Gravity dynamics in braneworlds with conical internal space

Antonios Papazoglou

Institut d'Astrophysique de Paris (IAP)

Outline

- Introduction to extra dimensions
- 6D models vs selftuning
- Gravity problem on conical branes
    - Gauss-Bonnet term inclusion and constraints
    - Regularization by lowering the codimension

On several works with:
Nilles, Tasinato, Lee, Papantonopoulos, Zamarias

Extra dimensions

- **Extra dimensions emerged in unification ideas**

- **First considered by Kaluza, Klein in the '20s**

  Tried to unify E/M and gravity

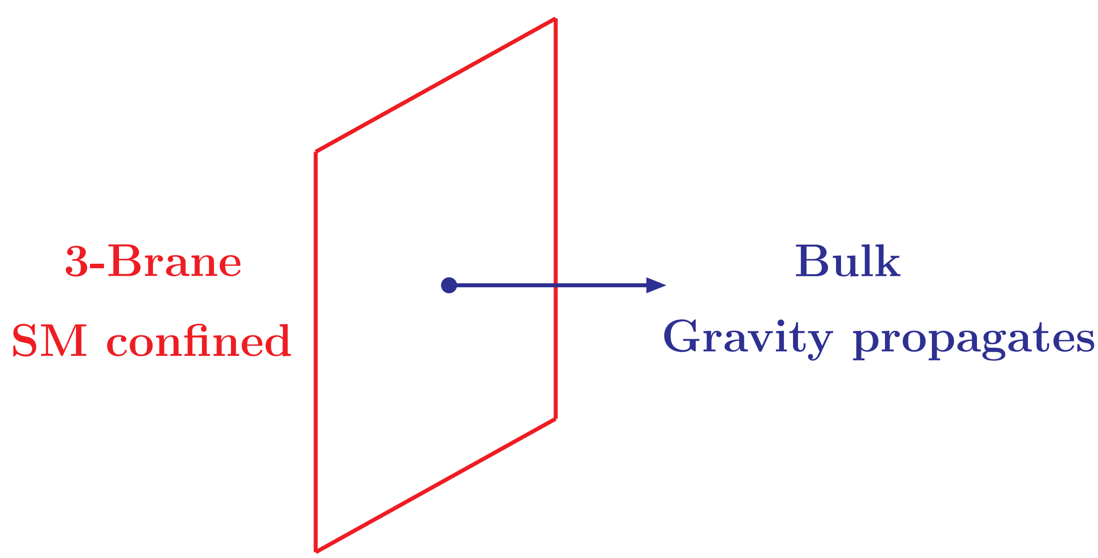
- **More seriously considered after the '70s**

  Discovery of string theory, consistent at $D = 10$

  ★ *Natural size* $R \sim M_{Pl}^{-1}$, *far from observable*

- **Entered in "hep-ph" in late '90s**

  Large extra dimension proposal, based on the concept of brane (discovered in early '90s in string theory)



★ *Constraints come only from the gravity sector*

⇒ *Can be large and observable!!!*

Brane world universes

- **Interesting for providing alternative explanations to long standing problems in physics**
  - **- Electroweak hierarchy problem**
  - **- Yukawa hierarchies**
  - **- Cosmological constant problem**

- **Example 1: Factorizable extra dimensions**

$$ds^2_{(4+n)} = g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n$$

The $4D$ and $(4+n)$ Planck masses are related as

$$M^2_{Pl} = R^n M^{2+n}$$

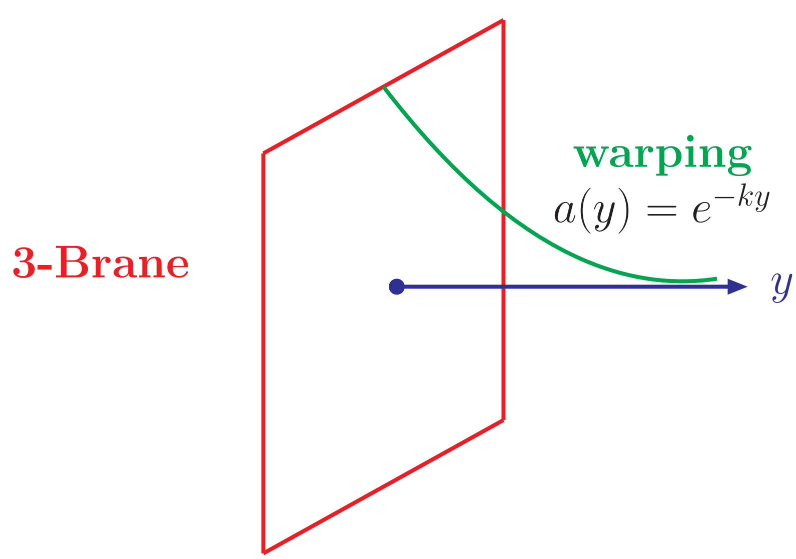
$M \sim TeV$ **is relevant for the hierarchy problem**

For $n = 2$ we have $R \sim mm$!

