Relativistic stars in f(R) gravity and Chameleon

Eugeny Babichev
APC University Paris VII

with D.Langlois

based on arxiv:0904.1382, arxiv:0911:1297

PLAN

- Introduction and motivations
- Action, equations of motion, spherically symmetric ansatz
- Chameleon model
- f(R) gravity
- Conclusions

INTRODUCTION
and
MOTIVATIONS

Introduction and motivation (I)

- A way to get acceleration of the Universe (Dark Energy):

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$$R \to R + \frac{R^2}{M^2}$$ inflation *Starobinsky'80*

$$R \to R - \frac{M^{2(n+1)}}{R^n}$$ Dark Energy *Carroll et.al.'03*

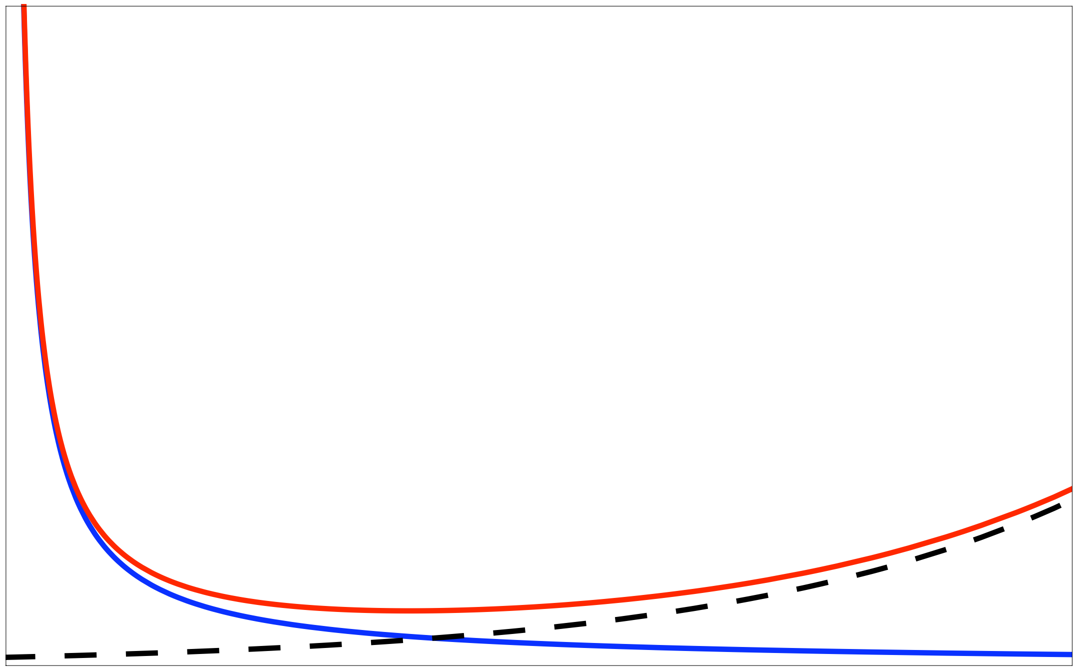
- Fast tachyonic instability, *Dolgov,Kawasaki'03*
- $\gamma = 1/2$ (need $\gamma = 1$) *Chiba'03*

Simple models of f(R) do not work !

Introduction and motivation (III)

- Chameleon effect in scalar-tensor theory, *Khoury,Weltman'03*

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\left(\nabla\phi\right)^2 - V(\phi)\right] + S_m[\Psi_m; e^{Q\phi/M_P}g_{\mu\nu}]$$



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in the effective potential

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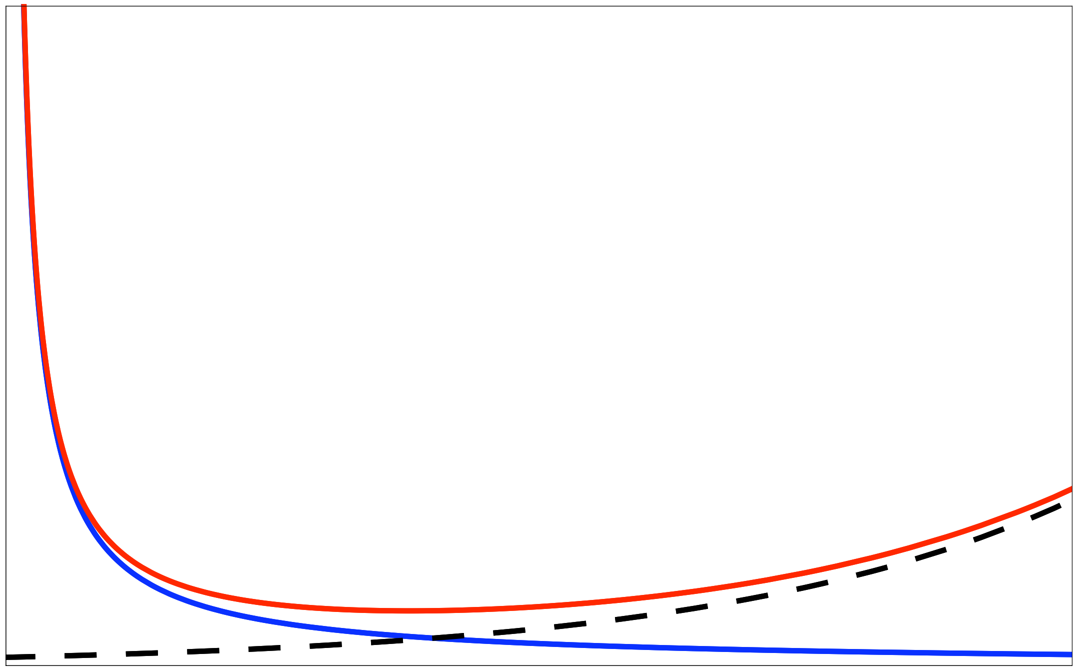

