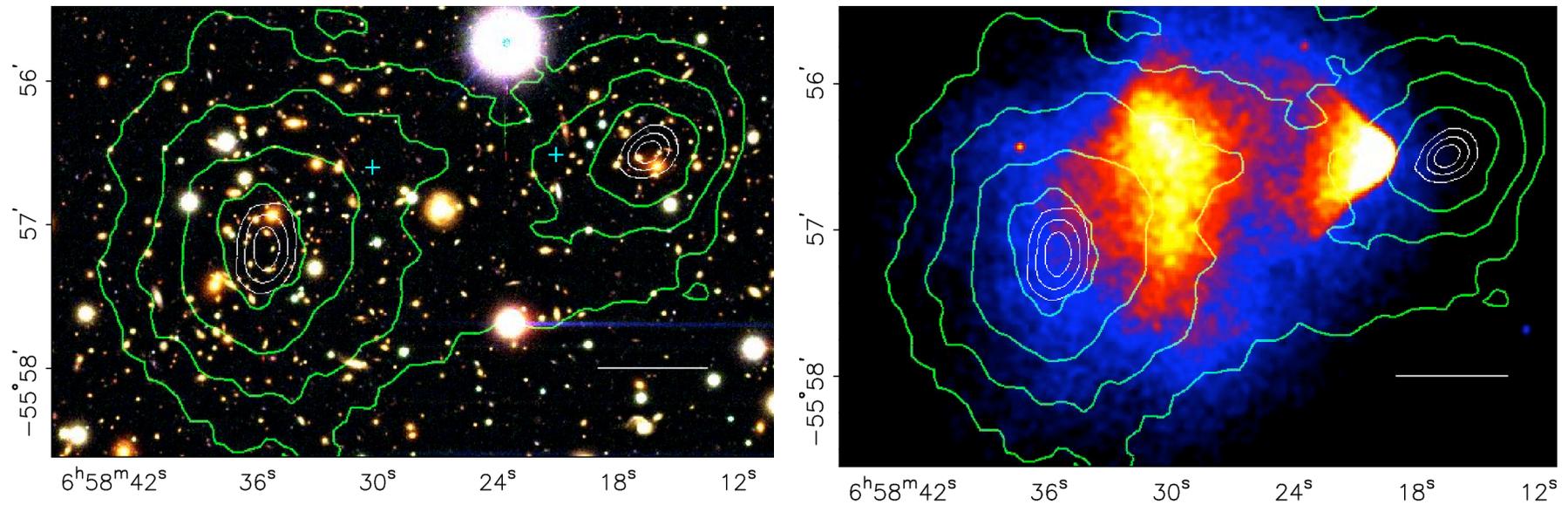
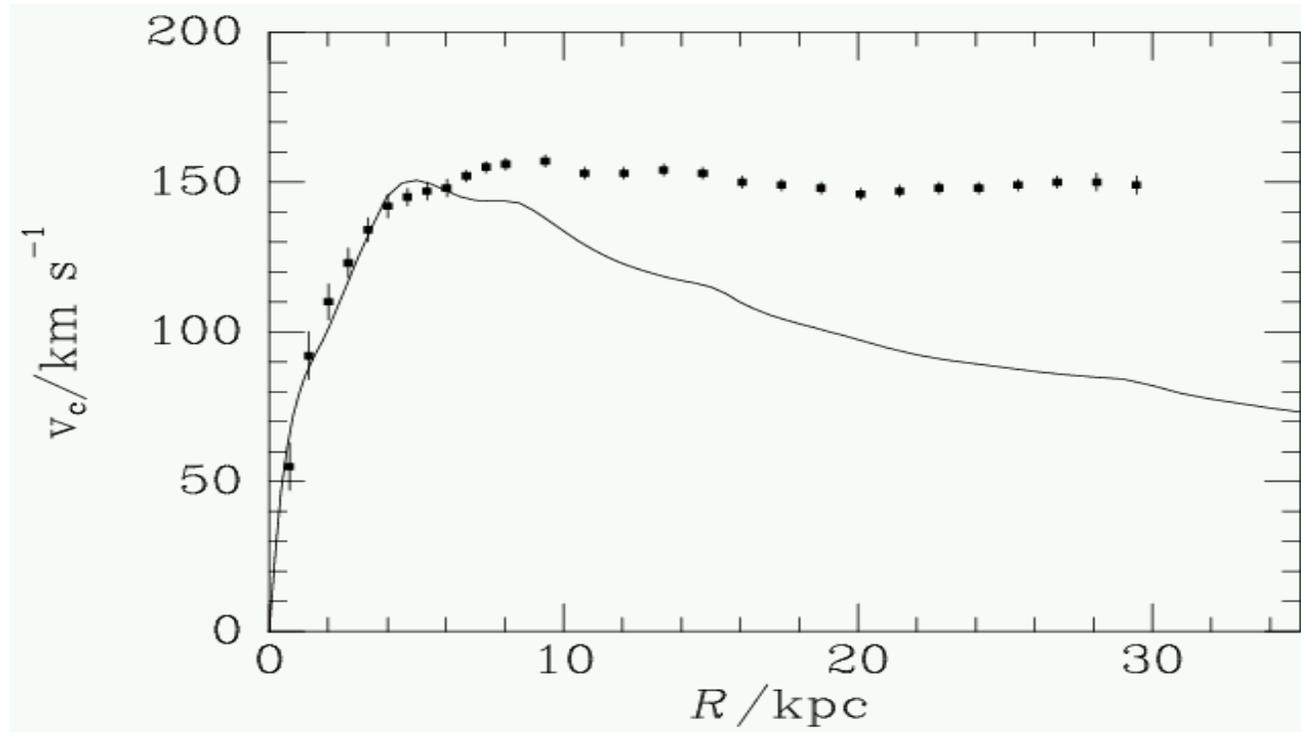