- **Example 2: Non-factorizable (warped) extra dimensions**

$$ds^2_5 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

For the Randall-Sundrum model, $a(y) = e^{-ky} \Rightarrow$ gravity is localised on the brane even for infinite y range



$5D$ theories

- **Most thoroughly studied case is braneworlds in 5D**

  One dim. $\perp$ to the brane $\equiv$ Codimension-1 brane

- **Einstein equation projected on the brane**

$$R^{(4)}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{(4)} = \frac{1}{M^3} T^{(B)}_{\mu\nu} + \{K^2\}_{\mu\nu} + \{C\}_{\mu\nu}$$

  - $K_{\mu\nu}$ relates to the brane matter through the junction conditions $K_{\mu\nu} \sim g'_{\mu\nu} \sim T^{(br)}_{\mu\nu}$

  - $\{C\}_{\mu\nu}$: bulk influence on brane dynamics

- **Cosmology example 1: Randall-Sundrum**

$$H^2 = \frac{1}{3M_{Pl}^2} \left[\rho + \frac{\rho^2}{2T} + \frac{C}{a^4} \right]$$

  Early time cosmology (for $\rho \gg T \sim M_{Pl}^4$) is $5D$

  Late time cosmology is $4D$

- **Cosmology example 2: Dvali-Gabadadze-Porrati**

$$H^2 \pm 2 \frac{M^3}{M_{Pl}^2} H = \frac{1}{3M_{Pl}^2} \rho$$

  Early time cosmology (for $H \gg M^3/M_{Pl}^2$) is $4D$

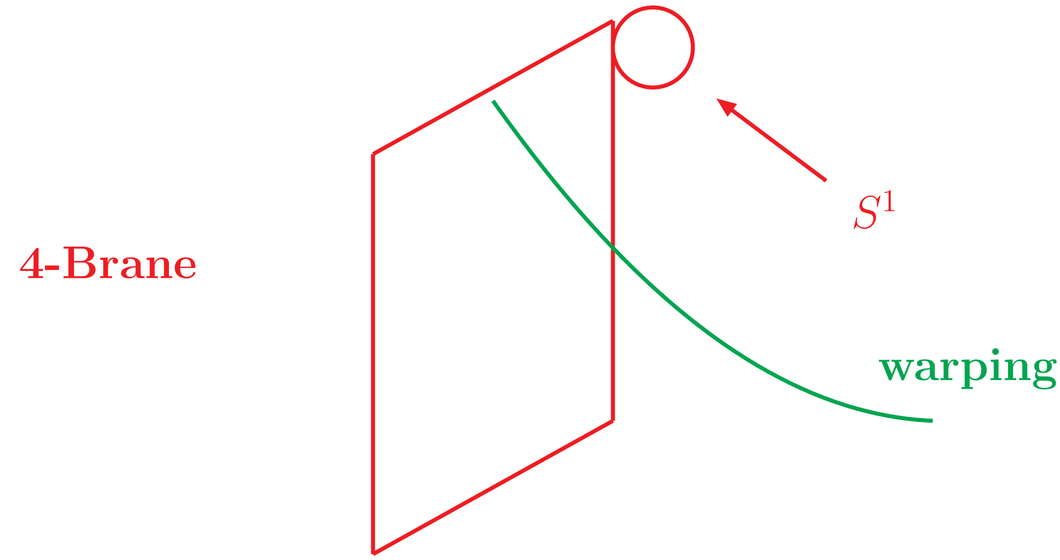
  Late time cosmology is $5D$ for "$+$" or dS for "$-$"

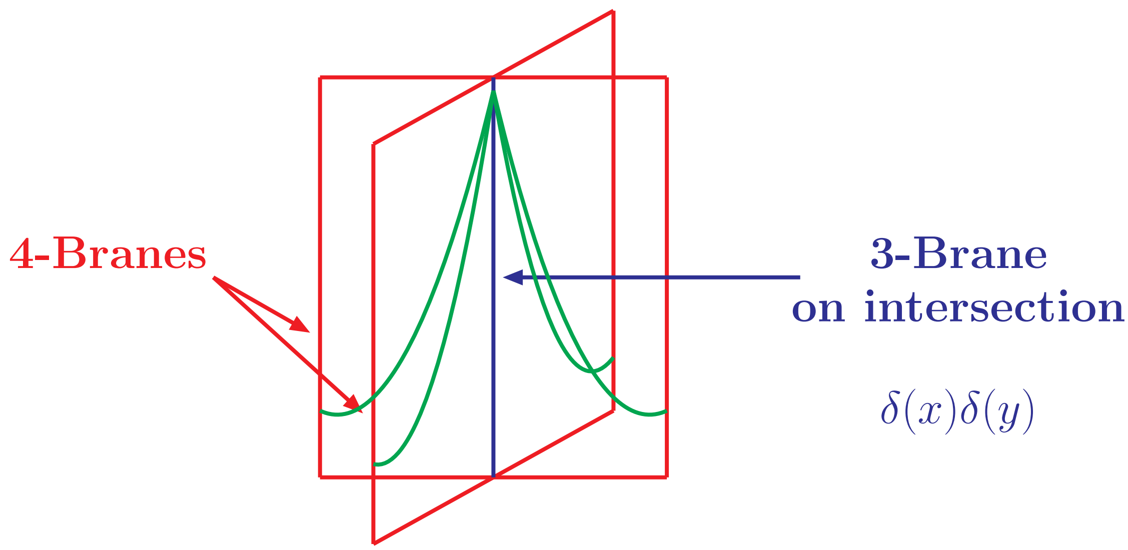
6D theories

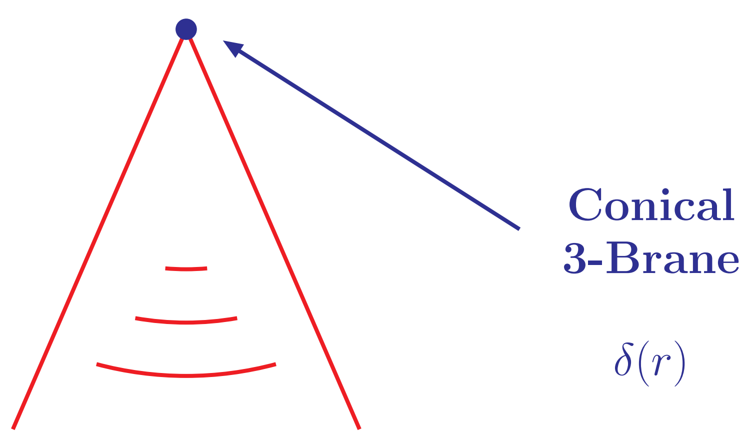
- **Less understood are 6D brane worlds**