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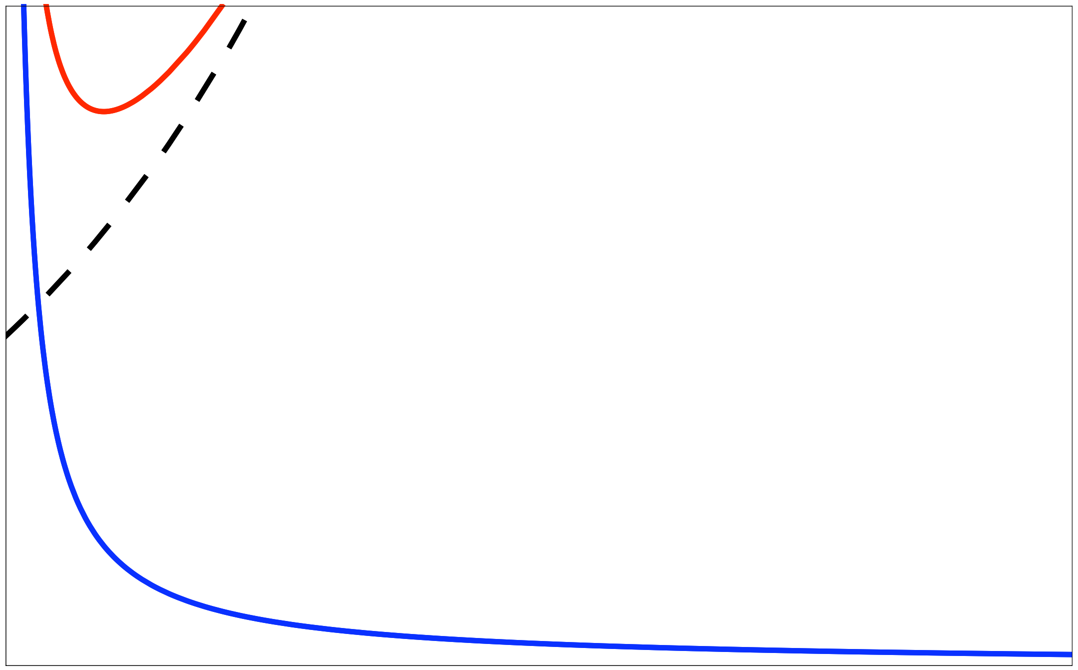
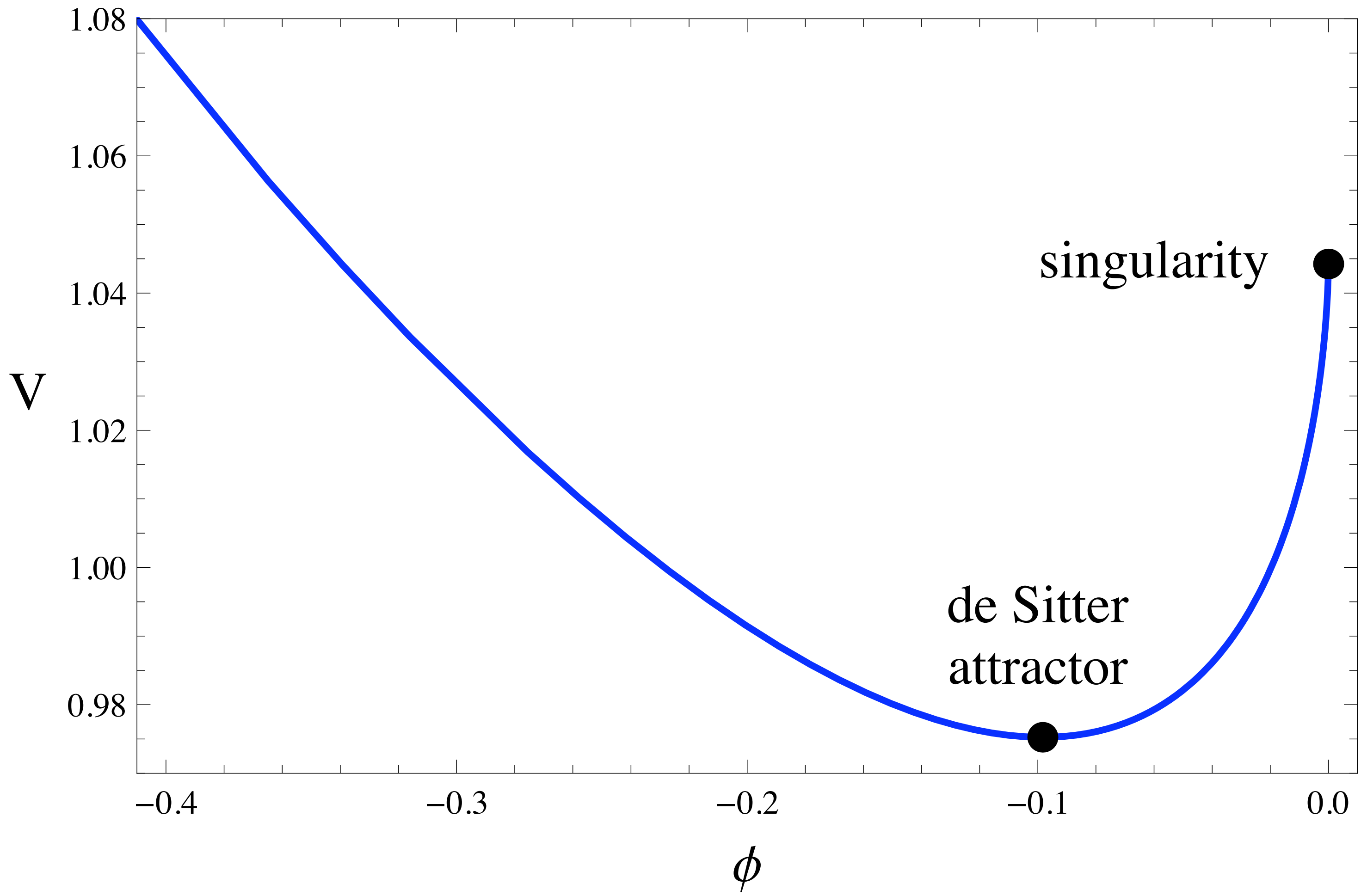


Introduction and motivation (IV)

- Curvature singularity problem - curvature singularity can be easily accessed for generic infrared modified f(R) theories (Frolov'08)



Introduction and motivation (V)

- No neutron stars for generic f(R) models [Starobinsky and Hu-Sawicky models] (Kobayashi&Maeda'08)
- No neutron stars for higher curvature modified f(R) models (Kobayashi&Maeda'08)
- No solutions for highly relativistic objects in the case of Chameleon field (Tsujikawa *et al.*'09)
- Instability associated with huge effective of s.d.f. and "fine-tuning problem" (Thongkool *et al.*'09)

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ACTION
and
EQUATIONS OF MOTION

action and eoms (I)

- action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\left(\nabla\phi\right)^2 - V(\phi)\right] + S_m\left[\Psi_m; \Omega^2(\phi)g_{\mu\nu}\right]$$

matter coupled to $\tilde{g}_{\mu\nu} = \Omega^2(\phi)g_{\mu\nu}$

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matter coupled to $\tilde{g}_{\mu\nu} = \Omega^2(\phi) g_{\mu\nu}$

- equations of motion,

$$R_{\mu\nu} - \frac{1}{2} R = M_P^{-2} \left[T^{(m)}_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu} \partial^\sigma\phi\partial_\sigma\phi - V g_{\mu\nu} \right]$$

$$\nabla_\sigma \nabla^\sigma \phi = -\frac{dV}{d\phi} - \frac{\Omega'}{\Omega} T^{(m)}$$

$$T^{(m)} \equiv g^{\mu\nu} T^{(m)}_{\mu\nu} = -\rho + 3P$$

action and eoms (II)

- static spherically symmetric ansatz,

$$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right)$$

$$e^{-\lambda} \equiv 1 - 2m/r$$

- energy-momentum conservation (in Jordan frame!),

$$\tilde{\nabla}_{\mu} \tilde{T}^{\ \mu}_{(m)\nu} = 0$$

$$\tilde{T}^{\mu}_{\nu} = \Omega^{-4} T^{\mu}_{\nu}, \quad \rho = \Omega^4 \tilde{\rho}, \quad P = \Omega^4 \tilde{P}$$

- equation of state closes the system of equations,

$$\tilde{P} = \tilde{P}(\tilde{\rho})$$

action and eoms (III)

tt component	$$m' = \frac{r^2}{2M_P^2}\left[\Omega^4\tilde{\rho} + \frac{1}{2}e^{-\lambda}\phi'^2 + V(\phi)\right],$$
rr component	$$\nu' = e^{\lambda}\left[\frac{2m}{r^2} + \frac{r}{M_P^2}\left(\frac{1}{2}e^{-\lambda}\phi'^2 - V(\phi)\right) + \frac{r\Omega^4\tilde{P}}{M_P^2}\right],$$
conservation	$$\tilde{P}' = -\frac{1}{2}\left(\tilde{\rho} + \tilde{P}\right)\left(\nu' + 2\frac{\Omega'}{\Omega}\phi'\right),$$
eq of state	$$\tilde{P} = \tilde{P}(\tilde{\rho}),$$
Klein-Gordon eq	$$\phi'' + \left(\frac{2}{r} + \frac{1}{2}(\nu' - \lambda')\right)\phi' = e^{\lambda}\left[\frac{dV}{d\phi} + \Omega^3\Omega'(\tilde{\rho} - 3\tilde{P})\right].$$

equation of state (I)

