Spherically symmetric solutions of massive gravity and the Goldstone picture

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INTRODUCTION

Motivation

Modification of gravity - a way to get acceleration of the Universe (geometrical Dark Energy).

Examples:

- f(R) theories of Gravity
- Models with extra-dimensions (e.g. Dvali-Gabadadze-Poratti model)

Main Idea: give a mass to a graviton with $m \sim 1/H_0$

Problems

Pathologies:

- Hamiltonian unbounded from below
- Ghosts
- Singular solutions

However:

- MG can be seen as a relatively simple toy model
- MG shares some properties with DGP mogel

Massive Gravity and Bi-Gravity theories

- Linear theory of MG (Quadratic action)

Fierz'39; Fierz&Pauli'39

$$S = \frac{M_P^2}{2}\int d^4x \left(\text{“}H\partial^2 H + \ldots\text{''} - \frac{m^2}{4}\left[H_{\mu\nu}H^{\mu\nu} - (H^\mu_\mu)^2\right]\right) + \int d^4x \frac{1}{2}T_{\mu\nu}H^{\mu\nu}$$

Kinetic term

Mass term

Coupling to matter

Non-linear generalization

$$\sqrt{-g}R[g] \qquad \mathcal{V}_{\rm int}[f, g] \qquad \sqrt{-g}\mathcal{L}_m[g]$$

Non-linear generalization

$$S = \int d^4x \, \left(\frac{M_P^2}{2} \sqrt{-g} \, R[g] + \mathcal{V}_{\rm int}[f,g] + \sqrt{-g} \, \mathcal{L}_m[g] \right)$$

- g is dynamical
- f is flat (non-dynamical)
- matter is coupled to g
- $\mathcal{V}_{int}[f,g]$ is a scalar density under common diffeomorphisms
- $\mathcal{V}_{int}[f,g]$ takes the PF term...

Examples:

$$\mathcal{V}^{(BD)}_{\rm int} = -\frac{1}{8} m^2 M_P^2 \, \sqrt{-f} \, H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau} \right)$$

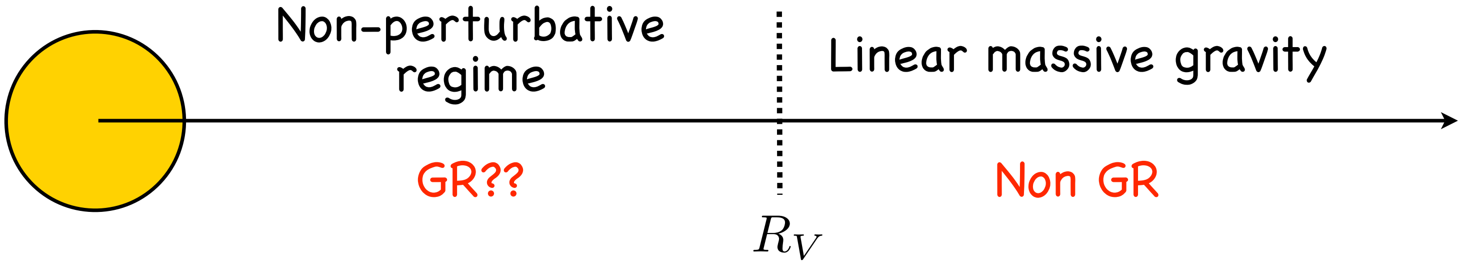
Boulware&Deser'72

$$\mathcal{V}^{(AGS)} = -\frac{1}{8} m^2 M_P^2 \sqrt{-g} \, H_{\mu\nu} H_{\sigma\tau} \left(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau} \right)$$

Arkani-Hamed, Georgi, Schwartz'03

$$H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$$

Static Spherically Symmetric Solutions



- New scale: the Vainshtein radius $R_V = \left(R_S\, m^{-4}\right)^{1/5}$ Vainshtein'72

➡ Is it possible to find a solution regular everywhere?

- Our approach is to study these questions in a specific limit: the decoupling limit

$$\begin{array}{r|l} M_P \to \infty & \Lambda \equiv (M_P m^4)^{1/5} \sim const \\ m \to 0 & T_{\mu\nu}/M_P \sim const \end{array}$$

Arkani-Hamed, Georgi, Schwartz'03

➡ Are there regular solutions in the DL?

➡ To what extend does the DL encode the physics of the full system?

STATIC SPHERICALLY SYMMETRIC SOLUTIONS OF MASSIVE GRAVITY

Metrics and Equations of Motion

- Bi-diagonal ansatz in the "Unitary" gauge:

$$
\begin{aligned}
g_{AB}dx^A dx^B &= -J(r)dt^2 + K(r)dr^2 + L(r)r^2 d\Omega^2 \\
f_{AB}dx^A dx^B &= -dt^2 + dr^2 + r^2 d\Omega^2
\end{aligned}
$$

- "Schwarzschild" gauge:

$$
\begin{aligned}
g_{\mu\nu}dx^\mu dx^\nu &= -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 && \text{Schwarzschild-like} \\
f_{\mu\nu}dx^\mu dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2 && \text{flat}
\end{aligned}
$$

- Equations of motion:

$$
\begin{aligned}
e^{\nu-\lambda}\left(\frac{\lambda'}{R} + \frac{1}{R^2}(e^\lambda - 1)\right) &= 8\pi G_N\left(T^g_{tt} + \rho e^\nu\right), \\
\frac{\nu'}{R} + \frac{1}{R^2}\left(1 - e^\lambda\right) &= 8\pi G_N\left(T^g_{RR} + P e^\lambda\right), \\
\nabla^\mu T^g_{\mu R} &= 0.
\end{aligned}
$$

solution far from source (I)