  Two dim. $\perp$ to the brane $\equiv$ Codimension-2 brane

- **In 6D there are two extra dimensions to hide**





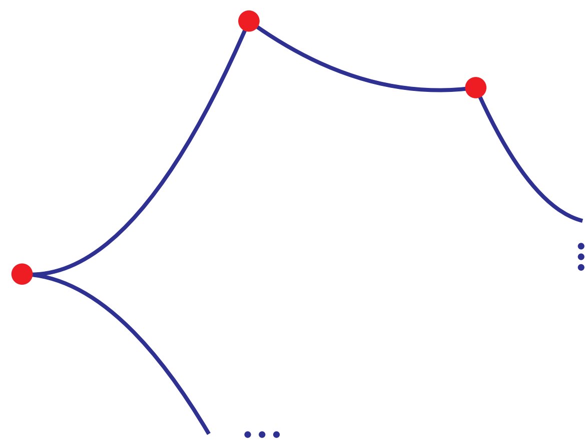


Conical branes

[H-P.Nilles,A.P.,G.Tasinato, hep-th/0309042]

- **Suppose a p-brane in D dimensions ($D = p + 1 + d$) with tension T_p. Then**

$$R = R^{(reg)} + R^{(sing)} \delta^{(d)}(r)$$



- **The singular part of the Einstein equations gives**

$$R^{(sing)} = \frac{p+1}{D-2} \, T_p$$

- **On shell value of the action**

$$S = \int d^D x (R^{(reg)} + \mathcal{L}^{(bulk)}) + \int d^{p+1} x (R^{(sing)} - T_p)$$

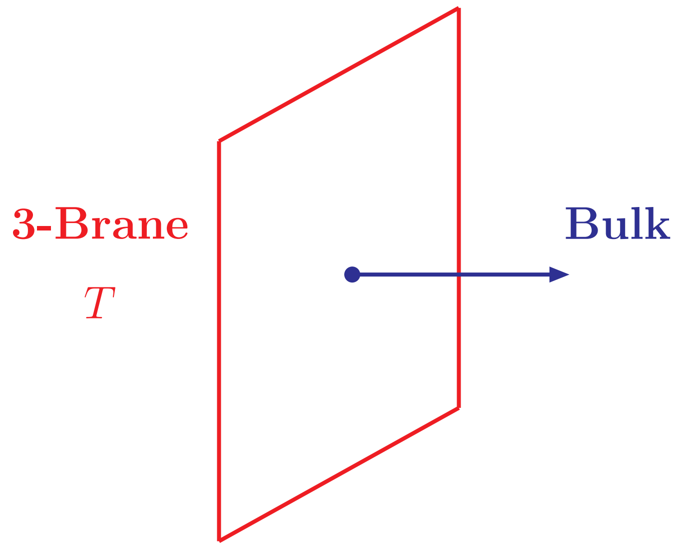
- **Cancellation of $R^{(sing)}$ and T_p happens automatically if**

$$\frac{p+1}{D-2} = 1 \qquad \Rightarrow \qquad D = (p+1) + 2$$

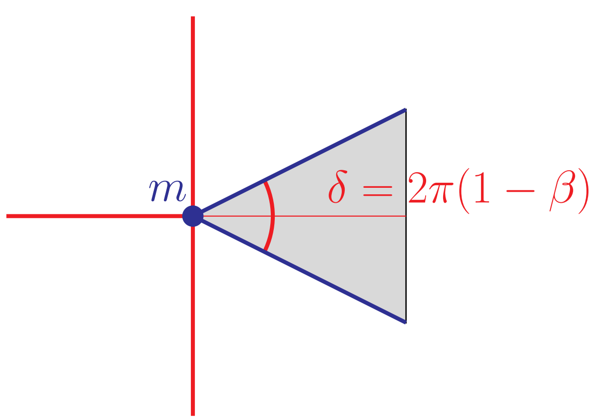
  i.e. **for a codimension-2 brane.**

- **For a $(p = 3)$-brane, $D = 6$**
- **Selftuning possible !!!**

Selftuning



- Selftuning model $\equiv$ Brane world model where
  1. The brane can be flat for any T
  2. No fine-tuning between T and other bulk quantities
- Solution to the cosmological constant problem, if in addition there are no fine-tuning between the bulk parameters
- Selftuning attempts in 5D models with codimension-1 branes failed (singularities, hidden finetuning)
- 6D attempts more promising (codimension-2 property)
- Origin of mechanism: in 1+2 dimensions, sources do not curve the space, but only introduce a deficit angle δ



