Tachyon entropy perturbations in brane inflation

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work with R. Brandenberger & A. Frey: **arXiv:0712.2178 [hep-th]**

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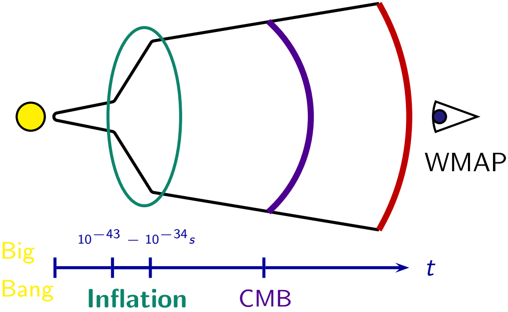


Inflation $\equiv$ accelerated expansion

$$\ddot{a} > 0$$

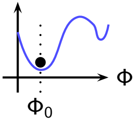
- solves Standard Big Bang Model problems
- predicts scale-invariant CMB radiation spectrum
- supported by WMAP satellite observations

Very early Universe physics was described by a **Grand Unified Theory**.

String Theory is a candidate – does it contain (predict) inflation ?

A promising start: String Theory contains lots of scalar fields, essential for the inflationary mechanism.

A challenging task: Does one field ϕ (or several, $\phi_1, \phi_2 \ldots$) have a suitable potential?



What kind of scalar fields?

- Coupling constants are determined by field VEV's, e.g. dilaton $g_{\mathrm{s}} = e^{\Phi_0}$.
- The stringy Universe has (at least) 10 dimensions.
- There are lowerdimensional hypersurfaces (*Dp* branes, *p*: # of spatial dimensions).

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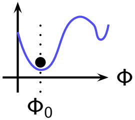

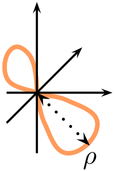

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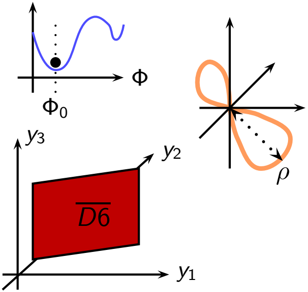
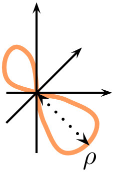


String Inflation

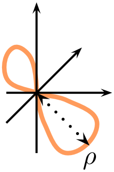
closed string mode
e.g. moduli

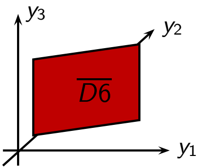


String Inflation

closed string mode
e.g. moduli

open string mode
e.g. brane position





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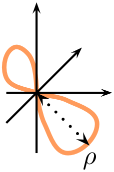
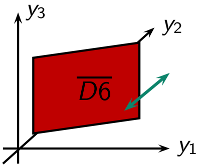


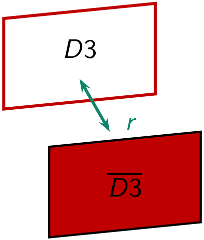


Brane Inflation

$D3-\overline{D3}$ inflation
in warped geometry

$D3-D7$ inflation

$D(p+3)$ wrapped
on a p-cycle



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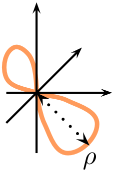

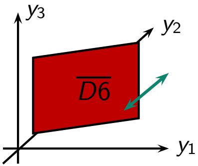

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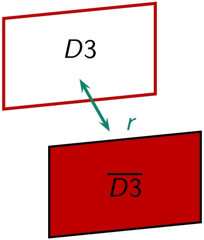


KKLMMT model

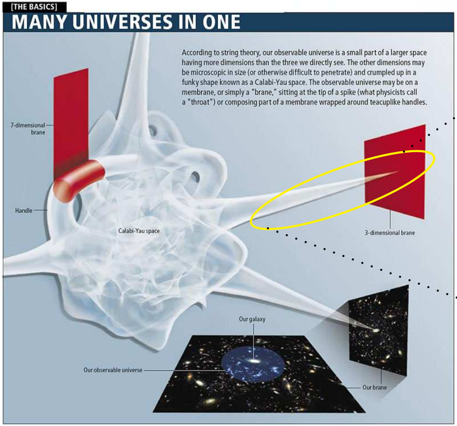
Deformed geometry fixes "form" of extra dim's (complex structure moduli).

