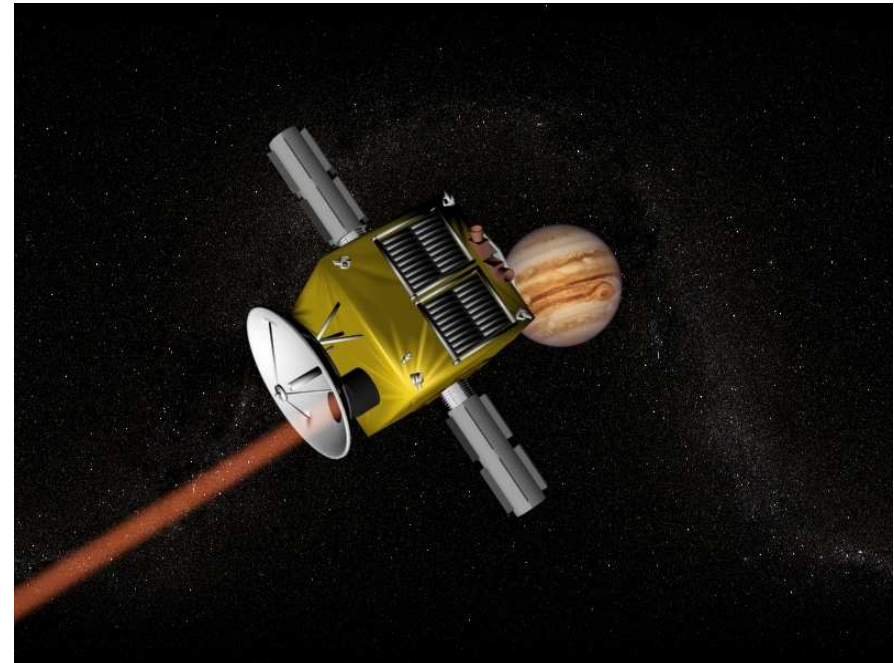
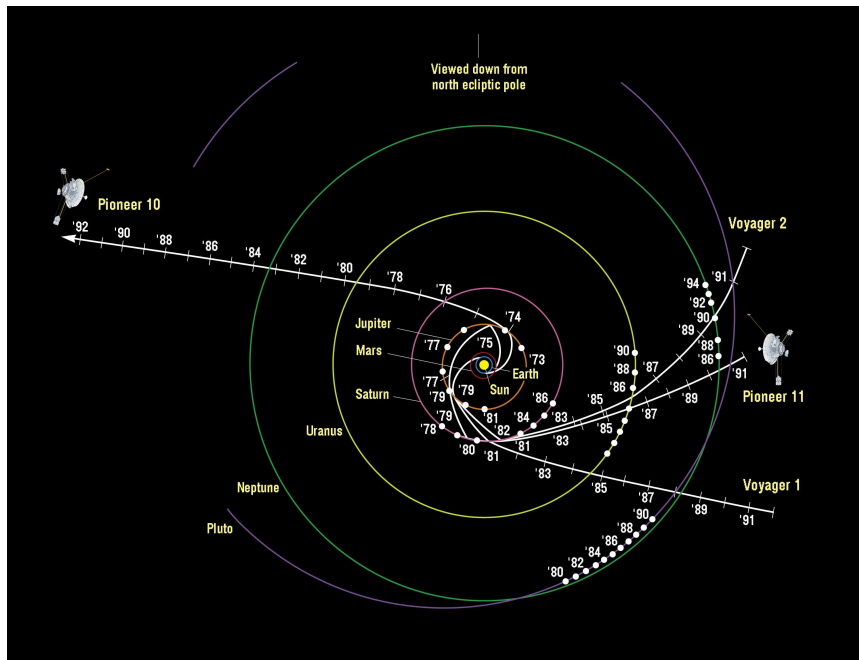


# Gravitation Modifiée à Grande Distance Tests dans le Système Solaire

Séminaire GReCO-SYRTE 09/04/2008



Gilles Esposito-Farese et Peter Wolf  
(remerciements à Serge Reynaud, LKB)

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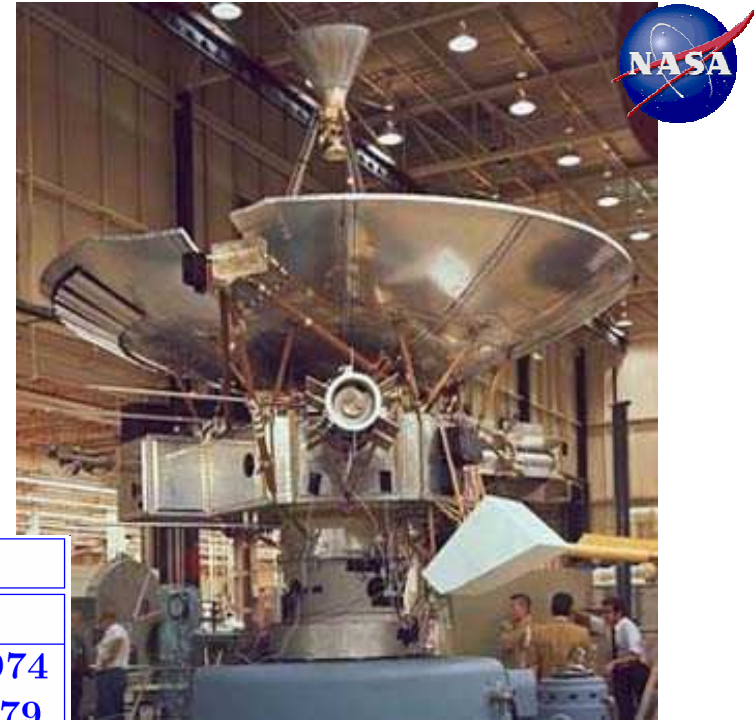
- Introduction
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## SAGAS:

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- Measurements and optimisation:
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# Pioneer: Introduction

- Gravity test performed by P10/P11 probes during their extended missions decided after the primary planetary objectives had been met



Pioneer 10: pre-launch testing

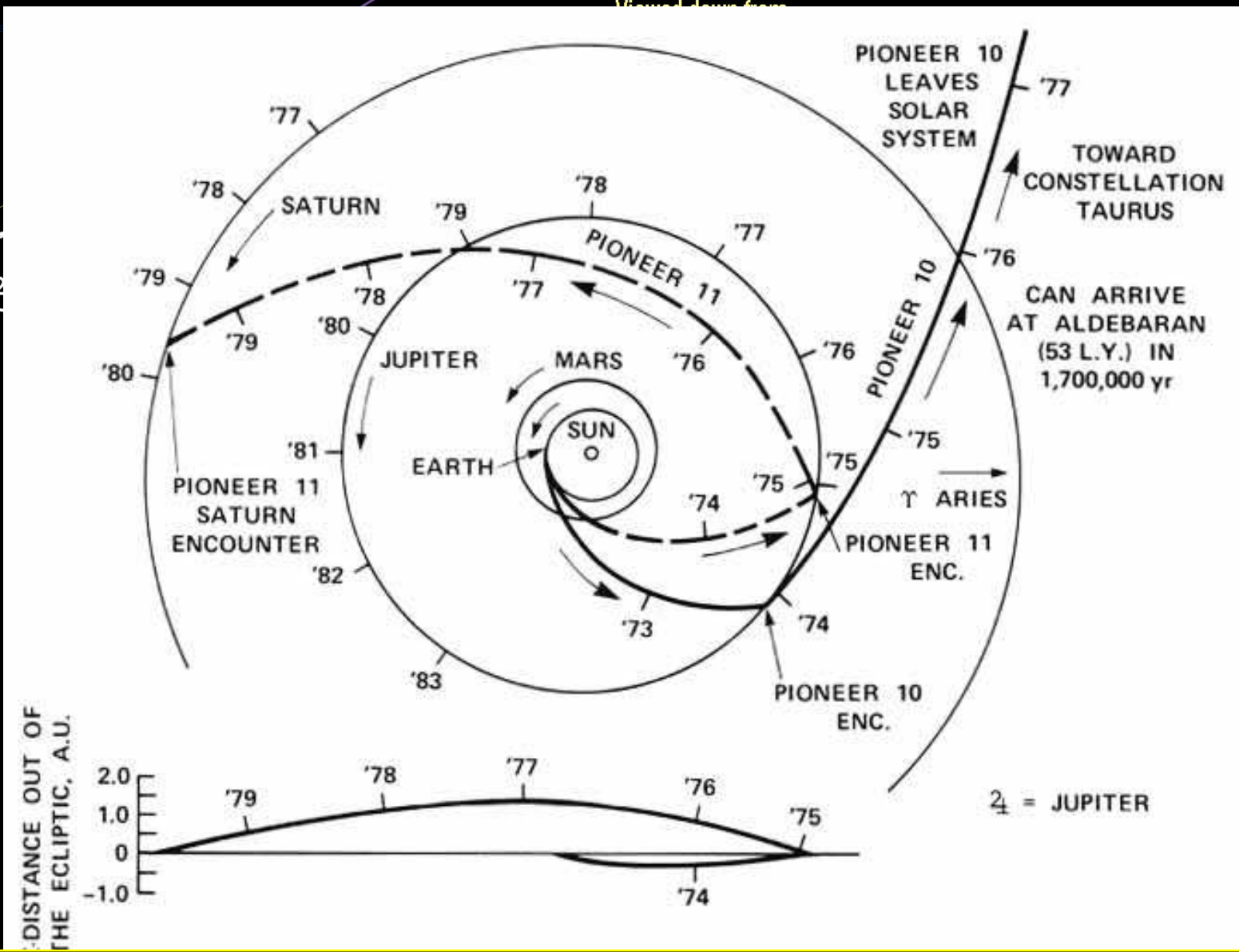
Courtesy : JPL @ NASA

| Agency: NASA             | Pioneer 10                       | Pioneer 11                                |
|--------------------------|----------------------------------|---|
| Launch                   | 2 March 1972                     | 5 April 1973                              |
| Planetary fly-bys        | Jupiter: 4 Dec 1973              | Jupiter: 2 Dec 1974<br>Saturn: 1 Sep 1979 |
| Last data point received | 27 Apr 2002<br>distance ~80.2 AU | 1 Oct 1990<br>distance ~30 AU             |