$$ds_2^2 = dr^2 + r^2 d\phi^2, \ \phi \in [0, 2\pi\beta)$$

or

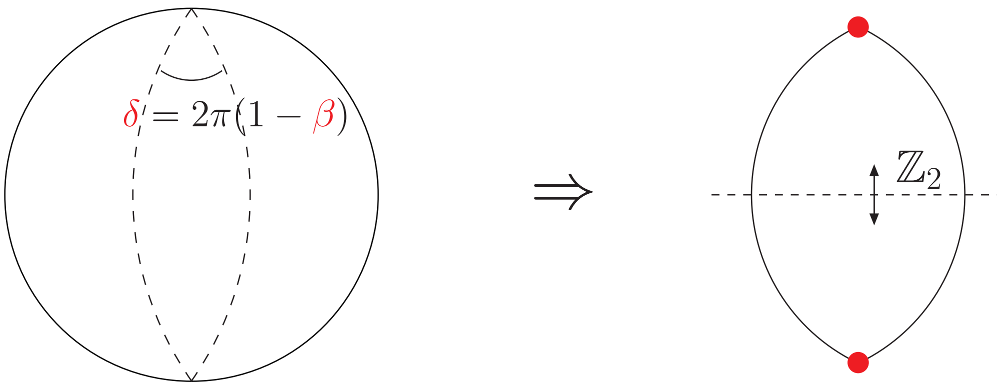
$$ds_2^2 = dr^2 + \beta^2 r^2 d\varphi^2, \ \varphi \in [0, 2\pi)$$

with $\beta = 4Gm$

$6D$ compactifications with conical branes

Flux compactification model

[Z.Horvath,L.Palla,E.Cremmer,J.Scherk, NPB127 (1977) 57]
[S.M.Carroll, M.M.Guica, hep-th/0302067]
[I.Navarro, hep-th/0302129]



- **Gravity + gauge field $F_{MN} = \partial_{[M}A_{N]}$ + bulk c.c. Λ**

$$S = \int d^6x\sqrt{-g_6}\left[\frac{1}{2}R_6 - \Lambda - \frac{1}{4}F^2_{MN}\right] - T\int d^4x\sqrt{-g_4}$$

- **Turning on the flux $F_{\theta\varphi} = f\,\epsilon_{\theta\varphi}$, the internal space is spontaneously compactified**

$$ds^2_6 = \eta_{\mu\nu}dx^\mu dx^\nu + R_0^2\left(d\theta^2 + \beta^2\sin^2\theta\, d\varphi^2\right)$$

with

$$\frac{1}{R_0^2} = f^2 \quad , \quad \Lambda = \frac{f^2}{2} \quad , \quad \beta = 1 - \frac{T}{2\pi}$$

- **T has no apparent relation to Λ or f**

  Selftuning ???

- **No, there is a flux quantization condition**

[Z.Horvath,L.Palla,E.Cremmer,J.Scherk, NPB127 (1977) 57]
[I.Navarro, hep-th/0305014]

$$f = \frac{N}{2eR_0^2\beta}$$

$$\Rightarrow \qquad N = \frac{2e}{\sqrt{2\Lambda}}\left(1 - \frac{T}{2\pi}\right)$$

- **This quantization condition ruins selftuning**

σ-model compactification model

[S.Radjbar-Daemi,V.Rubakov, hep-th/0407176]
[H.M.Lee,A.P., hep-th/0407208]

- **Instead of a gauge field + bulk c.c., one can use a 2d σ-model to compactify the internal space**

$$S = \int d^6x\sqrt{-g_6}\left[\frac{1}{2}R_6 - \frac{2\,\partial_M\Phi\partial^M\bar{\Phi}}{(1+|\Phi|^2)^2}\right] - T\int d^4x\sqrt{-g_4}$$

- **Obtain identical solution**

$$ds_6^2 = \eta_{\mu\nu}dx^\mu dx^\nu + R_0^2\left(d\theta^2 + \beta^2\sin^2\theta\, d\varphi^2\right)$$

$$\Phi = \left(\tan\frac{\theta}{2}\right)e^{i\beta\varphi} \qquad , \qquad \beta = 1 - \frac{T}{2\pi}$$

- **Now, the solution *is* selftuning**

Summary of motivations

1

Selftuning of the vacuum energy

2

Gravity in higher codimension defects
not still fully explored

3

$n = 2$ **submillimeter dimensions interesting**
for the hierarchy problem

Gravity on conical branes ???

[J.M.Cline,J.Descheneau,M.Giovannini,J.Vinet, hep-th/0304147]

- **For the static models we assumed** $T_{\mu\nu}^{(br)} = -T g_{\mu\nu} \delta^{(2)}(\vec{r})$

- **What if we have matter on the conical branes ???**

  e.g. cosmological fluid $T_{\mu\nu}^{(br)} = \mathrm{diag}(-\rho, P, P, P)\delta^{(2)}(\vec{r})$

- **General form of singularity**

$$\lim_{r \to 0} R \longrightarrow (\#_1)\ \frac{1}{r} + (\#_2)\ \delta^{(2)}(\vec{r})$$

- **Assumption: There is no singularity worse than conical**

$$(\#_1) = 0 \qquad \Rightarrow \qquad K_{\mu\nu} = 0$$

  Remaining singularity structure

$$E_{00}|_{\delta-part} = E_{ii}|_{\delta-part} \qquad \Rightarrow \qquad \rho = -P\ !$$

- **Ways out:**

  ★ **Keep assumption and complicate gravity dynamics so that the singularity structure is altered**

  ★ **Abandon assumption and regularize the brane around the singularity at** $r = 0$

Modifying gravity dynamics

[P.Bostock,R.Gregory,I.Navarro,J.Santiago, hep-th/0311074]

- **Modify the singularity structure of the equations of motion**

  $\Rightarrow$ **Addition of a bulk Gauss-Bonnet term**

$$\mathcal{S} = \frac{M_6^4}{2}\int d^6x\sqrt{G}\left[R^{(6)} + \alpha(R^{(6)\ 2} - 4R^{(6)\ 2}_{MN} + R^{(6)\ 2}_{MNK\Lambda})\right] + \int d^6x\mathcal{L}_{Bulk} + \int d^4x\mathcal{L}_{brane}\ \delta^{(2)}(\vec{r})$$