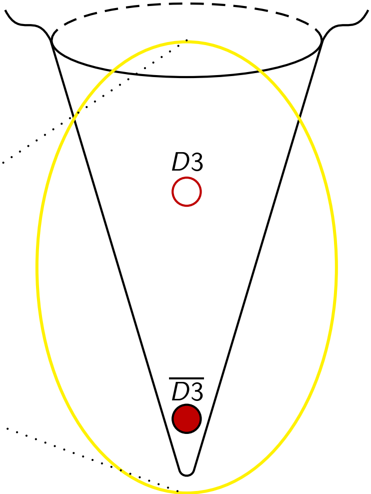
KKLT procedure fixes overall volume and lifts AdS to dS.

only r unfixed!

K*achru*, **K***allosh*, **L***inde*, **M***aldacena*, **M***cAllister*, **T***rivedi (2003)*

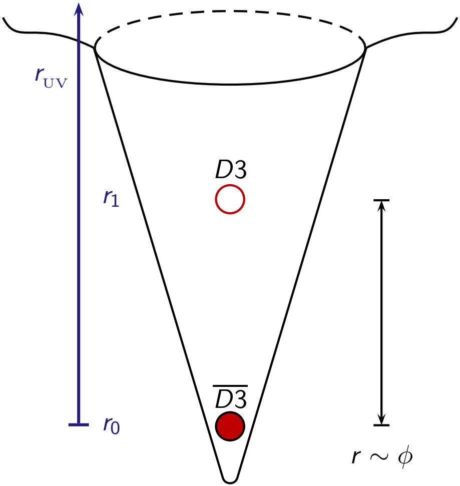
Inflation occurs while a brane moves towards an anti-brane in a 6d "warped throat".





C. Burgess & F. Quevedo in *Scientific American*, Nov 07

Candelas & de la Ossa (1990)



$$\mathbf{ds_{10}^2 = h^{-1/2}\boxed{ds_4^2} + h^{1/2}\boxed{ds_6^2}}$$

D3 brane

extra dim's "warped throat"

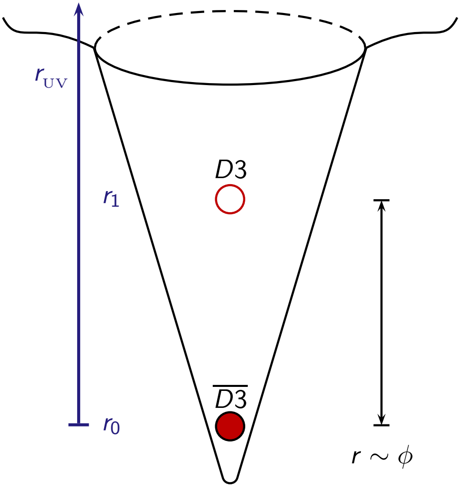
Inflaton is renormalized distance:

$$\phi = \sqrt{T_3}\, r$$

Potential from warp factor $h \sim \frac{1}{r^4}$:

$$\mathbf{V(\phi) = M^4 \left[1 - \left(\frac{\mu}{\phi}\right)^4\right]}$$

T_3: brane tension
M, μ: model parameters



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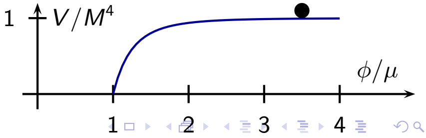

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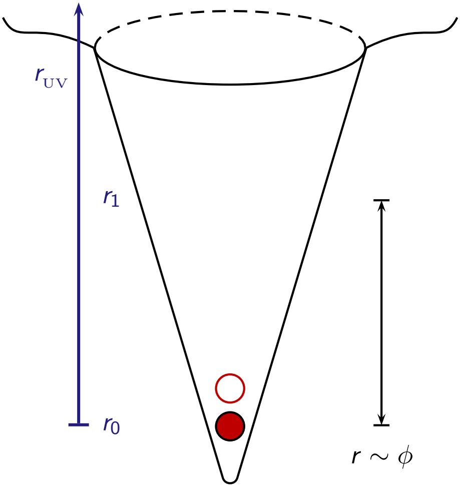