## Pioneer 10/11 as precisely navigated deep-space vehicles :

Spin-stabilization and design permitted acceleration sensitivity  $\sim 10^{-10}$  m/s<sup>2</sup>, unlike a Voyager-type 3-axis stabilization that was almost 50 times worse

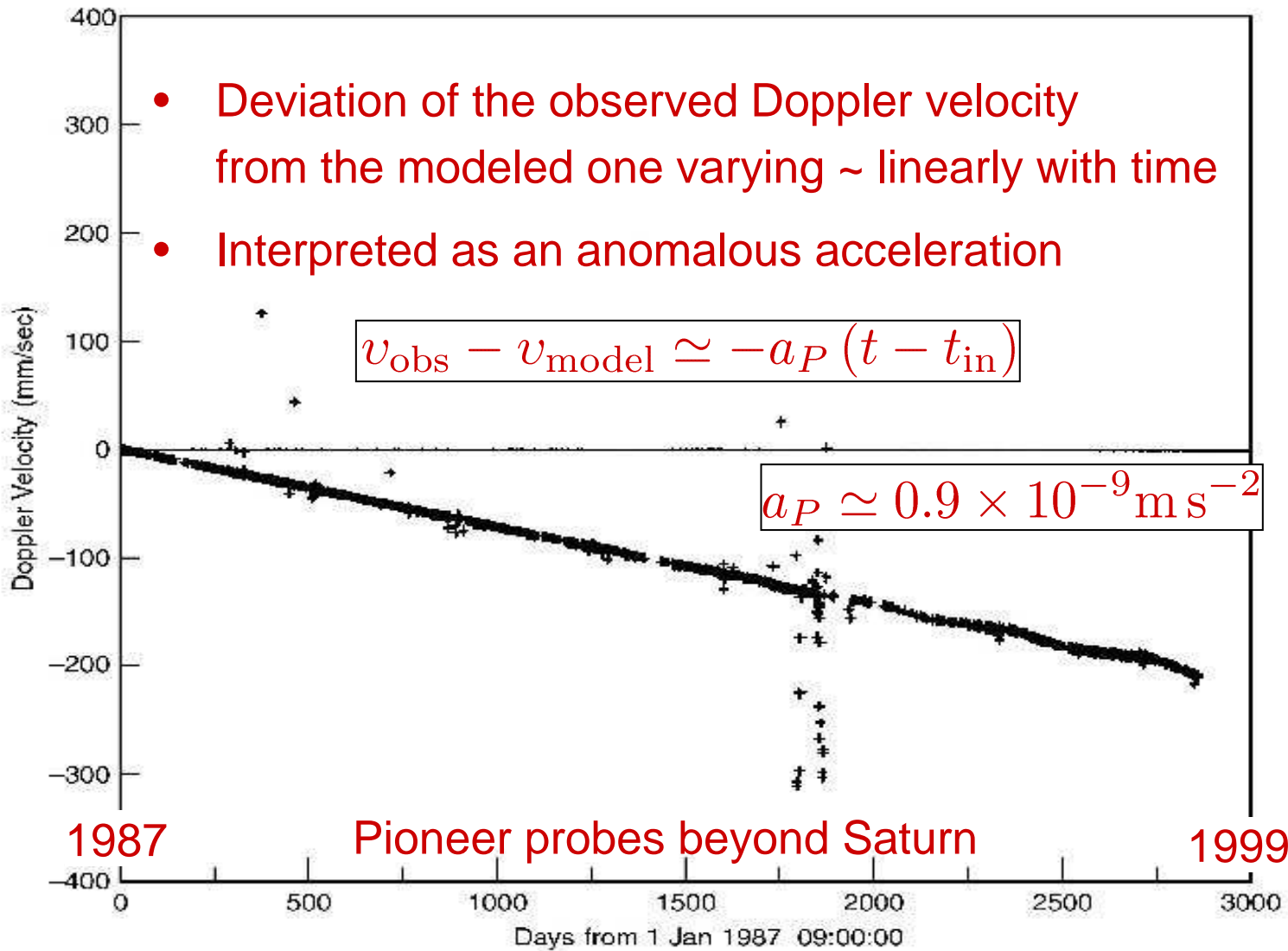
# The largest scaled gravity test ever carried out...



...and it failed to confirm the known laws of gravity !

# Anomalous Pioneer Acceleration

1 Hz is equivalent to 65 mm/s velocity



## Existing and recovered data

### Recent recovery effort (Planetary Society & JPL, NASA)

- ❑ Telemetry & Doppler data recovered from launch to the last data point

### Data used for the analysis in PRD 2002

- ❑ Pioneer 10 : **11.5 years**
  - Distance range: **40–70 AU**

- ❑ Pioneer 11 : **3.75 years**
  - Distance range: **22.4–31.7 AU**

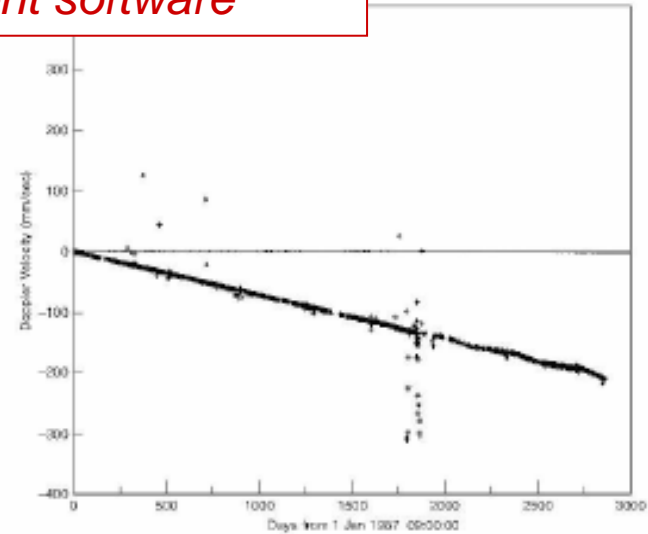
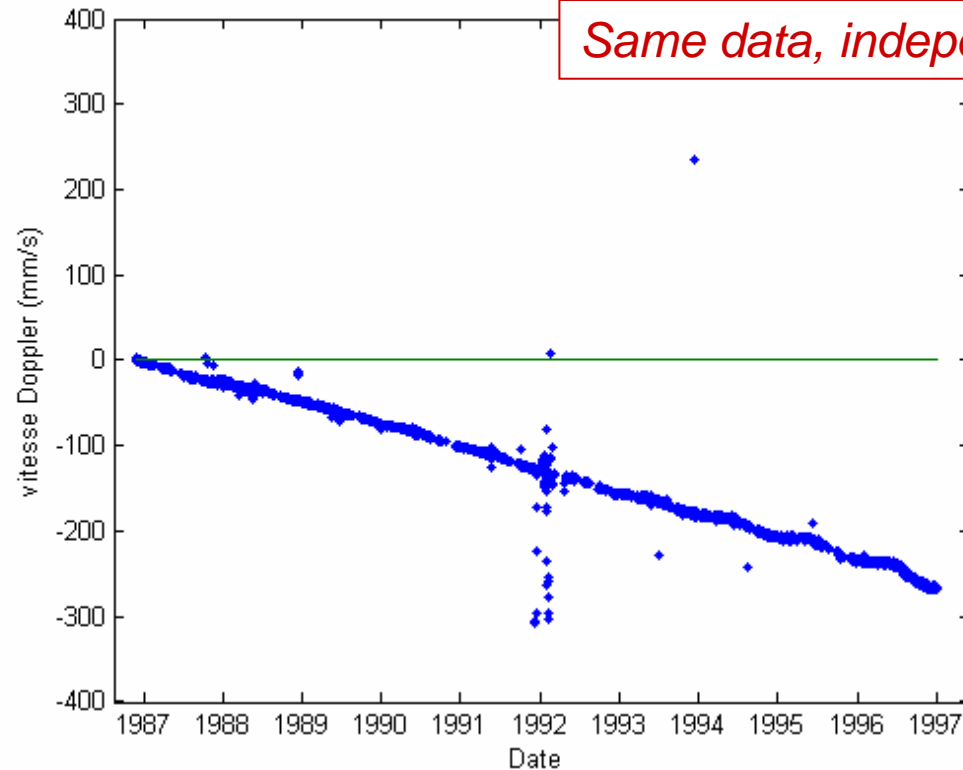
### Doppler data (soon) available

- ❑ Pioneer 10:
  - 1973-2002: ~ **30 years**
  - Distance range: **4–87 AU**
  - Jupiter encounter
  - ~60,000 data points, ~20GB
  - Maneuvers, spin, initial cond.