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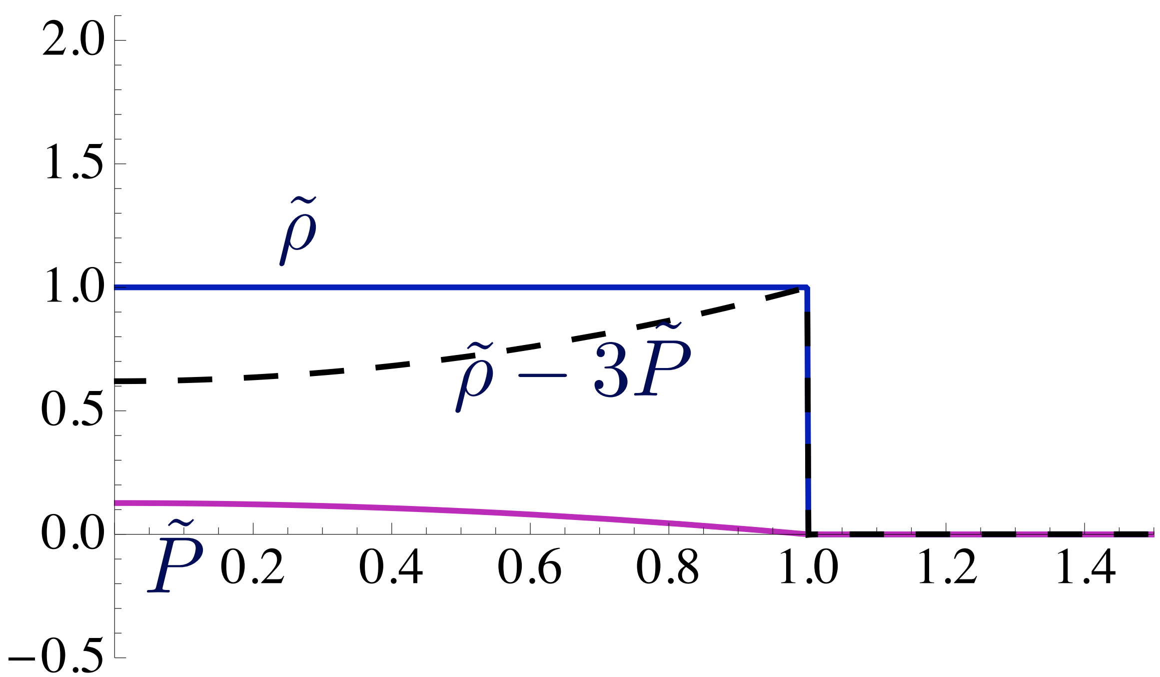

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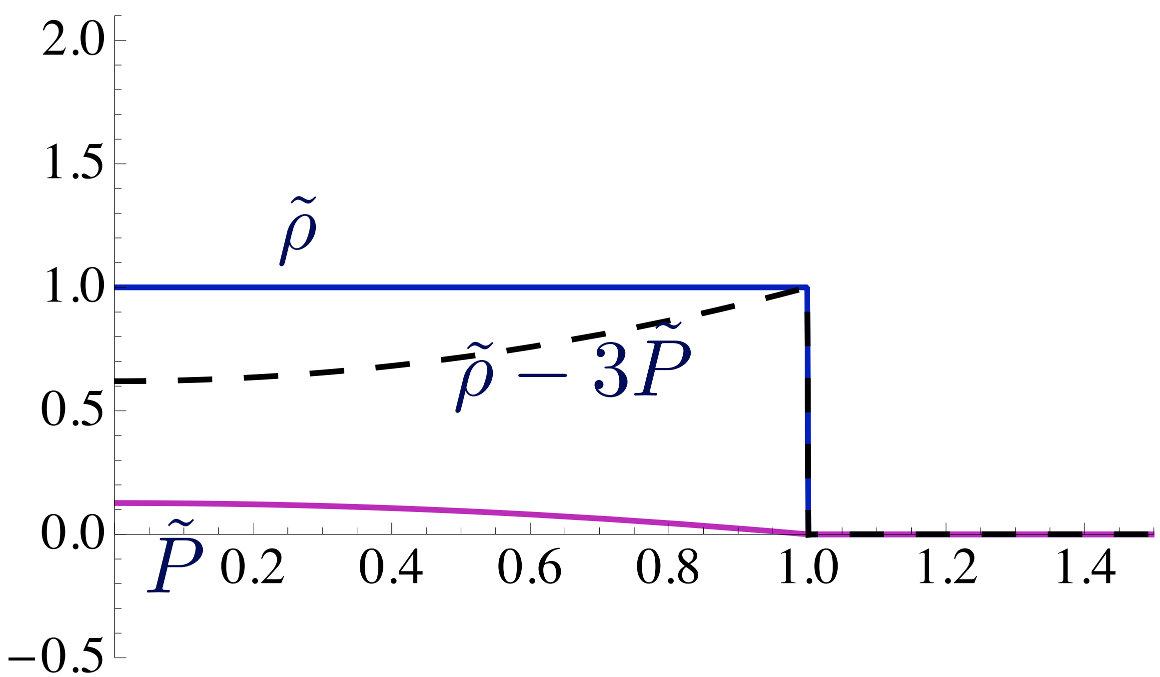
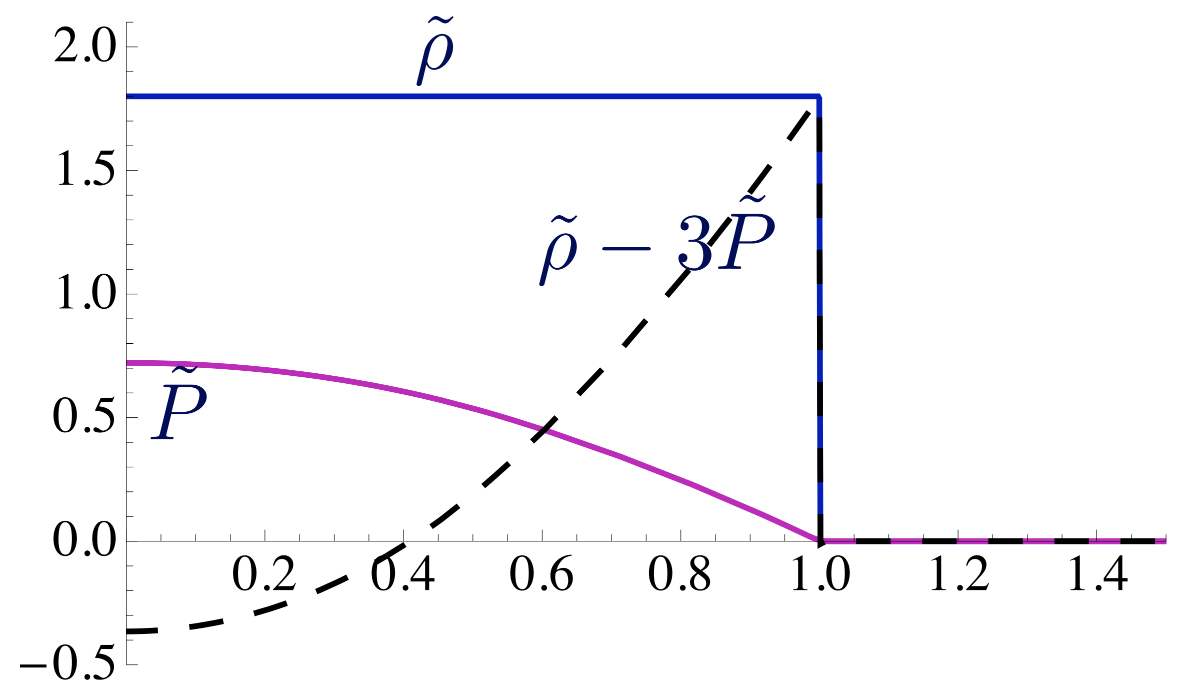


