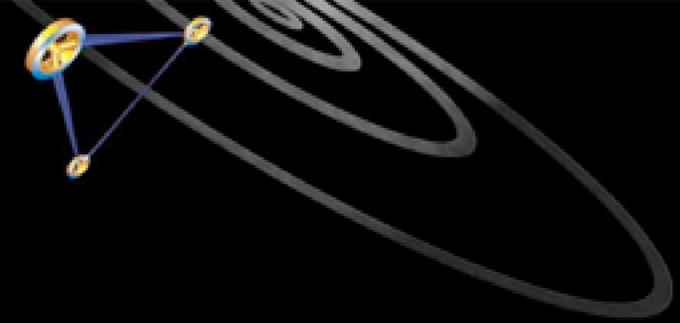
Antoine PETITEAU
AstroParticule et Cosmologie
Séminaire GReCO/IAP - 18 Septembre 2006



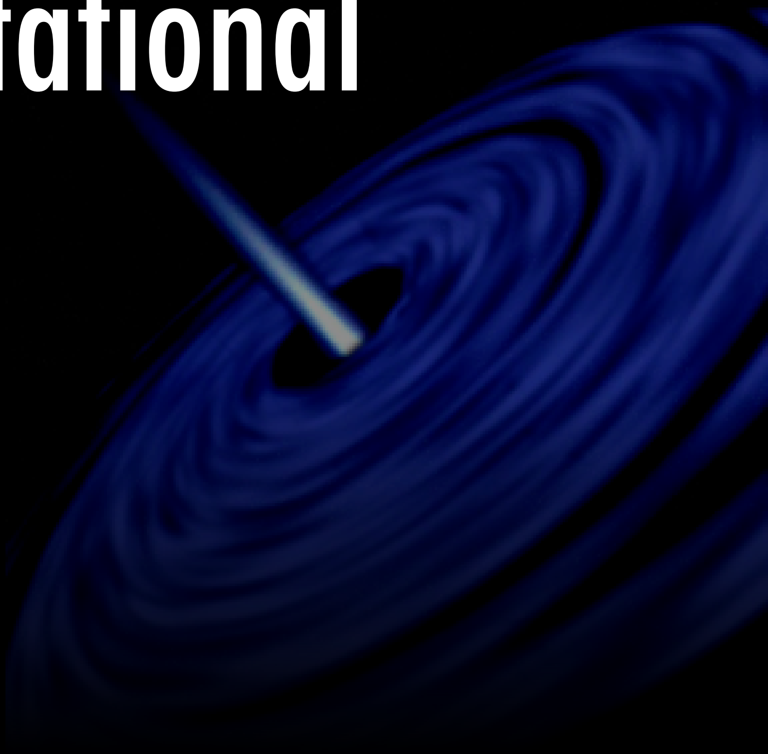
Outline

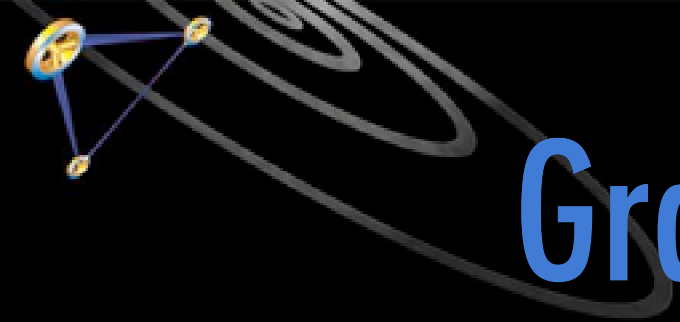
- What are gravitational waves ?
- Gravitational waves sources
  - Black Holes Binaries,
  - EMRIs,
  - Background (Galactic Confusion noise, cosmological background).
- How to detect gravitational waves ?
- Detecting Gravitational waves with LISA :
  - LISA configuration and gravitational waves detection,
  - Free fall in space,
  - Noises, Raw data and Time Delay Interferometry.
- LISACode : Simulating LISA
  - Description,
  - Result, Sensitivity.
- Data Analysis : Background, Time - Frequency.





What are the gravitational waves ?





Gravitational waves

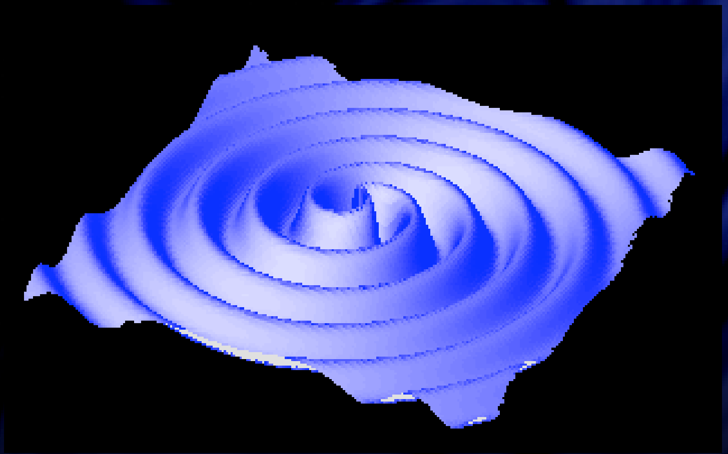
- *Newton* (1687) : Gravitation theory, its effect is instantaneous.

$$\boxed{\nabla^2\Phi = 4\pi G\rho}$$

- *Laplace* (1805) : If the propagation speed of gravity is limited, a binary system dissipates energy.

$$\boxed{\nabla^2\Phi - \frac{1}{c^4}\frac{\partial^2\Phi}{\partial t} = 4\pi G\rho}$$

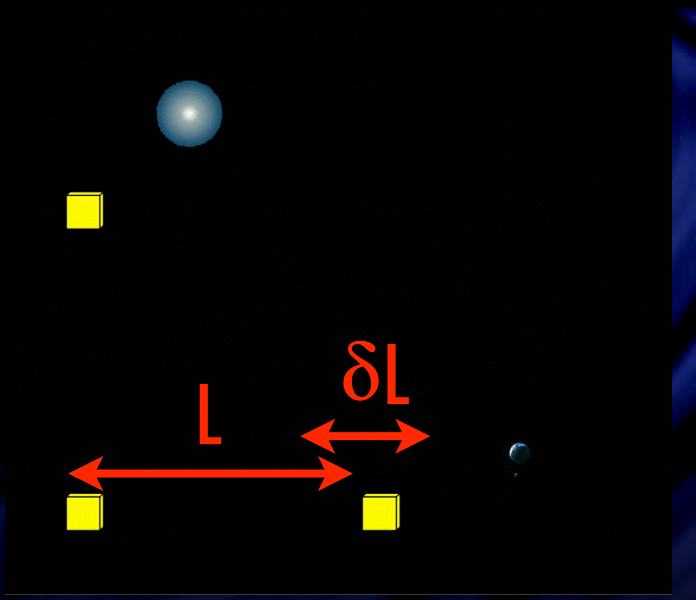
- *Einstein* (1905/1916) : Gravitational information is propagated at the speed of light, dissipation of energy by deformation of space-time : gravitational wave.



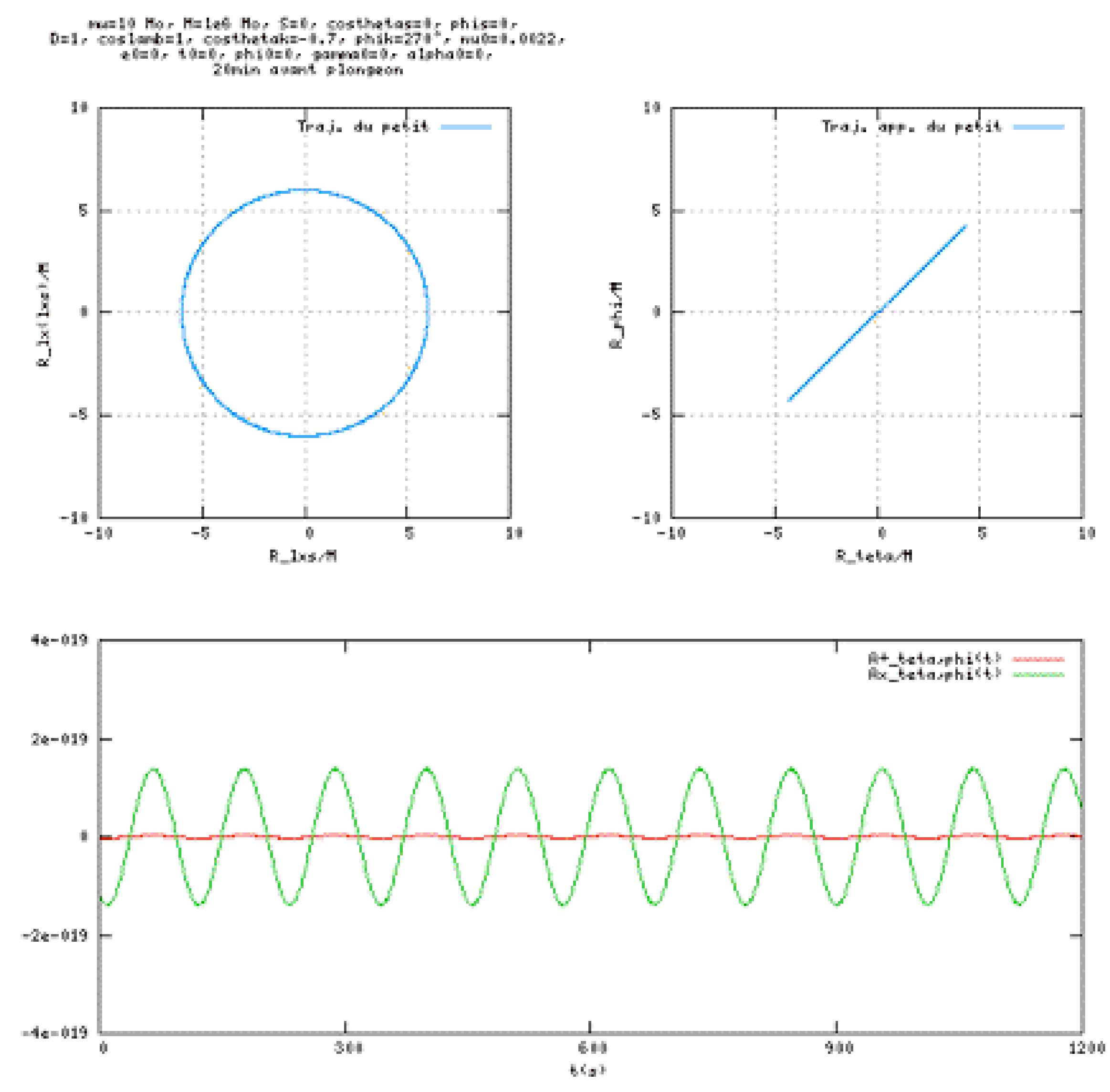
GWs characteristics

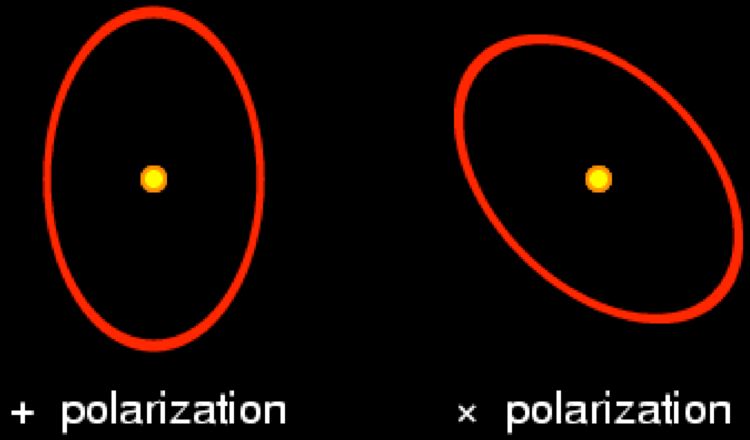
- Space-time deformation : a very small variation of distance
- Weak coupling, attenuation in 1/r.
- Transverse wave :
  - Strain perpendicular to the propagation,
  - Conservation of the surface.
- Two independent polarisation states : h_+ , h_x.

$\delta L < 10$ pm for $L \sim 10^9$ m !



Linear polarisation



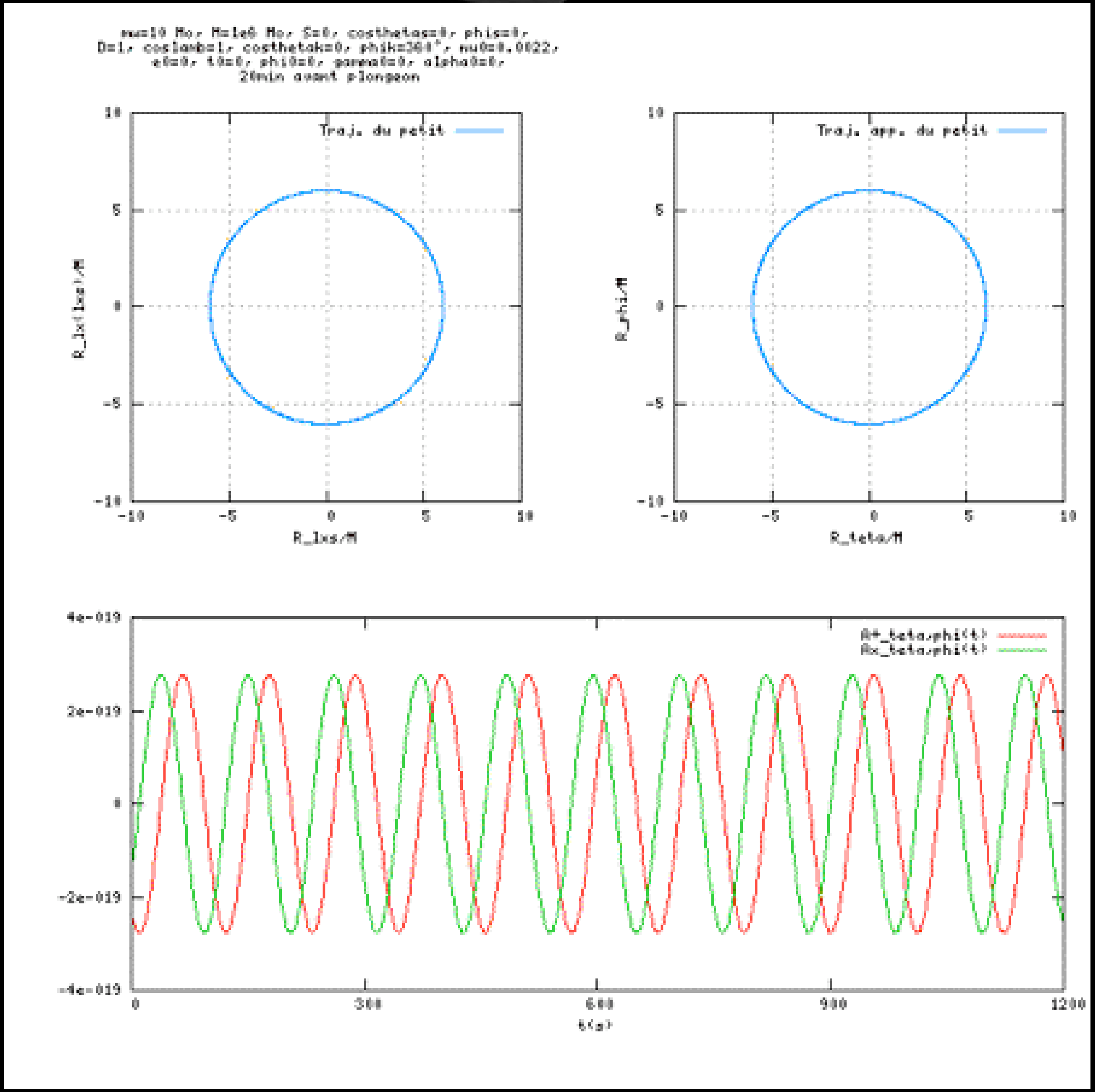


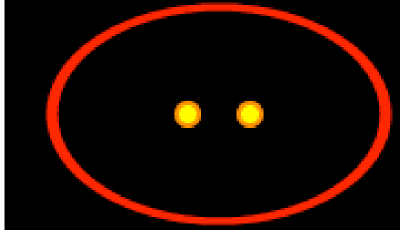
Components in phase, but not with the same amplitude.

Link with the source :

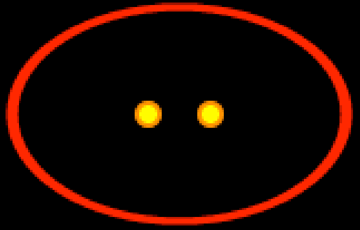
- Source reference (orbital momentum direction)
- Observer reference (source direction)
- Two polarisation components

Circular polarisation



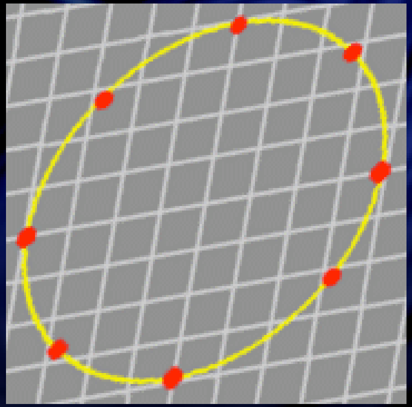


left polarization

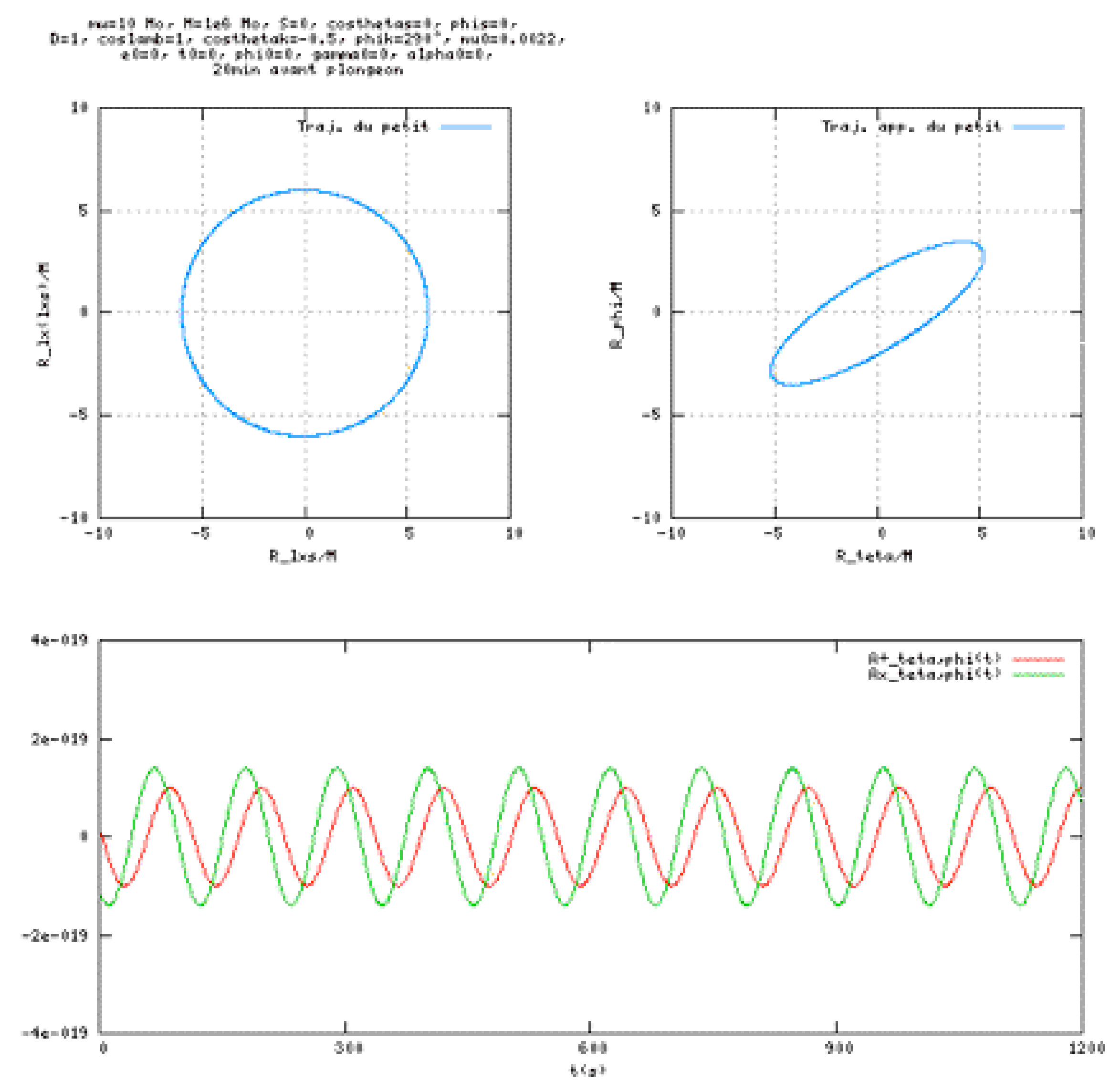


right polarization

Same amplitude but in quadrature.