# Modified Gravity and the Bullet Cluster



**Benoit Famaey (Université Libre de Bruxelles)**

# General Relativity



- Observe  $\rho_{\text{bar}}$  in galaxies derive  $\Phi_{\text{bar}}$   
 $(R|\nabla\Phi_{\text{bar}}|)^{1/2} = V_{\text{c bar}}$  too low in the  
galactic plane compared to observed  $V_c$   
 $\Rightarrow$  DARK MATTER HALO

- Concordance model: Assume GR and  $\Lambda$  to fit supernovae data (the cosmological constant or dark energy density  $c^4\Lambda/8\pi G$ )

$$R_{\alpha\beta} - 1/2 R g_{\alpha\beta} + \Lambda g_{\alpha\beta} = (8\pi G/c^4) T_{\alpha\beta}$$

- DM non-baryonic ( $\Omega_b \approx 0.05$ ,  $\Omega_m \approx 0.3$ ) and cold (CDM) i.e. massive particles (e.g. **neutralino**  $\sim 1\text{TeV}$ ) to grow hierarchical structure
- It **cannot be ordinary neutrinos**, too light ( $< 2.2\text{ eV}$ ) to form hierarchical structure, too light fermions to have a density comparable to DM densities in galaxies (colder than galaxy clusters). In standard cosmology  $\sum m < 0.6\text{ eV}$
- However, CDM (necessary in a GR Universe) is not without problems

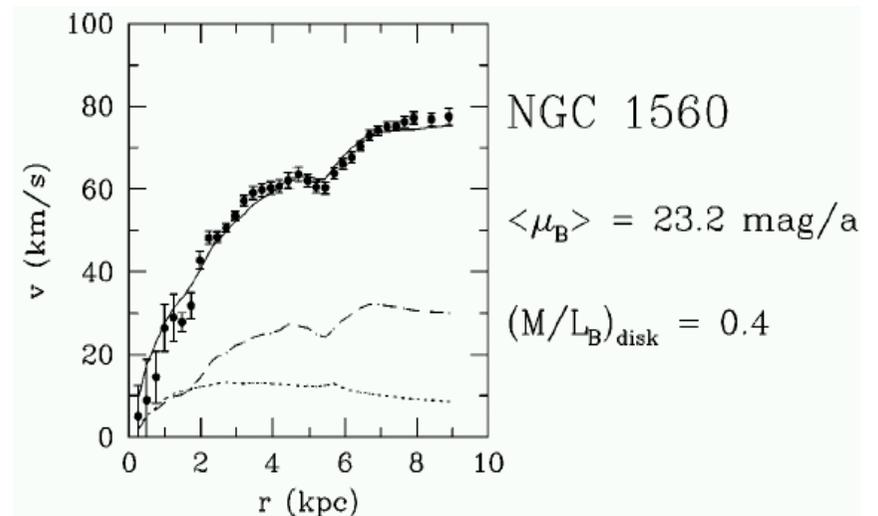
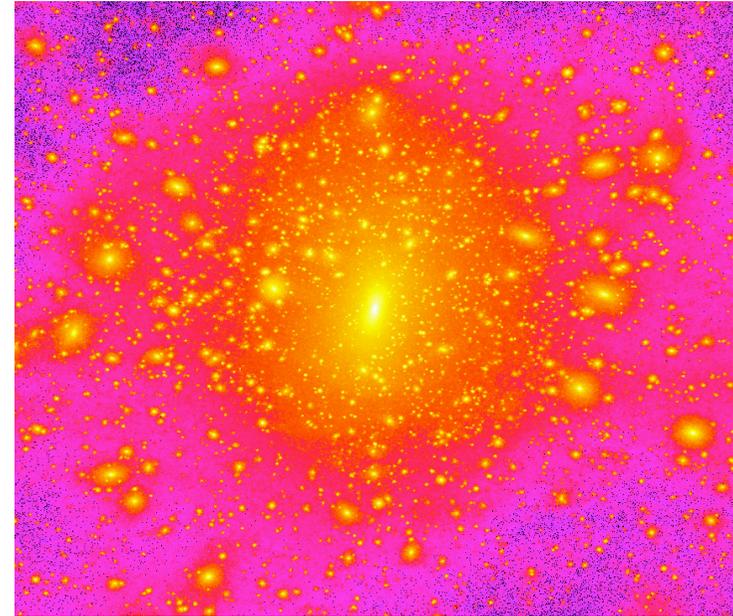
# CDM simulations

High resolution simulations of clustering CDM halos  
(e.g. Diemand et al.)

