Effect of peculiar motion in weak lensing

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Overview

- Description of the shear γ and the convergence κ.
- Peculiar velocities of galaxies add corrections to the standard formula for γ and κ. C. Bonvin, PRD 78, 123530 (2008)
  - The effect on the shear is second order, i.e. negligible in the calculation of the power spectrum.
  - The effect on the convergence is first order, hence it has an observable impact on the power spectrum.
- The consistency relation between κ and γ is modified by peculiar motion.

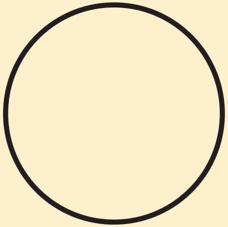
  We could use this to measure the peculiar velocities of galaxies.

Weak lensing

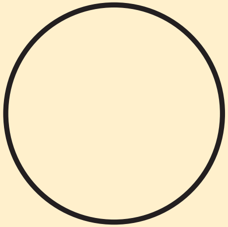
Lensing describes the deflection of light by the gravitational potential $\Rightarrow$ it is sensitive to the distribution of matter.

Lensing is a tool to map the large-scale structure of the Universe.

The distortion created by weak lensing can be split in two parts: the convergence and the shear.



Convergence κ



Shear γ

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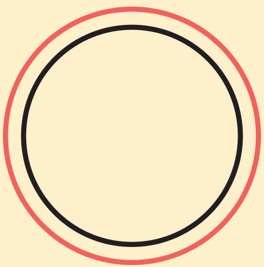

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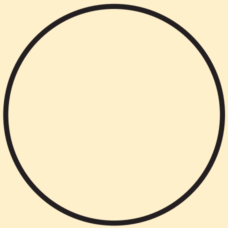

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Convergence κ Shear γ

Observations

Shear

The correlation between the ellipticity of galaxies is measured.

First measurements in 2000 : Bacon *et al.*, Kaiser *et al.*, Wittman *et al.*, L. van Waerbeke *et al.*. Recently: Fu *et al.* (2007), with CFHTLS.

Future measurements (DES, Pan-STARR, EUCLID, LSST...) should reach 1% accuracy.

Convergence

The intrinsic size of the galaxies is unknown. The number of galaxies in a unit solid angle at a given z and flux is known.

Recent measurements Scranton *et al.* (2005), Broadhurst *et al.* (2008) are in agreement with shear observations.

In the future the SKA plan to reach 1% accuracy, using redshift information.

Expression in term of Ψ

The shear and the convergence are related to the gravitational potential.

Schneider, Ehlers and Falco (1992)

$$\gamma_1 = (E_1^i E_1^j - E_2^i E_2^j)\partial_i \partial_j \hat{\Psi}_S \qquad \gamma_2 = 2E_1^i E_2^j \partial_i \partial_j \hat{\Psi}_S$$

$$\kappa = (E_1^i E_1^j + E_2^i E_2^j)\partial_i \partial_j \hat{\Psi}_S \equiv \Delta_\perp \hat{\Psi}_S$$

Integrated potential: $\displaystyle \hat{\Psi}_S = \int_{\eta_S}^{\eta_O} d\eta \frac{(\eta - \eta_S)(\eta_O - \eta)}{\eta_O - \eta_S} \Psi(\eta, \mathbf{x}(\eta))$

From the measurement of the shear and the convergence, it is possible to reconstruct the potential along the line of sight.

The shear and the convergence are not independent.

Consistency relation

In the flat sky approximation

$$\gamma(\boldsymbol{\beta}) = \gamma_1(\boldsymbol{\beta}) + i\gamma_2(\boldsymbol{\beta}) = \frac{1}{\pi}\int d^2\beta' D(\boldsymbol{\beta} - \boldsymbol{\beta}')\kappa(\boldsymbol{\beta}')$$

where

$$D(\boldsymbol{\beta}) = \frac{\beta_2^2 - \beta_1^2 - 2i\beta_1\beta_2}{|\boldsymbol{\beta}|^4}$$

The power spectra are equal $P_\gamma(k) = P_\kappa(k)$

In the all sky calculation

$$C_\ell^\kappa = \frac{\ell(\ell+1)}{(\ell+2)(\ell-1)} C_\ell^\gamma$$

Hu (2000)