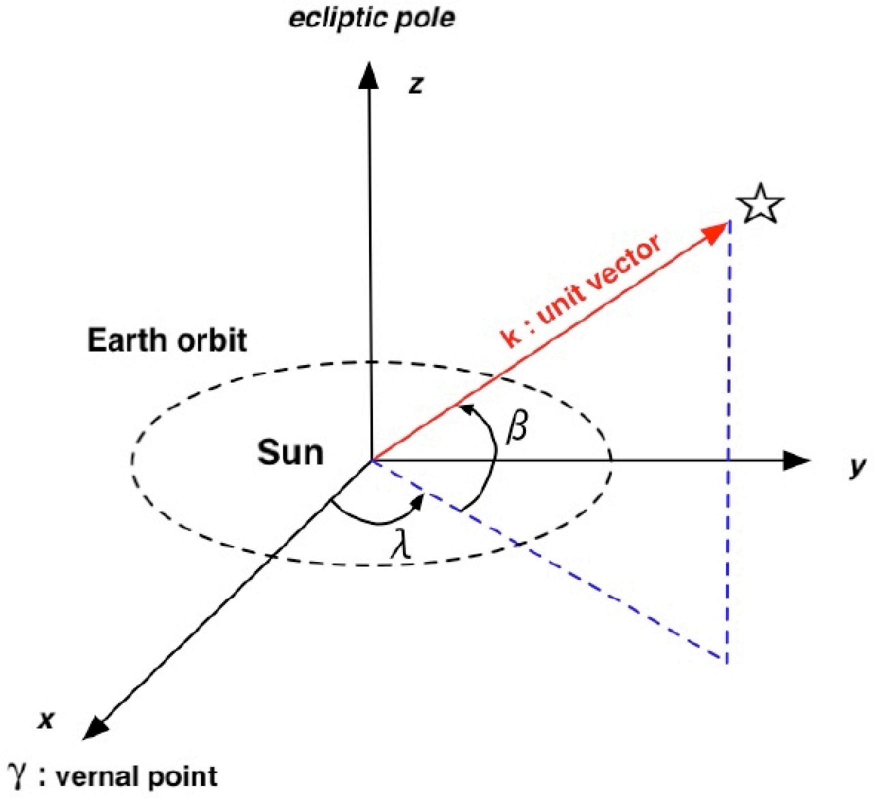
Elliptic polarisation

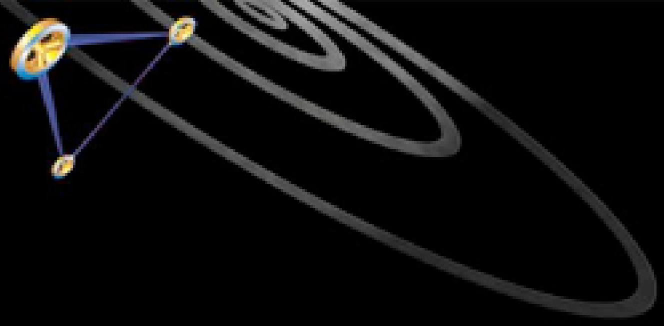


- In quadrature but not the same amplitude.
- Others cases with no polarisation

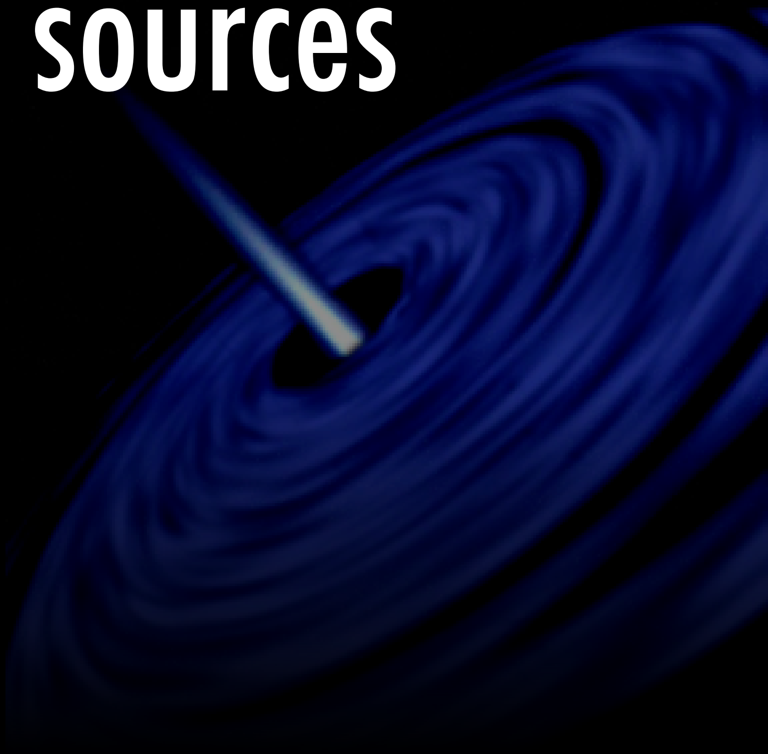
GWs description

- The gravitational wave can be characterised by the following parameters :
  - Source direction : λ and β .
  - Polarisation angle ψ
  - Component $h_+(t)$ and $h_\times(t)$ which evolved in time.

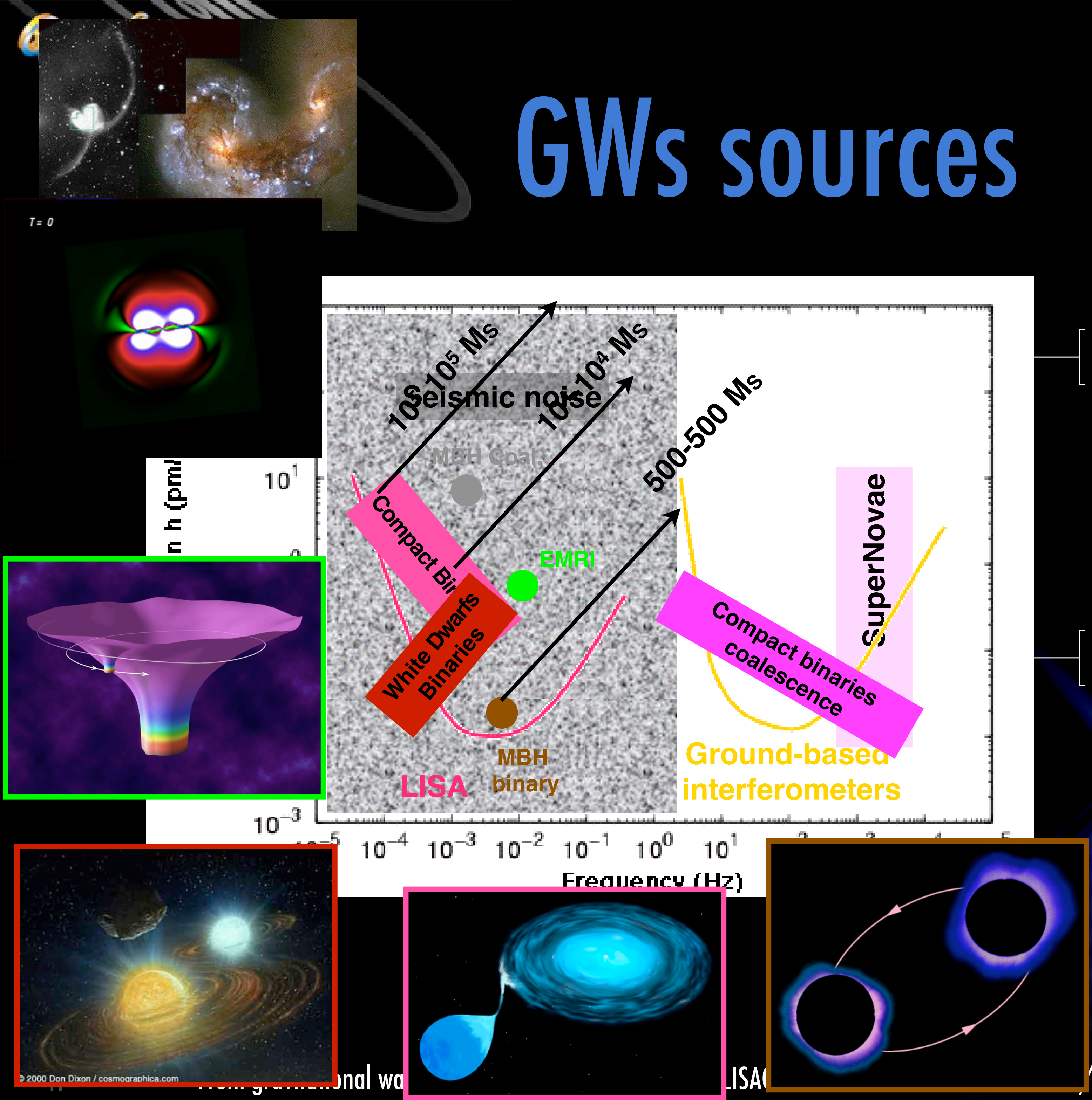




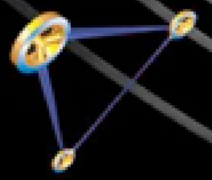
Gravitational waves sources



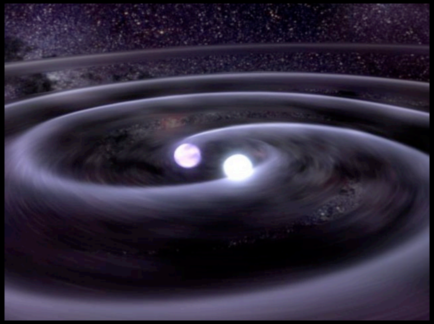
GWs sources



- Ground-based detectors : Low frequency wall due to the seismic noise
- In space, "no limit" : Low frequency objects : Neutron Stars binaries, Black Hole Binaries, Massive Black Hole Binaries, EMRIs, ...

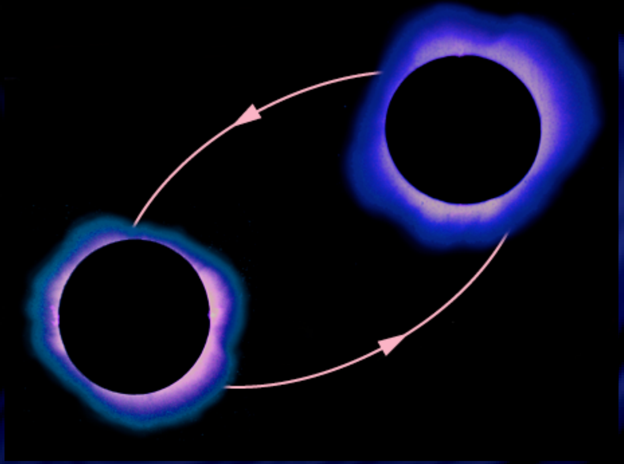


Binaries (similar masses)



- Low mass objects : 1 - 10^4 M_{Sun} :
  - Neutron Stars Binaries, Black Holes Binaries, ...

    ⇒ must be near : Galactic or Near Extra-Galactic.

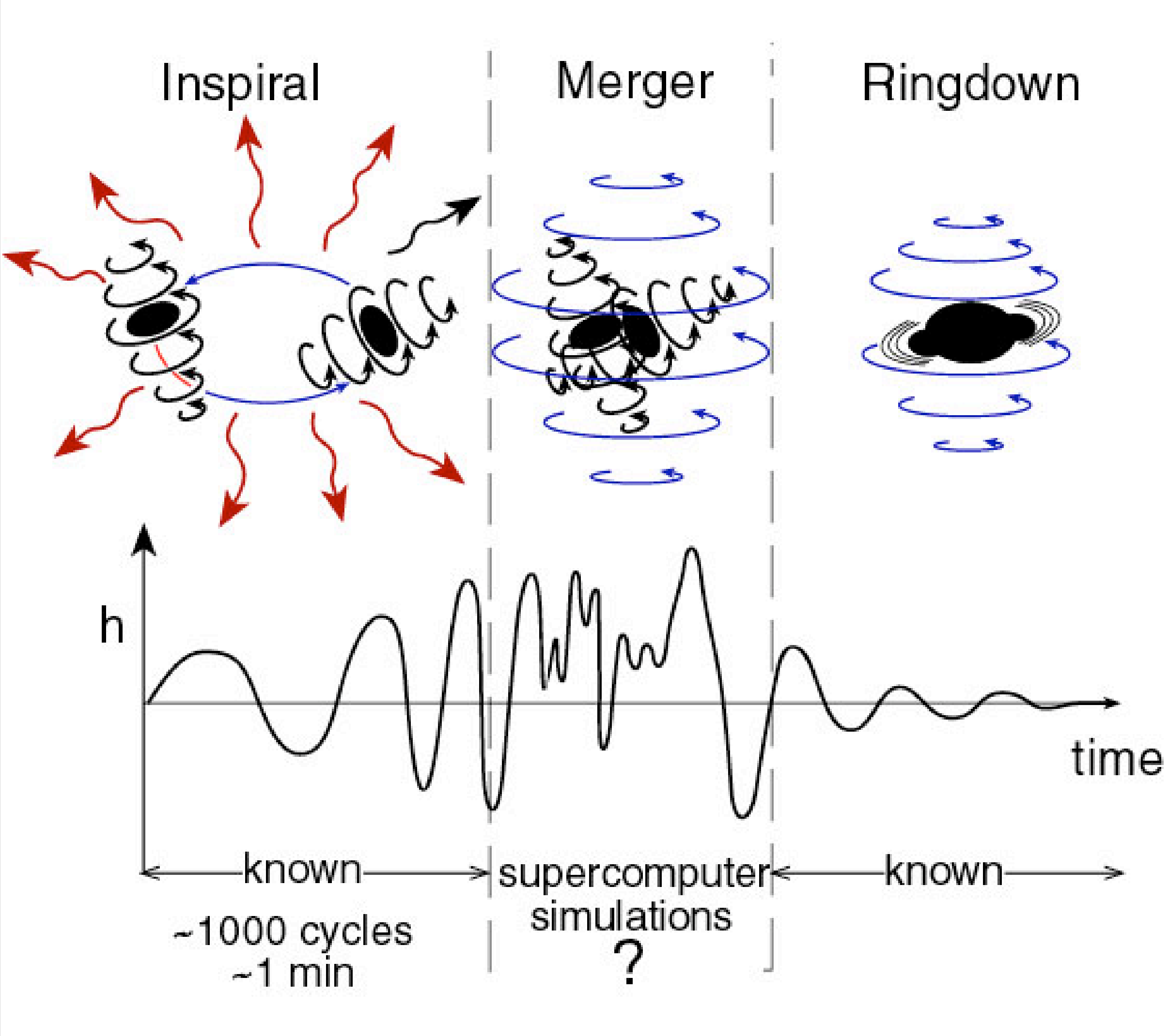


- Large mass object : 10^6 to 10^9 M_{Sun}
  - Supermassive Black Hole Binaries

    ⇒ Distant objects (over the whole universe)
  - Events rate : 0.1 to 100 per year !
  - For example, these binaries can be created when two galaxies merge.

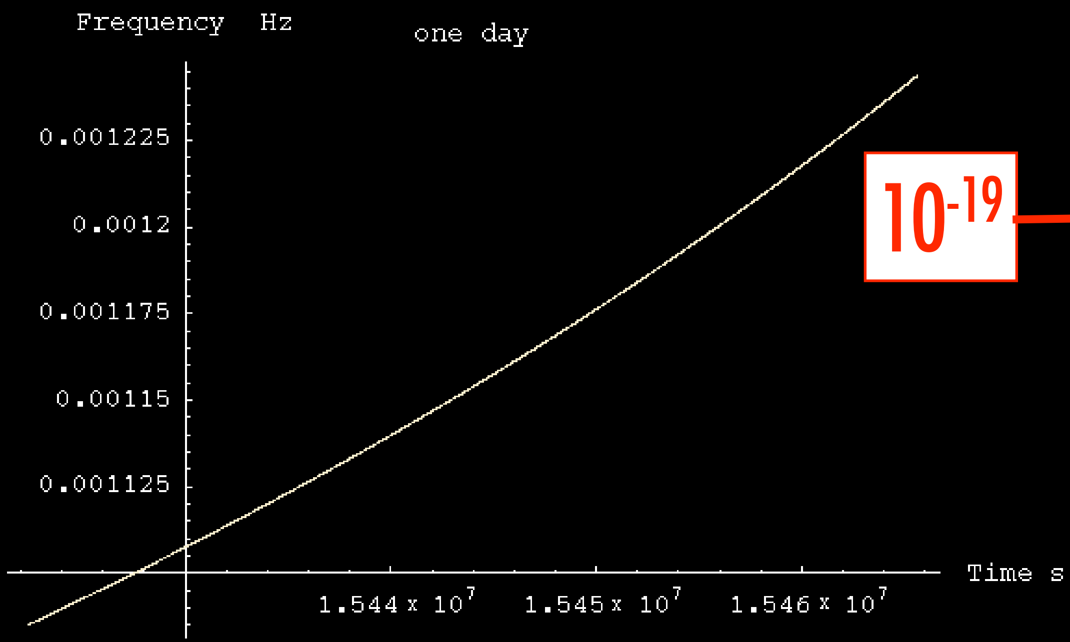
Binaries (similar masses)

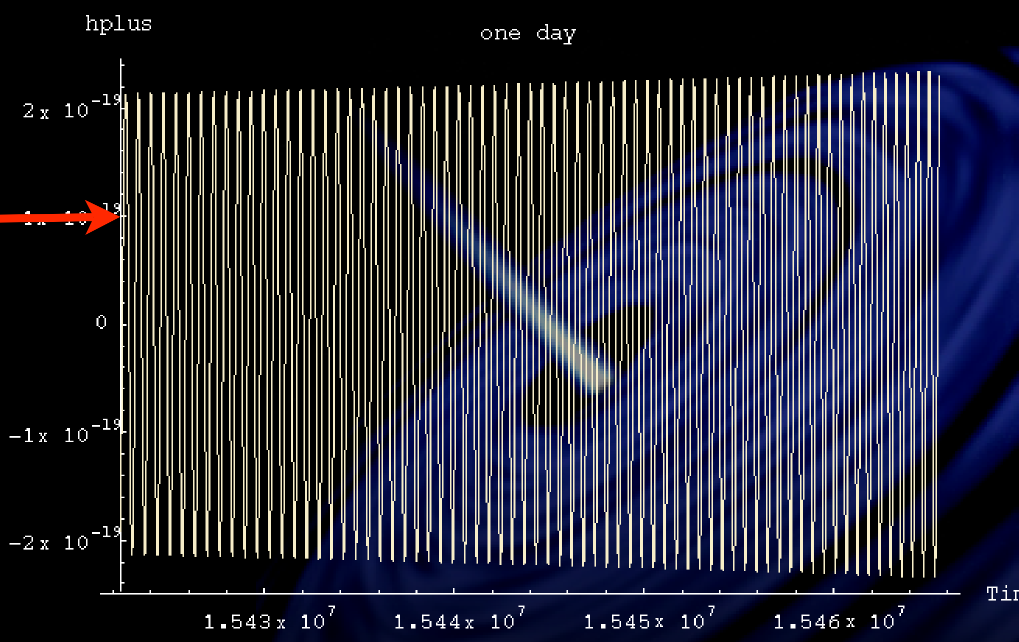
- PN model
- Close to coalescence, analytic model becomes too complex (high field gravity).
- Numerical relativity … in progress !