- ❑ Pioneer 11:
  - 1974-1994: ~ **20 years**
  - Distance range: **4–33 AU**
  - Jupiter & Saturn encounters
  - ~50,000 data points, ~15GB
  - Maneuvers, spin, initial cond.

- Ongoing Pioneer data re-analysis is an international effort : teams at work in US, Canada, Germany, France, Italy, Norway...

# Present Analysis Efforts and Results

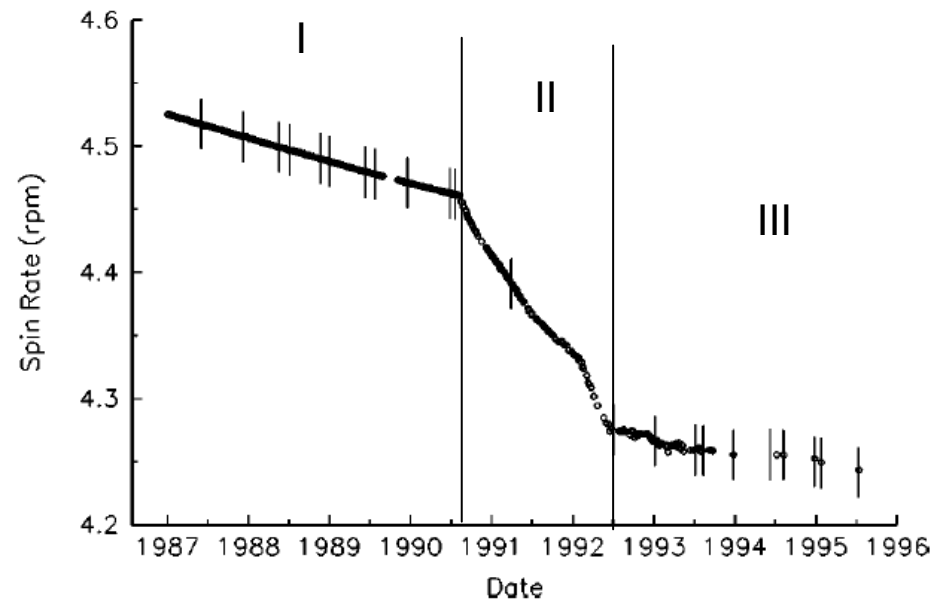


*J. Anderson et al, Phys. Rev. D  
65 (2002) 082004*

The presence of the anomaly is confirmed  
(its interpretation is still unknown) ...

Gilles Metris & Philippe Berio (OCA),  
Agnès Lévy (ONERA), Jean-Michel Courty (LKB)

# Present Analysis Efforts and Results



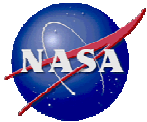
*J. Anderson et al, (2002)*

TAB. 2 – Constant anomalous acceleration ( $10^{-10} \text{ ms}^{-2}$ )

| Software            | P10(I)                            | P10(II)                           | P10(III)                          |
|---------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| ODP/Sigma           | $8.00 \pm 0.01$                   | $8.66 \pm 0.01$                   | $7.84 \pm 0.01$                   |
| CHASMP              | $8.22 \pm 0.02$                   | $8.89 \pm 0.01$                   | $7.92 \pm 0.01$                   |
| HELIOSAT            | $7.85 \pm 0.02$                   | $8.78 \pm 0.01$                   | $7.75 \pm 0.01$                   |
| <b>ODYSSEY</b>      | <b><math>8.33 \pm 0.04</math></b> | <b><math>9.23 \pm 0.03</math></b> | <b><math>7.56 \pm 0.02</math></b> |
| Residuals (mHz rms) | 8.1                               | 8.3                               | 8.9                               |

ODYSSEY fit over whole period:  $a_0 = (8.16 \pm 0.01) 10^{-10} \text{ ms}^{-2}$ ; resid.: 9.6 mHz  
 [courtesy: Metris, Berio, Lévy and Courty]





## Thermal modelling



- An anisotropy of a few % in the dissipated heat produces an acceleration  $\approx a_p$
- Dissipated power (RTG) decreases ( $\approx 8\%$  over 1987 – 1997), but constant  $a_p$
- Require high quality thermal models
- Need to reproduce measured temperatures, evolution etc... in terms of dissipated power
- Two active groups: JPL (USA), ZARM (Bremen, D)
- At present, try and model thermal acceleration at a given distance (25 AU)
- Strong dependence on “badly” known parameters (optical properties of surfaces, ageing, louver positions, payload heat dissipation, RTG properties and design,...)
- More phenomenologically, you can search for correlation between Doppler data and measured on-board power levels
- Spin data can support such correlations (anisotropic dissipation  $\Rightarrow$  torque)
- Work in progress

**Origin of the anomaly is still an open question...**

# SAGAS

(Search for Anomalous Gravitation with Atomic Sensors)

arXiv: 0711.0304, (2008)

## Quantum Physics Exploring Gravity in the Outer Solar System

> 70 participants from:

- **France:** SYRTE, IOTA, LKB, ONERA, OCA, LESIA, IMCCE, Université Pierre et Marie Curie Paris VI, Université Paul Sabatier Toulouse III
- **Germany:** IQO Leibniz Universität Hannover, ZARM, PTB, MPQ, Astrium, Heinrich Heine Universität Düsseldorf, Humboldt Universität Berlin, Universität Hamburg, Universität Ulm, Universität Erlangen
- **Great Britain:** National Physical Laboratory
- **Italy:** LENS, University of Firenze, INFN, INRIM, Università di Pisa, INOA Firenze, Politecnico Milano
- **Portugal:** Instituto Superior Técnico
- **Austria:** University of Innsbruck
- **Canada:** NRC
- **USA:** JPL, NIST, JILA, Global Aerospace Corp., Stanford University, Harvard University
- **Australia:** University of Western Australia

# Introduction

- Gravitation is well described by General relativity (GR).
  - GR is a classical theory, which shows inconsistencies with quantum field theory.
  - All unification models predict (small) deviations of gravitation laws from GR.
  - Gravity is well explored at small (laboratory) to medium (Moon, inner planets) distance scales.
  - At very large distances (galaxies, cosmology) some puzzles remain (galactic rotation curves, SNR redshifts, dark matter and energy, ....).
  - The largest distances explored by man-made artefacts are of the size of the outer solar system  $\Rightarrow$  carry out precision gravitational measurements in outer solar system.
- 
- Kuiper Belt ( $\approx 40$  AU,  $\approx 1000$  KBOs since 1992), the disk from which giant planets formed is largely unexplored.
  - Known mass ( $M_{\text{KB}} \approx 10^{-1} M_{\text{E}}$ ) about 100 times too small for in situ formation of KBOs.
  - KBO masses only inferred from albedo and density hypothesis ( $\Rightarrow$  uncertainty).
  - “In situ” gravitational measurements yields exceptional information on  $M_{\text{KB}}$ , overall mass distribution, and individual KBO masses (+ discover new KBOs ?)
  - Measurements during planetary fly by (Jupiter) can yield highly accurate determination of planetary gravity.

# SAGAS: Overview

## Payload:

1. Cold atom absolute accelerometer, 3 axis measurement of local non-gravitational acceleration.
2. Optical atomic clock, absolute frequency measurement (local proper time).
3. Laser link (frequency comparison + Doppler for navigation).

## Trajectory:

- Jupiter flyby and gravity assist ( $\approx 3$  years after launch).
- Reach distance of  $\approx 39$  AU (15 yrs nominal) to  $\approx 53$  AU (20 yrs, extended).

## Measurements:

- *Gravitational trajectory of test body (S/C):* using Doppler ranging and correcting for non-gravitational forces using accelerometer measurements.
- *Gravitational frequency shift of local proper time:* using clock and laser link to ground clocks for frequency comparison.