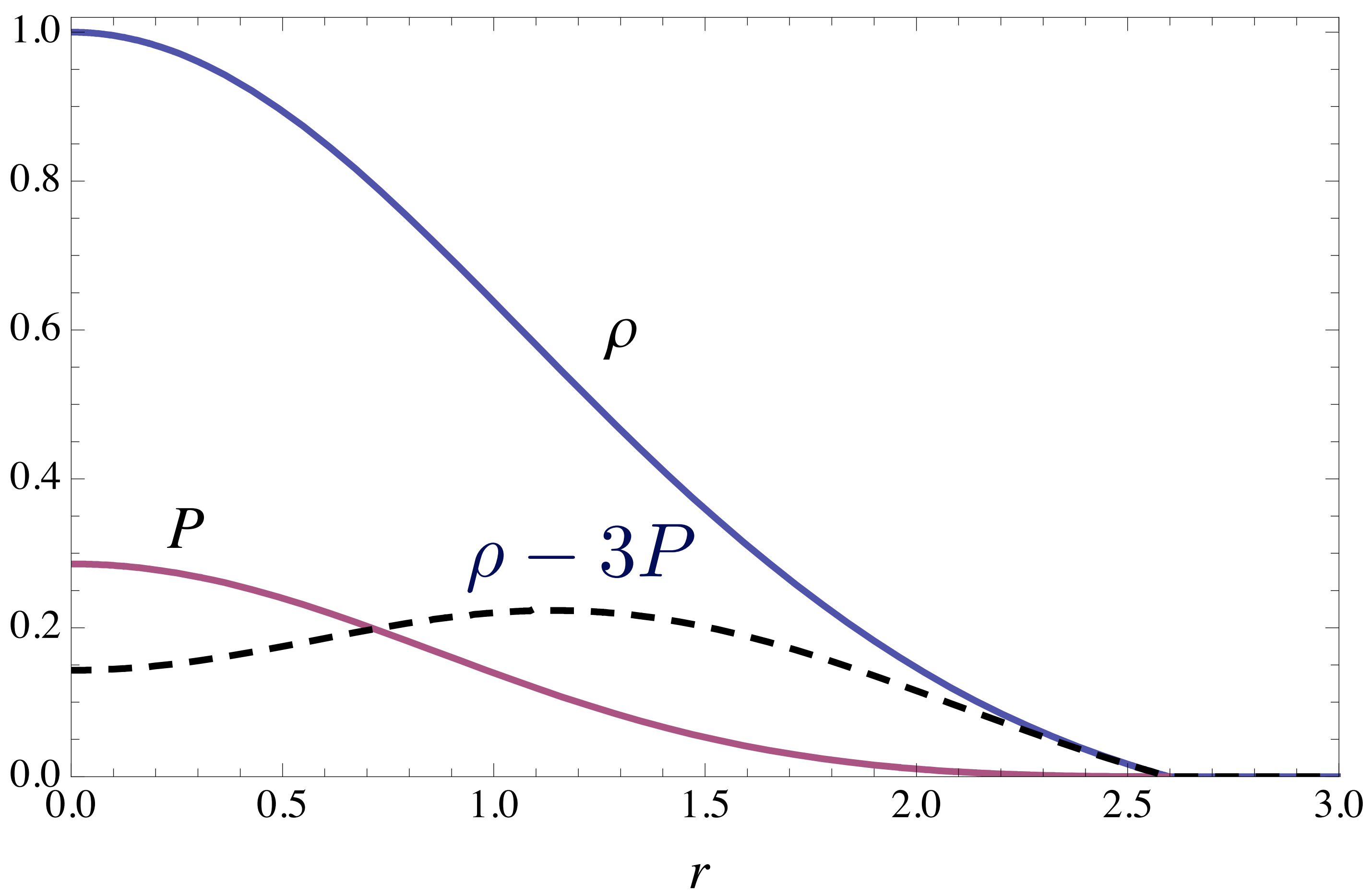


equation of state (II)

- For a realistic EoS of NS $\rho - 3P > 0$

$$\tilde{\rho}(\tilde{n}) = m_B \left(\tilde{n} + K \frac{\tilde{n}^2}{n_0} \right), \quad \tilde{P}(\tilde{n}) = K m_B \frac{\tilde{n}^2}{n_0},$$

$$m_B = 1.66 \times 10^{-27}\mathrm{kg},\ n_0 = 0.1\,\mathrm{fm}^{-1} \text{ and } K = 0.1.$$



$$\tilde{n}_c = 0.4\ \mathrm{fm}^{-3},$$
$$|\Phi_*| \simeq 0.25$$

instability for high-density star (I)

- constant density star in GR, the higher density the higher pressure in the center (constant density stars),

$$\rho - 3P < 0 \quad \text{for} \quad \Phi_* \equiv \frac{\mathrm{GM}}{\mathrm{r}_*} > \frac{5}{18}$$

$$\phi'' + \left(\frac{2}{r} + \frac{1}{2}(\nu' - \lambda')\right)\phi' = e^{\lambda}\left[\frac{dV}{d\phi} + \Omega^3\Omega_{\phi}(\tilde{\rho} - 3\tilde{P})\right].$$

Tachyon instability

- explains the results of Tsujikawa et.al'09, no solutions for the Chameleon for $\Phi > 0.3$

instability for high-density star (II)

- small perturbations, $\delta\phi(t,r,\theta,\phi) = \sum \delta\phi_{lm}(t,r) Y_{lm}(\theta,\phi)$,
- EoM for perturbations,

$$\ddot{\delta}\phi - e^{\nu-\lambda}\left[\delta\phi'' + \left(\frac{\nu'-\lambda'}{2} + \frac{2}{r}\right)'\right] + e^{\nu}\left[\frac{l(l+1)}{r^2} + m_{\rm eff}^2\right]\delta\phi = 0.$$

$$\omega^2 \sim k^2 + m_{\rm eff}^2$$

- assuming the length-wave is of order of the star radius,

$$\omega^2 \sim \frac{\alpha^2}{r_*^2} + 4\frac{Q^2}{M_P^2}(\tilde{\rho}_* - 3\tilde{P}) > 0$$