Binaries (similar masses) Post-Newtonian model

- L. Blanchet & al.
- Example for two Black Hole of 10^5 M_{Sun} at 1Gpc.

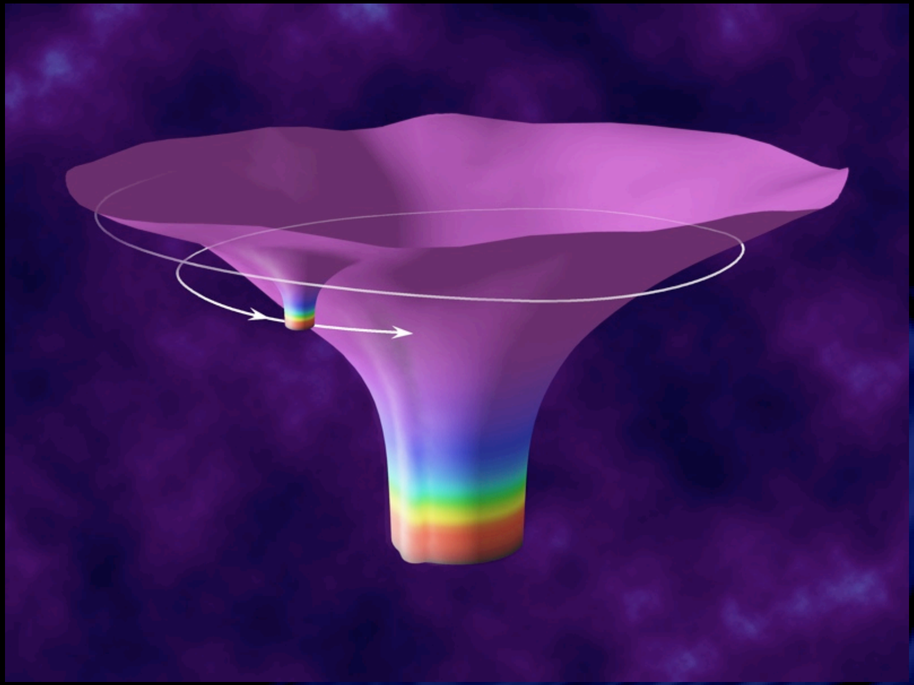




10^{-19}

EMRIs

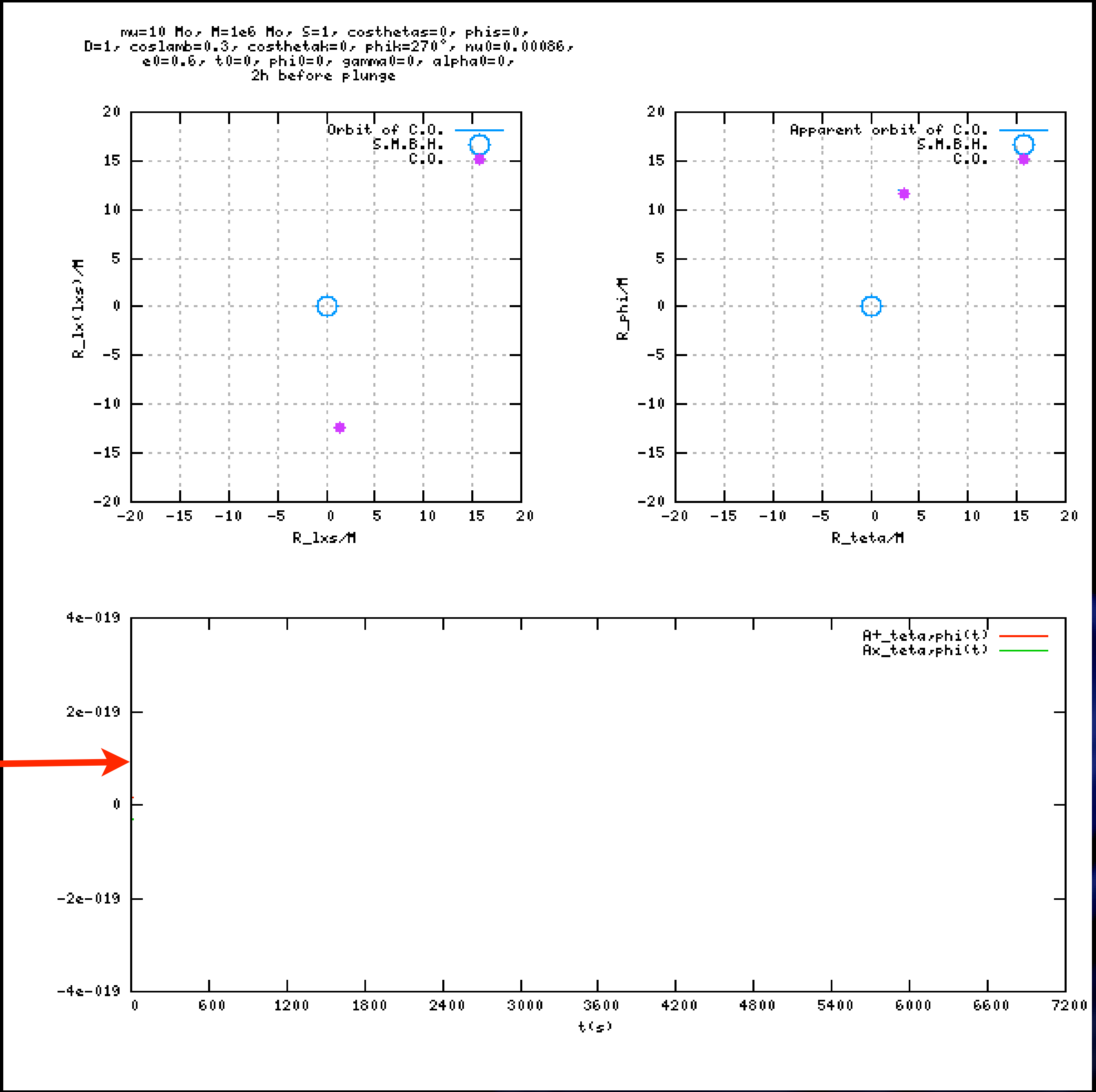
- Extreme Mass Ratio Inspiral
- SMBH with Star, White Dwarf, Neutron Star
- Complex objects : 14 parameters
- Possible EMRIs in the galactic centre
- Events rates (per year) :
  - 500 - 1000 for 10 M_{Sun} +10^6 M_{Sun},
  - 500 - 1000 for 0.6 M_{Sun} +10^6 M_{Sun},
  - 1 for 100 M_{Sun} +10^6 M_{Sun}



EMRIs

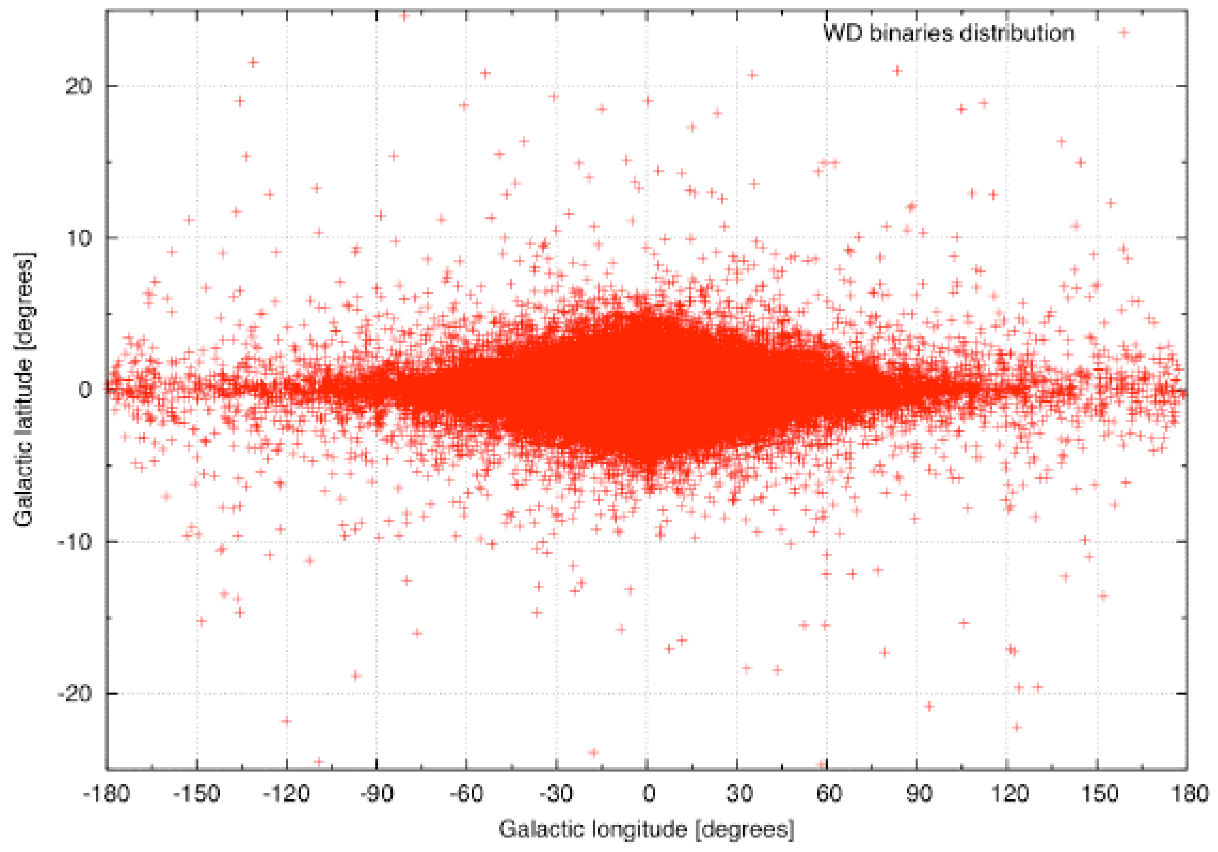
- GW evolution : model of Barack & Cutler, PRD 69 082005 (2004)
- Example :
  - 10 M_{Sun} +10^6 M_{Sun} ,
  - Spin = 1,
  - e = 0.6,
  - D = 1Mpc,
  - Position : λ=72,54° ,β=90° .

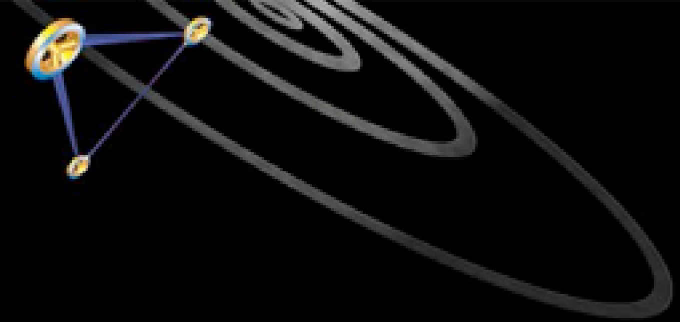
10^{-19}



Galactic binaries

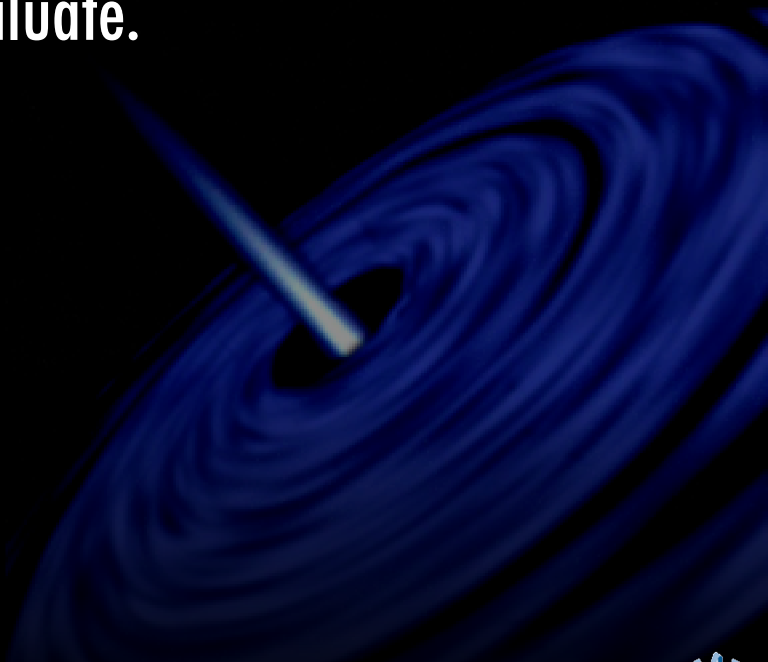
- There are many white dwarfs binaries systems in the Milky Way.
- Not detectable independently but the sum of these binaries give a background.
- Distribution :
  - Position
  - Distance
  - Frequency
  - Masses



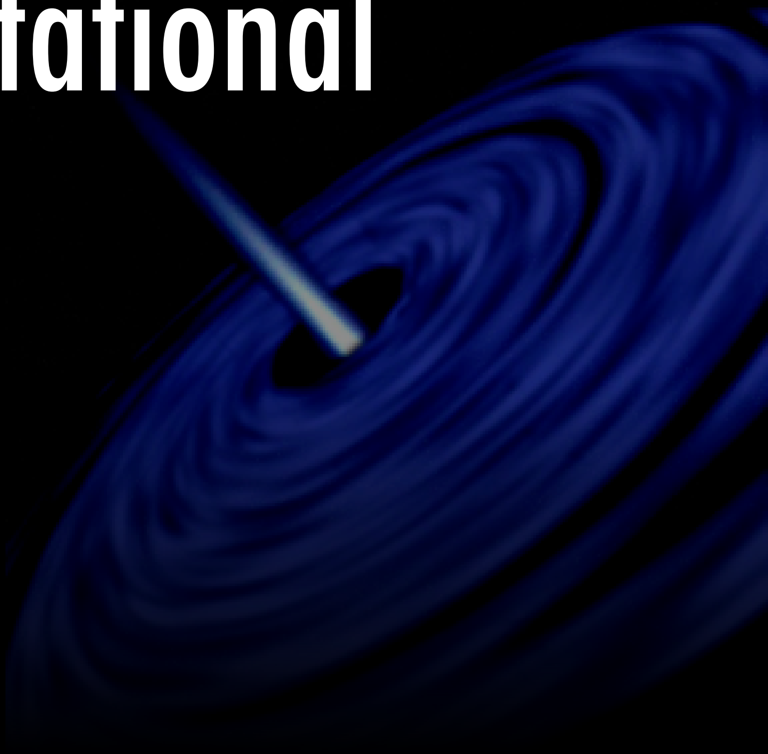


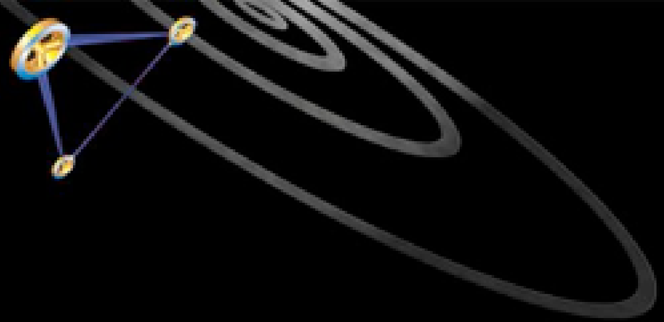
Others GWs

- Extra-galactic :
  - EMRIs, Neutron Star Binaries, Black Holes binaries,... too low to be detected independently.
  - Contribution to the background hard to evaluate.
- Cosmological background :
  - Very dependent on the model used.
  - Hypothetical detection ...



How to detect gravitational waves ?