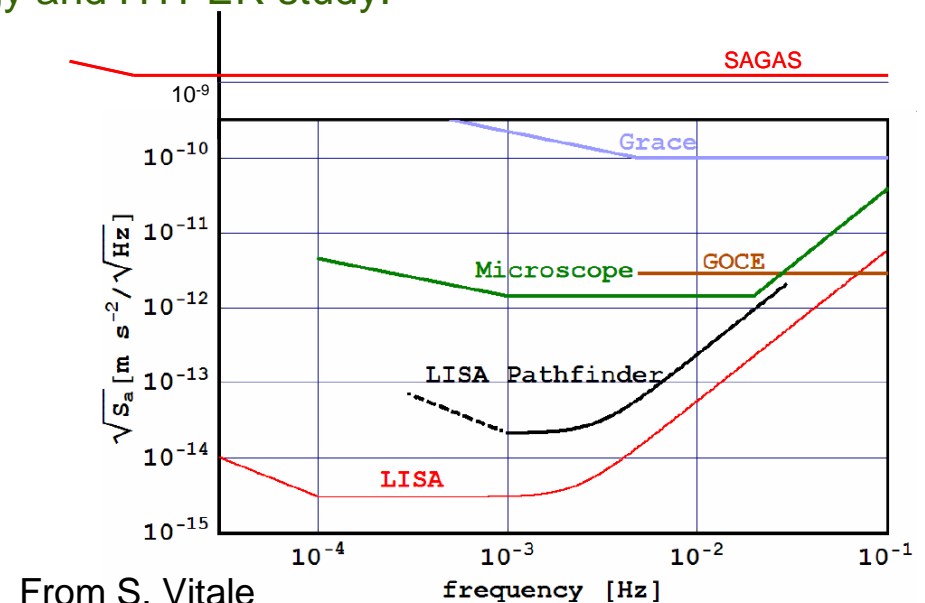
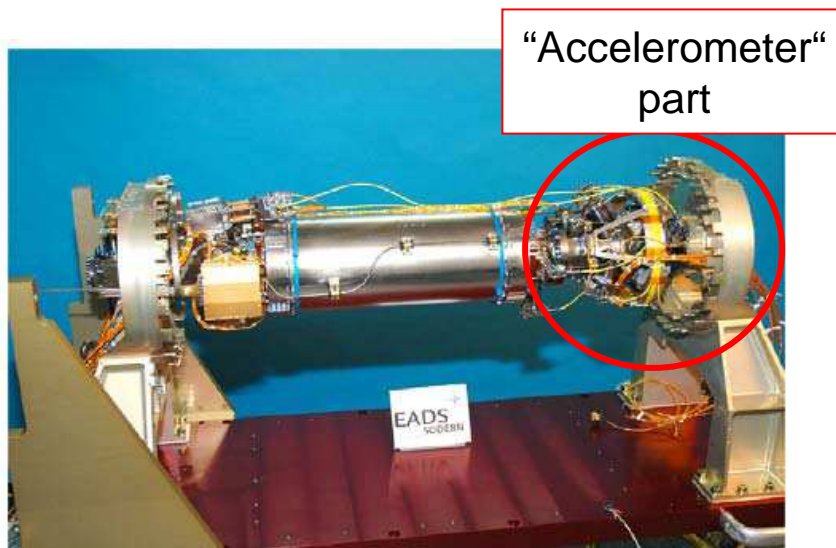
**$\Rightarrow$  Measure all aspects of gravity !**

# Science Objectives: Overview

| Science Objective            | Expected Result   | Comments   |
|------------------------------|---|--|
| Test of Universal Redshift   | $1 \times 10^{-9}$ of GR prediction   | $10^5$ gain on present   |
| Null Redshift Test           | $1 \times 10^{-9}$ of GR prediction   | $10^3$ gain  |
| Test of Lorentz Invariance   | $3 \times 10^{-9}$ to $5 \times 10^{-11}$<br>(IS or "time dilation" test)   | $10^2$ to $10^4$ gain<br>fct. of trajectory  |
| PPN test                     | $\delta(\gamma) \leq 2 \times 10^{-7}$  | $10^2$ gain<br>may be improved by orbit<br>modelling   |
| Large Scale Gravity          | - Fill exp. data gap for scale<br>dependent modif. of GR<br>- Identify and measure PA to $< 1\%$<br>per year of data                                | Different observation types and<br>large range of distances will allow<br>detailed "map" of large scale<br>gravity |
| Kuiper Belt (KB) Total Mass  | $\delta M_{KB} \leq 0.03 M_E$   | Dep. on mass distribution and<br>correlation of clock meas.  |
| KB Mass Distribution         | Discriminate between different<br>common candidates   | Will contribute significantly to<br>solution of the "KB mass deficit"<br>problem                                   |
| Individual KB Objects (KBOs) | Measure $M_{KBO}$ at $\approx 10\%$   | Depending on distance of closest<br>approach   |
| Planetary Gravity            | -Jupiter Gravity at $\leq 10^{-10}$<br>-Study Jupiter and its moons   | $10^2$ gain on present for Jupiter<br>idem for other planet in case of 2 <sup>nd</sup><br>fly-by                   |
| Variation of Fund. Const.    | $\delta\alpha/\alpha \leq (2 \times 10^{-9}) \delta(GM/rc^2)$   | <b>250-fold</b> gain on present  |
| Upper limit on Grav. Waves   | $\Omega_h \leq 10^{-5}$ @ $10^{-5}$ Hz<br>$h \leq 10^{-18}$ @ $10^{-6}$ to $10^{-3}$ Hz   | $10^3$ gain @ $10^{-6}$ to $10^{-3}$ Hz<br>Integration over one year   |
| Technology Development       | Develops S/C and ground segment technologies for wide use in future<br>missions (interplanetary timing, navigation, broadband<br>communication,...) |  |

# Payload: Accelerometer

- Atom interferometer, using laser cooled Cs atoms as “test masses”.
- Interrogation of atoms using Raman laser pulses in 3D (sequentially).
- Ground atom interferometers have uncertainties comparable to best “classical” methods,  $\approx 10^{-8} \text{ m/s}^2$ , limited by vibrations, Earth rotation, atmosphere, tides....
- In a quiet space environment, with possibility of long interrogation times (2 s) expect:  
 $\sqrt{S_a(f)} = 1.3 \cdot 10^{-9} \text{ m/s}^2 \text{ Hz}^{-1/2}$  (limited by RF stability, PHARAO quartz USO)  
Absolute accuracy (bias determination):  $5 \cdot 10^{-12} \text{ m/s}^2$ .
- “Classical” space accelerometers have  $\sqrt{S_a(f)} = 10^{-10} \text{ m/s}^2 \text{ Hz}^{-1/2}$  (GRACE), or better ( $10^{-12}$  GOCE,  $\mu$ SCOPE;  $10^{-15}$  LISA) with bias calibration at  $4 \cdot 10^{-11} \text{ m/s}^2$  (ODYSSEY).
- Based to a large extent on PHARAO technology and HYPER study.



## Payload: Optical Clock

- Single trapped ion optical clock, using Sr<sup>+</sup> with 674 nm clock transition.
- Other options kept open (Yb<sup>+</sup>, Ca<sup>+</sup>,...) subject to development of laser sources.
- Provides narrow and accurate laser:

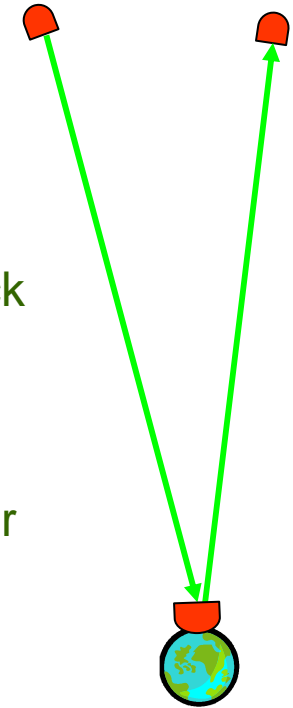
Stability:  $\sigma_y(\tau) = 1 \cdot 10^{-14} / \sqrt{\tau}$  ( $\tau$  = integration time in s)  
Accuracy:  $\delta y \leq 1 \cdot 10^{-17}$  in relative frequency ( $y = \delta f/f$ )

- Best ground trapped ion optical clocks show  $\sigma_y(\tau) = 3 \cdot 10^{-15} / \sqrt{\tau}$  and  $\delta y \leq 2 \cdot 10^{-17}$ .
- Challenge for SAGAS is not performance but space qualification and reliability.