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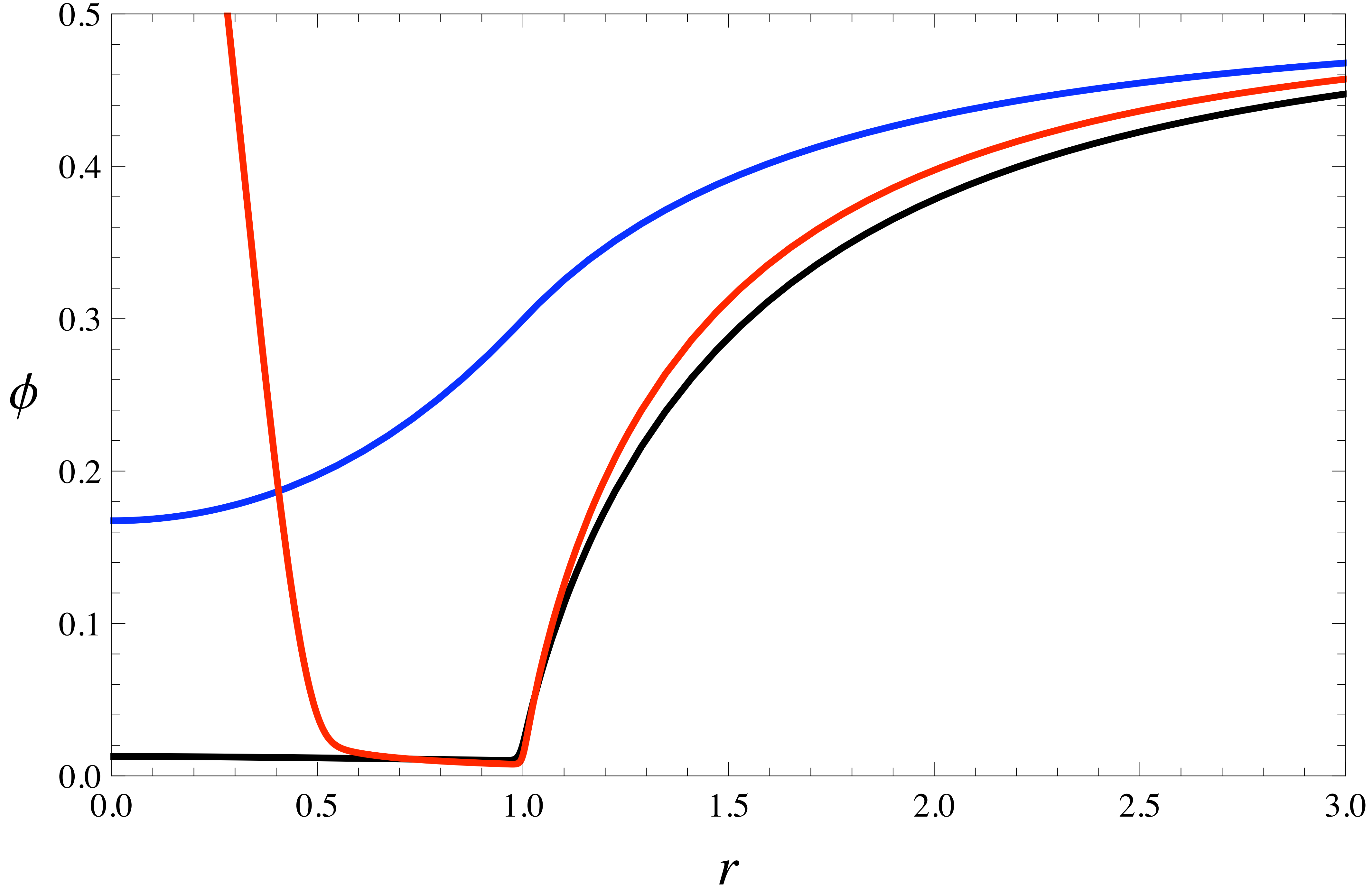
CHAMELEON

profile of s.f.



thin-shell effect (I)

- asymptotically, $$e^{\nu} \approx e^{-\lambda} \approx 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2 .$$
- In Jordan frame, $$\tilde{g}_{\mu\nu} = \exp(2Q\phi/M_P) g_{\mu\nu}$$
- Asymptotic behavior of scalar,

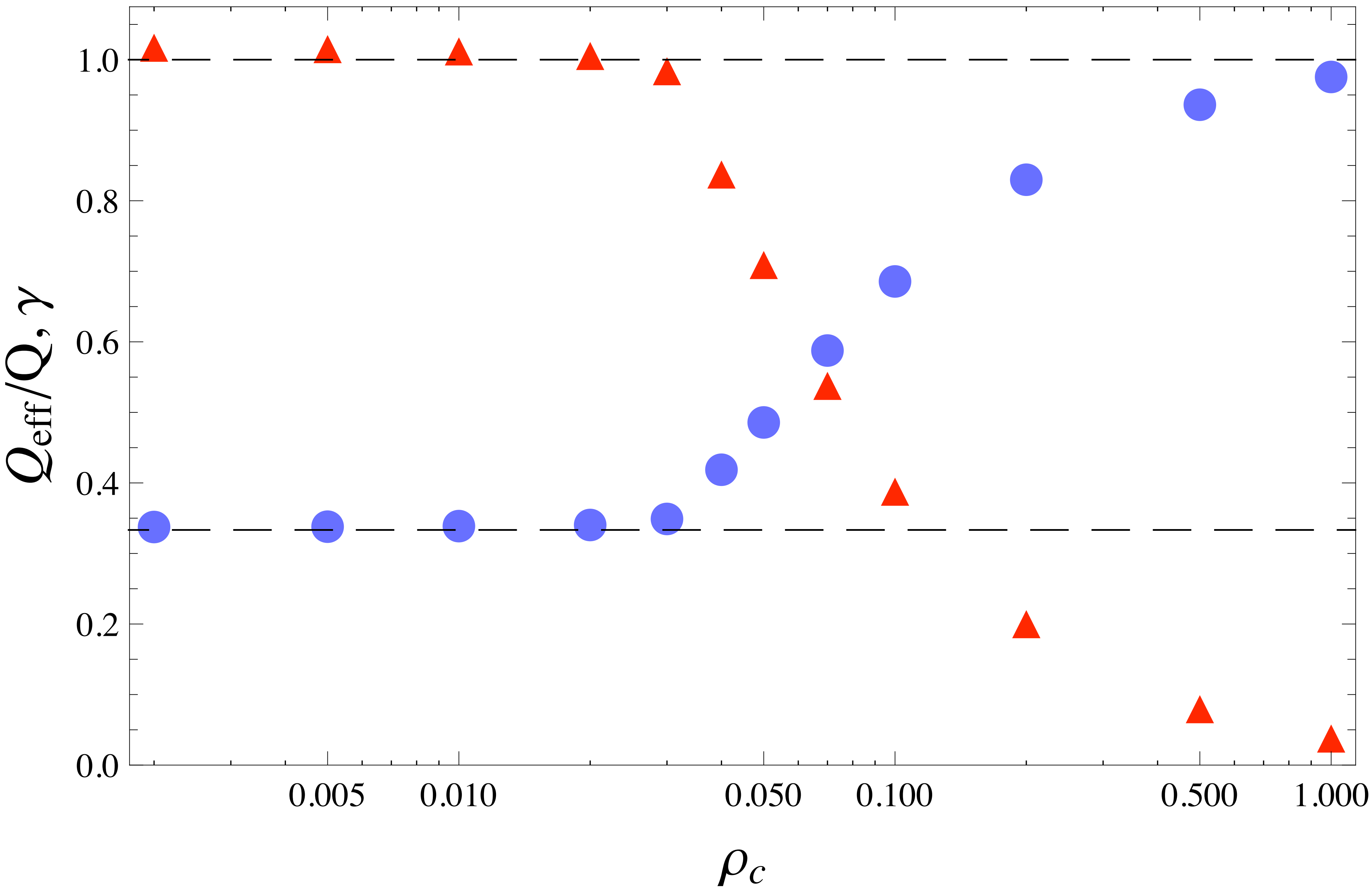
$$\phi \approx \phi_\infty + \frac{2GM}{r} Q_{\rm eff} e^{-m_{\rm eff} r}$$