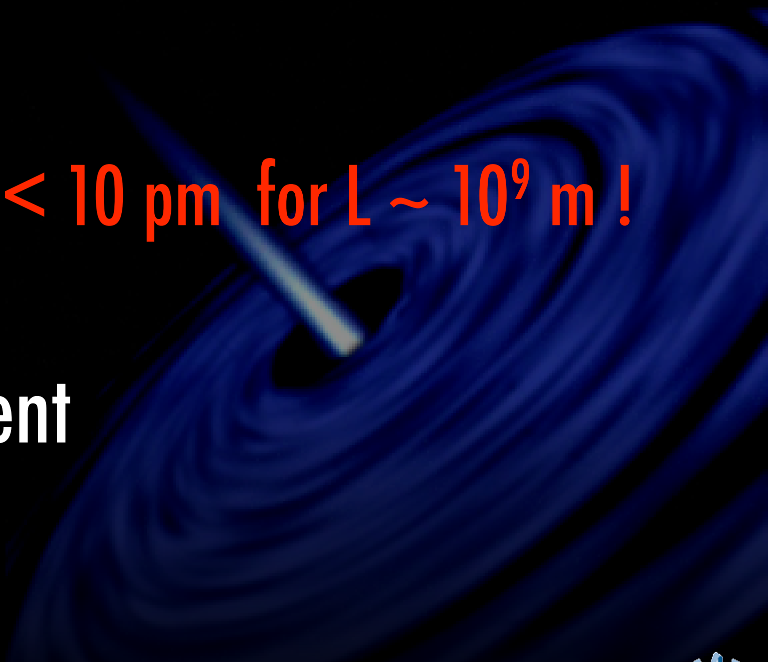




GW detection

- Spacetime deformation
- Relative distance variation

$$\frac{\delta L}{L} \propto h$$

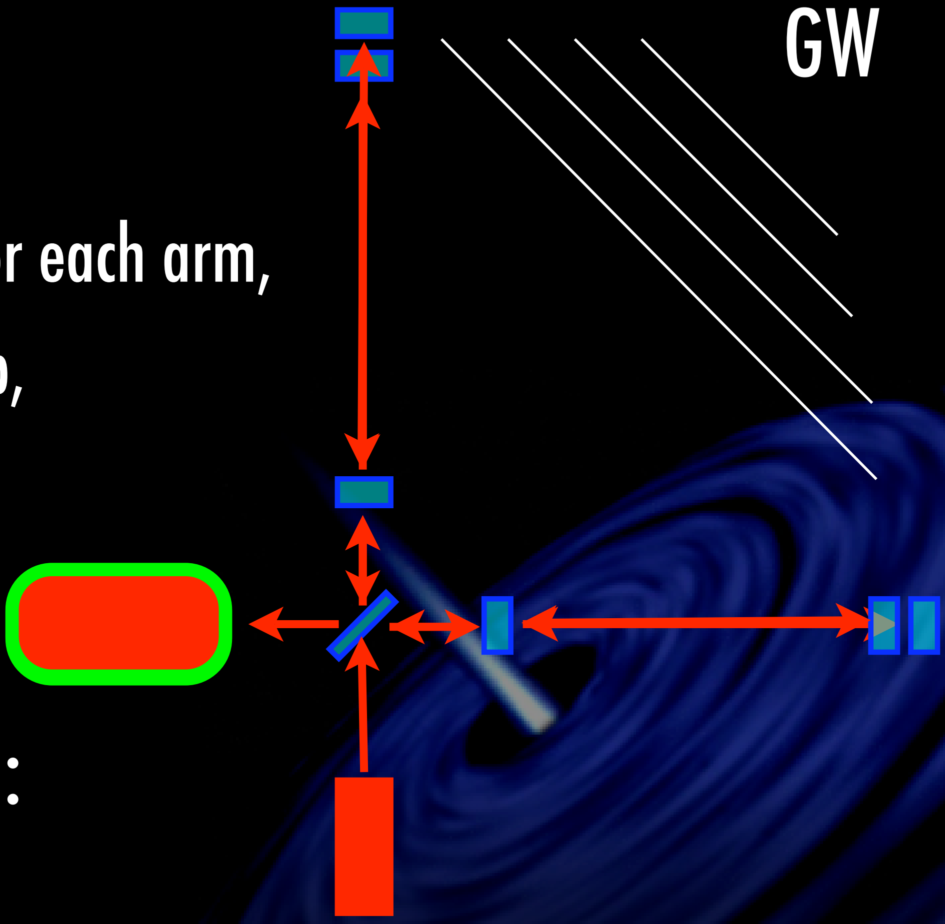


$\delta L < 10$ pm for $L \sim 10^9$ m !

- $\Rightarrow$ High precision distance measurement

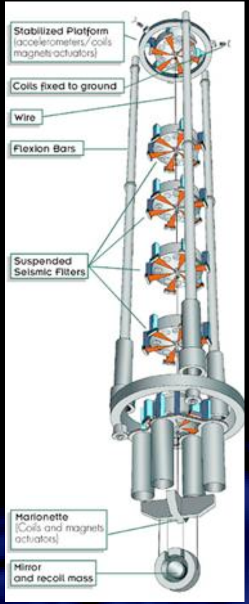
How to detect GWs ? Interferometers

- Large Michelson
  - One laser splitted into 2 beams, one for each arm,
  - Fixed optical length for each round trip,
  - Recombination
- GW changes relative optical length : luminosity variation ⇒ detection.

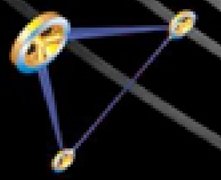


How to detect GWs ?
Ground-based Interferometers

- Ground-based detectors :
  - LIGO (USA - 3km & 4km),
  - VIRGO (Italy - 3km),
  - GEO (Germany - 600m)
  - TAMA (Japan - 300m)

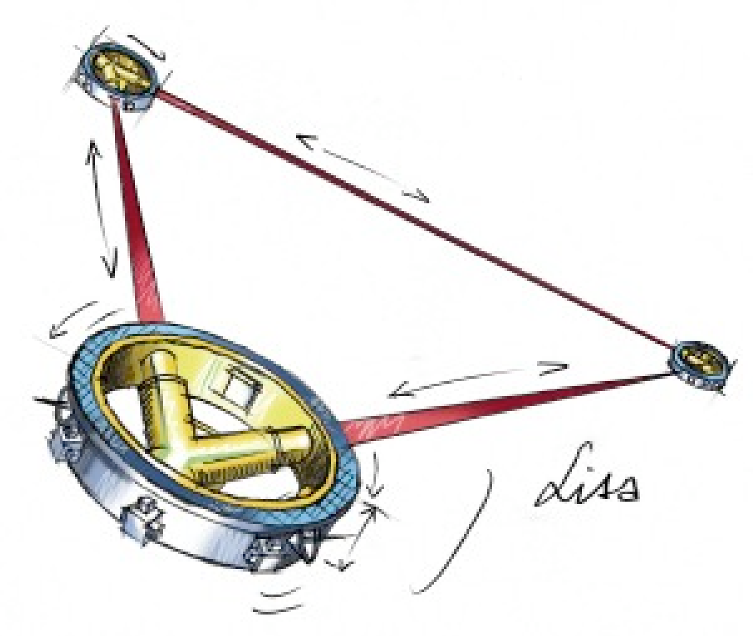


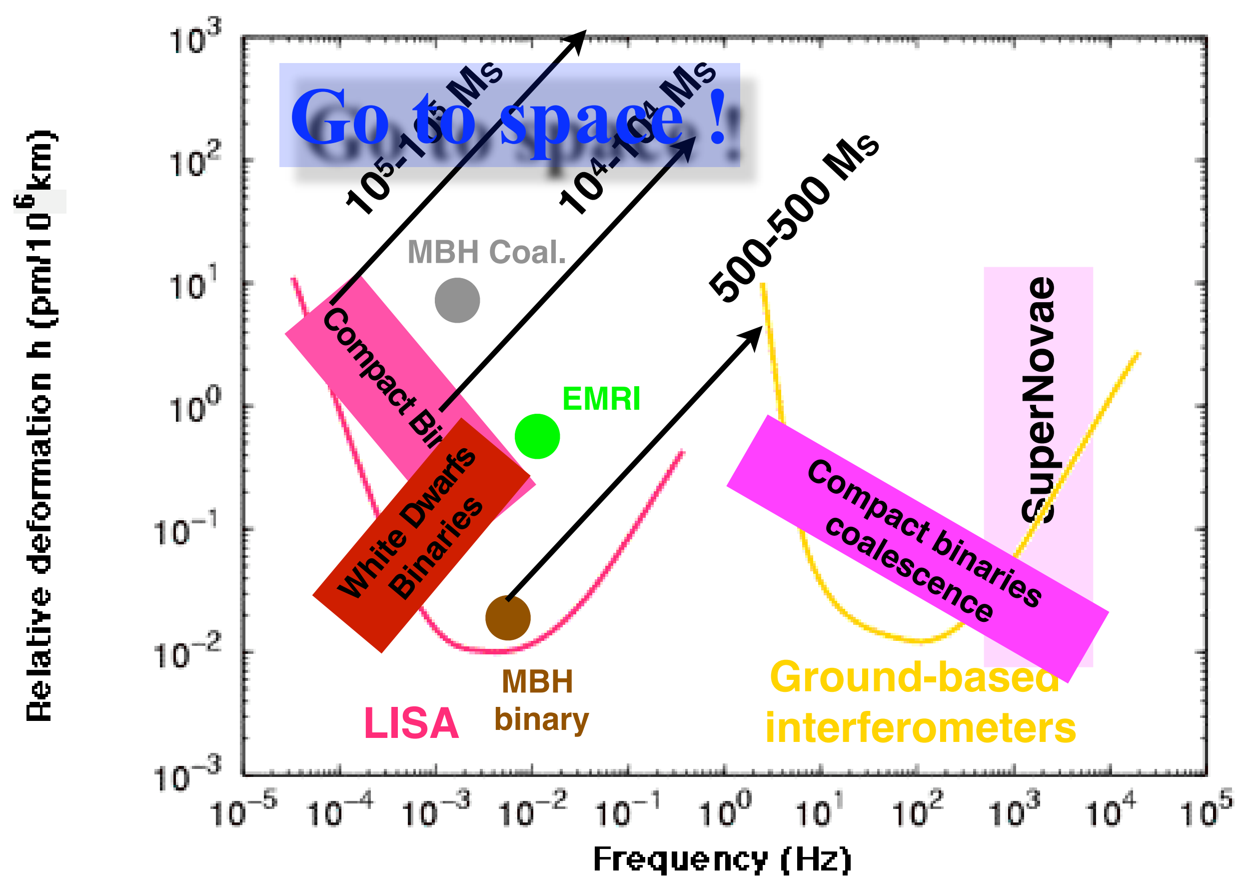
- Many noise sources : noise dominated data
  - Seismic noise : complex seismic filters



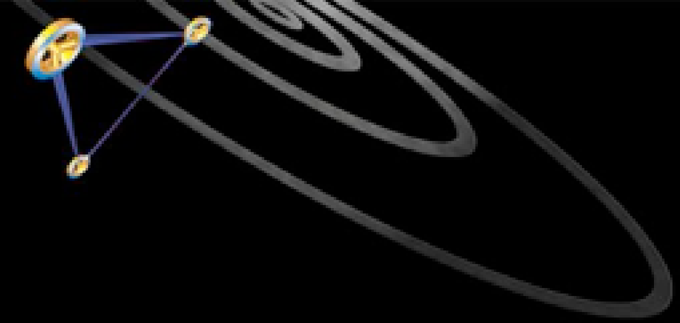
How to detect GWs ?
Low-frequency limit

No seismic noise and
no armlength limit





Go to space !

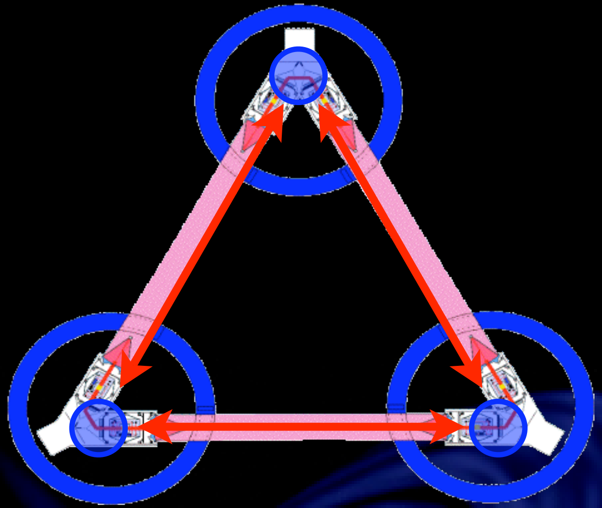


LISA :
Laser Interferometer Space Antenna

LISA : GWs detection

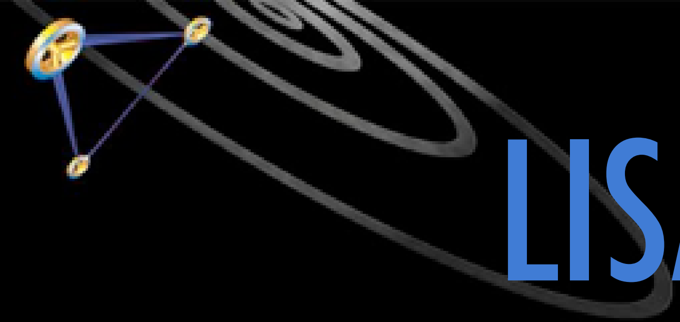
- Michelson in space : Possible link between the two mirrors : 3 spacecrafts in equilateral triangular formation.

- Armlength = 5.10^9 m to detect GWs at 10^{-4} - 10^{-1} Hertz.



⇒ 3 Michelson in space with one redundant

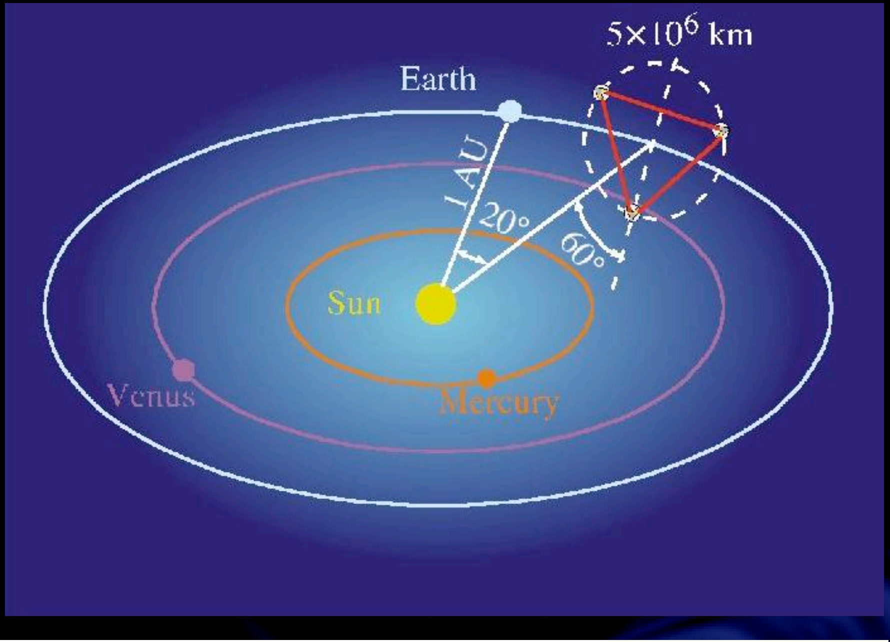
⇒ Polarisation information of GWs

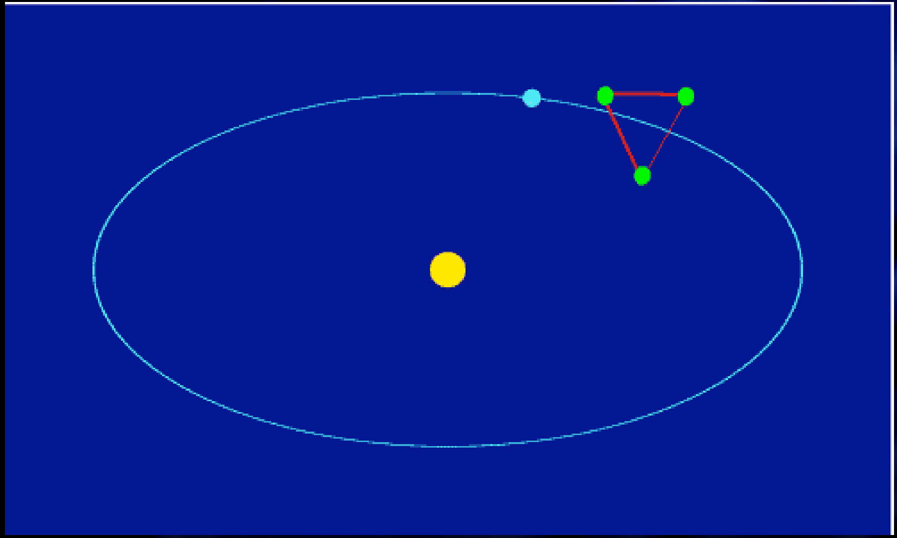


LISA : GWs detection

- 3 heliocentric orbits : spacecraft in free fall.
- LISA centre follows the Earth (-20°).
- Angle between LISA plane and ecliptic plane is 60°.
- Variation of LISA during the year

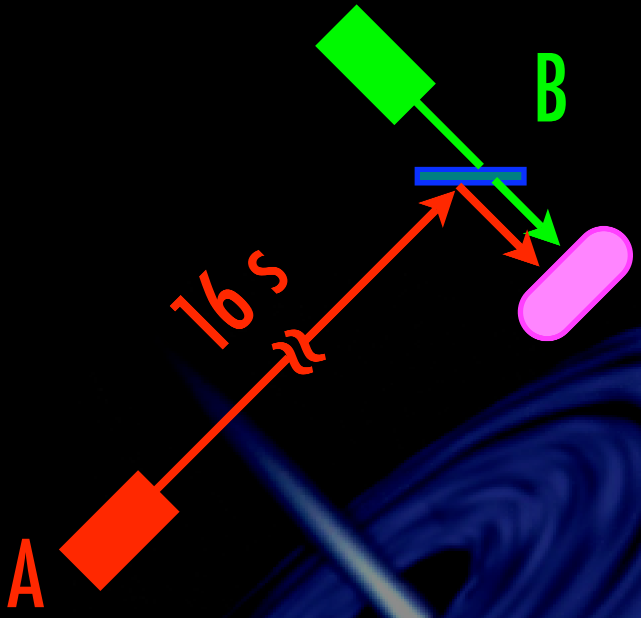
⇒ Directional information of GWs.

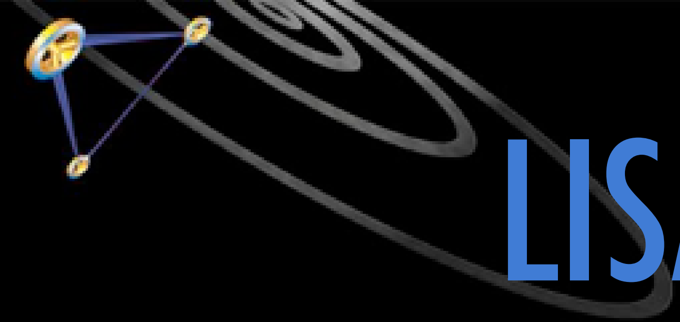




LISA : GWs detection

- Problem of 5.10^9 m : A laser beam cannot make a round trip because too much intensity is lost.
  - 70pW received for 1 Watt emitted.
- Measurement with one arm and two coherent lasers in phase :
  - Distant laser and local laser.
- 6 measurements.
- But lasers aren't perfects ... complex problem ...