## Payload: Optical Link

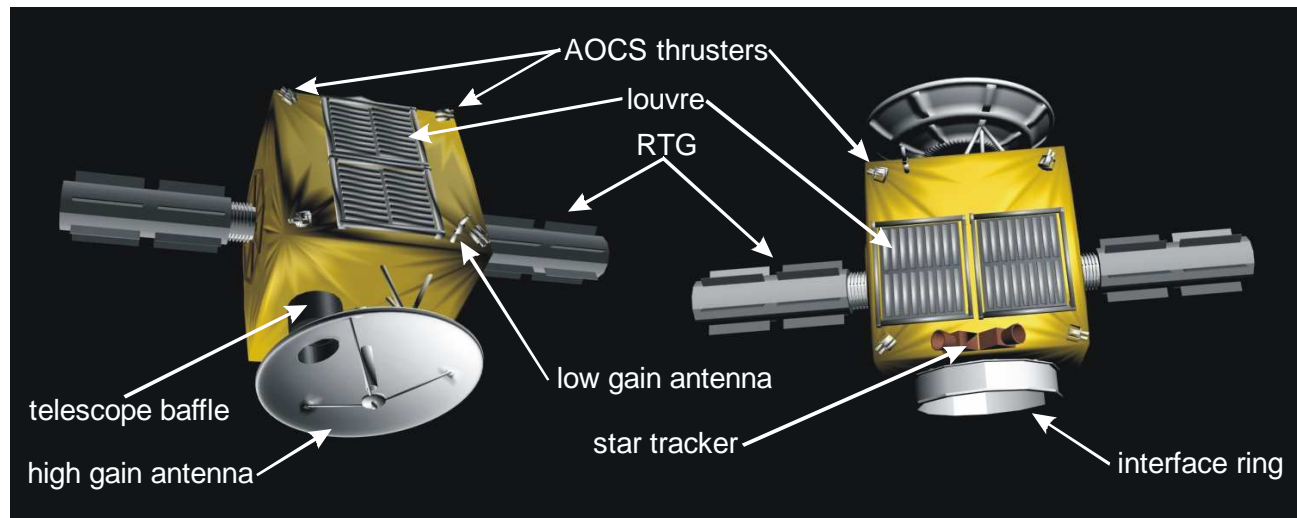
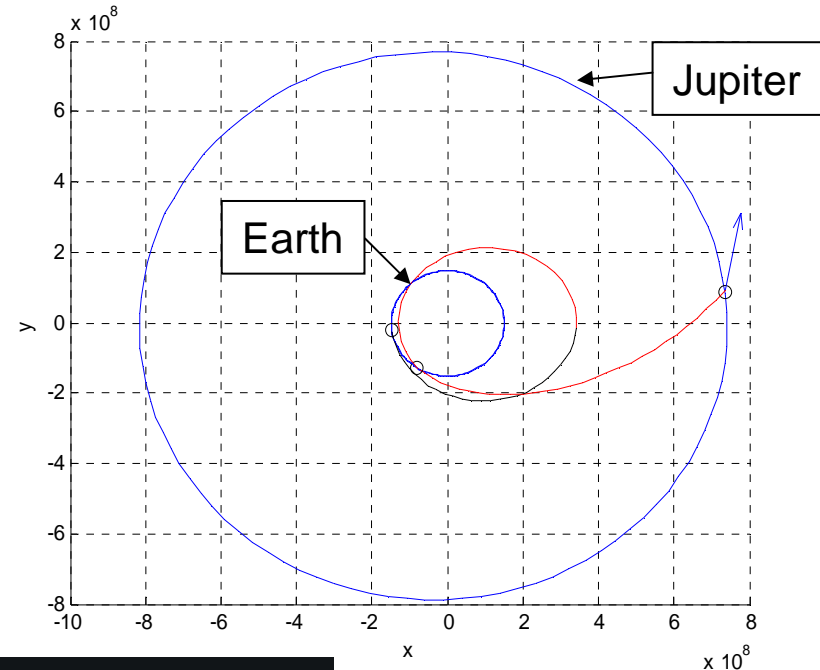
- Independent up and down link.
  - Heterodyne frequency measurement with respect to local laser.
  - Combine on board and ground measurements (asynchronous) for clock comparison (= difference) or Doppler (= sum).
  - 1 W emission, 40 cm telescope on S/C (LISA), 1.5 m on ground (LLR).
  - 22000 detected photons/s @ 30 AU. (LLR < 1 photon/s).
  - Takes full advantage of available highly stable and accurate clock laser and RF reference.
  - Technological challenges are pointing requirements (0.3") and laser availability and reliability (1 W @ 674 nm).
- 
- Pointing and signal acquisition is based on COROT performance (0.5") ensuring initial acquisition using the Earth's image as a target.
  - Two-way asynchronous laser link to MESSENGER spacecraft (0.16 AU, Mercury mission) achieved in 2005 [Science 2006].
  - Presently available "off the shelf" diode lasers + amplifiers provide 250 – 300 mW @ 674 nm, but upgrades expected.
  - Alternative is on-board femtosecond frequency comb  $\Rightarrow$  1064 nm link, for which space qualified lasers exist.





# Trajectory and Spacecraft

- Present baseline: Ariane 5 ECA + propulsion module;  $\Delta V$ -EGA + Jupiter GA @ 22.6 km/s 3 years after launch.
- 38 AU after 15 yrs (nominal), 53 AU after 20 yrs (extended).
- Can be shortened (- 2 yrs) by using larger launcher (Ariane 5 ECB, Atlas 5, Delta IV).
- Total: 950 kg, 390 W (incl. 20% margin).



# Fundamental Physics: Non-metric gravity

$$\text{In GR: } \frac{d\tau_S}{dt} - \frac{d\tau_G}{dt} = \frac{w_G - w_S}{c^2} + \frac{v_G^2 - v_S^2}{2c^2} + O(c^{-4})$$

- Gravitational frequency shift
- $w$  = Newtonian potential (determined from ephemerides)
- Test of LPI (part of equivalence principle)
- $10^{-9}$  measurement
- $10^5$  improvement on present knowledge (GP-A)
  
- Also tests for coupling between gravity and e-m interaction (variation of  $\alpha$  with grav. field).
- 250 fold improvement on present.

- 2nd order Doppler (Special Relativity)
- Ives-Stilwell test
- $10^2$  to  $10^4$  improvement on present (TPA in particle accelerator)
- Depends on signal propagation direction with respect to CMB anisotropy.

**Violation implies non - metric description of Gravitation**

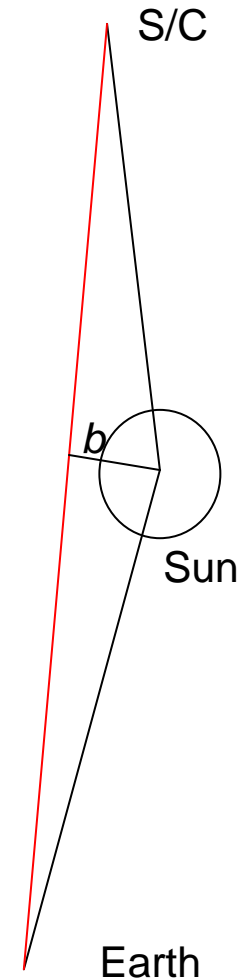
# Fundamental Physics: Metric gravity

$$\delta D_v(t) \approx \frac{d}{dt} \left[ 2(1+\gamma) \frac{GM}{c^3} \ln \left( \frac{4r_S r_G}{b^2} \right) \right] \approx -4(1+\gamma) \frac{GM}{c^3 b} \frac{db}{dt}$$

PPN parameter, in GR  $\gamma=1$

- Gravitational time delay (Shapiro delay)
- Large variation during occultation  $\Rightarrow$  effect on Doppler observable
- Test of metric theories (Parametrised Post-Newtonian framework)
- $10^{-7}$  to  $10^{-9}$  uncertainty on  $\gamma$
- $10^2$  to  $10^4$  improvement on present knowledge (Cassini)
- Well within region where some unification models predict deviations ( $10^{-5}$  to  $10^{-7}$ ).
- Takes advantage of laser and X-band (solar corona effect), and accelerometer (precise knowledge of S/C motion).
- Jupiter occultation allows for independent “test” (100 times less precise).

**Violation allows metric description of Gravitation  
but not GR**



# Fundamental Physics: Scale dependent gravity

Search for a deviation

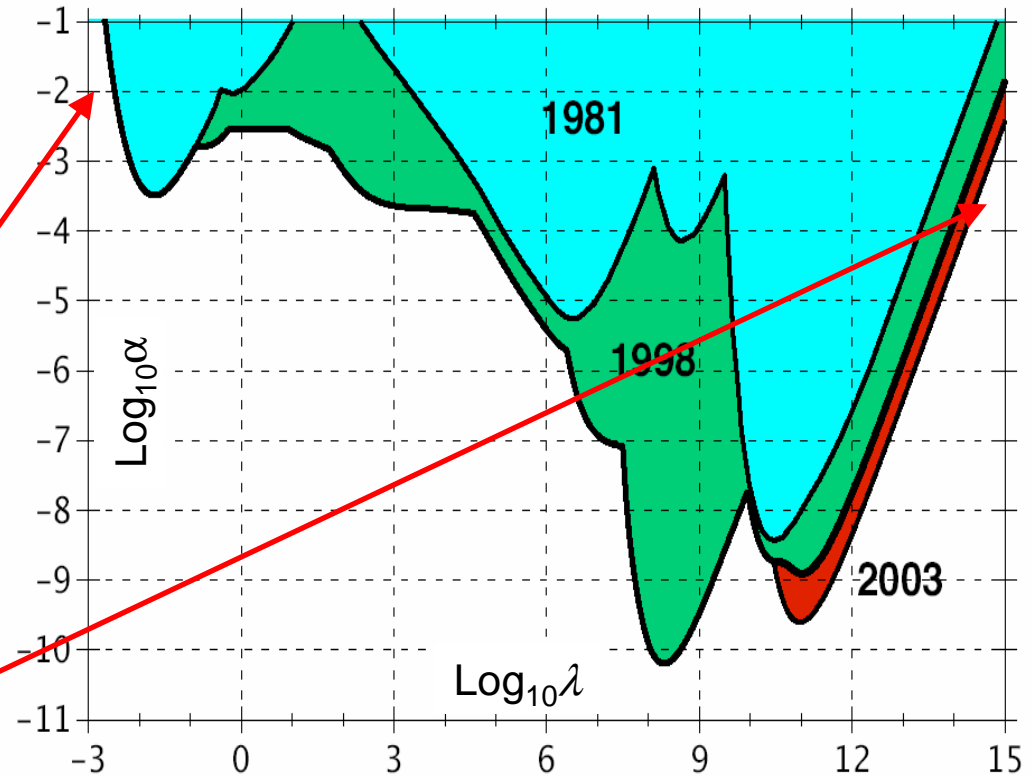
$$g_{00} = [g_{00}]_{\text{GR}} + \delta g_{00}$$