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branes collide $\longrightarrow$ reheating

background evolution:

$$\begin{aligned} \frac{8\pi G}{3}\rho &= H^2 \\ \ddot{\phi} + 3H\dot{\phi} + V_\phi &= 0 \end{aligned}$$

with $V(\phi) = M^4 \left[1 - \left(\frac{\mu}{\phi}\right)^4\right]$

In the CMB, we observe *perturbations* around this background:

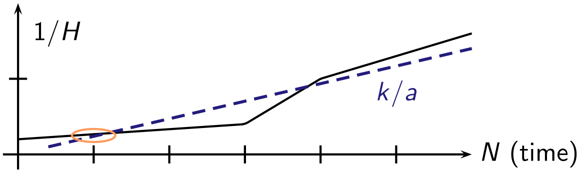
$$\phi = \phi_0 + \delta\phi$$

Comoving curvature perturbation $\mathcal{R}$:

$$\mathcal{R} = \frac{\delta\rho}{\rho} \approx \frac{HV_\phi(\phi_*)}{\dot{\phi}_*^2}$$

ϕ_*: field value when observable scales k left Hubble radius

$\mathcal{R}$ must match COBE normalisation: $\mathcal{R} \approx 10^{-5}$



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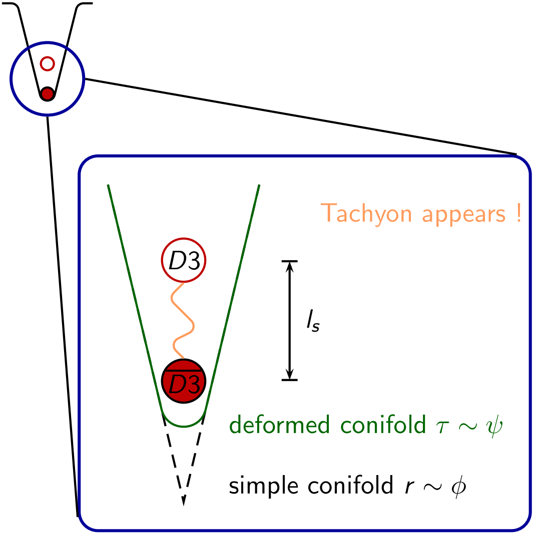
$\mathcal{R}$ must match COBE normalisation: $\mathcal{R} \approx 10^{-5}$

Hence COBE fixes the scale of inflation, here M!

For this normalization, $\mathcal{R}$ must not change for $k/a \to 0$!
"primary perturbations"



l_s: string length

<u>Zoom on the bottom of the throat</u>

When *proper* brane distance approaches l_s:

- Deformation: "new" field $\psi \sim \ln \phi$
- Tachyon appears: two fields at play!

$$\mathbf{T}, \psi$$

Study two fields' evolution in early reheating:

$$\phi < \phi_{\mathbf{strg}}$$

"inflaton" ψ, tachyon T

potential driving inflation:

$$V(\phi) = M^4 \left[1 - \left(\frac{\mu}{\phi}\right)^4\right]$$

valid up to $\phi_{\rm strg}$

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switch fields $\phi \to \psi \sim \ln \phi$

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potential during reheating:

$$\tilde{V}(\psi, T) = v_0^4 + \frac{1}{2} \underbrace{\left(-m_{\rm s}^2 + 4\pi g_{\rm s}\psi^2\right)}_{\text{"waterfall point"}} T^2$$

T starts rolling when $\psi = \psi_{\rm strg} = \frac{m_{\rm s}}{4\pi g_{\rm s}}$.