- Expansion in Newton's constant:

$$\lambda = \lambda_0 + \lambda_1 + \ldots \quad \text{etc., with} \quad \lambda_i, \nu_i, \mu_i \propto G_N^{i+1}$$

- Equations of motion:

$$\begin{aligned}
E_{tt} &\Rightarrow \frac{\lambda_0'}{R} + \frac{\lambda_0}{R^2} = -\frac{m^2}{2}(\lambda_0 + 3\mu_0 + R\mu_0') \\
E_{rr} &\Rightarrow \frac{\nu_0'}{R} - \frac{\lambda_0}{R^2} = \frac{m^2}{2}(\nu_0 + 2\mu_0) \\
\text{Bianchi} &\Rightarrow \frac{\lambda_0}{R^2} = \frac{\nu_0'}{2R}
\end{aligned}$$

$$\mu_0 \sim \frac{\lambda_0}{(mR)^2} \gg \lambda_0, \nu_0$$

$$\text{for} \quad mR \ll 1$$

solution:

$$\begin{aligned}
\lambda_0 &= \frac{C_1}{2R}, \\
\nu_0 &= -\frac{C_1}{R}, \\
\mu_0 &= \frac{1}{(mR)^2}\frac{C_1}{2R}
\end{aligned}$$

vDVZ discontinuity

solution far from source (II)

$$\boxed{\begin{aligned} \frac{\lambda_1'}{R} + \frac{\lambda_1}{R^2} &= -\frac{m^2}{2}(3\mu_1 + R\mu_1') \\ \frac{\nu_1'}{R} - \frac{\lambda_1}{R^2} &= m^2\mu_1 \\ \frac{\lambda_1}{R^2} &= \frac{\nu_1'}{2R} + Q(\mu_0), \end{aligned}}$$

1st order Eqs

$$\begin{aligned} Q(\mu) &= -\frac{1}{2R}\left\{3\alpha\left(6\mu\mu' + 2R\mu'^2 + \frac{3}{2}R\mu\mu'' + \frac{1}{2}R^2\mu'\mu''\right)\right. \\ &\quad \left. +\beta\left(10\mu\mu' + 5R\mu'^2 + \frac{5}{2}R\mu\mu'' + \frac{3}{2}R^2\mu'\mu''\right)\right\} \end{aligned}$$

solution:

$$\boxed{\begin{aligned} \nu &= -\frac{2}{3}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{n_1}{(mR)^4} + \mathcal{O}(R_S^3) \\ \lambda &= \frac{1}{3}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{l_1}{(mR)^4} + \mathcal{O}(R_S^3) \\ \mu &= \frac{1}{3(mR)^2}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{m_1}{(mR)^6} + \mathcal{O}(R_S^3) \end{aligned}}$$

relevant at

$$\boxed{R_V = \left(\frac{R_S}{m^4}\right)^{1/5}}$$

solution close to source

- Expansion in m: $$f(R) = \sum_{n=0}^{\infty} m^{2n} f_n(R)$$
- 0th order:

$$\lambda_0 = -\nu_0 = -\ln\left(1 - \frac{R_S}{R}\right)$$

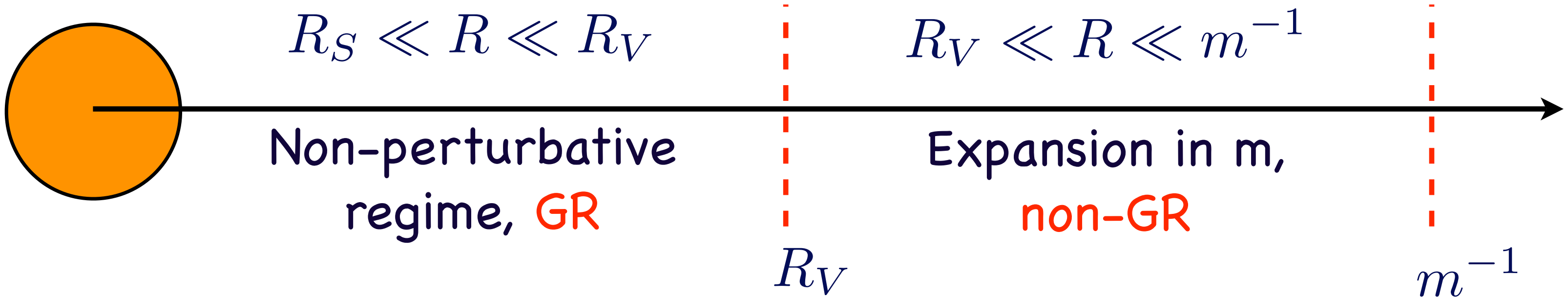
$$\mu_0 = m_0\sqrt{R_S/R} \gg \lambda_0, \nu_0$$

assume $R \gg R_S$

1st order solution

$$\boxed{\begin{aligned}
\nu &= -\frac{R_S}{R} + n_1 (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}\left(m^4\right) \\
\lambda &= \frac{R_S}{R} + l_1 (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}\left(m^4\right) \\
\mu &= m_0\sqrt{\frac{R_S}{R}} + m_1 (mR)^2 + \mathcal{O}\left(m^4\right)
\end{aligned}}$$