For example under the form  
of a Yukawa correction

$$\delta g_{00}(r) = 2\phi(r) \alpha \exp\left(-\frac{r}{\lambda}\right)$$

Windows remain  
open for deviations  
at short ranges

or long ranges



Courtesy : J. Coy, E. Fischbach, R. Hellings,  
C. Talmadge, and E. M. Standish (2003)

*The Search for Non-Newtonian Gravity*, E. Fischbach & C. Talmadge (1998)

# Fundamental Physics: Large scale gravity test (Pioneer example)

- Pioneer 10 and 11 data show unexplained almost constant Doppler rate ( $a_p \sim 8.7 \cdot 10^{-10} \text{ m/s}^2$ ) between 20 AU and 70 AU.

---

Some conventional and “new physics” hypotheses (non exhaustive):

C1: Non-gravitational acceleration (drag, thermal, etc...)

C2: Additional Newtonian potential (Kuiper belt, etc...)

C3: Effect on Pioneer Doppler (DSN, ionosphere, troposphere, etc...) that also effects SAGAS ranging (sum of up and down link) but not the time transfer (difference of up and down link).

C4: Effect on Pioneer Doppler that has no effect on SAGAS ranging or time transfer (eg. ionosphere  $\propto 1/f^2$ )

P1: Modification of the metric component  $g_{00}$  ("first sector" in Jaekel & Reynaud, Moffat...)

P2: Modification of the metric component  $g_{00}g_{rr}$  ("second sector" in Jaekel & Reynaud)

# Large scale gravity sensitivity (Pioneer example)

Orders of magnitude of measurable effect with 1 year of data, satellite on radial trajectory,  $v \sim 13$  km/s,  $r \sim 30$  AU,  $a_p \sim 8.7 \cdot 10^{-10}$  m/s<sup>2</sup> :

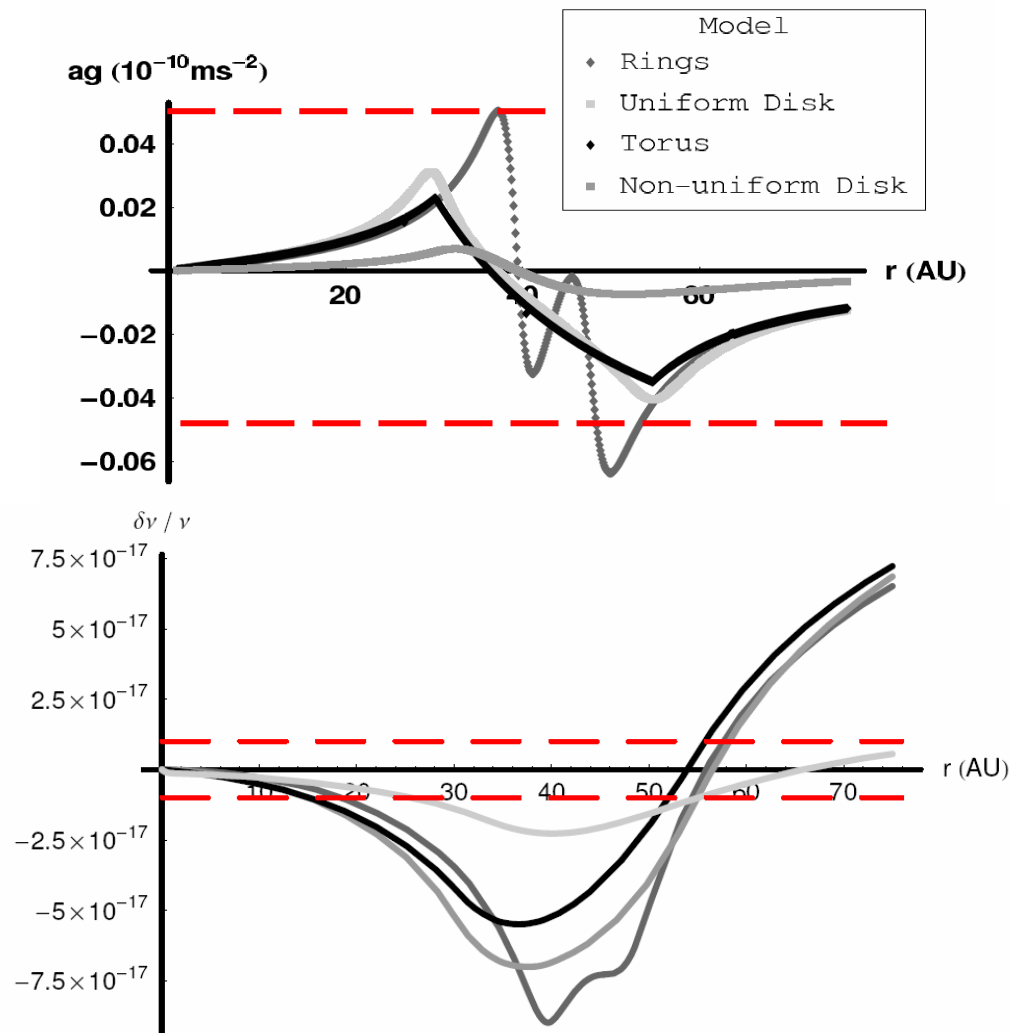
| Observable uncertainty | Acc. / ms <sup>-2</sup><br>( $5 \cdot 10^{-12}$ ) | Clock<br>( $1 \cdot 10^{-17}$ ) | Doppler<br>( $\leq 10^{-13}$ ) |
|------------------------|---|---------------------------------|--------------------------------|
| <b>C1</b>              | $8.7 \cdot 10^{-10}$                              | $4 \cdot 10^{-15}$              | $2 \cdot 10^{-10}$             |
| <b>C2</b>              | -   | $5 \cdot 10^{-14}$              | $2 \cdot 10^{-10}$             |
| <b>C3</b>              | -   | -                               | $2 \cdot 10^{-10}$             |
| <b>C4</b>              | -   | -                               | -                              |
| <b>P1</b>              | -   | $5 \cdot 10^{-14}$              | $2 \cdot 10^{-10}$             |
| <b>P2</b>              | -   | $-9 \cdot 10^{-14}$             | $2 \cdot 10^{-10}$             |

Accelerometer limitation

“-” = no anomaly effect

- All instruments show sensitivity of  $10^{-3}$  or better  $\Rightarrow$  measurement of “fine structure” and evolution with  $r$  and  $t$ , ie. rich testing ground for theories.
- Complementary instruments allow good discrimination between hypotheses
- C2 and P1 are phenomenologically identical (identical modification of Newtonian part of metric in  $g_{00}$ ) but precise measurement will allow “fine tuning”
- Longer data acquisition will improve most numbers

# Solar System Exploration: Kuiper Belt



Provided by O. Bertolami et al.

## Kuiper belt mass distribution models, with $M_{\text{KP}} = 0.3 M_{\text{E}}$

- Remnant of disc from which giant planets formed.
- Mass deficit problem (100 times less than expected from in situ formation of KB objects).
- Acceleration sensitivity insufficient to distinguish between models ( $\propto 1/r^2$ ).
- But clock well adapted for measurement of diffuse, large mass distributions ( $\propto 1/r$ ).
- Depending on distribution SAGAS can determine  $M_{\text{KB}}$  with  $\delta M_{\text{KB}} \approx 10^{-2} M_{\text{E}}$  to  $10^{-3} M_{\text{E}}$

# Solar System Exploration: KBOs and Planets

| Object                         | Semi major axis / AU | Estimated Mass/ $10^{21}$ kg | $\delta M/M$ @ 0.5 AU | $\delta M/M$ @ 0.2 AU |
|--------------------------------|----------------------|------------------------------|-----------------------|-----------------------|
| Pluto                          | 39.5                 | 13.05                        | 0.03                  | 0.005                 |
| (136108) 2003 EL <sub>61</sub> | 43.3                 | 4                            | 0.1                   | 0.02                  |
| (136472) 2005 FY <sub>9</sub>  | 45.8                 | 4                            | 0.1                   | 0.02                  |
| Quaoar                         | 43.4                 | 2                            | 0.2                   | 0.03                  |
| Ixion                          | 39.7                 | 0.6                          | 0.7                   | 0.1                   |

- Trajectory (accelerometer) more sensitive at distances  $< 1.2$  AU.
- Use trajectory to measure characteristics of individual objects, clock to subtract “background”.
- Possibility to discover new objects

| Planet  | $\delta GM/GM$ (present) | $r_C / \text{AU}$ |
|---------|--------------------------|-------------------|
| Jupiter | $2 \cdot 10^{-8}$        | 0.15              |
| Saturn  | $3 \cdot 10^{-8}$        | 0.1               |
| Uranus  | $2 \cdot 10^{-6}$        | 0.3               |
| Neptune | $2 \cdot 10^{-6}$        | 0.4               |

- Below  $r_C$  uncertainty from planet larger than measurement accuracy.
- Improve on present knowledge when sufficiently approaching planet.
- @ 0.01 AU achieve  $10^2$  to  $10^3$  improvement.
- Closest approach to Jupiter will be 0.004 AU  
 $\Rightarrow$  Improve knowledge on Jupiter, maybe others.