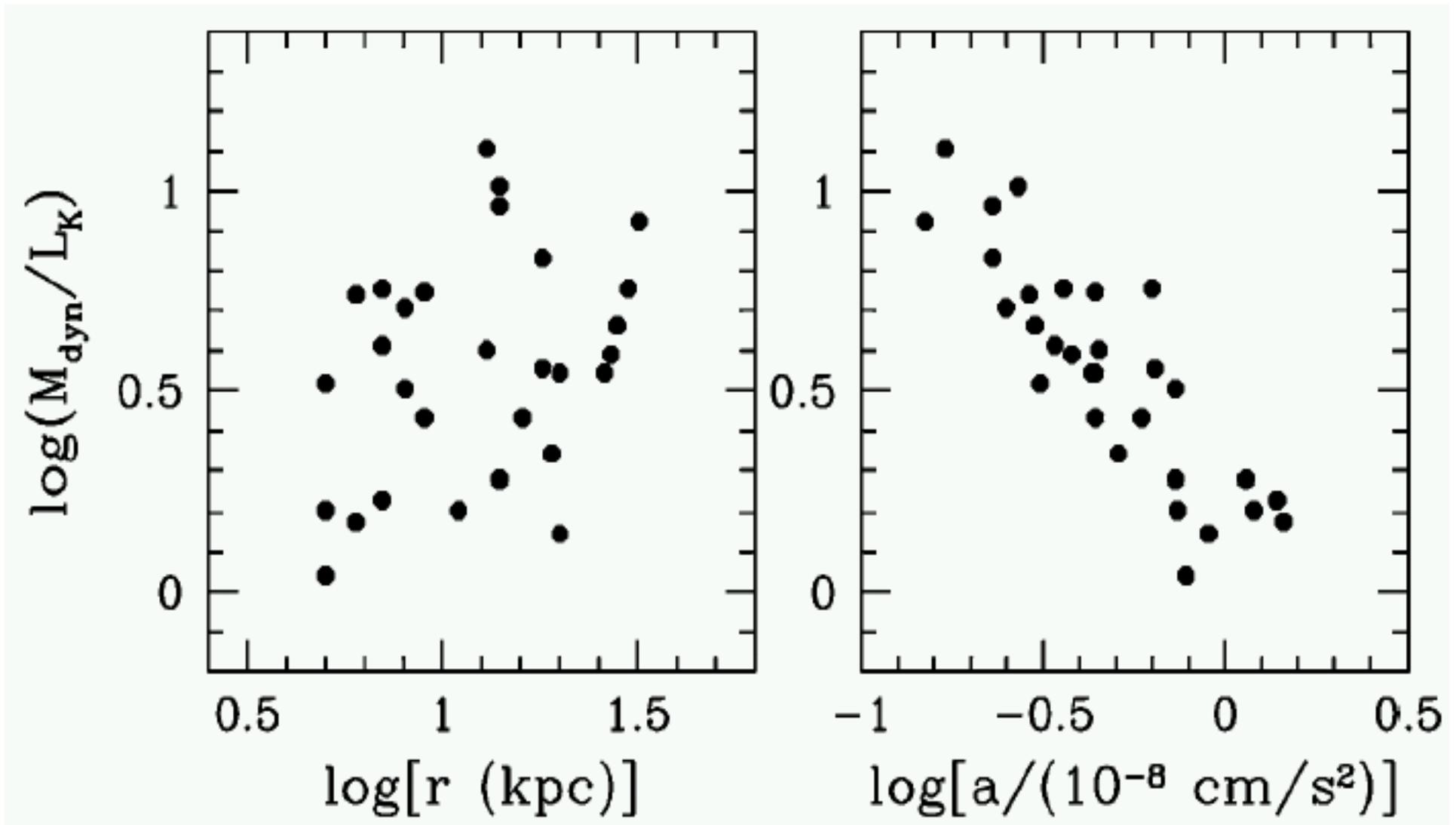
Central **cusp**  $\rho \propto r^{-\gamma}$ , with  $\gamma > 1$

Observed **neither in the Milky Way** (see [Famaey & Binney 2005](#)) **neither in LSB** **nor in HSB** (No present-day satisfactory solution)

**What is more:** wiggles of rotation curves follow wiggles of baryons!!



# The baryon-gravity relation



# Modification of Newtonian gravity

Suppose  $\mathbf{F} = -\nabla\phi$  where

$$\nabla^2\phi_N = 4\pi G\rho \quad \rightarrow \quad \nabla \cdot [\mu(|\nabla\phi|/a_0)\nabla\phi] = 4\pi G\rho$$

where

$$\mu(x) \rightarrow \begin{cases} 1 & \text{for } x \gg 1 \\ x & \text{for } x \ll 1 \end{cases}$$