LISA : GWs detection

- Travel from transmitter A to the receiver B :
  - L : armlength,
  - $\mathbf{n}$: arm vector,
  - $\mathbf{w}$: GW's propagation vector,
  - r_A , r_B : spacecraft position,
  - t_r : delayed time (~ proportionnal to L),

Beam phase :

$$\Phi(t) = 2\pi \nu_0 t_r$$

Beam frequency :

$$\nu(t) = \frac{1}{2\pi} \frac{d\Phi(t)}{dt}$$

- Measurement : relative laser frequency shift, like Doppler effect :

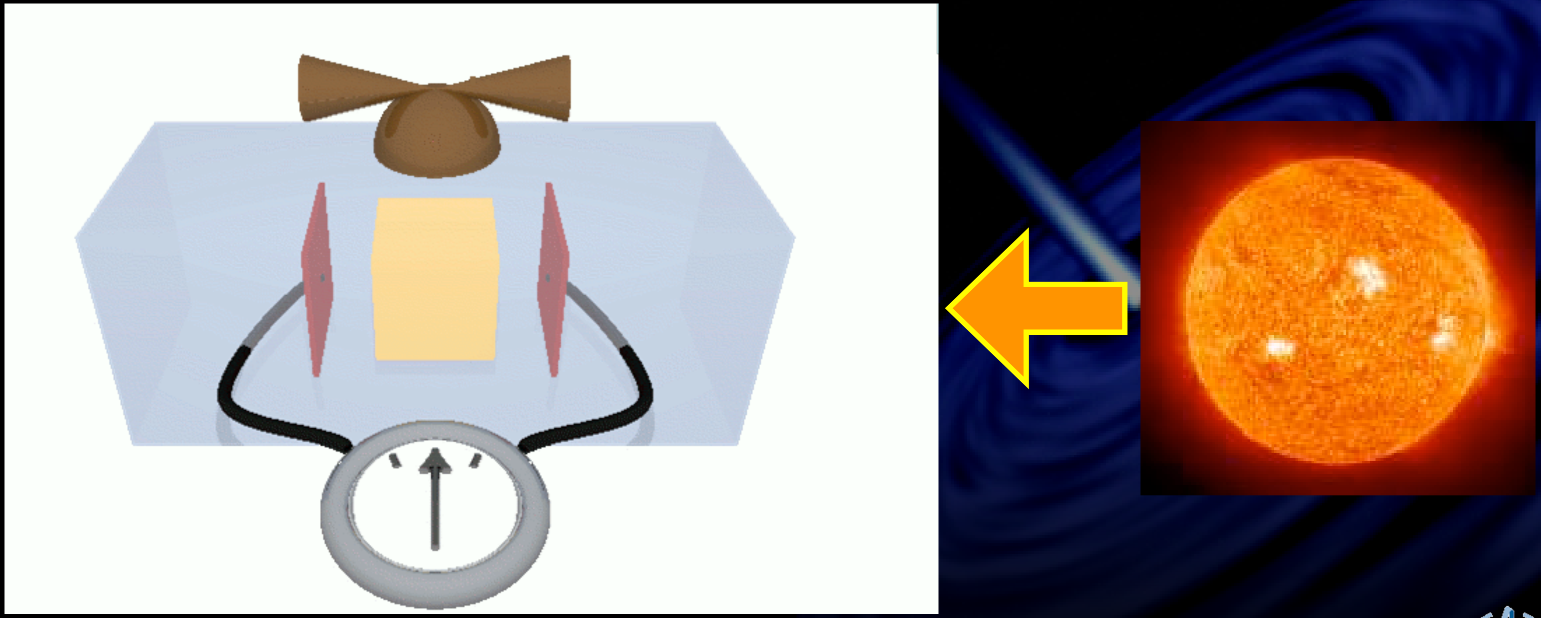
$$\boxed{\frac{\delta\nu}{\nu}(t) = \frac{1}{2(1-\mathbf{w}.\mathbf{n})} \left[H(t-\mathbf{w}.\mathbf{r}_B) - H(t-\mathbf{w}.\mathbf{r}_A - L) \right]}$$

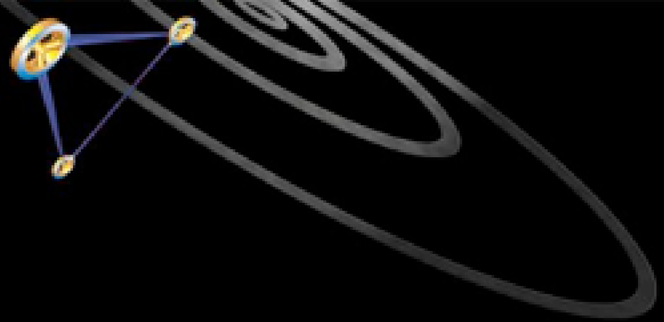
LISA : Free fall in space

- Spacecrafts must follow geodesics.
- BUT many others forces can act on spacecrafts :
  - Solar wind,
  - Radiation pressure,
  - …
- A “drag free system” is necessary.

LISA : Free fall in space

- Inertial masses are shielded from external forces : gravitational reference.
- The spacecraft is sensitive to external forces,
- The spacecraft follows one of the masses with micro-trusters.





LISA : Noises

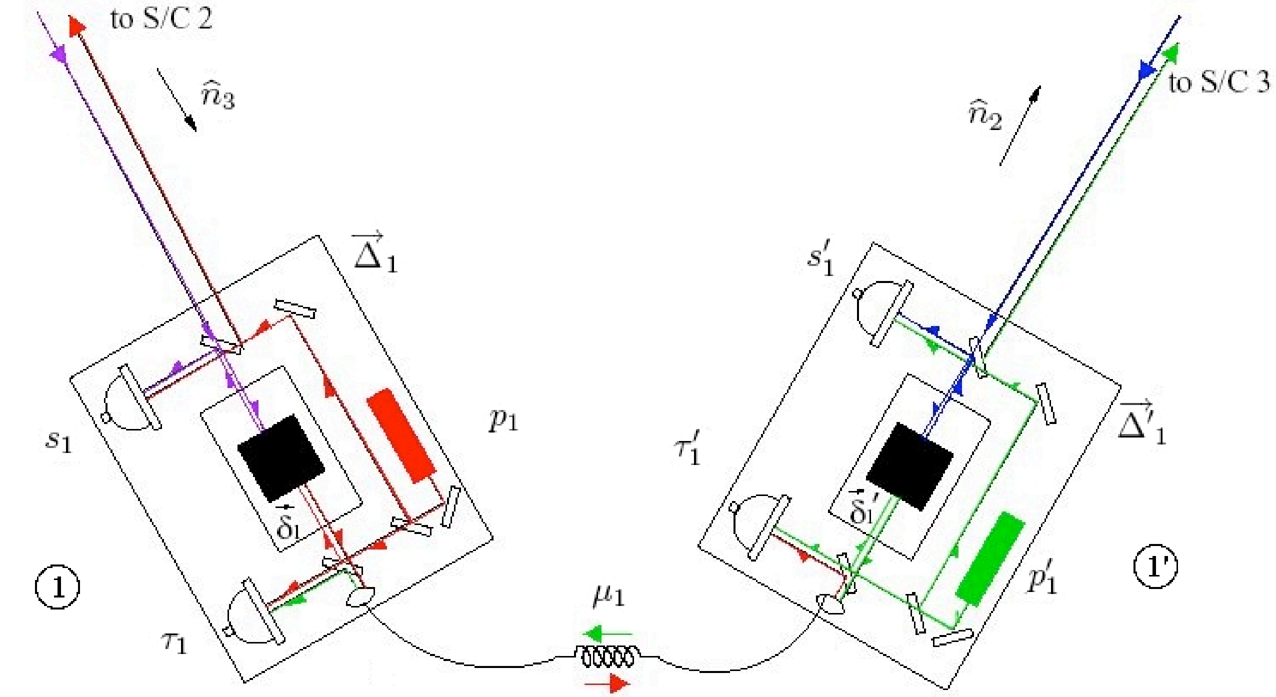
- ***Laser noise*** : Interference between two lasers which are not perfectly stable.
  - Power Spectral Density = 30 Hz.$Hz^{-1/2}$ $\equiv 10^{-13}$ $Hz^{-1/2}$ ($\delta\nu/\nu$ unit)

- ***Inertial mass noise*** : Imperfection of drag free system, time lag.
  - PSD = $3\mathrm{x}10^{-15}$ $m.s^{-2}.Hz^{-1/2}$ $\equiv 1.59\mathrm{x}10^{-24}$ $f^{-1}.Hz^{-1/2}$ ($\delta\nu/\nu$ unit)

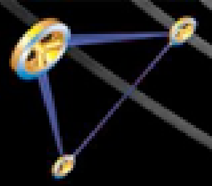
- ***Shot noise*** : Measurement noise on the photodiode.
  - PSD = $20\mathrm{x}10^{-12}$ $m.Hz^{-1/2}$ $\equiv 4.2\mathrm{x}10^{-19}$ $f.Hz^{-1/2}$ ($\delta\nu/\nu$ unit)

LISA : Measurements

- Beam circulation :
  - Between spacecrafts,
  - Between optical benchs of the same spacecraft.

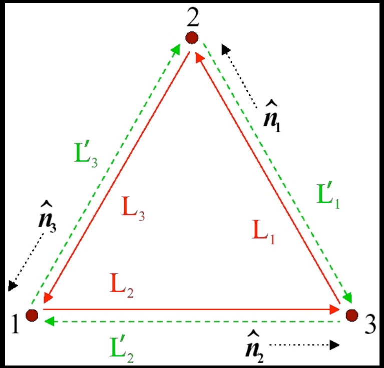


- Noises :
  - p_i : laser
  - Δ_i : optical bench
  - δ_i : inertial mass
- Measurements :
  - s_i : external + internal beam
  - τ_i : 2 benches' beams



LISA : Measurements

- Phase shift between the two beams measured by a phasemeter.
- Beams from an external spacecraft, are delayed :
  - delay operator D_i : $D_i\, x(t) = x(t-L_i/c)$
- The measurements :



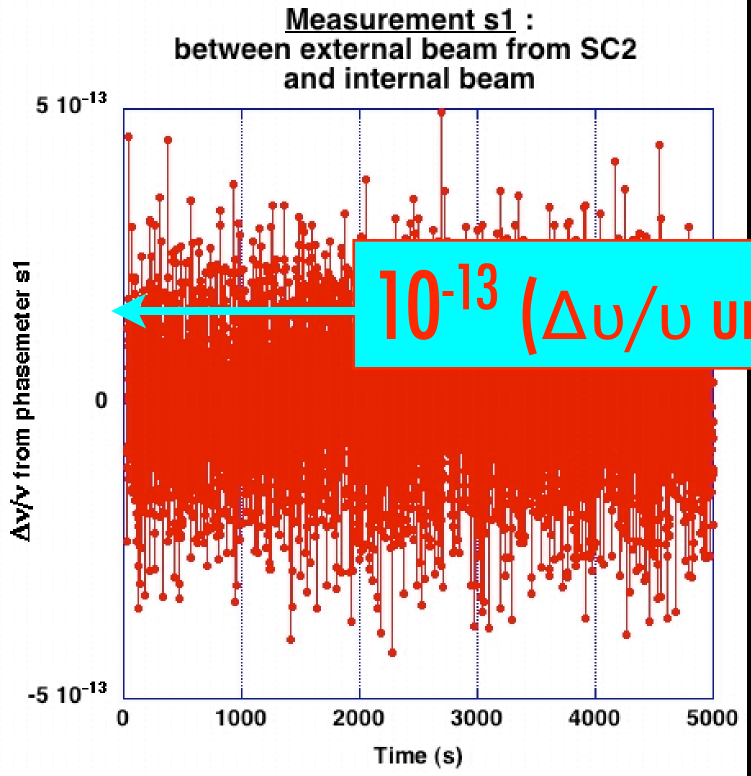
$$
\begin{aligned}
s_1 &= s_1^{GW} + \boxed{s_1^{ShotNoise}} + \boxed{D_3 p'_2} - \boxed{p_1} + \nu_0 \left(-2.\widehat{n}_3.\boxed{\overrightarrow{\delta}_1} + \widehat{n}_3 \boxed{\overrightarrow{\Delta}_1} + \widehat{n}_3.\boxed{D'_3 \overrightarrow{\Delta'_2}} \right) \\
\tau_1 &= p'_1 - p_1 - 2\nu_0 \widehat{n}_2.\left(\overrightarrow{\delta'}_1 - \overrightarrow{\Delta'}_1 \right) + \mu_1 \\
s'_1 &= {s'}_1^{GW} + {s'}_1^{ShotNoise} + D'_2 p_3 - p'_1 + \nu_0 \left(2.\widehat{n}_2.\overrightarrow{\delta'}_1 - \widehat{n}_2.\overrightarrow{\Delta'}_1 - \widehat{n}_2.D_2 \overrightarrow{\Delta}_3 \right) \\
\tau'_1 &= p_1 - p'_1 + 2\nu_0 \widehat{n}_3.\left(\overrightarrow{\delta}_1 - \overrightarrow{\Delta}_1 \right) + \mu_1
\end{aligned}
$$

With only the laser noise :

$$
\begin{aligned}
s_1 &= s_1^{GW} + D_3 p'_2 - p_1 \\
s'_1 &= {s'}_1^{GW} + D'_2 p_3 - p'_1
\end{aligned}
$$

LISA : Raw Data

- Raw data for each spacecraft :


Measurement s1 :
between external beam from SC2
and internal beam

10^{-13} (Δυ/υ unit)

GW :

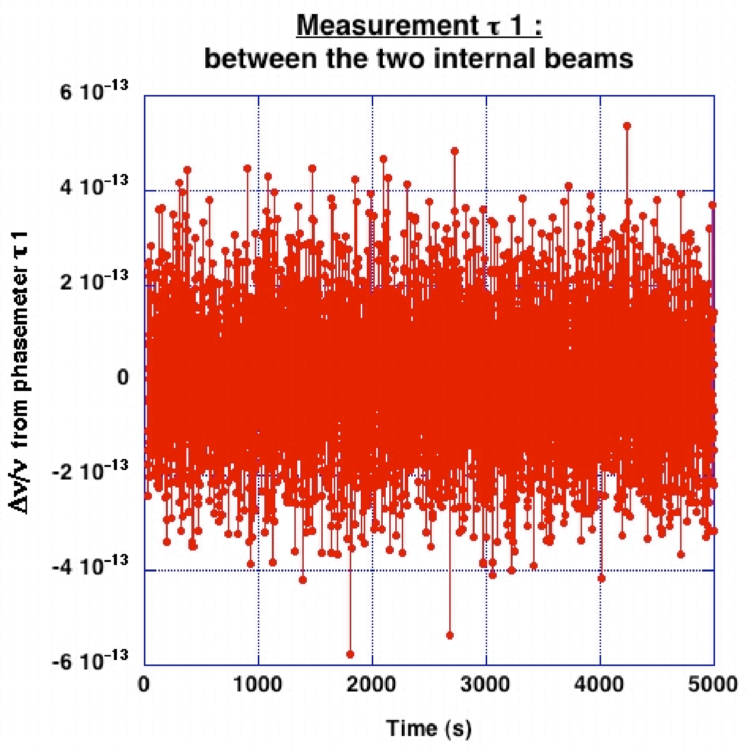
f = 10^{-3} Hz

$h_{+} = 10^{-21}$

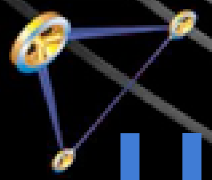
Time :

Step = 1 s

For 5000 s


Measurement τ 1 :
between the two internal beams

Noise dominated signal → Noise reduction method ...

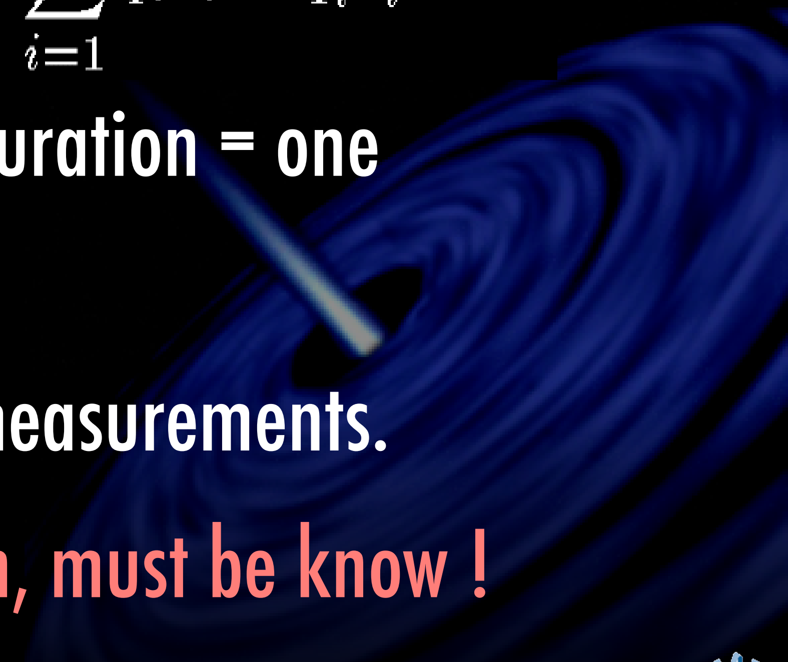


LISA : Time Delay Interferometry

Tinto & Durandhar, Revue *gr-qc/0409034* (2004)

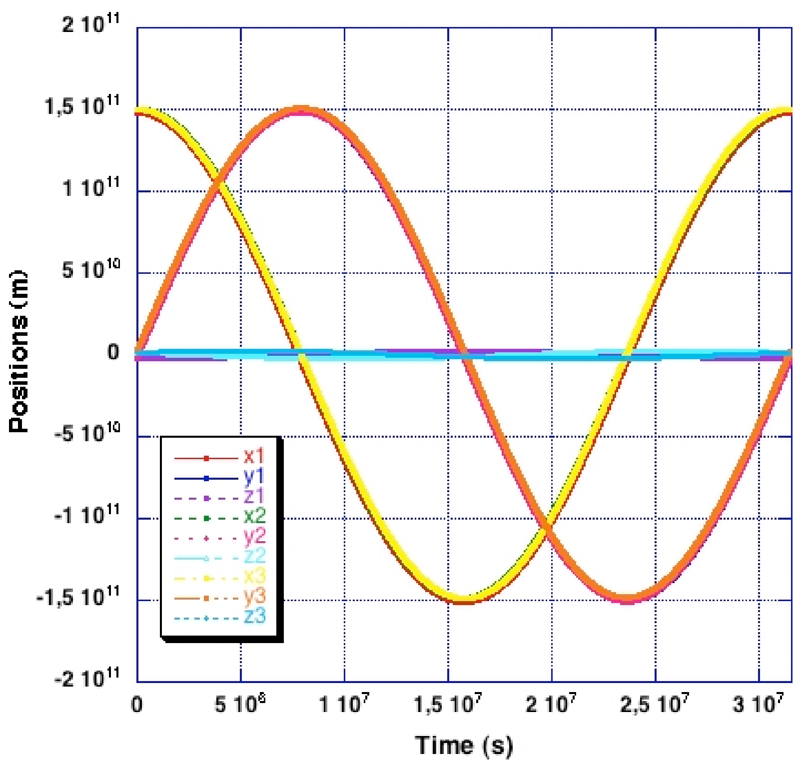
- Pre data analysis methods,
- Delayed measurement combinations to reduce noise,
- Find the $q_i(D_i)$, $q'_i(D_i)$ series, for $\sum_{i=1}^{3} q_i s_i + q'_i s'_i = 0$
- One serie = one generator = one configuration = one interferometer
- Application of delay operators on the measurements.