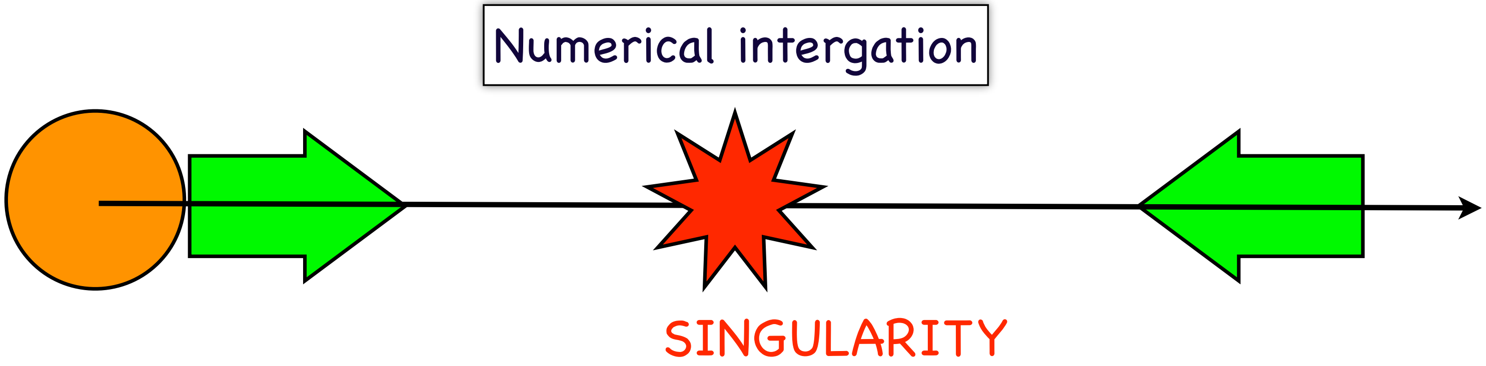
$$\boxed{R_V = \left(\frac{R_S}{m^4}\right)^{1/5}}$$



$R_S \ll R \ll R_V$
$R_V \ll R \ll m^{-1}$
Non-perturbative
regime, GR
Expansion in m,
non-GR
R_V
m^{-1}

Is it possible to match large R and small R (Vainshtein) solutions???

Damour, Kogan, Papazoglou'03



However: complicated problem. In order to study it, we take a limit which simplifies the system while keeping the most important non-linearities: the Decoupling Limit

➡ Are there regular solutions in the DL?
➡ To what extent does the DL encode the physics of the full system?

THE GOLDSTONE PICTURE
and
THE DECOUPLING LIMIT

The Stuckelberg mechanism (I)

Goal: to separate explicitly the various degrees of freedom (tensor, vector, scalar) of a massive field.

Example: the Proca's field $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu$

breaks gauge invariance

- Field redefinition $A_\mu \to A_\mu - \partial_\mu B$

$$\Rightarrow \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2(A_\mu - \partial_\mu B)(A^\mu - \partial^\mu B)$$

New gauge invariance:
$$\begin{array}{rcl} A_\mu & \to & A_\mu + \partial_\mu \Lambda \\ B & \to & B + \Lambda \end{array}$$

- "Unitary gauge": $B = 0$
- "Longitudinal gauge": $\partial_\mu A^\mu = 0$

2 DOF 1 DOF

$$\Rightarrow \mathcal{L} = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu - \frac{m^2}{2}A_\mu A^\mu - \frac{m^2}{2}\partial_\mu B \partial^\mu B$$

The Stuckelberg mechanism (II)

Arkani-Hamed, Georgi, Schwartz'03

- Massive spin-2 graviton: in the action

$$S = \frac{M_P^2}{2}\int d^4x \left(\sqrt{-g}R[g] - \frac{m^2}{4}\mathcal{V}\left[\mathbf{g^{-1}f}\right]\right) + S_m[g],$$

replace
$$\begin{aligned} f_{\mu\nu}(x) &\rightarrow f_{\mu\nu}(x) = \partial_\mu X^A(x)\partial_\nu X^B(x) f_{AB}\left(X(x)\right) \\ g_{\mu\nu}(x) &\rightarrow g_{\mu\nu}(x) \end{aligned}$$

the action is now invariant under both
$$\begin{aligned} X^A &\rightarrow X'^A \\ x^\mu &\rightarrow x'^\mu \end{aligned}$$

- "Unitary gauge": $X_0^A(x) \equiv \delta_\mu^A x^\mu$

- In non-unitary gauge, introduce the "Goldstone boson" π :

$$X^A(x) = X_0^A(x) + \pi^A(x).$$

The Goldstone boson expansion

- Scalar-vector decomposition: $\pi^A(x) = f^{AB}\left(A_B + \partial_B \phi\right)$
- Action in terms of $h_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$
- Shift: $h_{\mu\nu} = \hat{h}_{\mu\nu} - m^2 \eta_{\mu\nu}\phi$ and gauge fixing ➡ demixing h and ϕ

➡ $$S \supset \int d^4x \Big\{ M_P^2 \hat{h} \Box \hat{h} + \ldots + M_P^2 m^2 A \Box A + \ldots + M_P^2 m^4 \phi \Box \phi + \ldots \Big\}$$

Canonical normalization:
$$\begin{aligned} \tilde{h}_{\mu\nu} &= M_P \hat{h}_{\mu\nu}, \\ \tilde{A}^\mu &= M_P m A^\mu, \\ \tilde{\phi} &= M_P m^2 \phi. \end{aligned}$$

- Dominant higher order term: $\dfrac{(\partial^2 \tilde{\phi})^3}{\Lambda^5}$ with $\Lambda = \left(m^4 M_P\right)^{1/5}$

Decoupling Limit
$$\boxed{\begin{aligned} M_P &\to \infty \\ m &\to 0 \\ \Lambda &\sim const \\ T_{\mu\nu}/M_P &\sim const \end{aligned}}$$

Action for $\tilde{\phi}$ in the Decoupling limit

- The action for the scalar sector:

$$S = \frac{1}{2}\int d^4x \left\{\frac{3}{2}\tilde{\phi}\Box\tilde{\phi} + \frac{1}{\Lambda^5}\left[\alpha\,(\Box\tilde{\phi})^3 + \beta\,(\Box\tilde{\phi}\,\tilde{\phi}_{,\mu\nu}\,\tilde{\phi}^{,\mu\nu})\right] - \frac{1}{M_P}T\tilde{\phi}\right\}$$