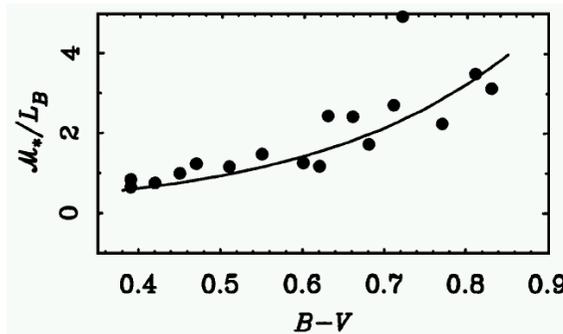
This is the B-M equation ([Bekenstein & Milgrom 1984](#))

Milgrom's formula exact in spherical and cylindrical symmetry and good approximation in a flat axisymmetric disk:

$$\mathbf{a} = -\nabla\Phi$$

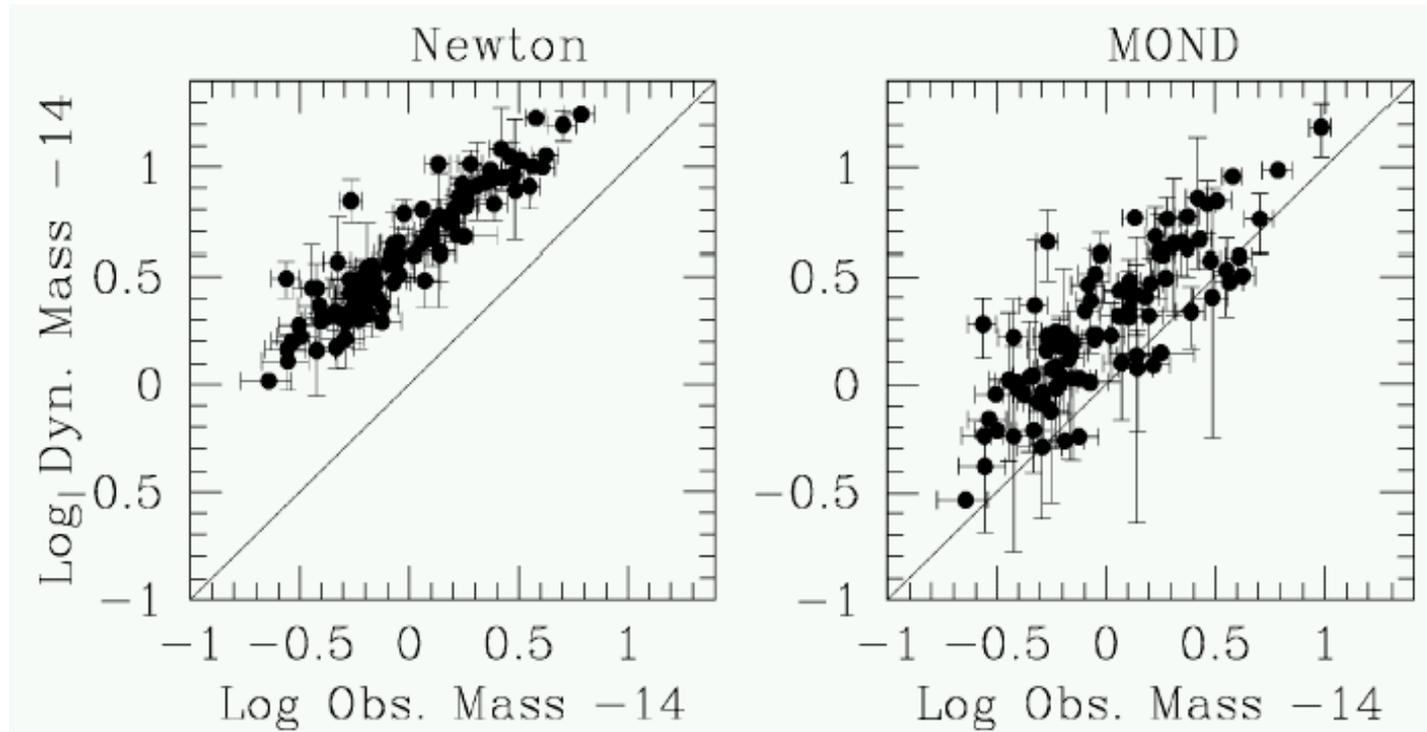
$$\mu(|\nabla\Phi|/a_0)\nabla\Phi = \nabla\Phi_N + \nabla \times \mathbf{K}$$

- Explains the RC wiggles following the baryons
- Tully-Fisher relation at small  $x$ :  $v^4 = GM_{\text{bar}}a_0$  (small observed scatter)
- Rotation curves of **HSB** (see e.g. Famaey, Gentile, Bruneton, Zhao [astro-ph/0611132](#))
- Rotation curves of **LSB** ( $\Sigma \ll a_0/G \Rightarrow g_N \ll a_0$ ), with high-discrepancy
- Fitted M/L ratios follow predictions of pop. synthesis models