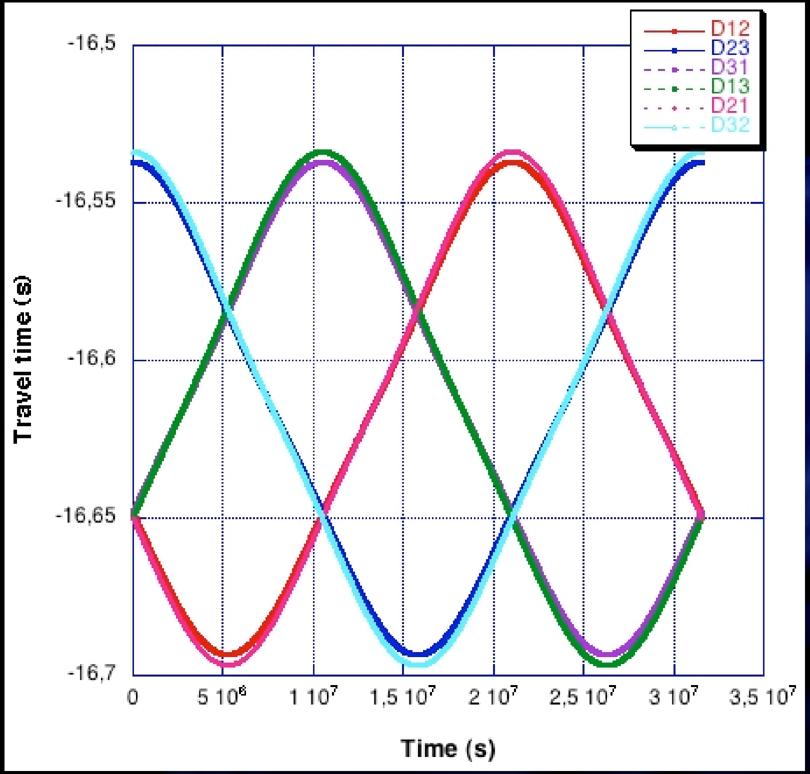
$\Rightarrow$ The delay, therefore the armlength, must be know !



Complex orbits

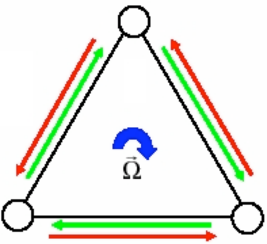
- The orbits are chosen to reduce the armlength variations, flexing.
- Sagnac effect : Triangle rotation induces variation in time delay on the same arm.



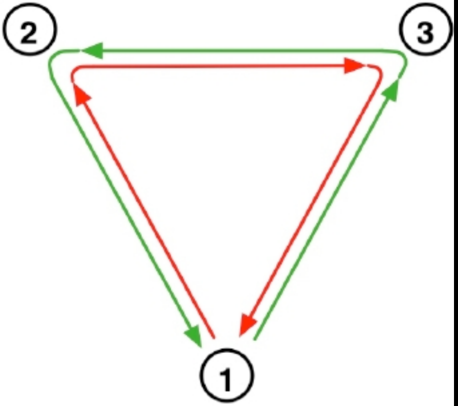


Time Delay Interferometry

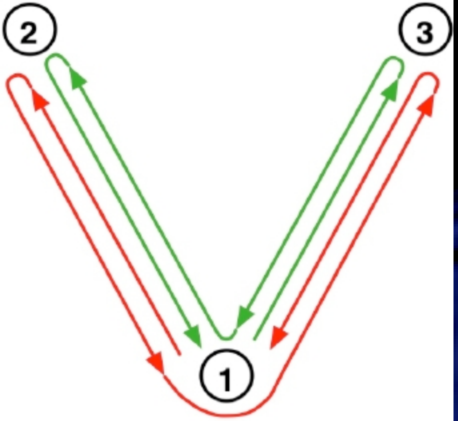
- Many groups of TDI generators
  - 1st generation : fixed LISA configuration.
  - 2nd generation : consideration of flexing and Sagnac effect.



- Geometric representation by beam loops :



Article : gr-qc/0504145 Vallisneri

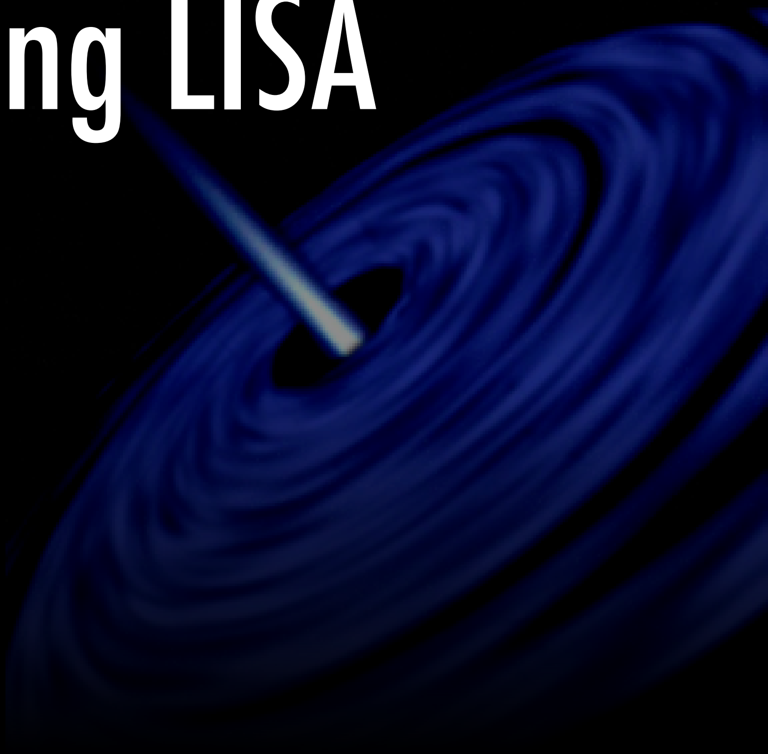


$$\begin{aligned}\alpha &= -s_1 - D_3\ s_2 - D_1 D_3\ s_3 \\ &\quad + s'_1 + D_{2'}\ s'_3 + D_{1'}\ D_{2'}\ s'_2 \\ &\simeq 0\end{aligned}$$

$$\begin{aligned}X &= -s_1 - D_3\ s'_2 - D_3 D_{3'}\ s'_1 - D_3 D_{3'} D_{2'}\ s_3 \\ &\quad + s'_1 + D_{2'}\ s_3 - D_{2'} D_2\ s_1 - D_{2'} D_2 D_3\ s_3 \\ &\simeq 0\end{aligned}$$

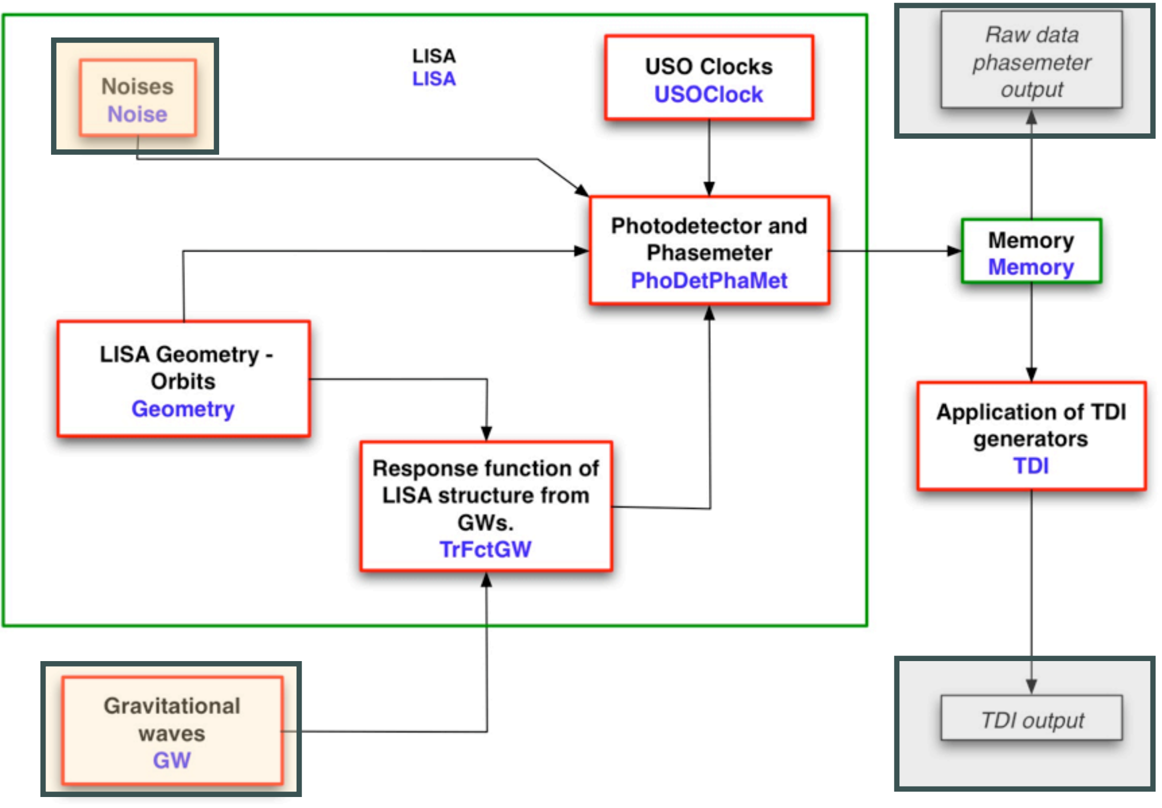
Changes the signal shape $\Longrightarrow$ Data analysis

LISACode : Simulating LISA



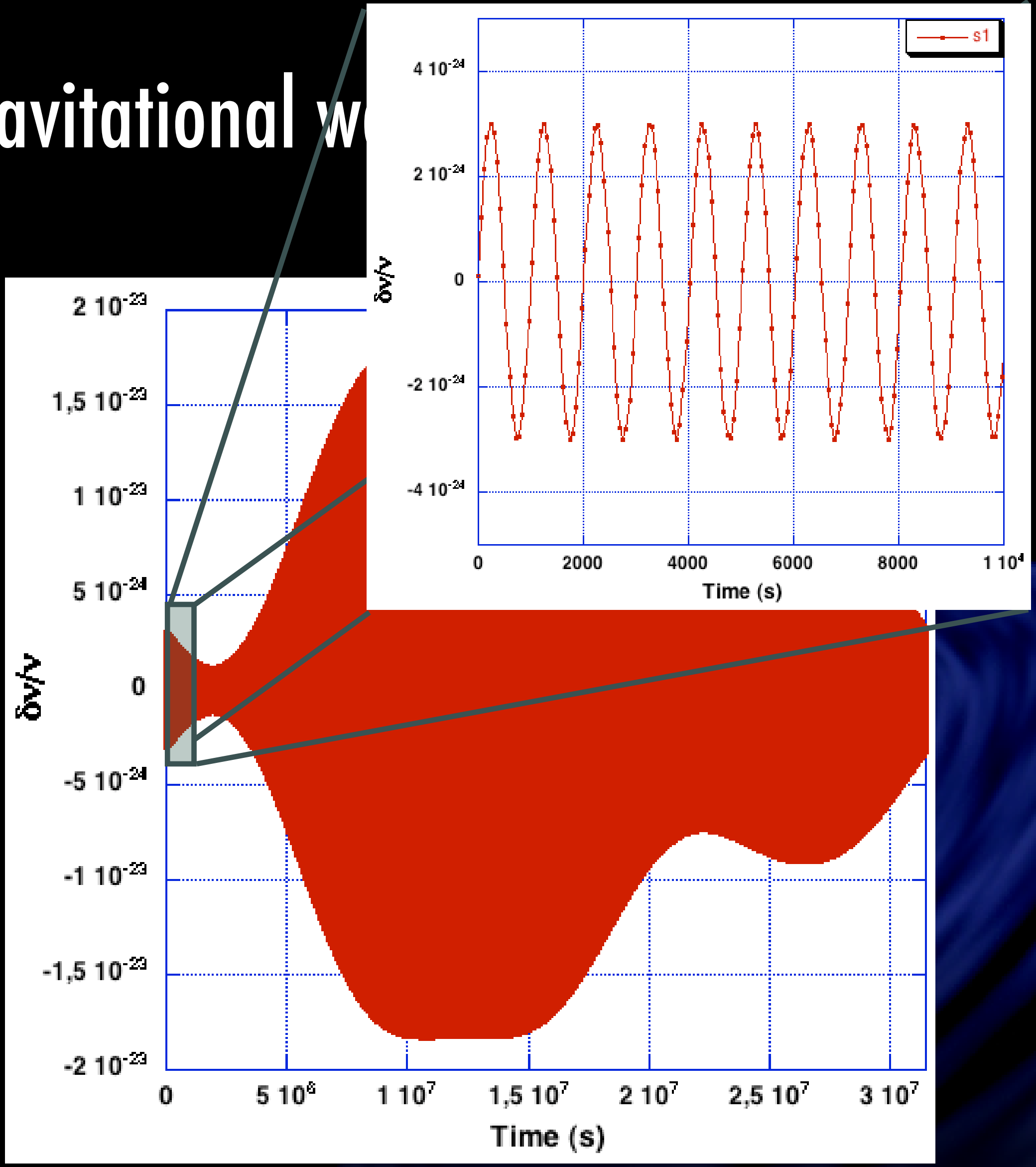
Structure of the code

- LISACode is a scientific simulator.
- Inputs : Gravitationnal Waves (and noise).
- Outputs : Time sequences : phasemetres and TDI



LISACode results : modulation

- Example of monochromatic gravitational w
  - $\lambda = 298°$, $\beta = 27°$,
  - $\psi = 228°$,
  - $f = 10^{-3}$ Hz,
  - $h_+ = 3.5\mathrm{x}10^{-22}$, $h_\times = 3.5\mathrm{x}10^{-22}$,
  - $\phi_{0h+} = 4.21$, $\phi_{0h\times} = 5.78$.
- Modulation of sinusoid.

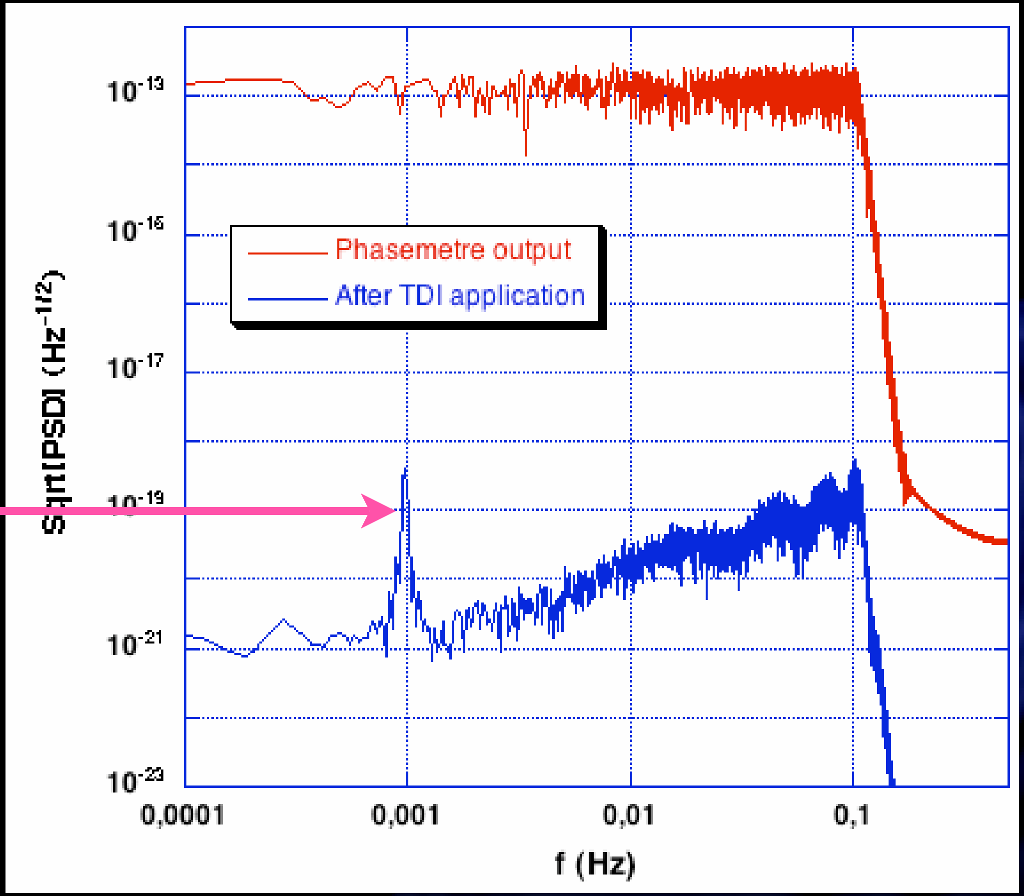


TDI : an example

- The laser noise is modeled by a bandwidth limited white noise at 30 Hz.Hz$^{-1/2}$.
- The application of TDI recovers the GW signal.

A Gw is
hidden in
there !

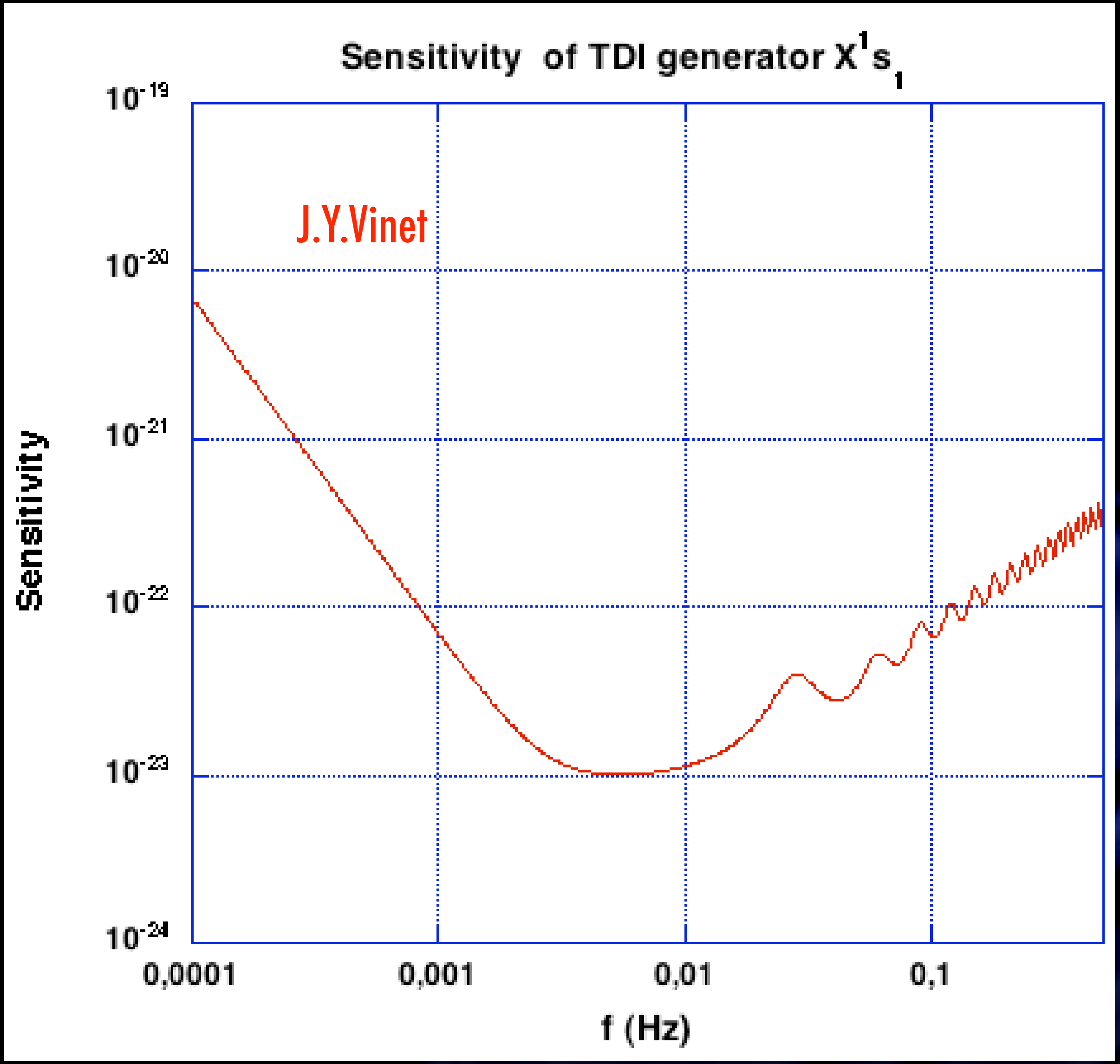
Here it is !