- **Metric ansatz**

$$ds^2 = g_{\mu\nu}(x,r)dx^\mu dx^\nu + dr^2 + L^2(x,r)d\theta^2$$

with $L = \beta(x)r + \mathcal{O}(r^3)$

- **Conical singularity conditions**

$$K_{\mu\nu} \sim g'_{\mu\nu}\big|_{r=0} = 0 \quad \text{and} \quad \beta = const.$$

- **The δ-function part of the $(\mu\nu)$ Einstein equations gives**

$$R^{(4)}_{\mu\nu} - \frac{1}{2}R^{(4)}g_{\mu\nu} = \frac{1}{M_{Pl}^2}\left[T^{(br)}_{\mu\nu} - \Lambda_4 g_{\mu\nu}\right]$$

with, $M_{Pl}^2 = 8\pi(1-\beta)\alpha M_6^4$ **and** $\Lambda_4 = -2\pi(1-\beta)M_6^4$

4D EQUATION WITH AN INDUCED Λ_4

Bulk & brane matter relations

[E.Papantonopoulos,A.P., hep-th/0501112]
[E.Papantonopoulos,A.P., hep-th/0507278]

- There is more information coming from the (rr) equation evaluated at $r=0$

$$R^{(4)} + \alpha[R^{(4)\ 2} - 4R^{(4)\ 2}_{\mu\nu} + R^{(4)\ 2}_{\mu\nu\kappa\lambda}] = -\frac{2}{M_6^2}T_r^{(B)r}$$

  - $R^{(4)}_{\mu\nu}$ and $R^{(4)}$ given by $T^{(br)}_{\mu\nu}$
  - $R^{(4)}_{\mu\nu\kappa\lambda}$ is arbitrary in general

- In several interesting cases $R^{(4)}_{\mu\nu\kappa\lambda}$ is also related to $T^{(br)}_{\mu\nu}$

  e.g. cosmological isotropic metric

$$ds^2 = -N^2(t,r)dt^2 + A^2(t,r)d\vec{x}^2 + dr^2 + L^2(t,r)^2 d\theta^2$$

  Then, the brane & bulk matter components are tuned to each other

- Different from the $5D$ brane cosmology

  In $5D$ $K_{\mu\nu} \neq 0$ on the brane

  $\Rightarrow$ Independence of brane matter from bulk matter

- Example: isotropic cosmology for $T^{(B)}_{MN} = -\Lambda_B G_{MN}$, $\rho = -\Lambda_4 + \rho_m$ and $P = \Lambda_4 + w\rho_m$

  - Brane matter tuning:

$$-\frac{\Lambda_B}{M_6^4} = \frac{\rho_m}{M_{Pl}^2}\left[\frac{1}{2}(3w-1) + \frac{2}{3}(3w+1)\alpha\frac{\rho_m}{M_{Pl}^2}\right]$$

  - We have $w = f(\rho_m)$!

Brane regularization

- These artificial constraints may be due to our insistance not to tolerate $1/r$ singularities
- Most physical procedure is to regularize the model via some core dynamics

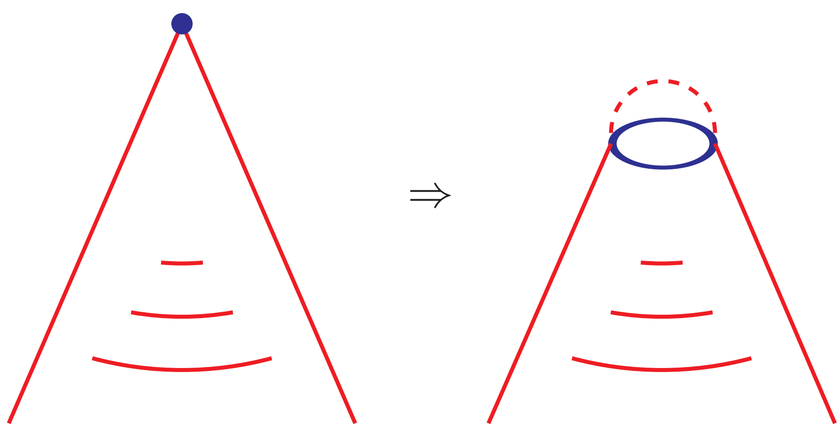
  A vortex solution in $6D$ is a thick 3-brane