- No discrepancy in giant **ellipticals** ([Milgrom & Sanders 2003](#))
- No discrepancy in nearby **globular clusters**  
(external field effect, breaks the strong equivalence principle)
- Local galactic escape speed from the Milky Way  
 $v_{\text{esc}} \sim 545$  km/s as observed ([Famaey, Bruneton & Zhao astro-ph/0702275](#))  
for an external field of order  $a_0/100 \sim H_0 \cdot 600$  km/s

- Dwarf spheroidals not too bad (large error bars)
- But... clusters of galaxies need dark matter, e.g. neutrinos or the missing baryons... we shall see that the bullet cluster implies that it must be collisionless



# Modifying GR?

Bekenstein (2004) proposed a **bi-metric multi-field** theory with a physical metric that couples with matter fields:

$$g'_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta} + U_{\alpha}U_{\beta}) - e^{2\phi}U_{\alpha}U_{\beta}$$

$$S = S_g + S_{\phi} + S_U + S_m$$

with a dynamical normalized **vector field**  $U_{\alpha}$   
(pointing in the time-direction for a quasi-static system)

and a **k-essence-like scalar field**  $\phi$  (with a free function in its lagrangian density depending on  $\nabla\phi$ ,  $\nabla\phi$ , and linked to the MOND  $\mu$ )

=> one can obtain **MOND**

- In a **quasi-static** system with a **weak** gravitational field:

$$g'_{00} = - (1-2\Phi)$$

$$\text{where } \Phi \equiv \Phi_N + \phi$$

$$g'_{ij} = (1-2\Phi) \delta_{ij}$$

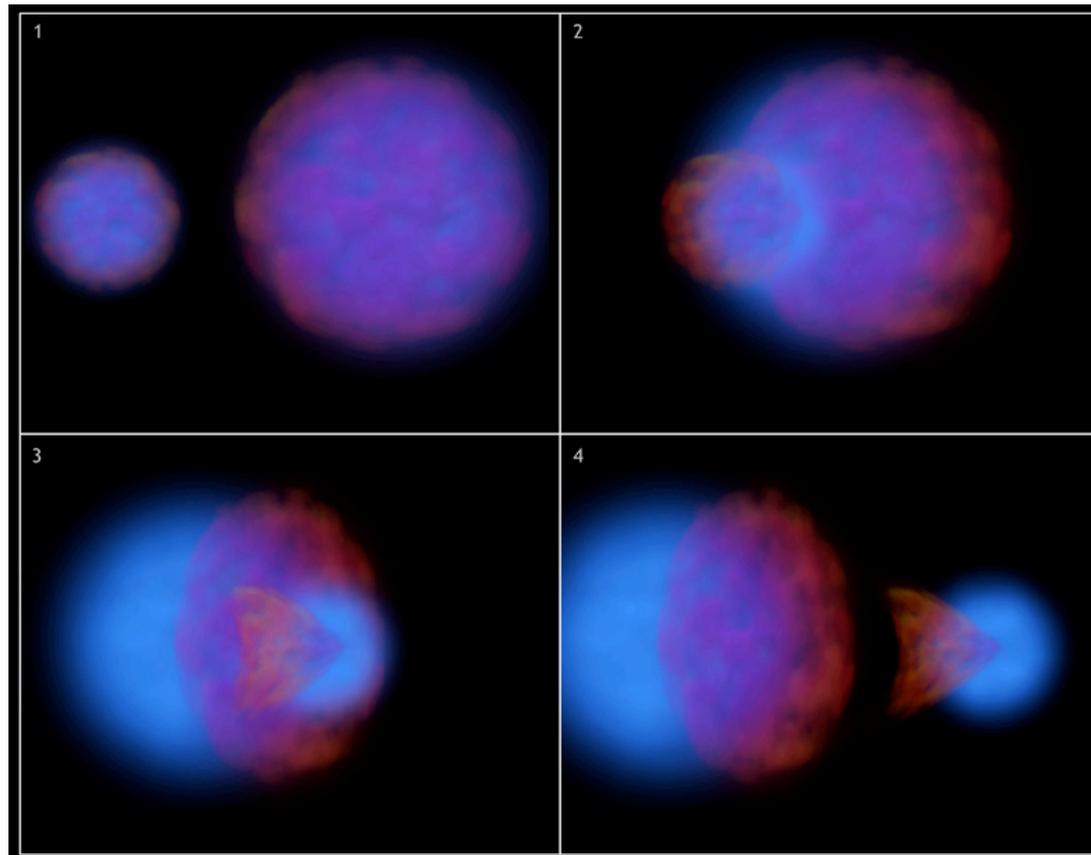
$$\text{where } \Phi \equiv \Phi_N + \phi$$

where  $\phi$  **obeys a MOND-like equation**, and plays the role of the dark matter potential (dynamics and lensing are governed by the **same** physical metric  $g'$ , MOND precisely recovered in spherical symmetry)