The LISA sensitivity curves : 1

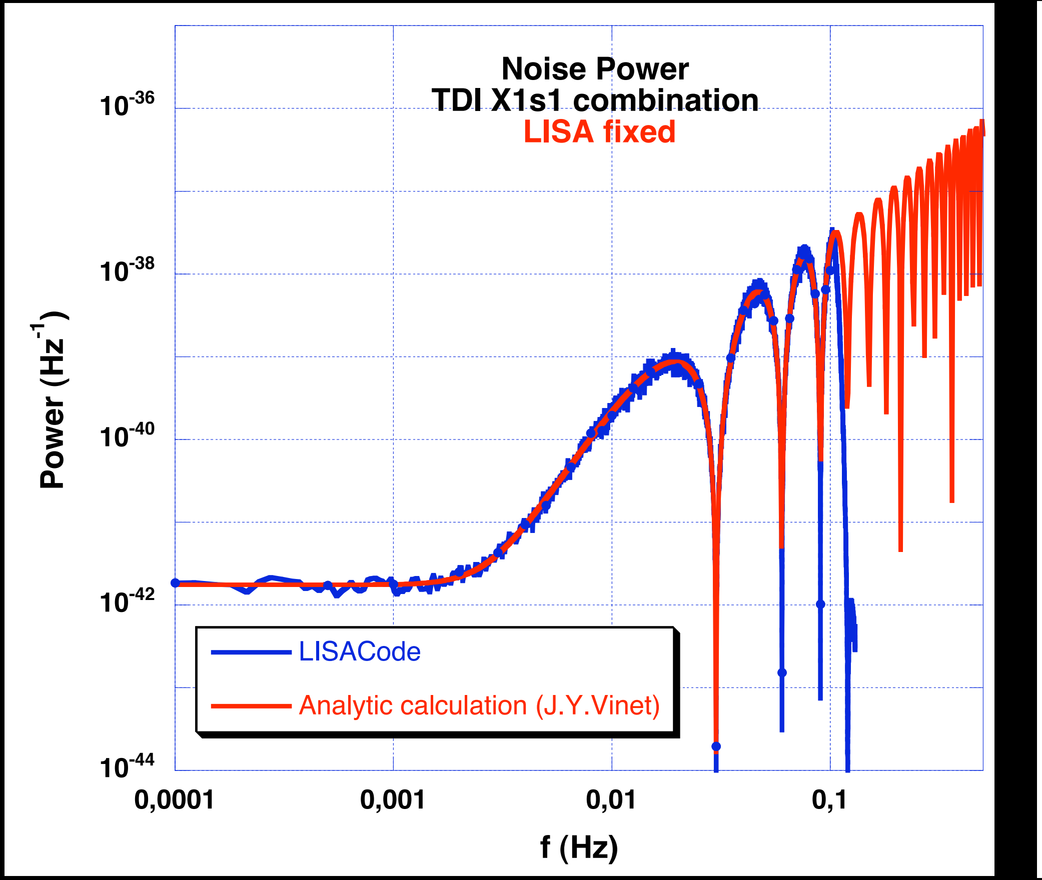
$$h = 5\sqrt{\frac{Noise}{Yr * Rep_{GW}}}$$

- Theoretical curve :
  - Isotropic distribution of sources
  - Without laser noise
  - Lisa is fixed : no flexing or Sagnac
  - TDI first generation
- GW amplitude h ≥ 10^{-23}.

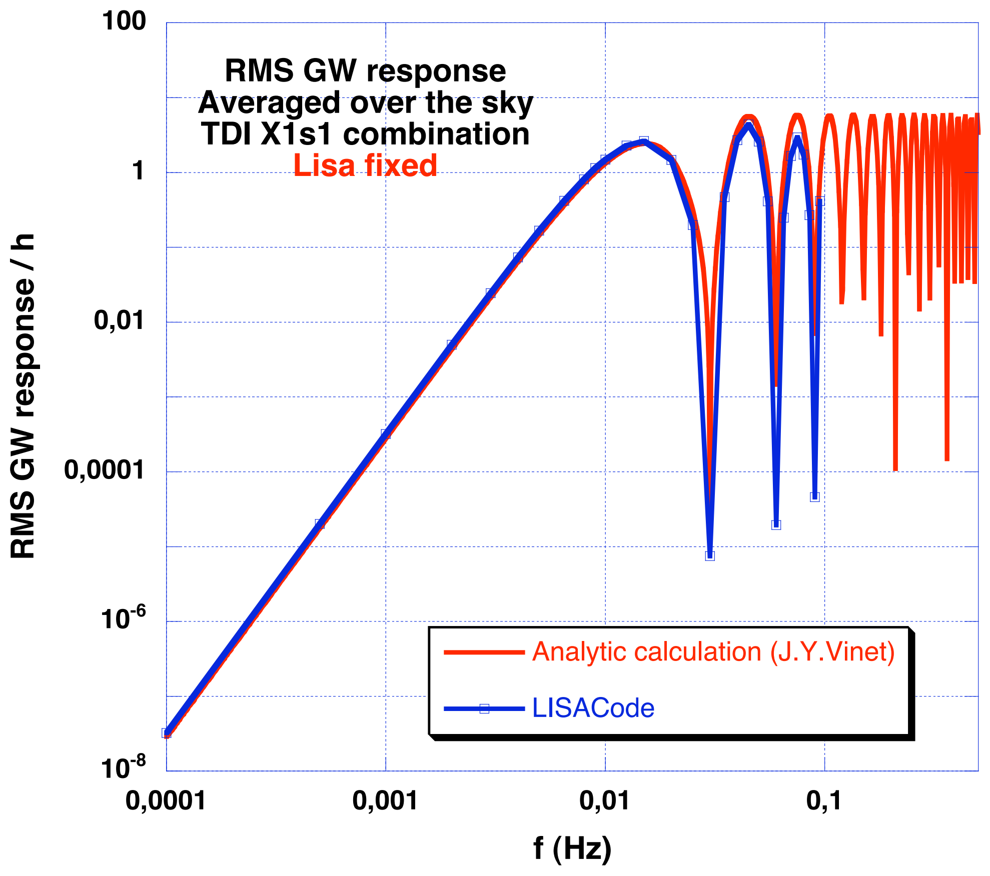


The LISA sensitivity curves : 1

Standard noises : inertial mass, optics and laser.



Isotropic distribution of sources

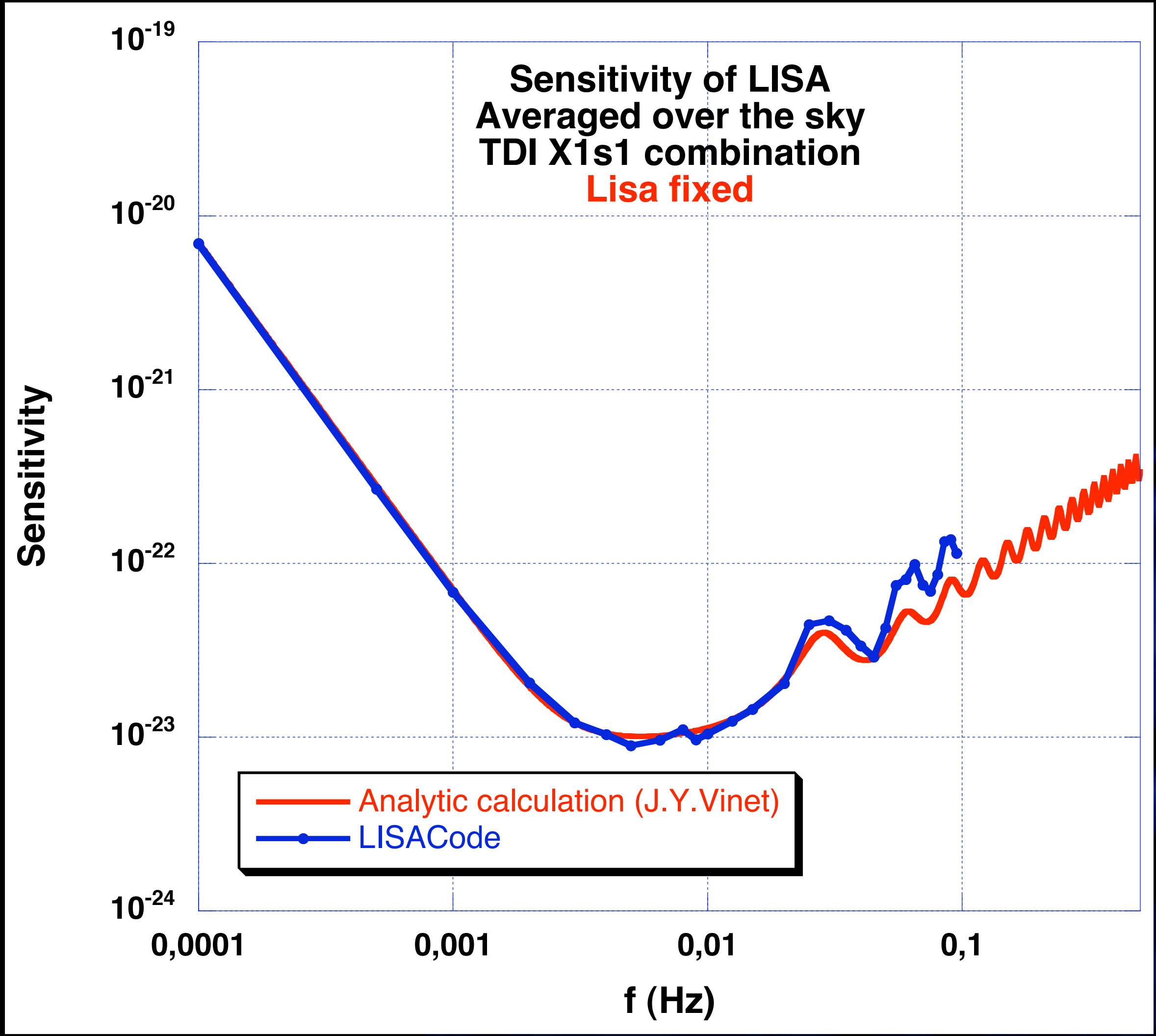


The LISA sensitivity curves : 1

Sensitivity

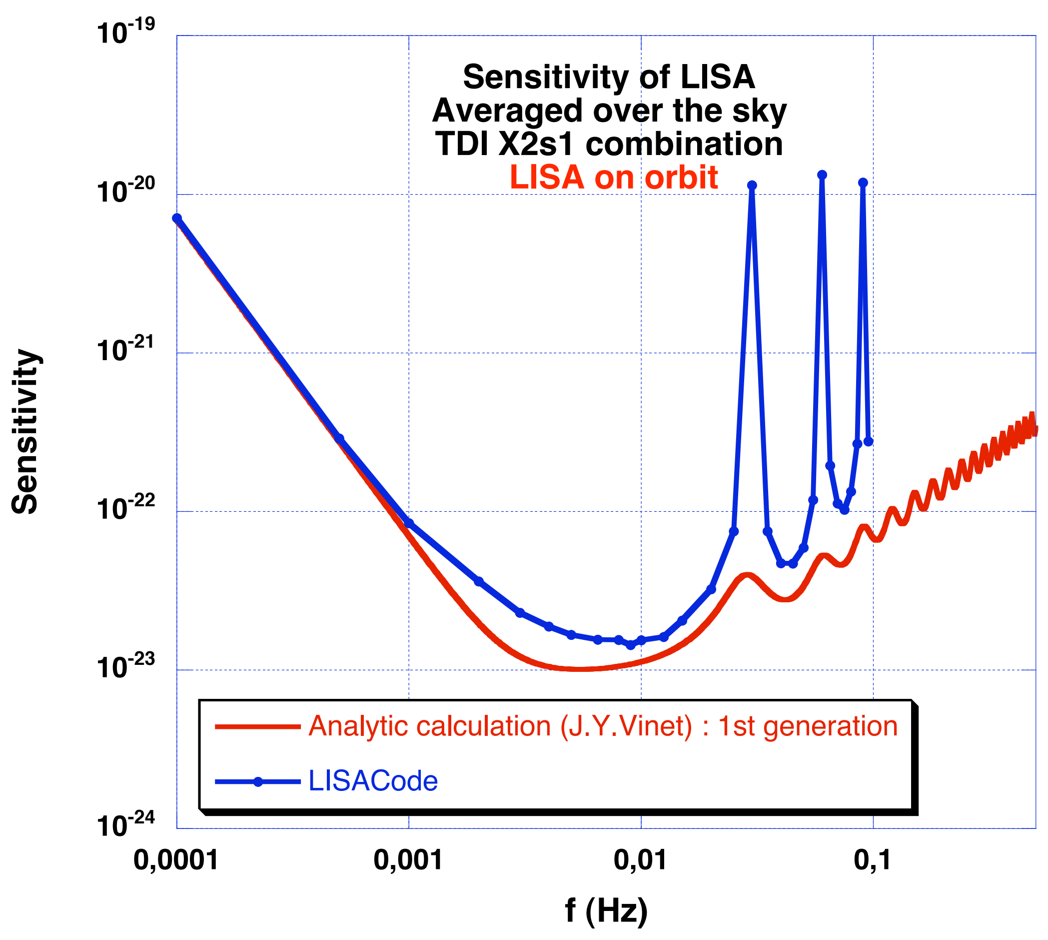
$$h = 5\sqrt{\frac{Noise}{Yr * Rep_{GW}}}$$

Validation of LISACode



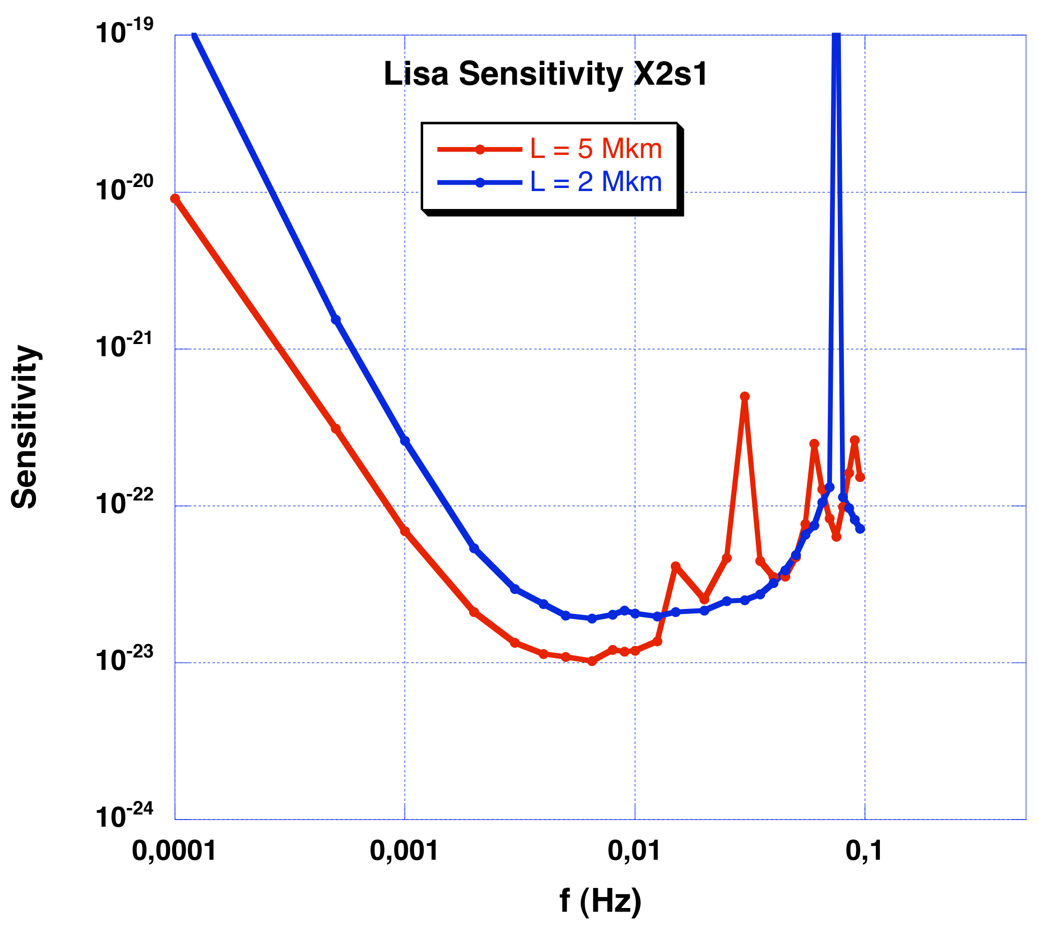
The LISA sensitivity curves : 2

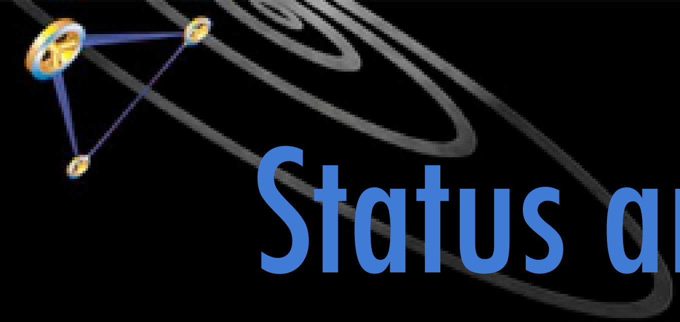
- Lisa on orbits : Sagnac + Flexing
- TDI 2nd generation



Modifying the armlengths.