$m_{\rm s}$: *local* string scale

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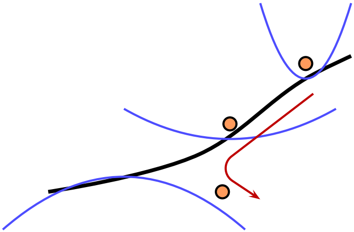
m_{s} : *local* string scale

ψ: "heir" of the inflaton field
initial values are set by $\phi_{\mathrm{strg}}, \dot{\phi}_{\mathrm{strg}}$

T: waterfall field of hybrid inflation
starts at rest, with small offset T_0



reheating potential: $\tilde{V} \sim \frac{1}{2}\left(-m_{\mathrm{s}}^2 + 4\pi g_{\mathrm{s}}\psi^2\right) T^2$

$$
\begin{aligned}
\ddot{T} + 3H\dot{T} + \tilde{V}_T &= 0 \\
\ddot{\psi} + 3H\dot{\psi} + \tilde{V}_\psi &= 0 \\
\frac{8\pi G}{3}\rho &= H^2
\end{aligned}
$$

neglect friction term

$\tilde{V}_T = \left(-m_{\mathrm{s}}^2 + 4\pi g_{\mathrm{s}}\psi^2\right) T$

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$\tilde{V}_T = \left(-m_{\rm s}^2 + \cancel{4\pi g_{\rm s}\psi^2}\right) T$

$$\mathbf{\ddot{T} - m_s^2 T = 0}$$

solution: $T(t) = T_0 \exp(m_{\rm s} t)$

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no further acceleration b/c

$\tilde{V}_\psi \simeq 2\pi g_s T_0^2 \psi$ small

$\psi \sim \ln\phi \rightarrow \dot{\boldsymbol{\psi}}_{\mathbf{strg}} \sim \dot{\boldsymbol{\phi}}_{\mathbf{strg}} / \boldsymbol{\phi}_{\mathbf{strg}}$

$\dot{\phi}_{\mathrm{strg}}$ **from slow-roll** reheating potential: $\tilde{V} \sim \frac{1}{2}\left(-m_\mathrm{s}^2 + 4\pi g_\mathrm{s}\psi^2\right) T^2$

$$\begin{aligned} \ddot{T} + 3H\dot{T} + \tilde{V}_T &= 0 \\ \ddot{\psi} + 3H\dot{\psi} + \tilde{V}_\psi &= 0 \\ \frac{8\pi G}{3}\rho &= H^2 \end{aligned}$$

neglect friction term

$\tilde{V}_T = \left(-m_\mathrm{s}^2 + \cancel{4\pi g_\mathrm{s}\psi^2}\right) T$

$$\ddot{\mathbf{T}} - \mathbf{m_s^2 T = 0}$$

solution: $T(t) = T_0 \exp(m_\mathrm{s} t)$

no further acceleration b/c

$\tilde{V}_\psi \simeq 2\pi g_\mathrm{s} T_0^2 \psi$ small

$\psi \sim \ln\phi \rightarrow \dot{\psi}_\mathbf{strg} \sim \dot{\phi}_\mathbf{strg}/\phi_\mathbf{strg}$

$\dot{\phi}_\mathrm{strg}$ **from slow-roll**

$\dot{T} = m_\mathrm{s} T$ **increases and catches up to** $\dot{\psi}$ **until** $m_\mathrm{s} T_\mathrm{eq} = \dot{\psi}_\mathrm{strg}$**.**

$T_0 \longrightarrow T_\mathrm{eq} \longrightarrow T$

Phase I	Phase II
$\dot{T} \ll \dot{\psi}_\mathrm{strg}$	$\dot{T} \gg \dot{\psi}_\mathrm{strg}$

Describe perturbations using comoving curvature $\mathcal{R} = \delta\rho/\rho$:

$$\dot{\mathcal{R}} = \frac{H}{\dot{H}} \frac{k^2}{a^2} \Psi$$

Gordon et al. (2001)

Ψ: longitudinal metric fluctuation, $\mathrm{d}s^2 = (1+2\Psi)\mathrm{d}t^2 - a^2(1-2\Psi)\mathrm{d}\vec{x}^2$

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in single field inflation: $\mathcal{R} \to$ const. when $k/a \to 0$

hence COBE normalization $\mathcal{R} \approx \frac{HV_\phi}{\dot{\phi}^2} \approx 10^{-5}$

Describe perturbations using comoving curvature $\mathcal{R}$:

$$\dot{\mathcal{R}} = \cancel{\frac{H}{\dot{H}}\frac{k^2}{a^2}\Psi} - \frac{2H}{\dot{\sigma}^2}\,\tilde{V}_s\,\delta s$$