- CMB (Skordis et al. 2006) needs a component of HDM, e.g. neutrinos  $m \sim 2\text{eV}$  (in order not to change the angular-distance relation by having too much acceleration) + good complement to dynamical mass estimated from temperature profiles in galaxy clusters (Aguirre et al. 2001, Sanders 2003, Pointecouteau & Silk 2005)
- The competitor of the  $\Lambda\text{CDM}$  model is thus the  $\mu\text{HDM}$  model bypassing CDM problems on galaxy scales

# The bullet cluster

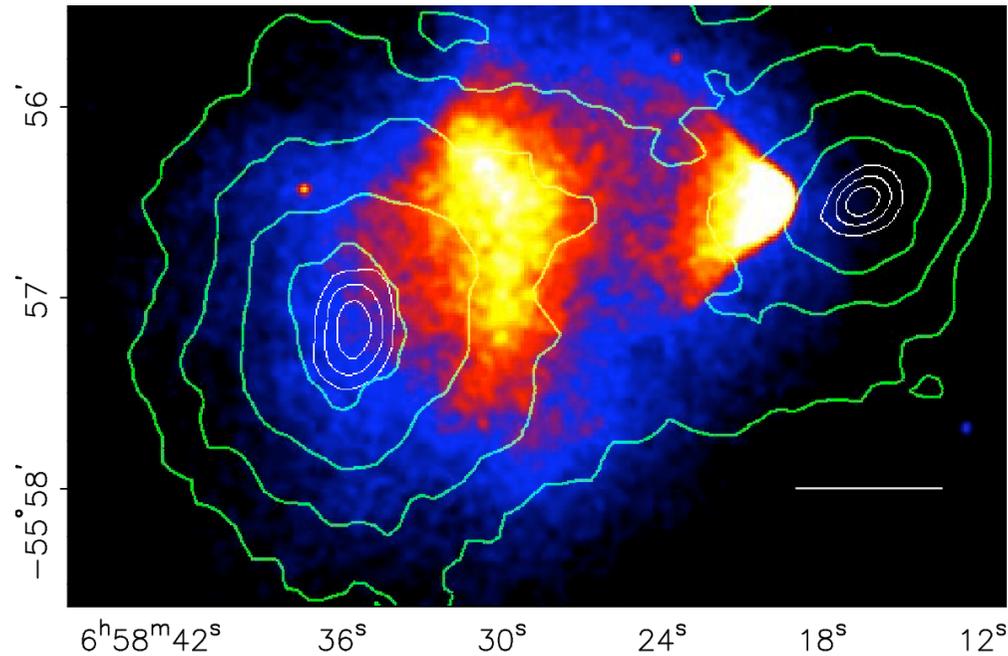
Merging galaxy cluster at a relative speed of 4700 km/s: a gigantic lab (1.4 Mpc for main axis) at a distance of 1Gpc ( $z=0.3$ ), separating the collisionless matter from the gas ( $10^{13}$  and  $2 \times 10^{13} M_{\text{sun}}$  of gas in the two clusters)



# Gravitational potential from weak lensing

- Weak lensing : deflection of light rays around a gravitational lens causes images of distant galaxies to appear **aligned (sheared) along the gradient of the gravitational potential** of the lens  
⇒ one can estimate the shear, and the convergence parameter  $\kappa(R)$  of the lens = divergence of the bending angle vector in the lens plane  $\alpha(R)$
- In any metric gravity theory there is a **linear** chain  
 $\Phi \rightarrow \mathbf{g} \rightarrow \alpha \rightarrow \kappa$
- In GR, there is an additional linear relation  $\rho \rightarrow \Phi$  , so the convergence  $\kappa(R)$  directly measures the projected surface density  $\Sigma(R)$
- In non-linear gravities,  **$\kappa$  can be non-zero where there is no projected matter** (Angus, Famaey & Zhao 2006)

# Convergence map of the bullet cluster



Clowe et al. (2006)

**Proof of DM? Proof of CDM??**

# Angus, Shan, Zhao, Famaey (ApJ 654 L13, astro-ph/0609125)

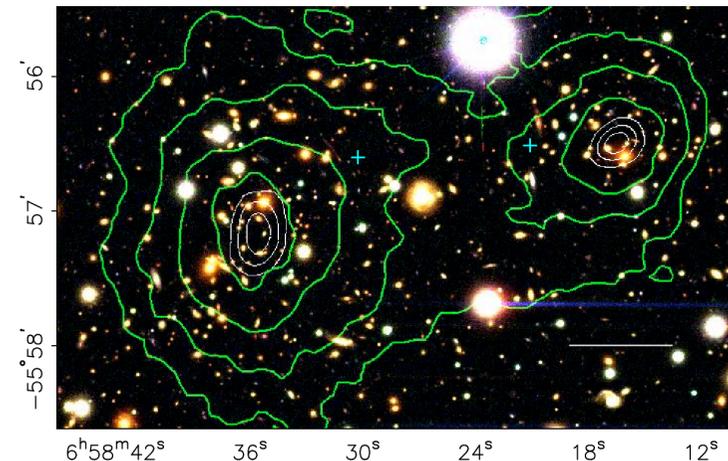
- Take parametric logarithmic potential  $\Phi(r)$

$$\Phi_i(r) = 1/2 v_i^2 \ln[1+(r/r_i)^2]$$

- Use  $\Phi_1, \Phi_2, \Phi_3, \Phi_4$  for the 4 mass components of the bullet cluster