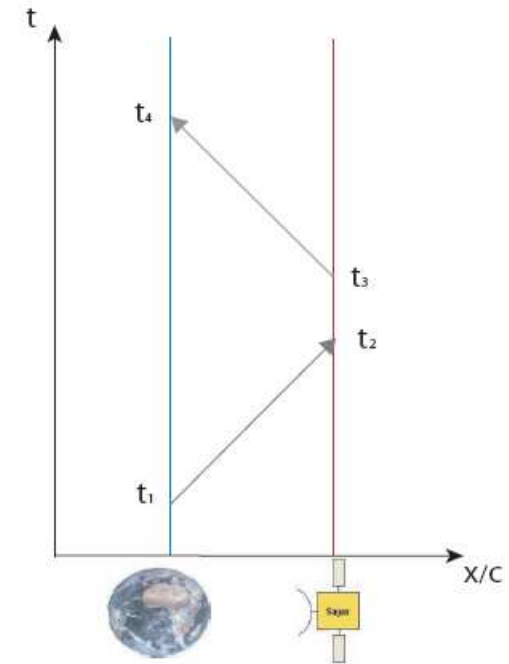
## Astronomy and Cosmology: Upper limits on low frequency grav. Waves (GW)

- SAGAS can be used to search for low frequency GW.
- Strain sensitivity depends on frequency and data combination strategy.
- Constrain cosmic stochastic GW background to about 3 orders of magnitude below present limits (Cassini), in the  $10^{-6}$  to  $10^{-3}$  Hz range.
- Could be further improved if extended to  $10^{-7}$  Hz (model for non-grav. accelerations?).
- For particular sources in the  $10^{-6}$  to  $10^{-3}$  Hz region can use template and optimal filtering. With one year data achieve  $h \leq 10^{-18}$ .
- Insufficient for expected sources (eg. for BHB expect  $h \leq 10^{-19}$ ).
- But may be useful for constraints on astrophysical models, and leaves door open for surprises.

# Measurements and Optimisation

(c.f. S. Reynaud et al., arXiv:0801.2896)

- Asynchronous two way link, i.e. independent up and down.
- Measure continuously incoming laser frequency with respect to local laser frequency.
- Combine S/C and ground measurements in post treatment, i.e. choose freely  $t_2$  and  $t_4$ .
- Many noise sources on “up” and “down” link are correlated (clocks, S/C and Earth motion, atmosphere, etc...).
- Signal (e.g. Shapiro, GW, planetary gravity, ...) affects “up” and “down” link differently  $\Rightarrow$  choose data combination to optimise signal to noise.



Ranging observable:

$$\tau_r \equiv -\frac{\tau_3^s - \tau_2^s}{2} + \frac{\tau_4^g - \tau_1^g}{2} \equiv \frac{\tau_d + \tau_u}{2}$$

$$\tau_u \equiv \tau_2^s - \tau_1^g, \quad \tau_d \equiv \tau_4^g - \tau_3^s$$

Timing observable:

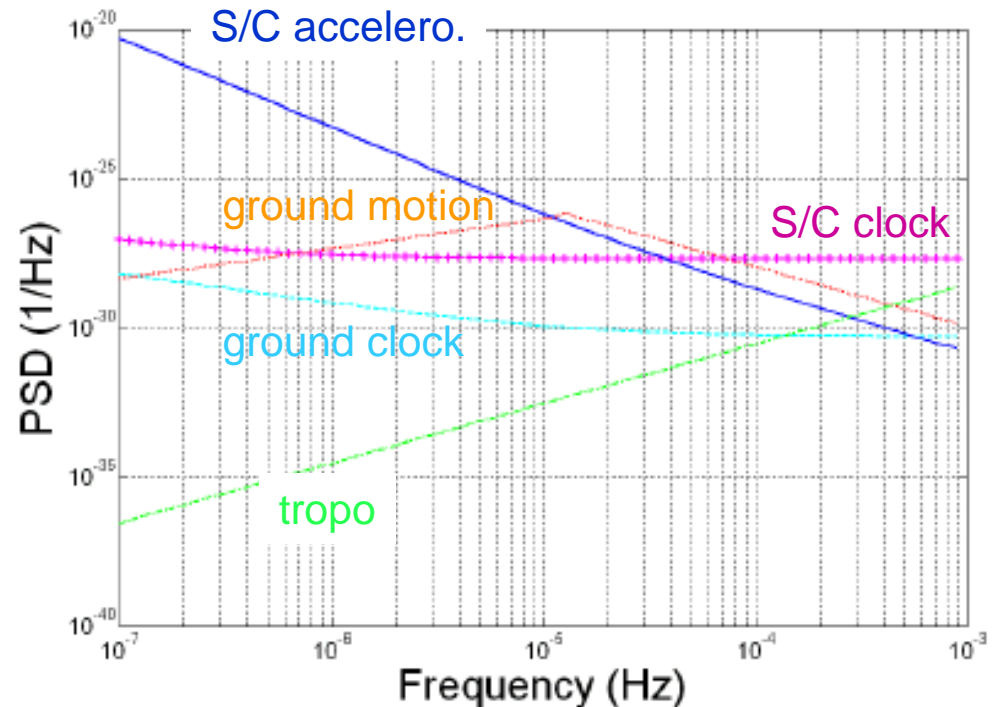
$$\tau_t \equiv \frac{\tau_3^s + \tau_2^s}{2} - \frac{\tau_4^g + \tau_1^g}{2} \equiv \frac{-\tau_d + \tau_u}{2}$$

SAGAS measures time derivatives (frequencies) i.e.  $y_{r/t} = d\tau_{r/t}/dt$

# Noise Sources

## Frequency noise on a one-way link:

- S/C accelerometer according to SAGAS specs.
- S/C clock according to SAGAS specs.
- Ground clock assumes some improvement in the near future (factor 6)
- Ground motion is best present gravimeter performance at high frequency, GNSS positioning at low freq.
- troposphere is best SLR models (mm noise level)



Power spectral densities (PSD) of noise sources on a one way Doppler link

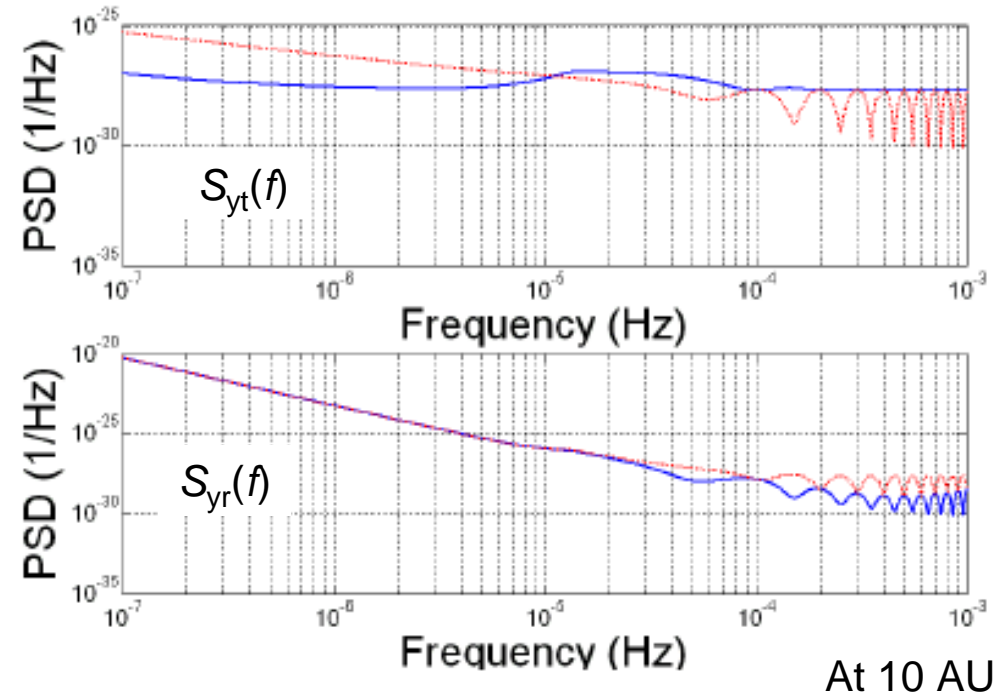
# Analysis Strategies

- The noise on up and down link are correlated in the two observables:

$$S_{y_r}(f) = \frac{1}{2} \left\{ (1 - \cos(2\pi f T_{14})) S_{y_g}(f) + (1 - \cos(2\pi f T_{23})) S_{y_s}(f) + (1 + \cos(2\pi f T_{23})) S_{v_{s/c}}(f) + (1 + \cos(2\pi f T_{14})) S_{v_{g/c}}(f) + (1 + \cos(2\pi f T_{14})) S_{y_{\text{tropo}}}(f) \right\}$$

$$S_{y_t}(f) = \frac{1}{2} \left\{ (1 + \cos(2\pi f T_{14})) S_{y_g}(f) + (1 + \cos(2\pi f T_{23})) S_{y_s}(f) + (1 - \cos(2\pi f T_{23})) S_{v_{s/c}}(f) + (1 - \cos(2\pi f T_{14})) S_{v_{g/c}}(f) + (1 - \cos(2\pi f T_{14})) S_{y_{\text{tropo}}}(f) \right\}$$

with  $T_{ij} = t_j - t_i$



$T_{23}=0$  (blue)