- Equation of Motion:

$$\nabla_\mu \left\{3\Lambda^5\nabla^\mu\tilde{\phi} + 3\alpha\nabla^\mu\left(\Box\tilde{\phi}\right)^2 + \beta\nabla^\mu\left(\tilde{\phi}_{;\delta\gamma}\right)^2 + 2\beta\nabla^\nu\left(\Box\tilde{\phi}\tilde{\phi}^{;\mu}{}_{\nu}\right)\right\} = \frac{\Lambda^5}{M_P}T$$

Can be integrated for $\tilde{\phi} = \tilde{\phi}(R)$ or $\tilde{\phi} = \tilde{\phi}(t)$

Spherically Symmetric case:

$$\boxed{\begin{aligned} &3\,\frac{\tilde{\phi}'}{R} + \frac{2}{\Lambda^5}\left\{3\alpha\left(-4\frac{\tilde{\phi}'^2}{R^4} + 2\frac{\tilde{\phi}'\tilde{\phi}''}{R^3} + 2\frac{\tilde{\phi}''^2}{R^2} + 2\frac{\tilde{\phi}'\tilde{\phi}^{(3)}}{R^2} + \frac{\tilde{\phi}''\tilde{\phi}^{(3)}}{R}\right) + \right.\\ &\qquad \left. +\beta\left(-6\frac{\tilde{\phi}'^2}{R^4} + 2\frac{\tilde{\phi}'\tilde{\phi}''}{R^3} + 4\frac{\tilde{\phi}''^2}{R^2} + 2\frac{\tilde{\phi}'\tilde{\phi}^{(3)}}{R^2} + 3\frac{\tilde{\phi}''\tilde{\phi}^{(3)}}{R}\right)\right\} \\ &\qquad\qquad = -\frac{1}{R^3}\int_0^R d\tilde{R}\,\tilde{\rho}\left(\tilde{R}\right)\,\tilde{R}^2 \end{aligned}}$$

Relation between ϕ and μ

- μ is defined via the gauge transformation

$$\begin{aligned} f_{AB}dX^A dX^B &= -dt^2 + dr^2 + r^2 d\Omega^2 \\ \rightarrow f_{\mu\nu}dx^\mu dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)} dR^2 + e^{-\mu(R)} R^2 d\Omega^2 \end{aligned}$$

- While the Stuckelberg field X is defined such that:

$$f_{\mu\nu}dx^\mu dx^\nu = \left[\partial_\mu X^A(x)\partial_\nu X^B(x) f_{AB}\left(X(x)\right)\right] dx^\mu dx^\nu$$

- This corresponds to the Stuckelberg field:

$$X^A \equiv x^A + f^{AB}\partial_B \phi = \left(t, R\, e^{-\frac{\mu(R)}{2}}, \theta, \phi\right) \Leftrightarrow \phi' \equiv \partial_R \phi = R\left(e^{-\frac{\mu(R)}{2}} - 1\right)$$

EOM for μ in the Decoupling Limit

$$\phi' \equiv \partial_R \phi = -R\left(1 - e^{-\frac{\mu(R)}{2}}\right) \sim -\frac{R\mu}{2} + \frac{R\mu^2}{8} + \ldots$$

- Rescaling: $\tilde{\phi} = M_P m^2 \phi \quad , \quad \tilde{\mu} = M_P m^2 \mu$
- In the Decoupling Limit: $\boxed{\tilde{\mu} = -\frac{2}{R}\tilde{\phi}'}$
- Equations of Motion:

Linear term

$$3\frac{\tilde{\phi}'}{R} + \frac{2}{\Lambda^5}\left\{3\alpha\left(-4\frac{\tilde{\phi}'^2}{R^4} + 2\frac{\tilde{\phi}'\tilde{\phi}''}{R^3} + 2\frac{\tilde{\phi}''^2}{R^2} + 2\frac{\tilde{\phi}'\tilde{\phi}^{(3)}}{R^2} + \frac{\tilde{\phi}''\tilde{\phi}^{(3)}}{R}\right) + \right.$$
$$\left. +\beta\left(-6\frac{\tilde{\phi}'^2}{R^4} + 2\frac{\tilde{\phi}'\tilde{\phi}''}{R^3} + 4\frac{\tilde{\phi}''^2}{R^2} + 2\frac{\tilde{\phi}'\tilde{\phi}^{(3)}}{R^2} + 3\frac{\tilde{\phi}''\tilde{\phi}^{(3)}}{R}\right)\right\}$$
$$= -\frac{1}{R^3}\int_0^R d\tilde{R}\,\tilde{\rho}(\tilde{R})\,\tilde{R}^2$$

Q=quadratic, second order differential operator

Source term

SPHERICALLY SYMMETRIC SOLUTIONS IN THE DECOUPLING LIMIT

EOM in the Decoupling Limit

- The Decoupling Limit:
$$\begin{aligned} M_P &\to \infty \\ m &\to 0 \\ \Lambda \equiv (M_P m^4)^{1/5} &\sim const \\ T_{\mu\nu}/M_P &\sim const \end{aligned}$$
$$\begin{aligned} \tilde{\nu} &\equiv M_P \nu \\ \tilde{\lambda} &\equiv M_P \lambda \\ \tilde{\mu} &\equiv m^2 M_P \mu \\ \tilde{\rho} &= \rho / M_P \end{aligned}$$

- From the Eqs. of the full system we get in DL:

$$\left.\begin{aligned} \frac{\tilde{\lambda}'}{R} + \frac{\tilde{\lambda}}{R^2} &= -\frac{1}{2}(3\tilde{\mu} + R\tilde{\mu}') + \tilde{\rho} \\ \frac{\tilde{\nu}'}{R} - \frac{\tilde{\lambda}}{R^2} &= \tilde{\mu} \\ \frac{\tilde{\lambda}}{R^2} &= \frac{\tilde{\nu}'}{2R} + \frac{Q(\tilde{\mu})}{\Lambda^5} \end{aligned}\right\}$$