Gordon et al. (2001)

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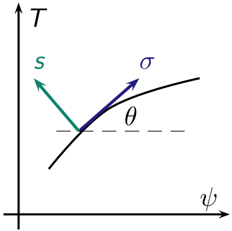
in single field inflation:	$\mathcal{R} \to$ const. when $k/a \to 0$
in two field inflation:	on large scales, $\mathcal{R}$ sourced by δs !
	normalization to COBE affected!
	"secondary perturbations"



$$\boxed{\dot{\mathcal{R}} \approx -\frac{2H}{\dot{\sigma}^2}\,\tilde{V}_s\,\delta s}$$

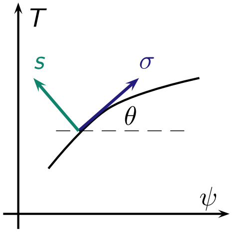
$\dot{\sigma}$: "adiabatic" field velocity
δs: "entropy fluctuation" $\perp \sigma$
$\tilde{V}_s$: "entropy potential" gradient

Study the growth of δs: [*Gordon et al. (2001)*]

$$\delta\ddot{s} + 3H\delta\dot{s} + \left(\frac{k^2}{a^2} + \tilde{V}_{ss} + 3\frac{\tilde{V}_s^2}{\dot{\sigma}^2}\right)\delta s = \frac{1}{2\pi G}\frac{\dot{\theta}}{\theta}\frac{k^2}{a^2}\Psi$$

where

$$\begin{aligned}
\dot{\sigma} &= \sqrt{\dot{T}^2 + \dot{\psi}^2} \\
\tilde{V}_s &= \frac{\dot{\psi}}{\dot{\sigma}}\,\tilde{V}_T - \frac{\dot{T}}{\dot{\sigma}}\,\tilde{V}_\psi \\
\tilde{V}_{ss} &= \frac{\dot{\psi}^2}{\dot{\sigma}^2}\,\tilde{V}_{TT} - 2\frac{\dot{T}\dot{\psi}}{\dot{\sigma}^2}\,\tilde{V}_{\psi T} + \frac{\dot{T}^2}{\dot{\sigma}^2}\,\tilde{V}_{\psi\psi}
\end{aligned}$$



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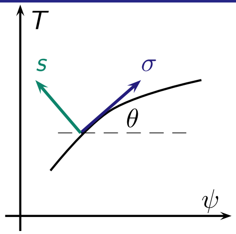

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\end{aligned}$$

on large scales $k/a \to 0$:

$$\delta\ddot{s} + 3H\delta\dot{s} + \underbrace{\left(\tilde{V}_{ss} + 3\frac{\tilde{V}_s^2}{\dot{\sigma}^2}\right)}_{=m^2_{\mathrm{entropy}}}\delta s \approx 0$$

Need $m^2_{\rm entropy} < 0$ for exponential growth: $\tilde{\mathbf{V}}_{\mathbf{ss}} < \mathbf{0}$, $\frac{|\tilde{\mathbf{V}}_{\mathbf{ss}}|}{3\tilde{\mathbf{V}}_{\mathbf{s}}^2/\dot{\sigma}^2} > \mathbf{1}$

Phase I: catch up
$T_0 < T < T_{\rm eq}$

$$\begin{aligned} \dot{\sigma} &= \sqrt{\dot{T}^2 + \dot{\psi}^2} \\ \tilde{V}_s &= \frac{\dot{\psi}}{\dot{\sigma}}\,\tilde{V}_T - \frac{\dot{T}}{\dot{\sigma}}\,\tilde{V}_\psi \\ \tilde{V}_{ss} &= \frac{\dot{\psi}^2}{\dot{\sigma}^2}\,\tilde{V}_{TT} - 2\frac{\dot{T}\dot{\psi}}{\dot{\sigma}^2}\,\tilde{V}_{\psi T} \\ &\quad + \frac{\dot{T}^2}{\dot{\sigma}^2}\,\tilde{V}_{\psi\psi} \end{aligned}$$

$$\mathbf{T}_{\mathbf{eq}} = \frac{\dot{\psi}_{\mathbf{strg}}}{\mathbf{m}_{\mathbf{s}}}$$