in the thin-shell regime $Q_{\rm eff}$ is strongly suppressed

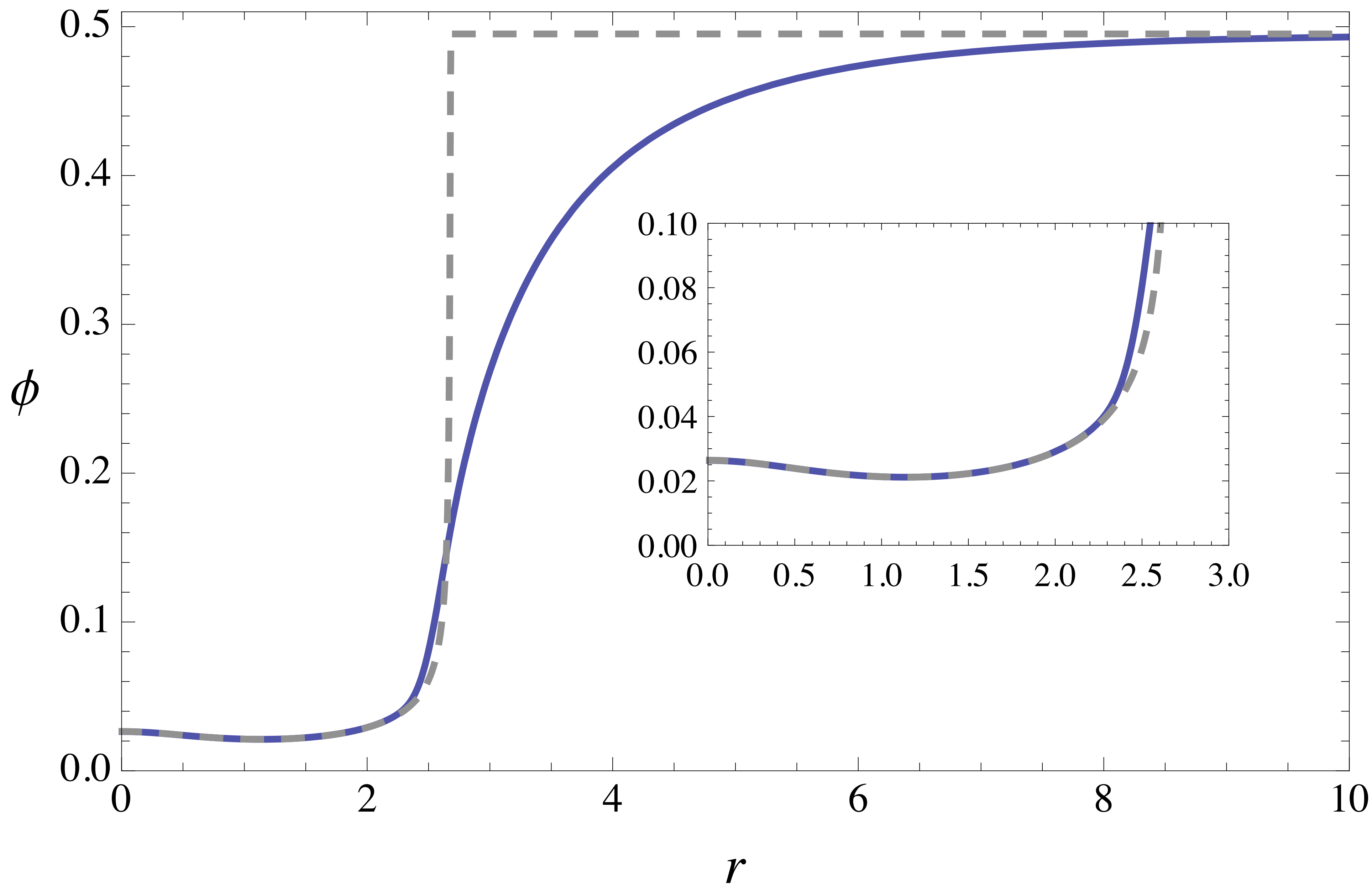
$$\tilde{g}_{tt} \approx e^{2Q\phi_\infty} \left[1 - (1 - 2QQ_{\rm eff}) \frac{2GM}{r} - \frac{\Lambda r^2}{3} \right],$$
$$\tilde{g}_{rr}^{-1} \approx e^{-2Q\phi_\infty} \left[1 - (1 + 2QQ_{\rm eff}) \frac{2GM}{r} - \frac{\Lambda r^2}{3} \right],$$

$$\tilde{\gamma} = \frac{1 - 2QQ_{\rm eff}}{1 + 2QQ_{\rm eff}}$$

thin-shell effect (II)



numerical solution (polytropic eos)



$f(R)$

f(R) gravity

- Jordan frame:

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-\tilde{g}}\, f(\tilde{R}) + S_m[\Phi_m; \tilde{g}_{\mu\nu}],$$

$$\phi = \sqrt{\frac{3}{2}} M_P \ln f_{,\tilde{R}}$$

$$g_{\mu\nu} = \Omega^{-2}\, \tilde{g}_{\mu\nu}, \qquad \Omega^{-2} = f_{,\tilde{R}} = \exp[\sqrt{\frac{2}{3}}\, \phi / M_P]$$

- Einstein frame:

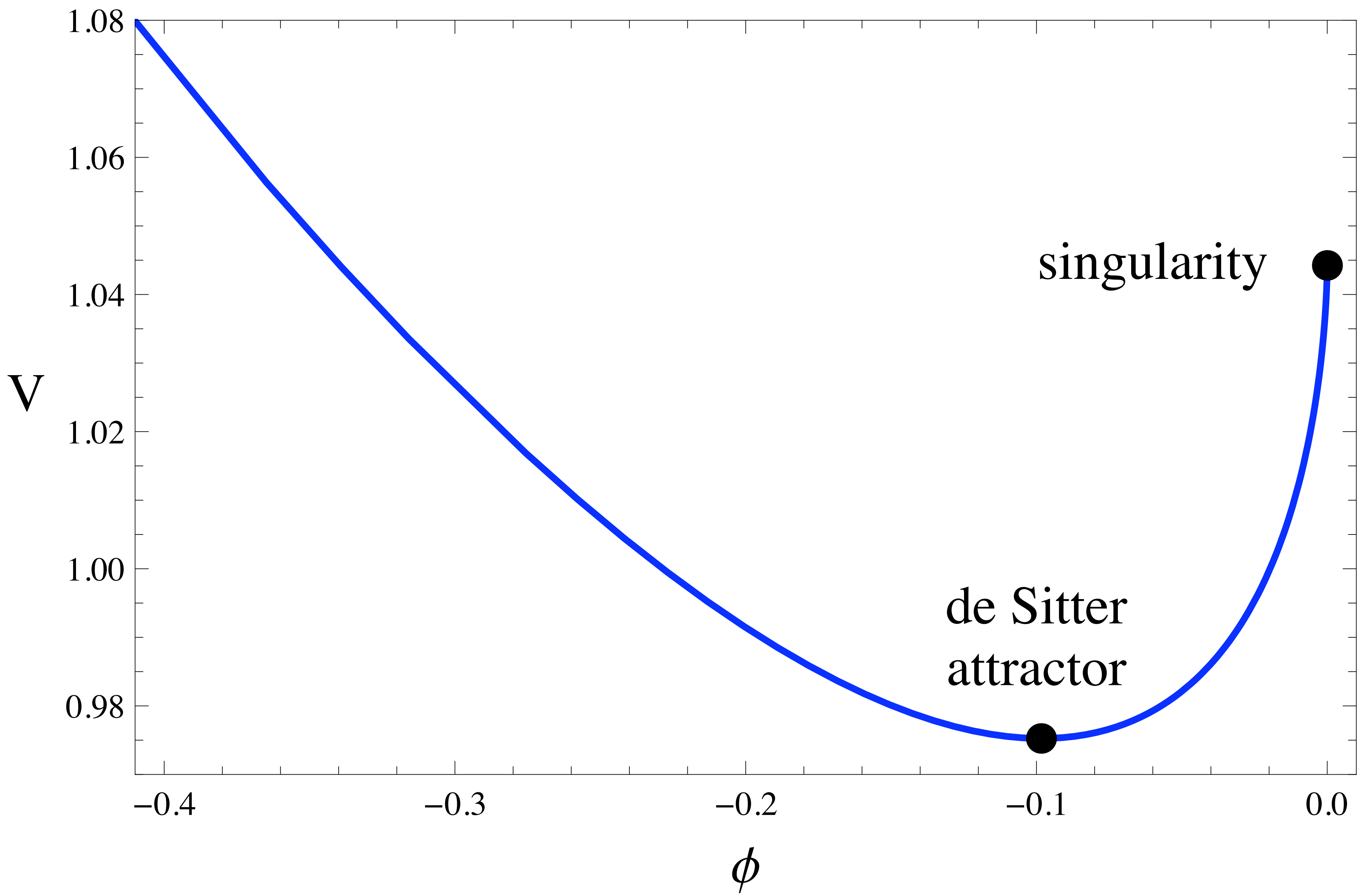
$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right] + S_m[\Phi_m; \Omega^2 g_{\mu\nu}]$$