➡ Equation for μ only:

$$\boxed{\frac{2}{\Lambda^5} Q(\tilde{\mu}) + \frac{3}{2}\,\tilde{\mu} = \frac{1}{R^3}\int_0^R d\tilde{R}\,\tilde{\rho}\left(\tilde{R}\right)\,\tilde{R}^2}$$

with $$Q(\mu) = -\frac{1}{2R}\left\{3\alpha\left(6\mu\mu' + 2R\mu'^2 + \frac{3}{2}R\mu\mu'' + \frac{1}{2}R^2\mu'\mu''\right) + \beta\left(10\mu\mu' + 5R\mu'^2 + \frac{5}{2}R\mu\mu'' + \frac{3}{2}R^2\mu'\mu''\right)\right\}$$

Rescaled variables in DL

- **Rescaled variables:**

$$\boxed{a \equiv R_V m = (R_S m)^{1/5}}$$

$$\begin{aligned} \xi &\equiv R/R_V \\ w(\xi) &\equiv a^{-2}\,\mu \\ v(\xi) &\equiv a^{-4}\,\nu \\ u(\xi) &\equiv a^{-4}\,\lambda \end{aligned} \qquad \begin{aligned} \rho_a &\equiv 4\pi \frac{R_V^3}{M}\rho, \\ P_a &\equiv 4\pi \frac{R_V^3}{M} P \end{aligned}$$

$$\left.\begin{aligned} \frac{\dot u}{\xi} + \frac{u}{\xi^2} &= -\frac{1}{2}(3w + \xi \dot w) + \rho_a \\ \frac{\dot v}{\xi} - \frac{u}{\xi^2} &= w \\ \frac{u}{\xi^2} &= \frac{\dot v}{2\xi} + Q(w) \end{aligned}\right\}$$

➡ Equation for μ only:

$$\boxed{2\,Q(w) + \frac{3}{2}w = \frac{1}{\xi^3}}$$

with $$Q(w) = -\frac{1}{2}\left\{3\alpha\left(\frac{\xi}{2}\dot w \ddot w + \frac{3}{2}w\ddot w + 2\dot w^2 + \frac{6w\dot w}{\xi}\right) + \beta\left(\frac{3\xi}{2}\dot w \ddot w + \frac{5}{2}w\ddot w + 5\dot w^2 + \frac{10w\dot w}{\xi}\right)\right\}.$$

$$2Q(w) + \frac{3}{2}\,w = \frac{1}{\xi^3}$$

close to source

far from source

Vainshtein solution,

$$2Q(w) + \cancel{\frac{3}{2}w} = \frac{1}{\xi^3}$$

$$w \propto \frac{1}{\sqrt{\xi}}, \quad (\nu = -\lambda)$$

pertutbative regime,

$$\cancel{2Q(w)} + \frac{3}{2}\,w = \frac{1}{\xi^3}$$

$$w \to \frac{2}{3\xi^3}, \quad (\nu = -2\lambda)$$

Another solution!

$$2Q(w) + \cancel{\frac{3}{2}}w = \cancel{\frac{1}{\xi^3}}$$

$$Q(w) = 0 \quad \Rightarrow \quad w \propto \frac{1}{\xi^2}, \quad (\nu = -\lambda)$$

Cauchy problem and initial conditions

Is it enough to know the asymptotic behavior at infinity?

NOTE: EOM is singular at $\xi = \infty$

$$2Q(w) + \frac{3}{2}\, w = \frac{1}{\xi^3}$$

$$w(\infty) \to \frac{2}{3\xi^3}$$

$\Rightarrow$ Unique solution??

YES

$$\begin{cases} w'' + w = \frac{1}{\xi} \\ w(\infty) \to \frac{1}{\xi} \end{cases}$$

$$w = \frac{1}{\xi} - \frac{2}{\xi^3} + \ldots$$

NO

$$\begin{cases} -w'' + w = \frac{1}{\xi} \\ w(\infty) \to \frac{1}{\xi} \end{cases}$$

$$w = C_1 \exp(-\xi) + \frac{1}{\xi} - \frac{2}{\xi^3} + \ldots$$

Example 1: the BD potential (I)

$$\mathcal{V}^{(BD)}[\mathbf{g}^{-1}\mathbf{f}] = \sqrt{-f}\; h_{\mu\nu}h_{\sigma\tau}\left(f^{\mu\sigma}f^{\nu\tau} - f^{\mu\nu}f^{\sigma\tau}\right)$$

Boulware, Deser'72

$$\boxed{2\left(\frac{\dot{w}^2}{4} + \frac{w\ddot{w}}{2} + 2\frac{w\dot{w}}{\xi}\right) + \frac{3}{2}w = \frac{1}{\xi^3}}$$

- Asymptotic behavior at infinity: linear theory

$$\cancel{2\left(\frac{\dot{w}^2}{4} + \frac{w\ddot{w}}{2} + 2\frac{w\dot{w}}{\xi}\right)} + \frac{3}{2}w = \frac{1}{\xi^3} \qquad \Rightarrow w(\xi) \sim w_\infty(\xi) \equiv \frac{2}{3\,\xi^3}$$

- Formal series expansion around $w_\infty(\xi)$

$$w(\xi) = \sum_{k=0}^{\infty} \frac{w_k}{\xi^{3+5k}} = \frac{2}{3\xi^3} - \frac{4}{3\xi^8} + \frac{1024}{27\xi^{13}} + \frac{712960}{243\,\xi^{18}} + \ldots$$

- Linearization around $w_\infty(\xi)$

$$w = w_\infty + \delta w \;\Rightarrow\; \delta w'' + \frac{\delta w'}{\xi} + \frac{9}{4}\xi^3\delta w = -\frac{3}{\xi^5}$$

~~two oscillatory modes~~ ➡ unique solution?