⇒ Parametric convergence  $\kappa(R)$

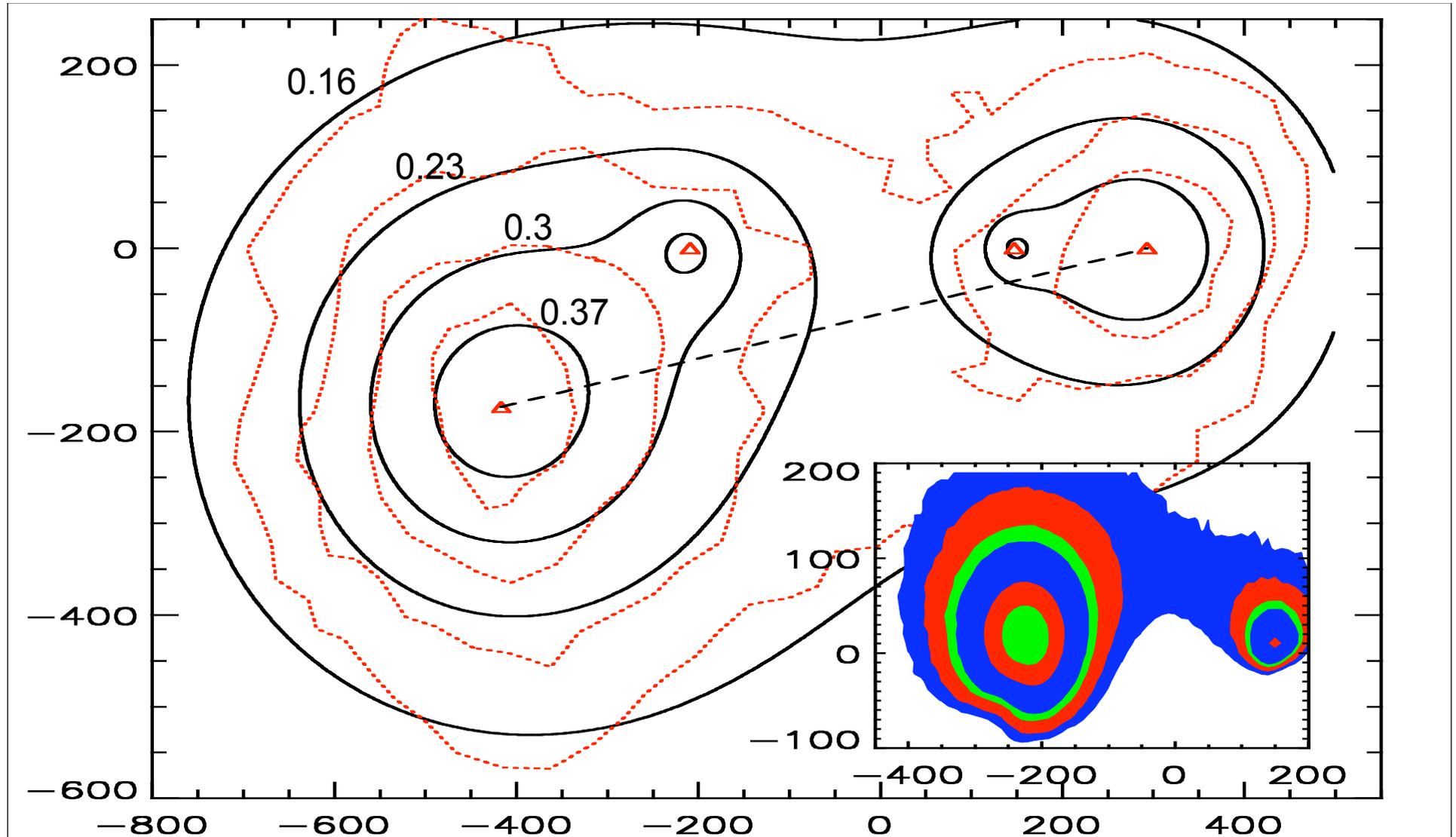
- $\chi^2$  fitting the 8 parameters on 233 points of the original convergence map