$$V = M_P^2 \frac{\tilde{R} f_{,\tilde{R}} - f}{2 f_{,\tilde{R}}^2}$$

model

$$f(\tilde{R}) = R_0 \left[x - \lambda \left(1 - \left(1 + x^2\right)^{-n}\right)\right], \quad x \equiv \frac{\tilde{R}}{R_0}$$

Starobinsky'07



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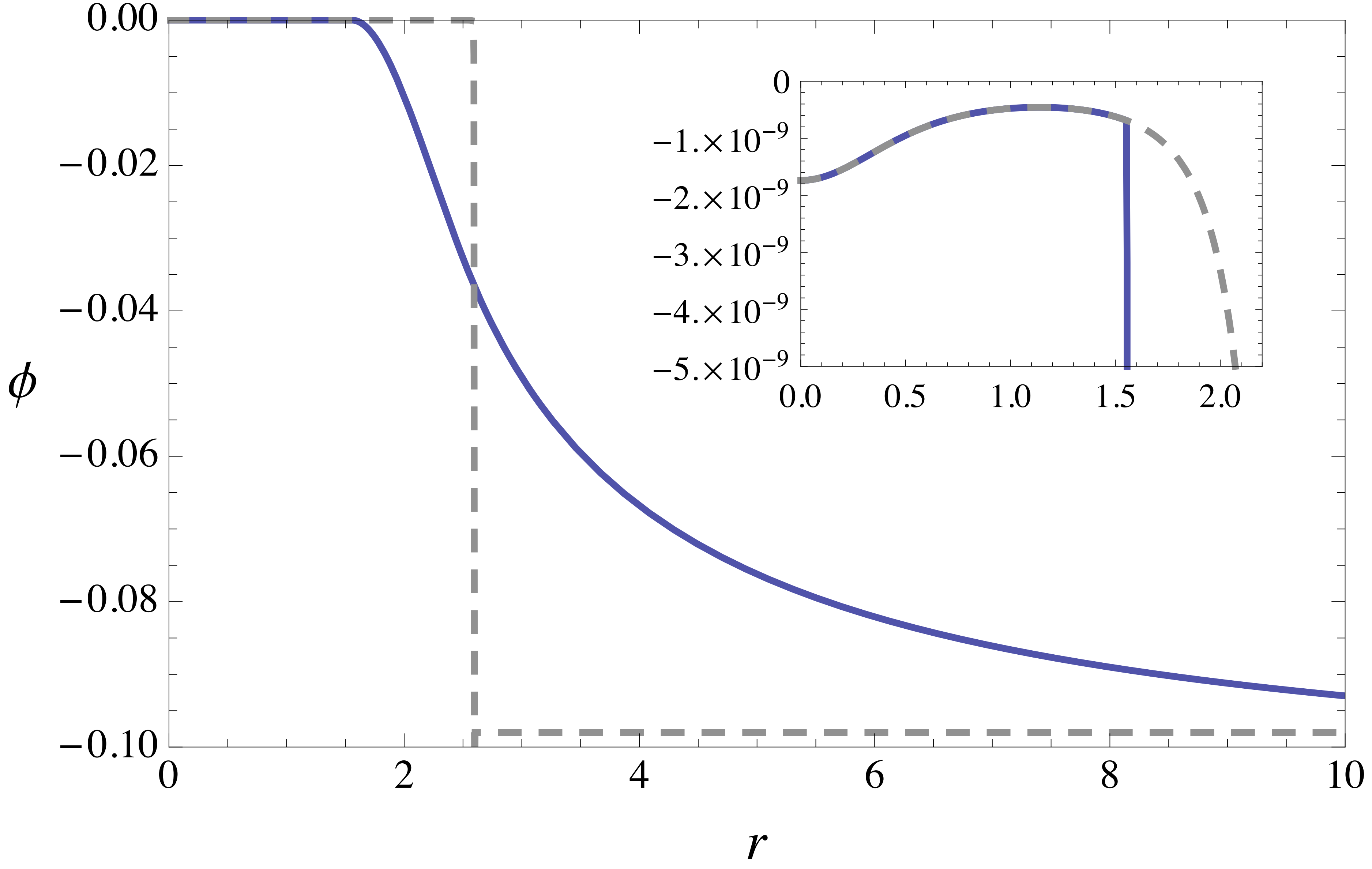