Example 1: the BD potential (II)

$$\boxed{2\left(\frac{\dot{w}^2}{4}+\frac{w\ddot{w}}{2}+2\frac{w\dot{w}}{\xi}\right)+\frac{3}{2}w=\frac{1}{\xi^3}}$$

- Small distance behavior: no Vainshtein scaling

$$2\left(\frac{\dot{w}^2}{4}+\frac{w\ddot{w}}{2}+2\frac{w\dot{w}}{\xi}\right)+\cancel{\frac{3}{2}}w=\frac{1}{\xi^3}\quad\Rightarrow \text{try}\quad w(\xi)\sim\frac{A}{\sqrt{\xi}}$$

➡ Imaginary solution!

- Another scaling is possible:

$$2\left(\frac{\dot{w}^2}{4}+\frac{w\ddot{w}}{2}+2\frac{w\dot{w}}{\xi}\right)+\cancel{\frac{3}{2}}w=\cancel{\frac{1}{\xi^3}}\quad\Leftrightarrow\quad Q(w)=0\quad\Leftrightarrow\quad w(\xi)\sim\frac{A}{\xi^2}$$

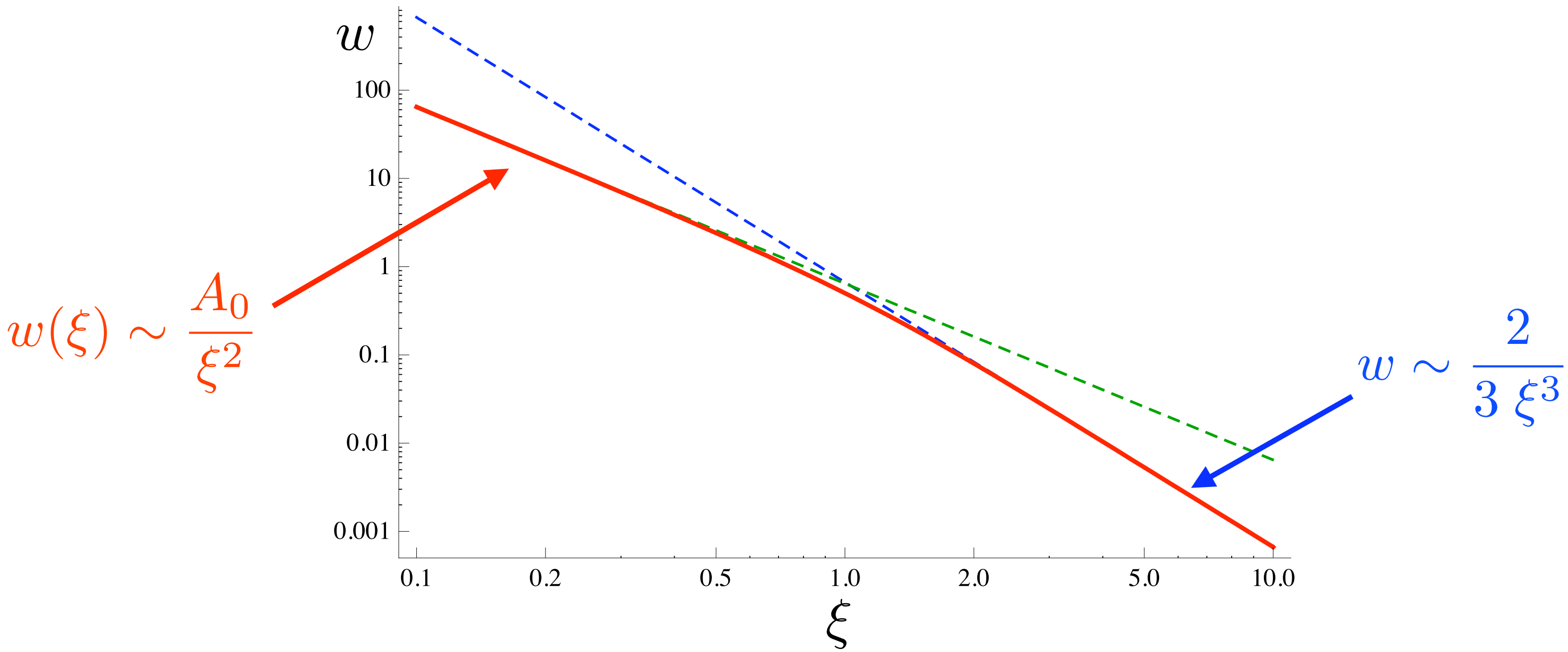
2 free constants

$$\boxed{w(\xi)=\frac{A_0}{\xi^2}+\sum_{n=1}^{\infty}\sum_{k=0}^{n}w_{n,k}\,\xi^n(\ln\xi)^k}$$

$$w(\xi)=\frac{A_0}{\xi^2}+\frac{3A_0B_0+\ln\xi}{3A_0}\,\xi-\frac{3}{8}\,\xi^2$$
$$+\frac{1-6A_0B_0-54{A_0}^2B_0^2-(2-36A_0B_0)\ln\xi-6\ln^2\xi}{216A_0^3}\,\xi^4+O\left(\xi^5\right)\ .$$

Example 1: the BD potential (III)

$$\boxed{2\left(\frac{\dot{w}^2}{4}+\frac{w\ddot{w}}{2}+2\frac{w\dot{w}}{\xi}\right)+\frac{3}{2}w=\frac{1}{\xi^3}}$$



➡ Unique regular solution (A_0 and B_0 fixed), is it a good one?