Shear and convergence

These relations are not completely general.

$$\gamma_1 \;=\; (E_1^i E_1^j - E_2^i E_2^j)\partial_i \partial_j \hat{\Psi}_S \qquad \gamma_2 = 2E_1^i E_2^j \partial_i \partial_j \hat{\Psi}_S$$

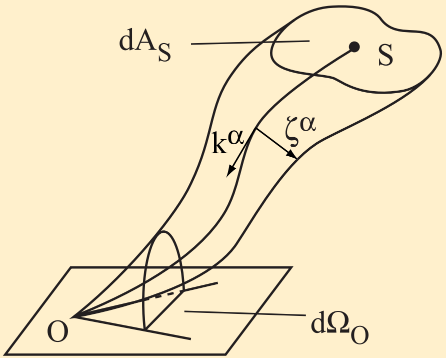
$$\kappa = (E_1^i E_1^j + E_2^i E_2^j)\partial_i \partial_j \hat{\Psi}_S \equiv \Delta_\perp \hat{\Psi}_S$$

Extra terms contribute to γ and κ.

The terms involving the peculiar velocities of galaxies are not negligible.

They modify the consistency relation between the shear and the convergence.

The Jacobi map



k^α photon direction
ξ^α connection vector

Sachs (1961)

$$\frac{D^2\xi^\alpha(\lambda)}{D\lambda^2} = R^\alpha_{\beta\gamma\delta}k^\beta k^\gamma \xi^\delta$$

Solution: $\xi^\alpha_S = J^\alpha_\beta(S,O)\delta\theta^\beta_O$

- $\xi^\alpha_S \perp v^\alpha_S, k^\alpha_S$: describes the surface of the source.
- $\delta\theta^\alpha_O \perp v^\alpha_O, k^\alpha_O$: describes the angle of observation.
- J^α_β, Jacobi map: describes the deformation of light.

Question: how do we extract κ and γ from J^α_β ?

Reduction of $J^{\alpha}_{\ \beta}$

Seitz, Schneider and Ehlers (1994)

We choose a basis at O: $\left[v^{\alpha}_{O}(\lambda_O),\ n^{\alpha}_{O}(\lambda_O),\ E^{\alpha}_{1}(\lambda_O),\ E^{\alpha}_{2}(\lambda_O)\right]$

with $n^{\alpha}_{O} = \frac{1}{\omega_O} k^{\alpha}(\lambda_O) - v^{\alpha}_{O}$

$$\delta\theta^{\alpha}_{O} = \theta_1 E^{\alpha}_{1}(\lambda_O) + \theta_2 E^{\alpha}_{2}(\lambda_O)$$

We parallel transport the basis at S. $\xi^{\alpha}(\lambda) k_{\alpha}(\lambda) = 0 \ \Rightarrow$

$$\begin{aligned}
\xi^{\alpha}_{S} &= \xi_1 E^{\alpha}_{1}(\lambda_S) + \xi_2 E^{\alpha}_{2}(\lambda_S) + \xi_0 \left[n^{\alpha}_{O}(\lambda_S) + v^{\alpha}_{O}(\lambda_S)\right] \\
&= \xi_1 E^{\alpha}_{1}(\lambda_S) + \xi_2 E^{\alpha}_{2}(\lambda_S) + \frac{\xi_0}{\omega_O} k^{\alpha}(\lambda_S)
\end{aligned}$$

Reduction of J^{α}_{β}

$$\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_0 \end{pmatrix}(\lambda_S) = \begin{pmatrix} \hat{J}^i_j(\lambda_O, \lambda_S) \end{pmatrix} \cdot \begin{pmatrix} \theta_1 \\ \theta_2 \\ 0 \end{pmatrix}(\lambda_O)$$

In the usual calculation, ξ_0 is neglected

Seitz, Schneider and Ehlers (1994)

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \frac{(\eta_O - \eta_S)}{1+z} \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \cdot \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

This is correct if $v^{\alpha}_S = v^{\alpha}_O(\lambda_S)$

However, usually the peculiar velocities are different.

$\Rightarrow$ We need to consider the 3x3 matrix to describe correctly the deformation of the source.