issues

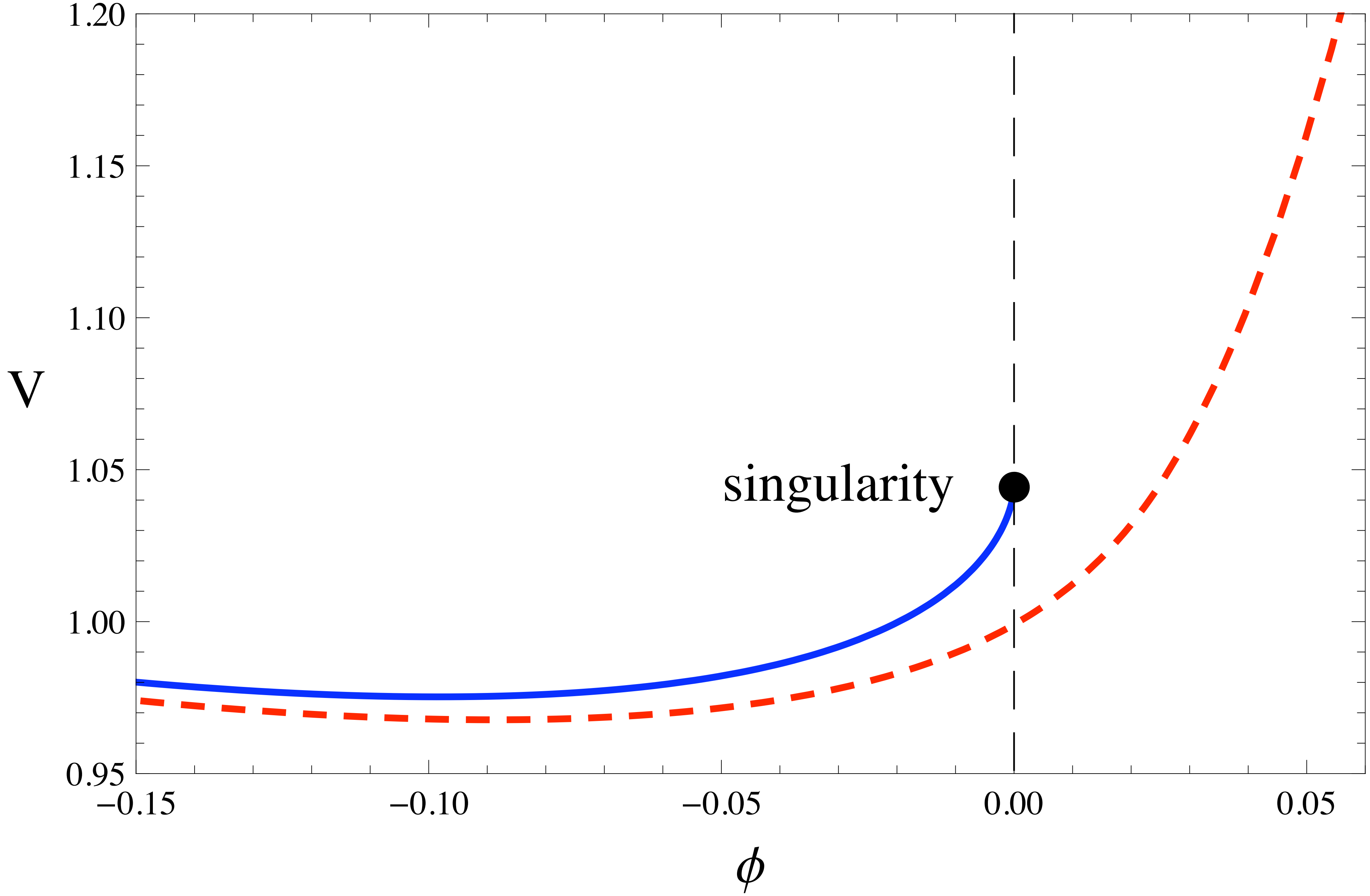
- Inside NS scalar field sits near by the curvature singularity, numerical procedure becomes challenging.
- What happens if the dynamics is included? Is it possible to reach the curvature singularity?
- Singularity is problematic for cosmology

Appleby et al.'09

solution to singularity problem

$$f(R) \to f(R) + \beta R^2$$

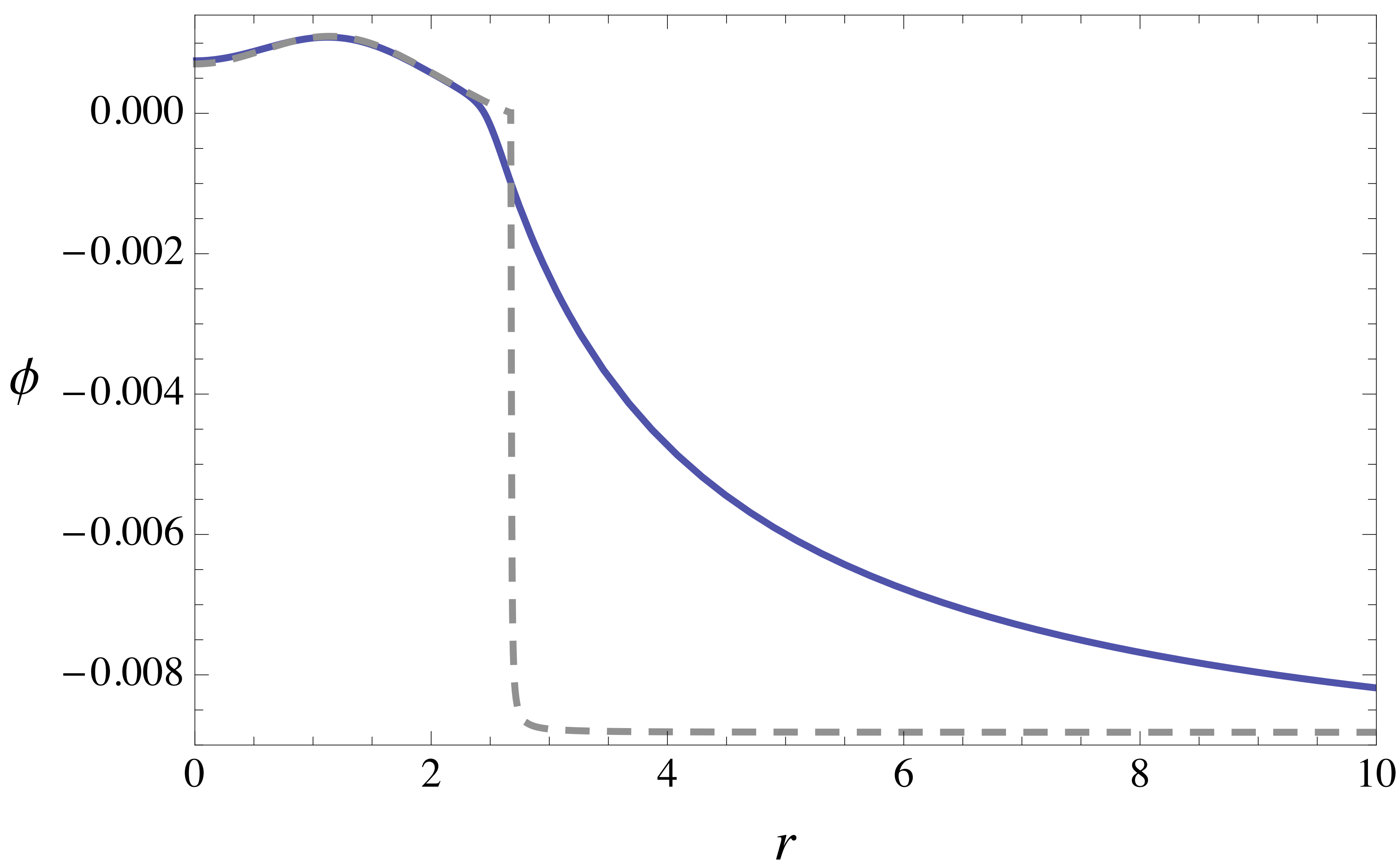
Starobinsky'07



Cured f(R) gravity

$$f(\tilde{R}) = (1-c)\tilde{R} + c\,\epsilon \ln\left[\frac{\cosh(\tilde{R}/\epsilon - b)}{\cosh b}\right] + \frac{\tilde{R}^2}{6M^2}, \quad \epsilon \equiv \frac{R_0}{b + \ln(2\cosh b)}$$

Appleby et al.'09



conclusions

- Static spherically symmetric configurations exist in f(R) gravity and chameleon-like scalar-tensor theories in the relativistic regime, in particular neutron stars exists (was confirmed in part recently by Upadhye&Hu).
- No solutions for stars with large regions having $\rho - 3P < 0$.
- Tachyon instability is generic if EoS for matter $\rho - 3P < 0$.
- Possible issues for dynamically evolving systems in f(R) with singularity
- Solution for possible issues is to cure the curvature singularity