Example 2: the AGS potential (I)

$$\mathcal{V}^{(AGS)}[\mathbf{g^{-1}f}] \quad = \quad \sqrt{-g}\, h_{\mu\nu} h_{\sigma\tau} \left(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau} \right)$$

Arkani-Hamed, Georgi, Schwartz'72

$$\boxed{-2\left(\frac{\dot{w}^2}{4} + \frac{w\ddot{w}}{2} + 2\frac{w\dot{w}}{\xi}\right) + \frac{3}{2}w = \frac{1}{\xi^3}}$$

- Asymptotic behavior at infinity: linear theory

$$\cancel{2\left(\frac{\dot{w}^2}{4} + \frac{w\ddot{w}}{2} + 2\frac{w\dot{w}}{\xi}\right)} + \frac{3}{2}w = \frac{1}{\xi^3} \qquad \Rightarrow w(\xi) \sim w_\infty(\xi) \equiv \frac{2}{3\,\xi^3}$$

- Linearization around $w_\infty(\xi)$

$$w = w_\infty + \delta w \;\Rightarrow\; \delta w'' + \frac{\delta w'}{\xi} - \frac{9}{4}\xi^3 \delta w = -\frac{3}{\xi^5}$$

1 decreasing mode
+ 1 ~~exploding mode~~ ➡ solutions isn't unique?

NB: the exploding mode makes the numerical integration tricky!

$$w(\xi) = \sum_{k=0}^{\infty} \frac{w_k}{\xi^{3+5k}} = \frac{2}{3\xi^3} + \frac{4}{3\xi^8} + \frac{1024}{27\xi^{13}} + \frac{712960}{243\,\xi^{18}} + \ldots$$

Example 2: the AGS potential (II)

$$\boxed{-2\left(\frac{\dot{w}^2}{4}+\frac{w\ddot{w}}{2}+2\frac{w\dot{w}}{\xi}\right)+\frac{3}{2}w=\frac{1}{\xi^3}}$$

- Small distance behavior: there is Vainshtein solution

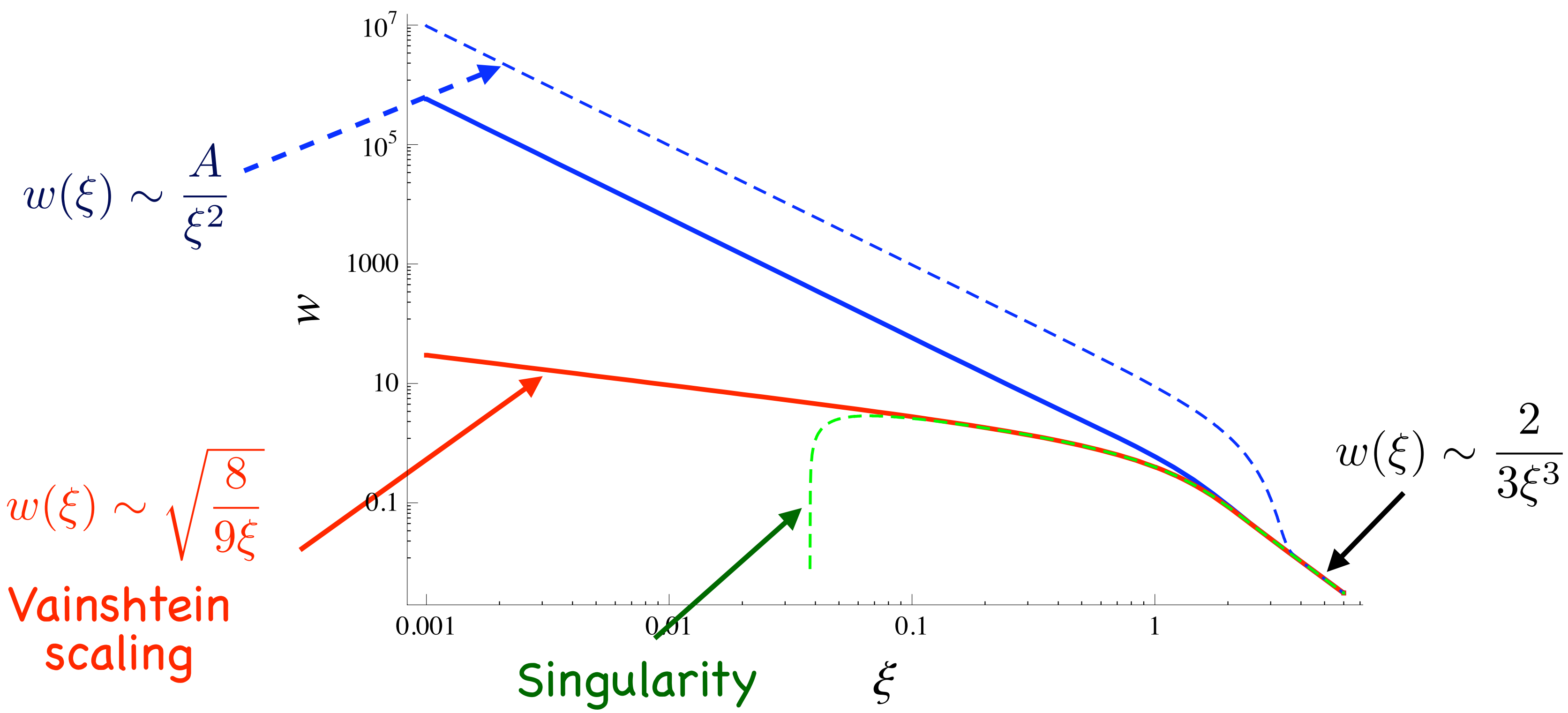
$$2\left(\frac{\dot{w}^2}{4}+\frac{w\ddot{w}}{2}+2\frac{w\dot{w}}{\xi}\right)+\cancel{\frac{3}{2}}w=\frac{1}{\xi^3} \quad\Rightarrow\quad w(\xi)=\sqrt{\frac{8}{9\,\xi}}$$

- Another scaling is possible:

$$2\left(\frac{\dot{w}^2}{4}+\frac{w\ddot{w}}{2}+2\frac{w\dot{w}}{\xi}\right)+\cancel{\frac{3}{2}}w=\cancel{\frac{1}{\xi^3}} \quad\Leftrightarrow\quad Q(w)=0 \quad\Leftrightarrow\quad w(\xi)\sim\frac{A}{\xi^2}$$

Example 2: the AGS potential (III)

NUMERICS



Example 2: the AGS potential (III)

How to choose the correct solution?