  Studing matter on a vortex is complex

- Simplest possibility: lowering codimension

  Codimension-2 $\Rightarrow$ Codimension-1

- Cut space and replace with ring + smooth cap

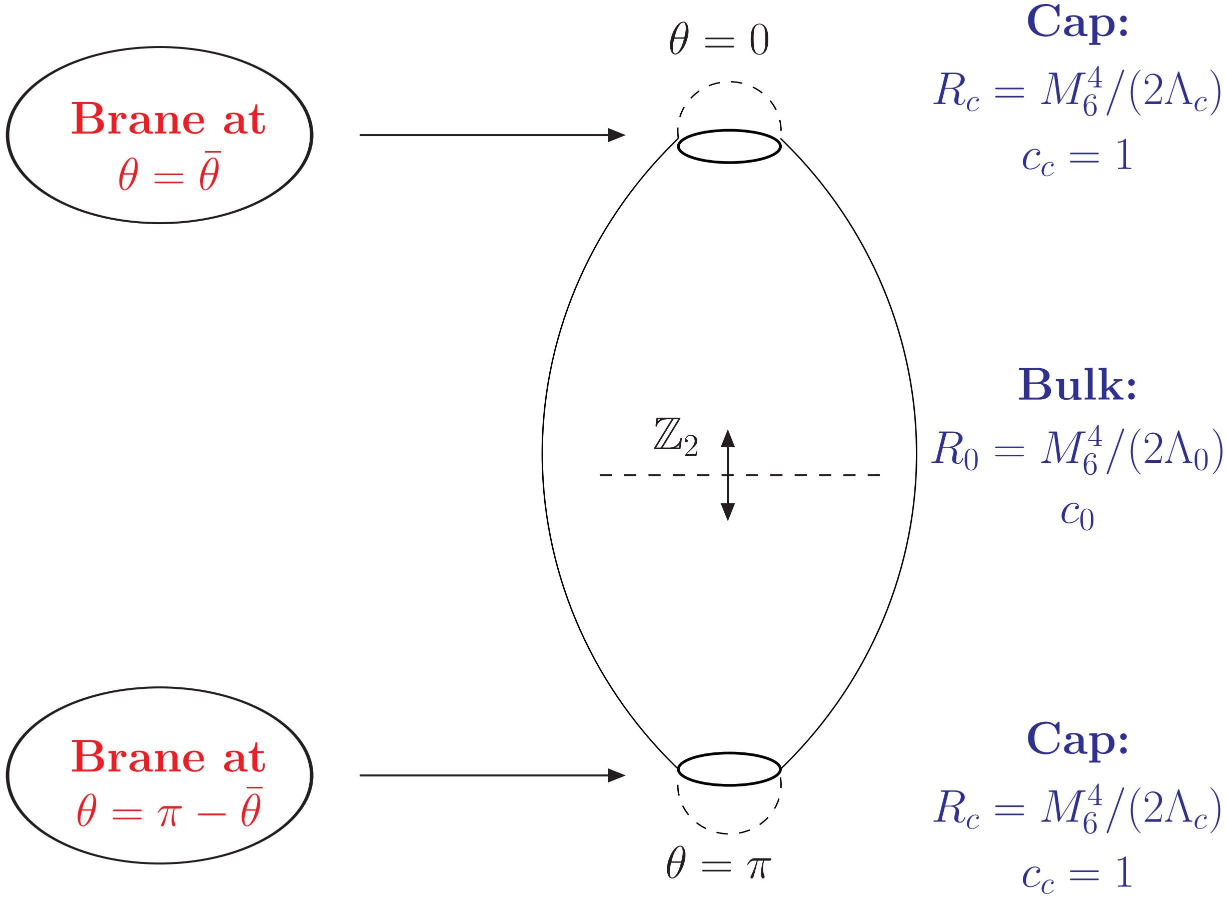


- Important ingredients to make this work
  - Choice of cap
  - Brane dynamics
- The limit where the radius of the 4-brane shrinks to zero is a conical 3-brane

"Rugby-ball" regularization

[M.Peloso,L.Sorbo,G.Tasinato, hep-th/0603026]

- **Example: flux compactification model**



$$
\begin{aligned}
ds^2 &= \eta_{\mu\nu} dx^\mu dx^\nu + R_\#^2 (d\theta^2 + c_\#^2 \sin^2\theta \; d\varphi^2) \\
F_{\theta\varphi} &= c_\# \; R_\# \; M^2 \cos\theta
\end{aligned}
$$

- continuity fixes $R_c = c_0 R_0$

- quantization condition $N = 2c_0 R_0 M^2 e$**,** $N \in \mathbf{Z}$

What kind of ring ?

- **Junction conditions for this particular geometry dictate** $T^{(br)}_{\varphi\varphi} = 0$ **and** $T^{(br)}_{\mu\nu} \sim \eta_{\mu\nu}$
- **Brane action proposal**

$$S_{br} = - \int d^5x \sqrt{-\gamma} \left(\lambda + \frac{v^2}{2} (\tilde{D}_{\hat{\mu}} \sigma)^2 \right)$$

with $\tilde{D}_{\hat{\mu}}\sigma = \partial_{\hat{\mu}}\sigma - eA_{\hat{\mu}}$, $\quad \hat{\mu} = (\mu, \varphi)$

- **Origin: Higgs phase** $H = v\ e^{i\sigma}$**, when the radial (heavy) part is integrated out**
- **Scalar field solution**

$$\sigma = n_{\pm}\varphi \quad , \quad n_{\pm} \in \mathbf{Z}$$

- **Furthermore the junction conditions determine** v, λ **as functions of** $\bar{\theta}$ **and relate the quantum numbers**

$$n_{\pm} = \pm \frac{N}{2}$$

Gravity for matter perturbations

- **Suppose that** $T_{\mu\nu} \to T_{\mu\nu} + \delta T_{\mu\nu}$ **(also for** $T_{\varphi\varphi}$**)**
- **The theory is Brans-Dicke with heavy scalar, decoupling in the conical limit (**$\bar{\theta} \to 0$**)**

$$R^{(4)}_{\mu\nu} = \frac{1}{M^2_{Pl}} \left[\delta T_{\mu\nu} - \frac{1}{2} \delta T \eta_{\mu\nu} \right] + (1 - \cos\bar{\theta}) F(\beta, \bar{\theta}, \partial_\mu) \left(\frac{1}{3} \delta T - \delta T^{\varphi}_{\varphi} \right)$$

- **The brane bending mode diverges in the conical limit (strong couping)**

Can we do more than that ?