$T_{14}=0$  (red)

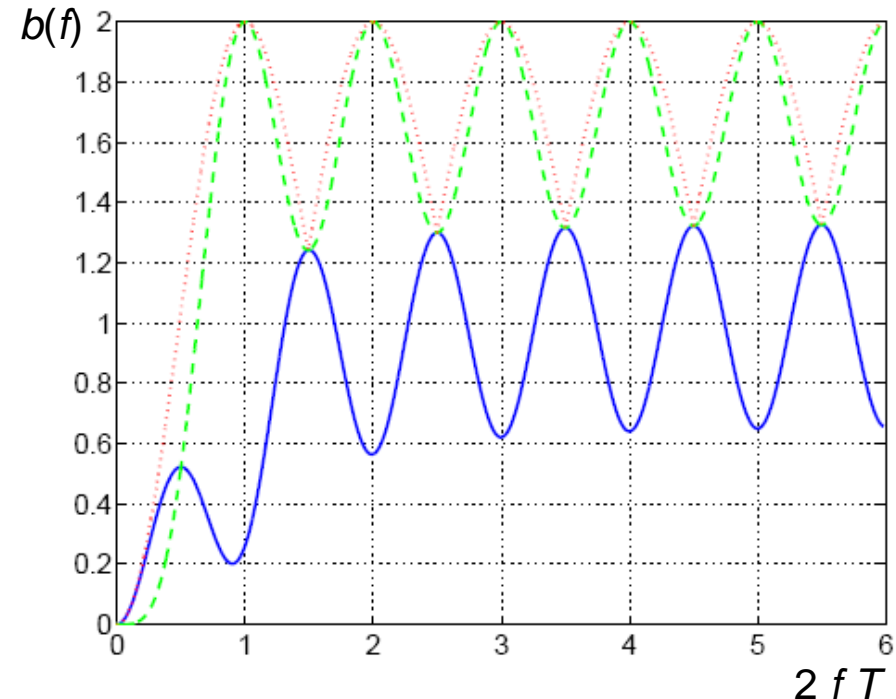
⇒ fine-tune observable by choosing  $T_{23}$  in order to optimise S/N (cf. TDI in LISA)

# Example: Stochastic GW background

Sensitivity of each observable to stochastic GW backgrounds can be calculated by integrating GW signal along up and down link paths:

$$\Omega_{GW}(f) = \frac{16\pi^2 f^3}{3H_0^2} \frac{S_{y_{r/t}}(f)}{b_{r/t}(f)}$$

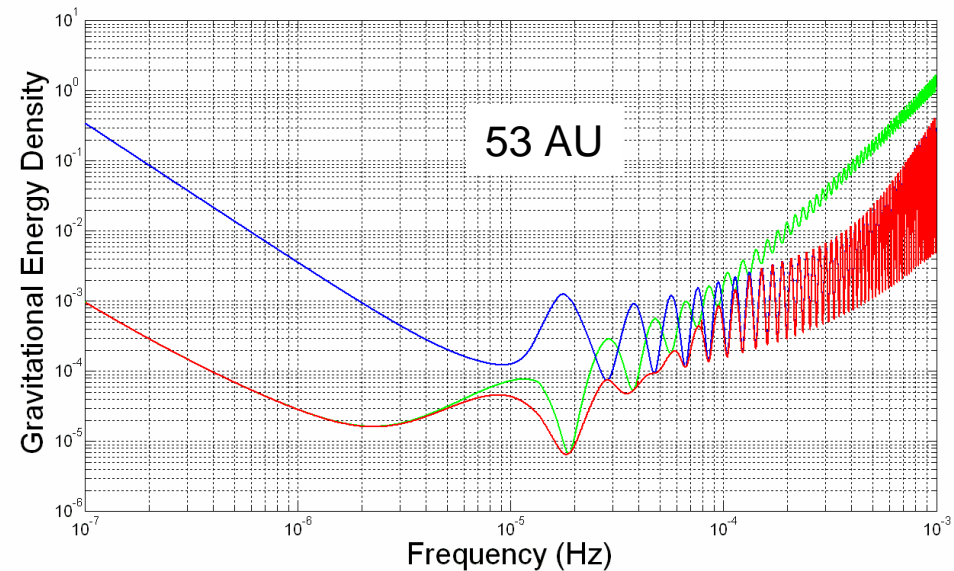
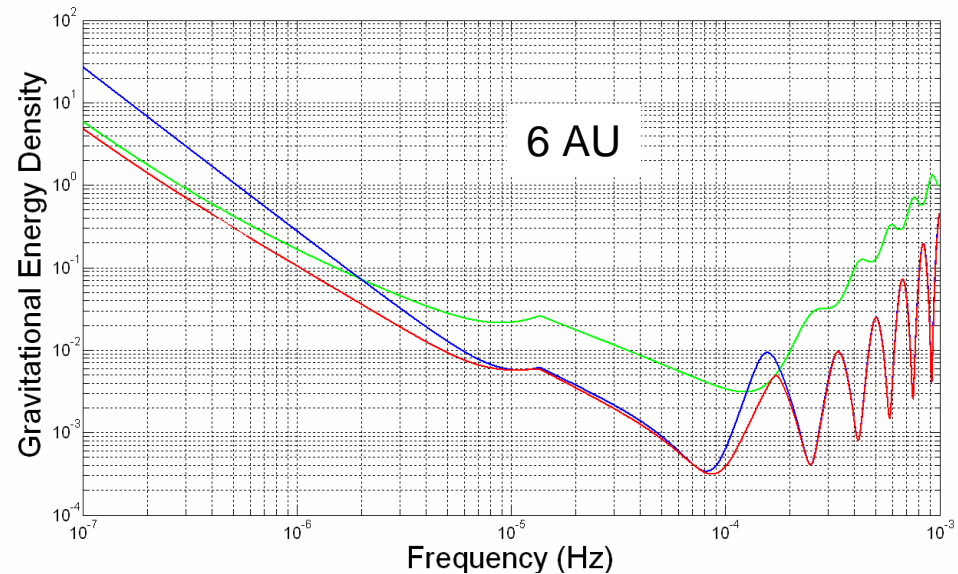
- At low frequency or short distance  $b_r$  is larger than  $b_t$ . No longer the case at arbitrary  $f$  or  $T$ .
- Optimum combination determined by optimising  $S/b$ .
- Classical Doppler ranging limited to  $S_{yr}$  and  $b_r$  with  $T_{23}=0$ .



$b_r$  with  $T_{23}=0$  (blue),  $b_t$  with  $T_{23}=0$  (green) and  $b_{opt}$  (red) as function of frequency and  $T=D/c$

# Example: Stochastic GW background

- at large distance asynchronous two way measurements allow up to two orders of magnitude gain over “classical” Doppler ranging with same fundamental noises.
- resulting limits are  $> 10^3$  times more stringent than best present limits (Cassini) in the  $10^{-6}$  -  $10^{-3}$  Hz range.
- same methods can be applied to optimise other measurements (eg. Shapiro, planetary gravity, etc...).
- can accommodate changing noise levels, in post-treatment optimisation (e.g. in quiet regions natural S/C motion noise may be smaller than accelerometer noise)



$\Omega_r$ (blue),  $\Omega_t$ (green) and  $\Omega_{opt}$ (red)

# Conclusion

**The two Pioneer S/C show an anomalous Doppler rate (“acceleration”)**

- several independent analysis of the same data confirm  $a_p$
- several thermal modelling efforts are in progress
- large amount of new data retrieved and (soon) available

**Origin of  $a_p$  still an open question**

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**SAGAS offers a unique possibility for a mission combining equally attractive objectives in fundamental physics and solar system exploration.**

- **Potential for a major discovery in fundamental physics and major contribution to constraining theoretical models (well beyond only  $a_p$ ).**
- Kuiper Belt (KB) potentially holds clues for planetary formation processes, and gives rise to fundamental questions (mass deficit?).
- **Major contribution to the understanding of planetary formation in the solar system, with potential for new discoveries (KB mass, new KBOs).**
- Onboard clock with asynchronous two-way link allows for large versatility in data analysis  
⇒ possibility to optimize S/N for given science objective.