Phase II: fast T
$T > T_{\rm eq}$

recall: $\tilde{\mathbf{V}} \sim \frac{1}{2}\left(-\mathbf{m}_{\mathbf{s}}^2 + 4\pi\mathbf{g}_{\mathbf{s}}\psi^2\right)\mathbf{T}^2$

Need $m^2_{\text{entropy}} < 0$ for exponential growth: $\mathbf{\tilde{V}_{ss} < 0}$, $\frac{|\tilde{\mathbf{V}}_{ss}|}{3\tilde{\mathbf{V}}_s^2/\dot{\sigma}^2} > \mathbf{1}$

Phase I: catch up
$T_0 < T < T_{\text{eq}}$

$\dot{\sigma} \approx \dot{\psi}$

$\tilde{V}_s \approx \tilde{V}_T$

$\tilde{V}_{ss} \approx \tilde{V}_{TT}$

$$\begin{aligned}
\dot{\sigma} &= \sqrt{\dot{T}^2 + \dot{\psi}^2} \\
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&\quad + \frac{\dot{T}^2}{\dot{\sigma}^2}\,\tilde{V}_{\psi\psi}
\end{aligned}$$

$$\mathbf{T_{eq}} = \frac{\dot{\psi}_{\mathbf{strg}}}{\mathbf{m_s}}$$

Phase II: fast T
$T > T_{\text{eq}}$

recall: $\tilde{\mathbf{V}} \sim \frac{1}{2}\left(-\mathbf{m_s^2} + 4\pi \mathbf{g_s}\psi^2\right)\mathbf{T^2}$

Need $m^2_{\text{entropy}} < 0$ for exponential growth: $\tilde{\mathbf{V}}_{\mathbf{ss}} < \mathbf{0}$, $\frac{|\tilde{\mathbf{V}}_{\mathbf{ss}}|}{\mathbf{3}\tilde{\mathbf{V}}_{\mathbf{s}}^{\mathbf{2}}/\dot{\sigma}^{\mathbf{2}}} > \mathbf{1}$

Phase I: catch up
$T_0 < T < T_{\text{eq}}$

$$\dot\sigma \approx \dot\psi$$

$$\tilde V_s \approx \tilde V_T$$

$$\tilde V_{ss} \approx \tilde V_{TT}$$

$$\frac{|\tilde V_{ss}|}{3\tilde V_s^2/\dot\sigma^2} \approx \frac{\dot\psi^2}{3m_s^2 T^2} > 1$$

growth while

$T < T_{\text{eq}}/\sqrt{3}$!

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$$\mathbf{T_{eq}} = \frac{\dot\psi_{\rm strg}}{\mathbf{m_s}}$$

Phase II: fast T

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$$\dot\sigma \approx \dot T$$

$$\tilde V_s \approx -\tilde V_\psi$$

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recall: $\tilde{\mathbf{V}} \sim \frac{1}{2}\left(-\mathbf{m}_s^2 + 4\pi \mathbf{g}_s \psi^2\right)\mathbf{T}^2$

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$T > T_{\rm eq}$

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$\tilde V_{ss} > 0$, so always $m^2_{\rm entropy} > 0$!

no growth!

recall: $\mathbf{\tilde V \sim \frac{1}{2}\left(-m_s^2 + 4\pi g_s\psi^2\right)T^2}$

δs grows while $T_0 < T < T_{\rm eq}/\sqrt{3}$, neglect friction:

$$\delta\ddot{s} + \underbrace{m^2_{\rm entropy}}_{\approx -m_{\rm s}^2}\delta s \approx 0$$

Solution: $\delta s(t) = \delta s_0 \exp\left(m_{\rm s} t\right)$, same as tachyon! $T(t) = T_0 \exp\left(m_{\rm s} t\right)$

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Solution: $\delta s(t) = \delta s_0 \exp\left(m_{\rm s} t\right)$, same as tachyon! $T(t) = T_0 \exp\left(m_{\rm s} t\right)$
Induced growth of $\mathcal{R}$ on large scales:

$$\dot{\mathcal{R}} \approx -\frac{2H}{\dot{\sigma}^2} \tilde{V}_s \, \delta s \approx \frac{2H}{\dot{\psi}^2} m_{\rm s}^2 T \delta s$$