- With  $\mu(x) = 1$  ( $\rightarrow$  GR), or e.g.  $\mu(x) = x/(1+x)$ , get enclosed  $M(r)$ :

$$4\pi GM(r) = \int \mu(|\nabla\Phi|/a_0) \partial\Phi/\partial r dA$$

# The fit

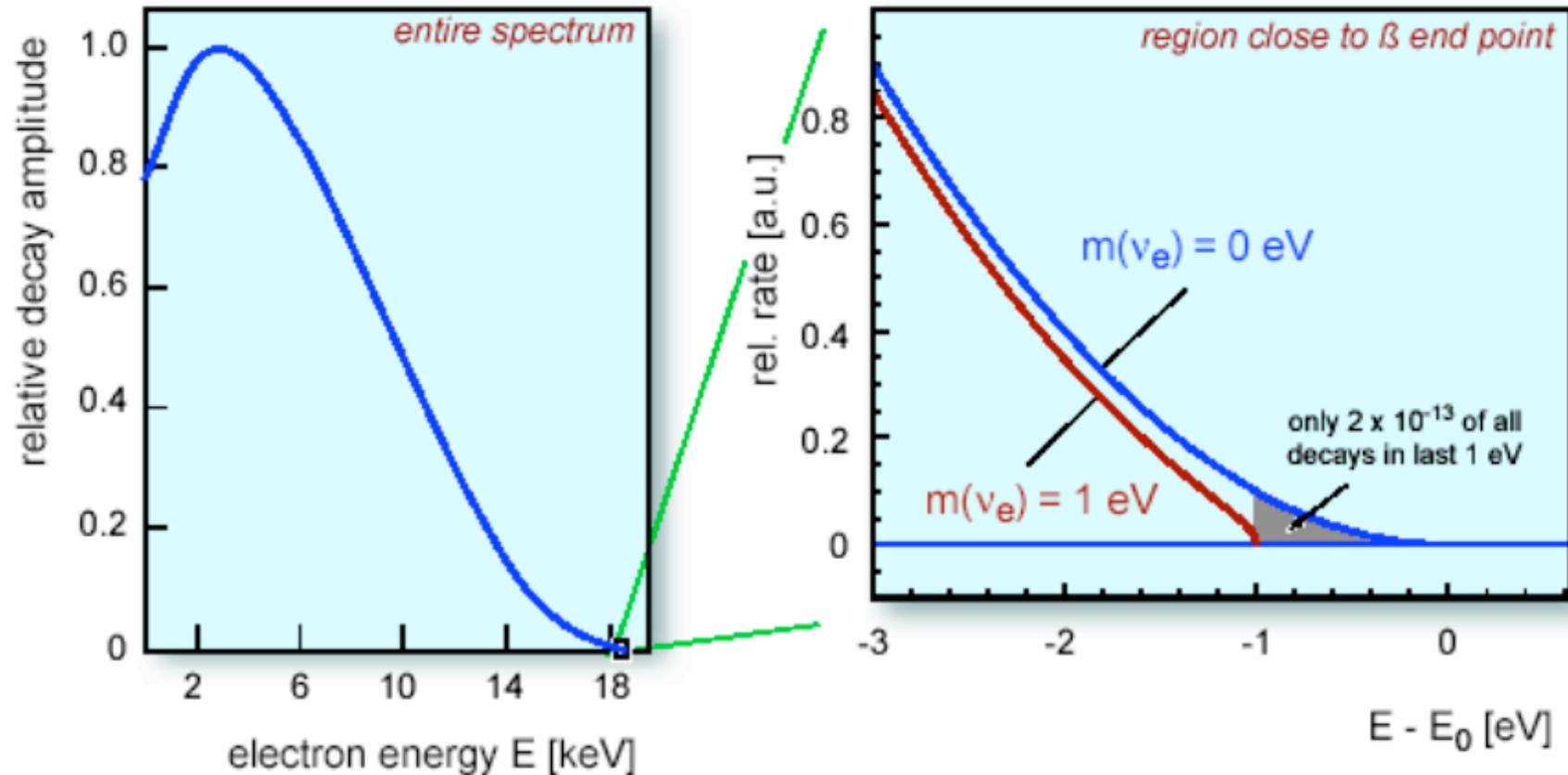


# Enclosed mass in MOND

- Collisionless:gas ratio within 180kpc of the galaxy and gas centers of the main cluster is **2.4:1**, instead of **1:8** for the ratio of observed collisionless baryons to X-ray gas  $\Rightarrow$  proof of DM... but does this exclude **MOND**?
- The central densities of the collisionless matter in **MOND** are compatible with the maximum density of 2eV neutrinos! ( $\sim 10^{-3} M_{\text{sun}}/\text{pc}^3$  in clusters)  
 $\Rightarrow$  does not exclude  $\mu\text{HDM}$

# Mass of electron neutrino

$\beta$ -decay of tritium ( ${}^3\text{H}$ ) into Helium 3 ion + electron + neutrino:



# Conclusion

- Unseen matter in GR or MOND must be collisionless, but BC doesn't rule out the MOND+neutrinos paradigm (note that this collisionless DM does not HAVE to be neutrinos). Seems somewhat ungainly, but don't forget the baryon-gravity relation in galaxies + velocity of the bullet cluster (Angus & McGaugh in prep.)
- If  $m_\nu \sim 2$  eV are discovered (active or sterile), it is a problem in standard cosmology while one could consider it as a successful "prediction" of MOND, then there could really be something fundamental about MOND/TeVS!