- Let's include source and ask for no conical singularity!

$$u(\xi) = \frac{1}{\xi} - \frac{1}{2}\xi^2 w(\xi)$$

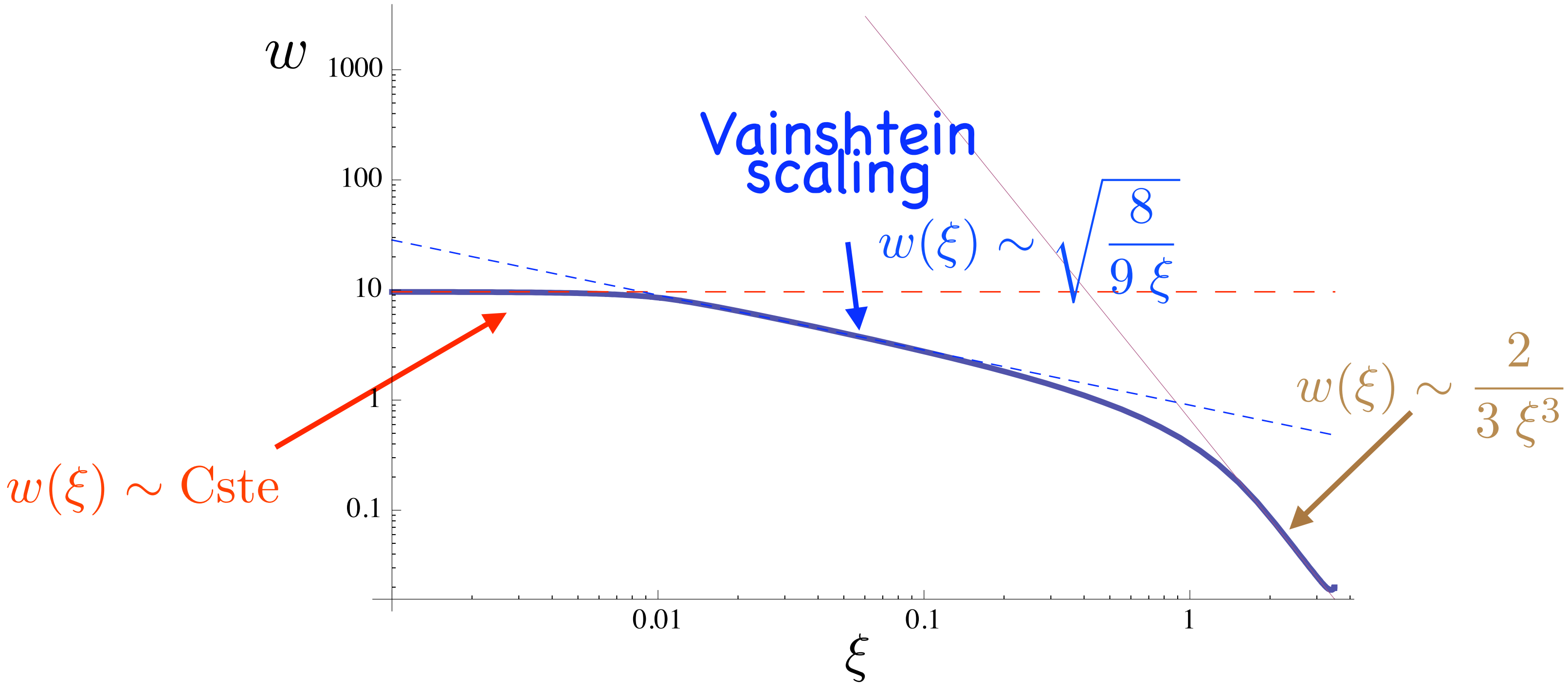
$$\Downarrow$$

$$\lambda(R) \sim \frac{1}{R} - R^2 w(\xi)$$

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$

- Thus we require $u(0) = \lambda(0) = 0 \;\Rightarrow\; w(\xi) < O\left(\frac{1}{\xi^2}\right)$ for $\xi \to 0$

Example 2: the AGS potential (IV)



Only Vainshtein solution gives good behavior at 0!

➡ the scaling $w(\xi) \sim \dfrac{A}{\xi^2}$ gives a conical singularity for both AGS and BD potentials!!!

Conclusion (I)

- It is possible to obtain the decoupling limit in the case of static spherically symmetric ansatz.
- This decoupling limit corresponds to DL in the Goldstone picture.
- In the non-linear regime, apart from the Vainshtein scaling there is another scaling (for some potentials), which can be smoothly extended to an asymptotically flat solution and is associated with zero modes of the non-linearities appearing in the decoupling limit.
- For BD potential the unique regular solution exists, which interpolates between asymptotically flat solution and the new scaling solution. However, the solution contains conical singularity.
- For AGS potential a family of solutions exists containing the new scaling solution with an arbitrary constant and Vainshtein-like solution as an asymptotic. The requirement of no-conical singularity at zero chooses uniquely the Vainshtein-like solution.

Conclusion (II)

- Our plan for future: It would be interesting to study the solutions of the full system.