Total growth from integrating away:

$$\mathbf{\Delta\mathcal{R} \approx \frac{H}{m_s}\frac{\delta s_0}{T_0}}$$

Initial values given by the same quantum fluctuations: $\boldsymbol{\delta s_0 / T_0 \sim \mathcal{O}(1)}$

$\mathcal{R} = \frac{\delta\rho}{\rho}$ **primary**

$\Delta\mathcal{R} \approx \frac{H}{m_{\rm s}} \frac{\delta s_0}{T_0}$ **secondary**

$H/m_{\rm s}$ determines impact of secondary perturbations!

$\Delta\mathcal{R} \ll \mathcal{R}$: COBE normalization as usual

$\Delta\mathcal{R} \approx \mathcal{R}$ or $\Delta\mathcal{R} \gg \mathcal{R}$: normalization to secondary perturbations

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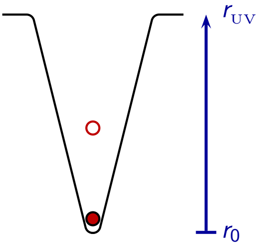
How to determine H/m_s?

Functions of background geometry:

$$m_s^2 = \frac{2h_0^{-1/2}}{\alpha'}$$

$$H^2 \approx \frac{8\pi G}{3} V = \frac{8\pi G}{3} M^4$$

$$M^4 = \frac{4\pi^2 v \phi_0^4}{\mathcal{N}}$$



$\phi = \sqrt{T_3} r$

$h_0 \sim \frac{r_{\rm UV}^4}{r_0^4}$

$\alpha' \sim l_s^2$, l_s: string length

T_3: brane tension

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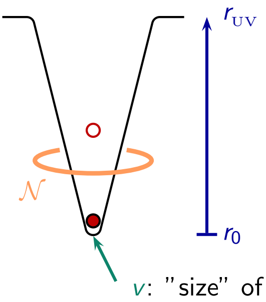

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Example: parameters of the original KKLMMT proposal
$T_3 \approx 10^{-3} m_{\rm Pl}^4$, $g_{\rm s} = 0.1$, $\alpha' m_{\rm Pl}^2 \approx 6.4$, $\mathcal{N} = 160$, $v = 16/27$

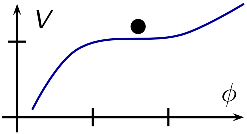
The resulting **secondary** perturbation amplitude is

$$\Delta\mathcal{R} \approx 10^{-5} \frac{\delta s(0)}{T_0} \approx 10^{-5}$$

Secondary perturbations of same order as primary ones!

- COBE normalization is affected (compare "curvaton" scenario).
- Issue can be worse for other parameter sets.
- Calculation found a lower limit.

Caveats



- Inter-brane potential is a toy model!
  Baumann et al. (2007)
  $V(\phi)$ more likely of "inflection-point" type.
- Slow-roll for original inflaton ϕ used until ϕ_{strg}.
  Could be fast-rolling or oscillating at bottom.
- Canonic kinetic terms: fields & velocities below local string scale.
- Need $T_0 < T_{\mathrm{eq}}$ to trigger effect.
  If $V(\phi)$ has an inflection point, T is very massive at waterfall point and is deflected from equilibrium only slightly later.

Conclusions

- $D3 - \overline{D3}$ inflation ends when tachyon appears (brane annihilation).
- Tachyon T and ex-inflaton ψ generate entropy perturbations δs.
- δs grows exponentially for a certain range $T_0 < T < T_{\mathrm{eq}}$.

It follows that

- Induced secondary perturbations $\Delta\mathcal{R}$ can be as important as primary ones $\mathcal{R} = \delta\rho/\rho$. $\Rightarrow$ COBE normalization different?
- Issue present over a wide range of parameter space.

Outlook

- Toy model, lots of simplifications to eliminate!
- Potential $V(\phi)$ should be different, initial tachyon offset T_0?
- Interaction with second order effects.