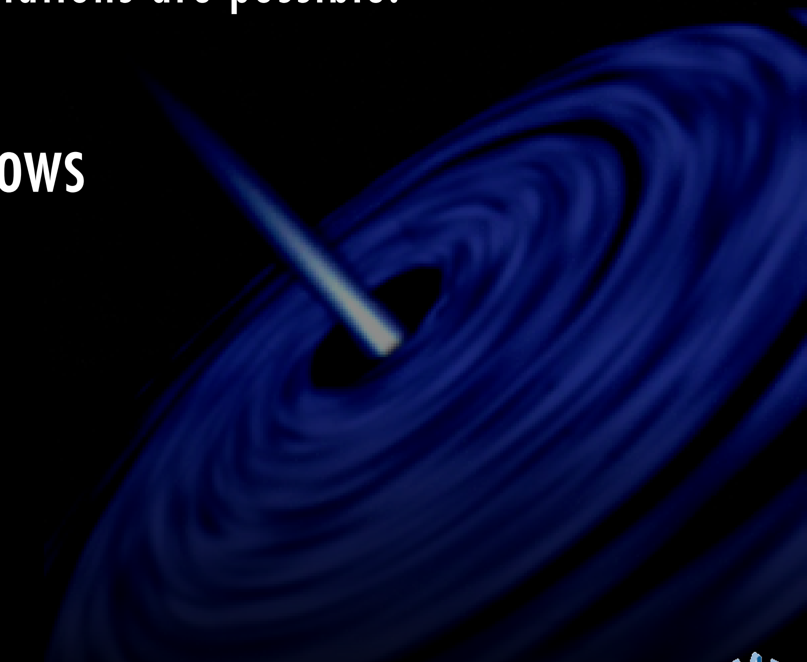
- Analysis of noises : Only the shot noise varies with L





Status and Evolution of the code

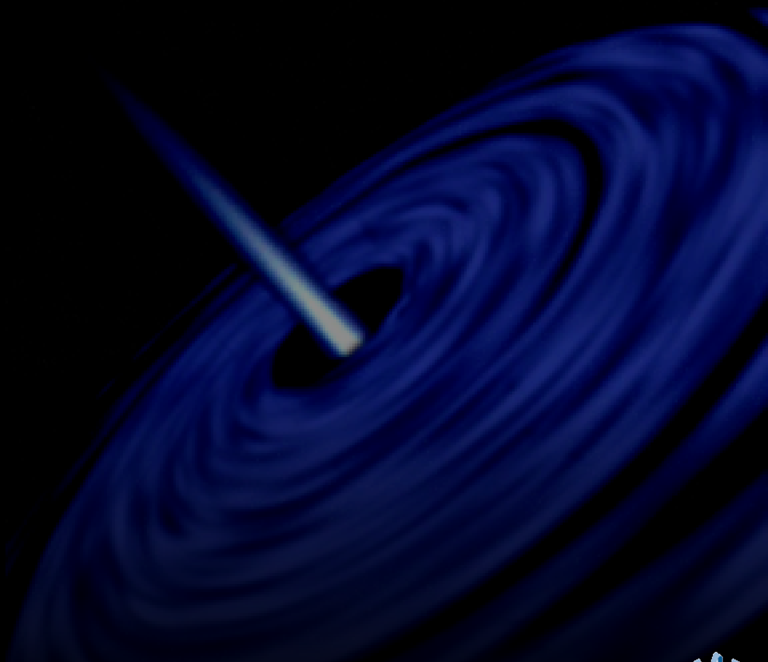
- LISACode is finalized : present version 1.2
  - GW : monochromatic, binaries, input files
  - Orbits
  - Noise : Laser, inertial mass, shot noise.
  - Phasemeter : filtering and sampling.
  - TDI : 1st and 2nd generation. non standard combinations are possible.
- Execute on most platforms : Mac, Unix, Windows
- The future ...
  - Galactic confusion noise (finalized)
  - more GW types : MBHB, EMRIs
  - ...

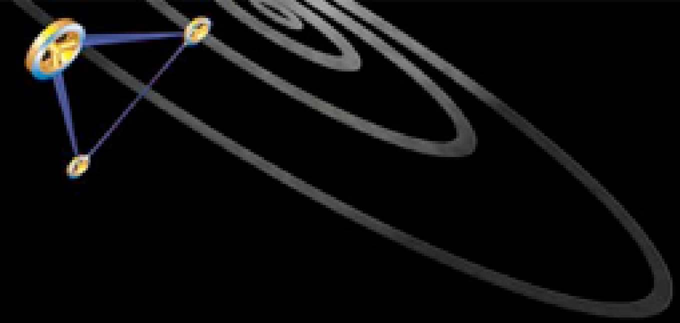


Another Lisa Simulator ?

- One : “Synthetic LISA”,
- Comparing Codes.

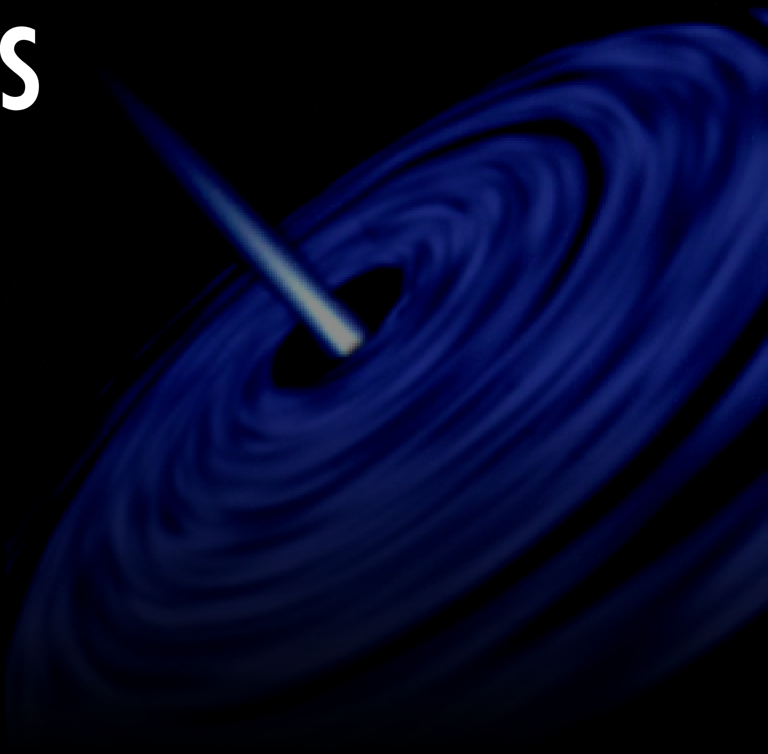
- Thanks are due to M.Tinto and M.Vallisneri





Data Analysis

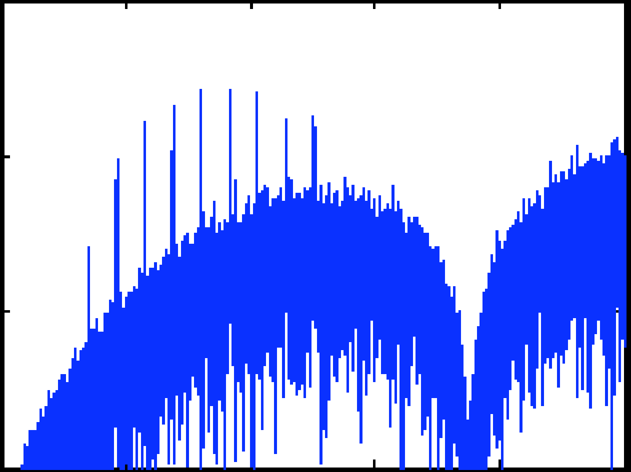
with LISACode !



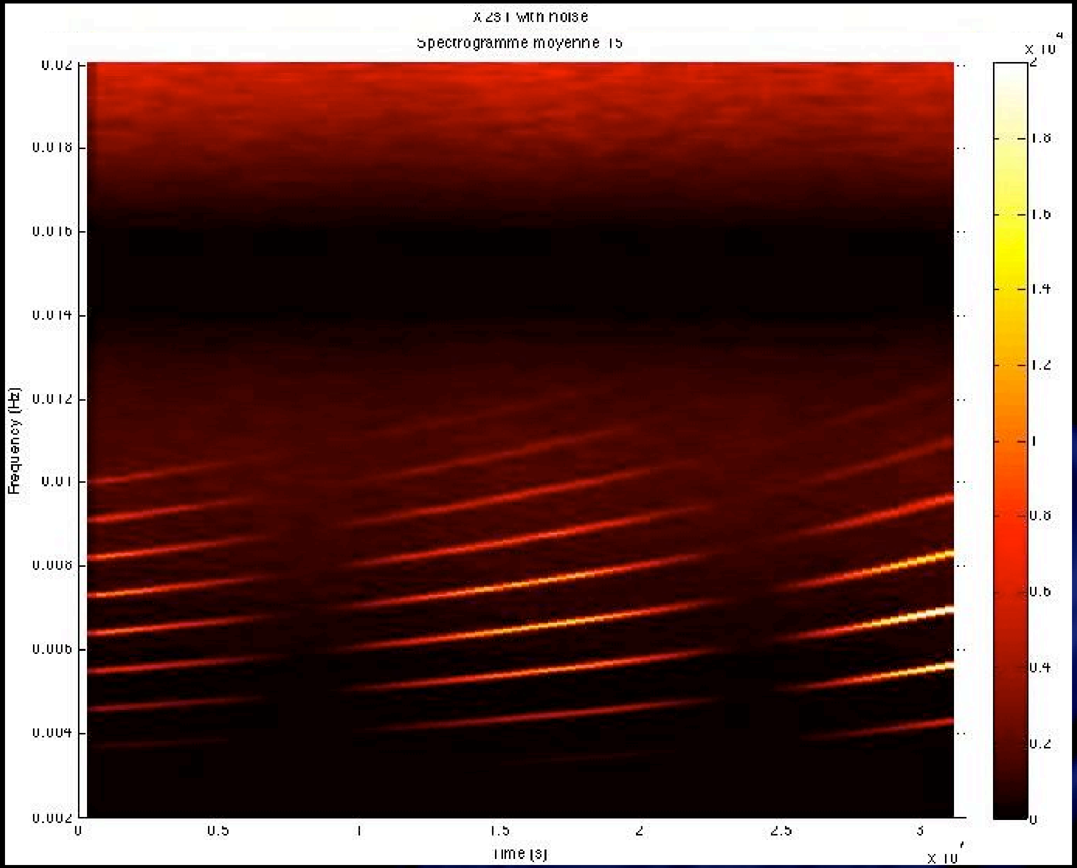
Time frequency analysis EMRIs

- Study of Gair & Wen EMRI :
  - Masses : 10 - 10^6
  - Spin : 0.8
  - Initial eccentricity : 0.4
  - Position : λ=72,54° ,β=90°

Distance : 100 Mpc

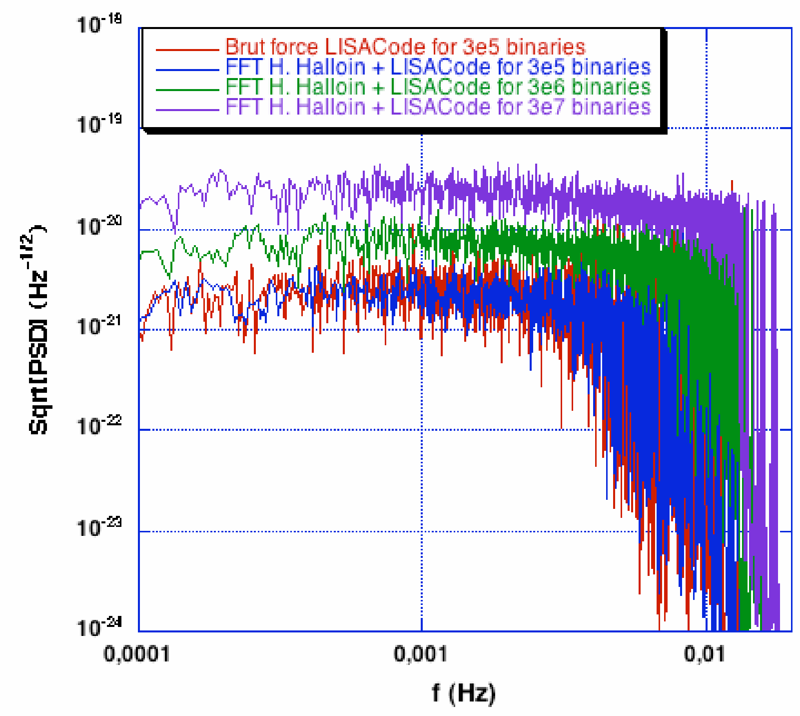


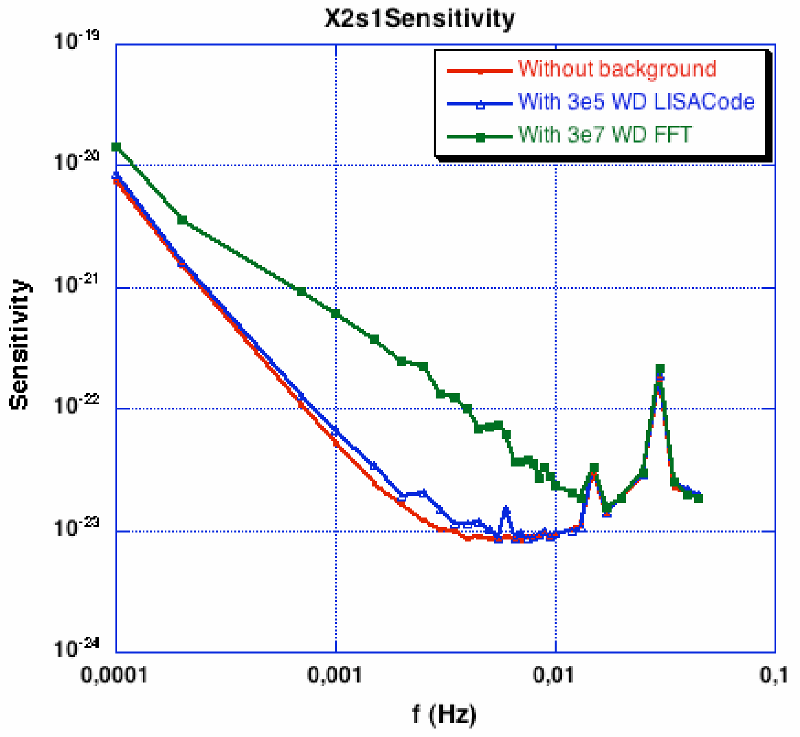
TDI X2s1 with noise se

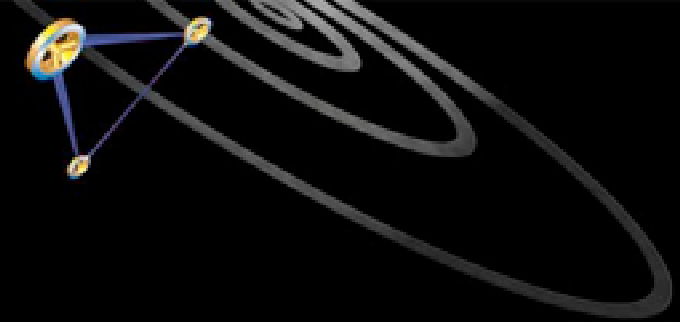


The Galactic confusion noise

- 3x10^7 Whites dwarfs binaries.
- Simulation by two methods :
  - Brut force with all binaries in LISACode input.
  - FFT model of signal in LISACode “noise input”.

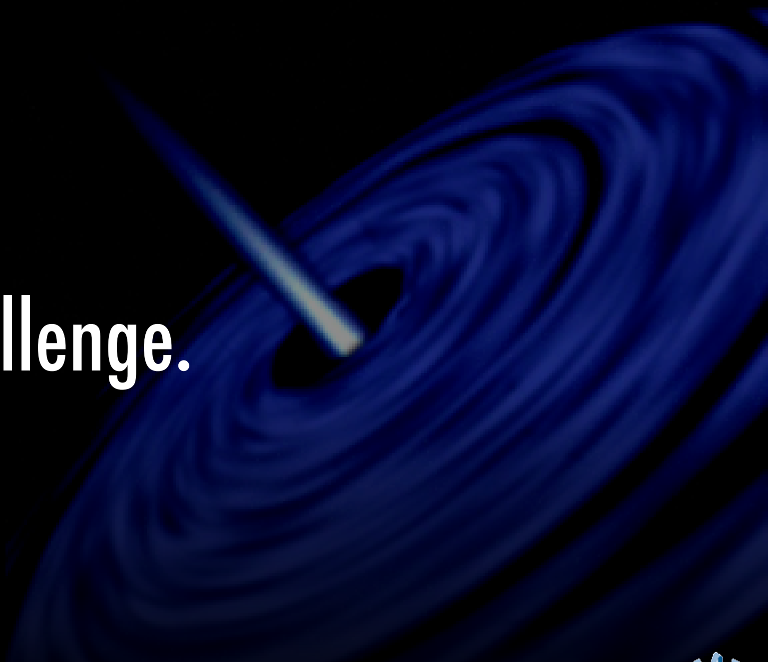


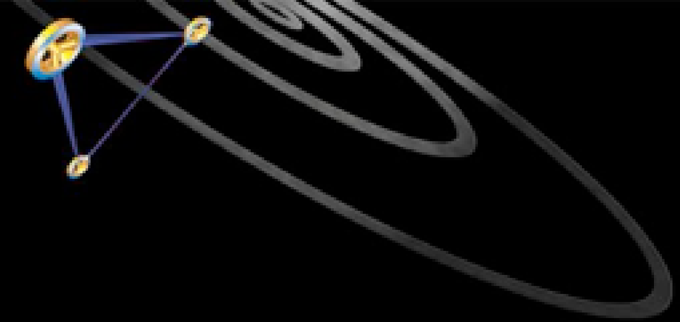




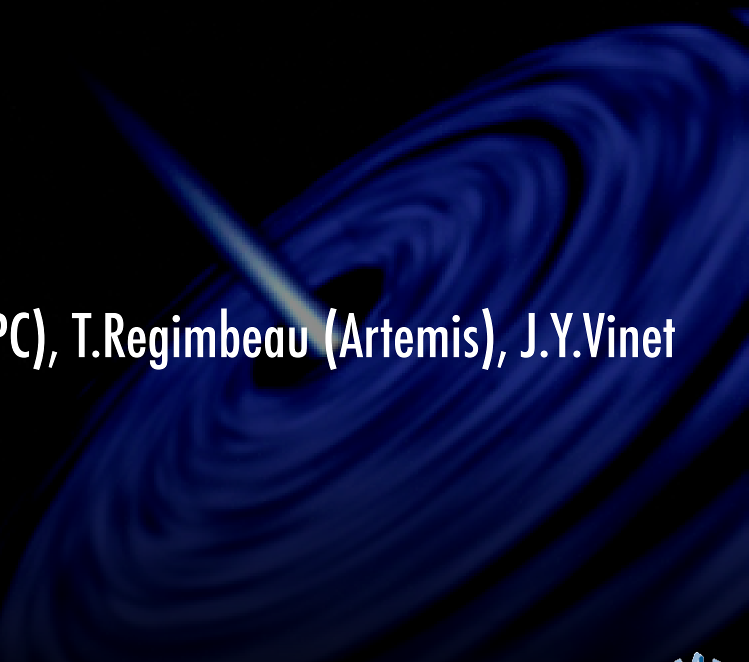
Summary

- LISACode is a complex scientific simulator.
- Real sensitivity of LISA :
  - Study of instrument limitations (interaction with technological developments).
- Realistic data output.
- Participation to the Mock LISA Data Challenge.
- Indispensable tool for data analysis.





Thank you



- G.Auger (APC), H.Halloin (APC), S.Pireaux (Artemis), E.Plagnol (APC), T.Regimbeau (Artemis), J.Y.Vinet (Artemis), G. Trap (APC trainee)
- G. Faye(GReCO), L. Blanchet(GReCO)
- M. Tinto(JPL), M.Vallisneri(JPL).