- **If we wish to check selftuning, we should have in mind that the quantum contributions of the brane fields are of the order of the tension of the 4-brane**
- **One should be able to discuss brane motion**
- **Simplest case: mirage cosmology**

  ★ **Use the static bulk sections we know and see what cosmology the brane motion induces on the brane**

  [A.Kehagias,E.Kiritsis,hep-th/9910174]

- **Suppose a static bulk metric**

$$ds^2_{(d)} = A^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + B_{mn}(r)dx^m dx^n$$

A brane moving as $X^r = \mathcal{R}(t)$ induces a cosmology

$$ds^2_{(d-1)} = -[A^2(\mathcal{R}(t)) - \dot{\mathcal{R}}^2(t)]dt^2 + A^2(\mathcal{R}(t))d\vec{x}^2 + B_{mn}(\mathcal{R}(t))dx^m dx^n$$

$$\Rightarrow \quad ds^2_{(d-1)} = -d\tau^2 + A^2(\mathcal{R}(\tau))d\vec{x}^2 + B_{mn}(\mathcal{R}(\tau))dx^m dx^n$$

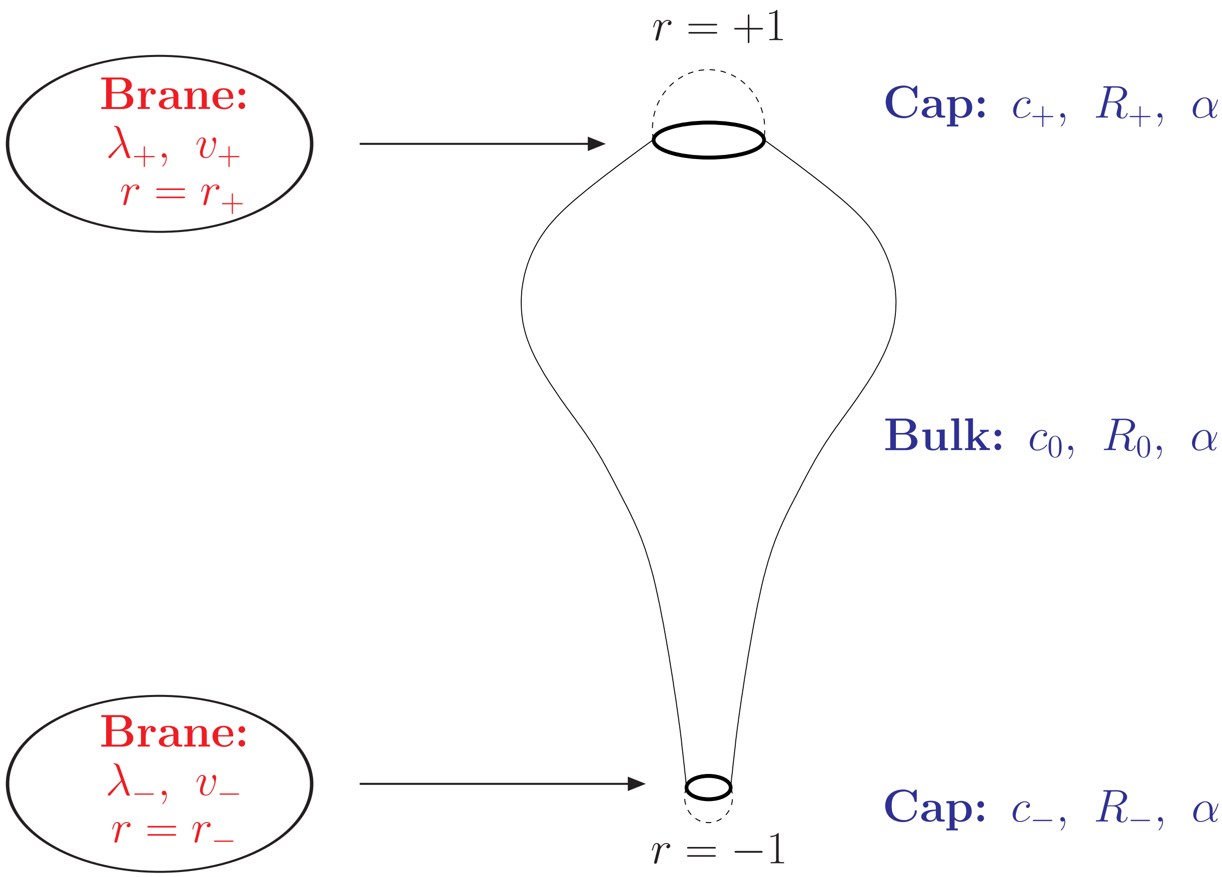
- **Need warping to have induced $4D$ cosmology**
- **The work is devided into two parts**

  ★ **Find regularization for the warped analogues of the "football" solutions**

  ★ **Study mirage cosmology in these backgrounds**

Warped model regularization

[E.Papantonopoulos,A.P.,V.Zamarias, hep-th/0611311]



- **Warped solution with codimension-2 known (Wick-rotated 6d Reissner-Nordström BH)**

[H.Yoshiguchi,S.Mukohyama,Y.Sendouda,S.Kinoshita, hep-th/0512212]

- **Use this to build the regular solution**

$$
\begin{aligned}
ds_6^2 &= z^2\eta_{\mu\nu}dx^\mu dx^\nu + R_\#^2\left[\frac{dr^2}{f} + c_\#^2 f\ d\varphi^2\right] \\
\mathcal{F}_{r\varphi} &= -c_\# R_\# M^2 S(\alpha)\cdot\frac{1}{z^4}
\end{aligned}
$$

with $R_\# = M^4/(2\Lambda_\#)$**,** $c_\pm = 1/X_\pm(\alpha)$**,** $R_\pm = c_0 R_0 X_\pm$ **and**

$$
\begin{aligned}
f &= \frac{1}{5(1-\alpha)^2}\left[-z^2 + \frac{1-\alpha^8}{1-\alpha^3}\cdot\frac{1}{z^3} - \alpha^3\frac{1-\alpha^5}{1-\alpha^3}\cdot\frac{1}{z^6}\right] \\
z &= [(1-\alpha)r + 1 + \alpha]/2
\end{aligned}
$$

Ring dynamics

- **Use again a scalar (Goldstone-like) field**

$$S_{br} = -\int d^5x\sqrt{-\gamma}\left(\lambda + \frac{v^2}{2}(\tilde{D}_{\hat{\mu}}\sigma)^2\right)$$

with $\tilde{D}_{\hat{\mu}}\sigma = \partial_{\hat{\mu}}\sigma - eA_{\hat{\mu}}$**,** $\quad \hat{\mu} = (\mu, \varphi)$

- **Scalar field solution**

$$\sigma = n_{\pm}\varphi \quad , \quad n_{\pm} \in \mathbf{Z}$$

- **From the junction conditions we determine** $v_{\pm}$**,** $\lambda_{\pm}$ **as functions of** $r_{\pm}$ **and relate the quantum numbers**

$$n_{\pm} = \pm\frac{N}{2}w_{\pm}(\alpha) \quad , \quad n_{+} - n_{-} = N$$

and

$$w_{+}(\alpha) = \frac{2}{(1-\alpha^3)}\left[\frac{5(1-\alpha^8)}{8(1-\alpha^5)} - \alpha^3\right]$$

- **Restriction of warping** α**, quantum number** N
  - **- Cannot have warped solutions for** $N \leq 4$
  - **- First warped solution for** $N = 5$**,** $n_{+} = 3$**,** $\alpha \approx .44$

- **Regularization scheme for** N**'s,** α**'s other than permitted breaks down**

Supersymmetric model (Salam-Sezgin)

[G.W.Gibbons,R.Guven,C.N.Pope, hep-th/0307238]
[C.P.Burgess,F.Quevedo,G.Tasinato,I.Zavala,hep-th/0408109]

- **Repeating the regularization procedure for the known warped solutions we find**

$$n_{\pm} = \pm\frac{N}{2}$$

- **No restriction of the warping** α

Brane motion

[E.Papantonopoulos,A.P.,V.Zamarias, hep-th/0703xxx]

- **Let one of the branes (*e.g.* the upper) move between the static bulk and the static cap with position**

$$X^r = \mathcal{R}(t)$$

[A non-trivial embedding of the time coordinate is also required for the continuity of the induced metric]

- **Induced metric**

$$ds^2 = -z^2 \left(1 - \dot{\mathcal{R}}^2 \frac{R_0^2}{f z^2}\right) dt^2 + z^2 d\vec{x}^2 + c_0^2 R_0^2 f d\phi^2$$

$$\Downarrow$$

$$ds^2 = -d\tau^2 + a^2(\tau) d\vec{x}^2 + b^2(\tau) d\phi^2$$

- **Hubble rates of a and b related**

$$H_b = \frac{z f'}{2 f z'} H_a$$

Close to the would-be conical singularity

$$\text{if} \quad H_a > 0 \quad \Rightarrow \quad H_b < 0$$

- **But we always have $H_a \sim \mathcal{O}(1) H_b$!**

In this regime, the influence of the extra dimension is significant

- **Include brane matter**

$$T_{\mu}^{(br)\ \nu} = \mathrm{diag}(-\rho, P, P, P, \hat{P})$$

- **General coupling of the gauge field to brane matter with**

$$\frac{\delta \mathcal{L}_{matt}}{\delta A^{\hat{\kappa}}} = (0, \vec{0}, \hat{L})$$

- **Two junction conditions determine $\hat{P}$, $\hat{L}$**

- **The remaining two junction conditions are the Friedmann and acceleration equations**

  $e.g.$ **the Friedmann equation is**

$$A(a)\sqrt{1 + H_a^2 R_0^2 B(a)} \left(\frac{1}{R_0} - \frac{1}{R_+} \sqrt{\frac{1 + H_a^2 R_0^2 B(a)}{1 + H_a^2 R_+^2 B(a)}} \right) = \frac{\rho_{tot}}{M^4}$$

  with $A,\ B$ functions of $z,\ f$

- **For $H_a R_0 \ll 1$ and $B \sim \mathcal{O}(1)$**

$$H_a^2 = G_{eff}(\alpha)\frac{\rho_{tot}}{3} + C(\alpha)$$

  with

$$\frac{\dot{G}_{eff}}{G_{eff}} \sim \mathcal{O}(1) H_a$$

- **The influence of the contraction of the ring is significant**

- **Any mechanism related to this brane motion (vacuum energy relaxation) should be operative at times before $e.g.$ the QCD phase transition**

- **There is energy flow from the brane to the bulk**

$$\frac{d\rho_{tot}}{d\tau} + 3(\rho + P)H_a + (\rho + \hat{P})H_b = -F(a, H_a)$$

- **Is $4D$ standard cosmology recovered in any region???**

- **We have made the assumption of brane motion in a static bulk**

- **If the bulk is also time dependent, a $4D$ regime could be obtained**

- *e.g.* **The perturbative analysis of the "rugby-ball" regularization, resulted in a linearized $4D$ Einstein equation with corrections due to a rather massive scalar**

- **Possible scenario:**

  $\Rightarrow$ **Early cosmology with static bulk settling the vacuum energy**

  $\Rightarrow$ **Late cosmology with time dependent bulk giving rise to standard expansion**

Conclusions

- Study of 6D models with codimension-2 branes interesting

  selftuning, codimension-2 effects, hierarchy problem

- General problem with gravity on them

  - either conical singularities + modified bulk gravity
  - or general singularities + regularization

- Regularization of singularities, *e.g.* by lowering the codimension more promising way forward

- Simplest scenario of brane motion could only be operative at early times not to contradict observations

- The bulk “isotropically”- warped background could be the reason of difficulties