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$$\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_0 \end{pmatrix} (\lambda_S) = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix} \cdot \begin{pmatrix} \theta_1 \\ \theta_2 \\ 0 \end{pmatrix} (\lambda_O)$$

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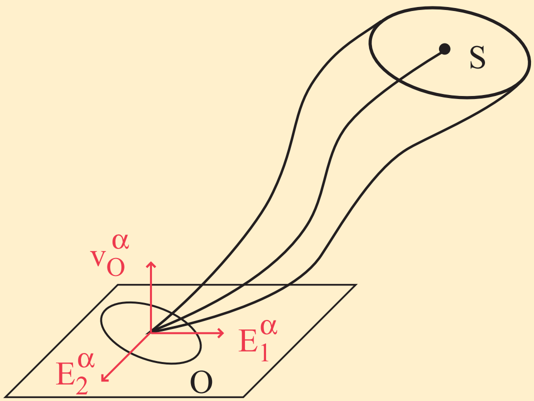
Difference between v_O^α and v_S^α



Basis at the observer:

$$\Big(v_O^\alpha(\lambda_O), n_O^\alpha(\lambda_O), E_1^\alpha(\lambda_O), E_2^\alpha(\lambda_O)\Big)$$

First difference with respect to the usual calculation.

This generates an additional shear component, that is second order in the velocity. Therefore it is negligible in the calculation of the power spectrum.

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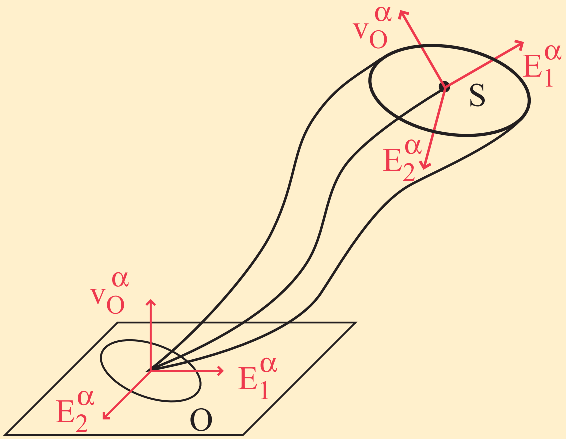

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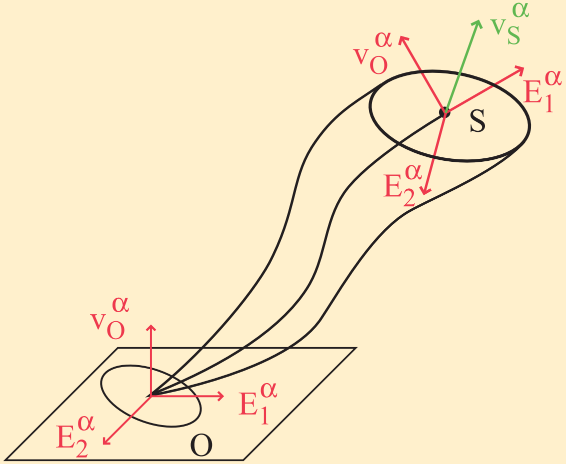

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Calculation of $\hat{J}^i{}_j$

$$g_{\mu\nu}dx^\mu dx^\nu = -a^2(1+2\Psi)d\eta^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

We calculate J^α_β at first order in the metric perturbation Ψ.

We extract $\hat{J}^i{}_j(\eta_S, \mathbf{n})$.

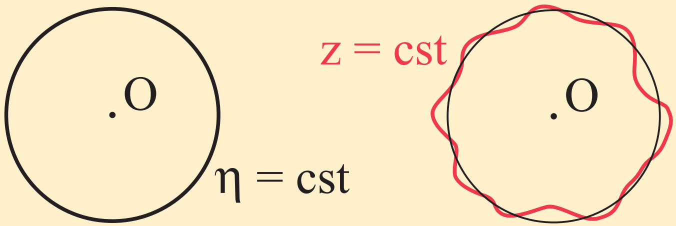
Since η_S is not a measurable quantity, we have to write $\hat{J}^i{}_j$ as a function of the redshift z_S.

$\Rightarrow$ It generates an additional first order contribution to $\hat{J}^i{}_j$, because the observed redshift is perturbed.

This is the second difference from the standard calculation: usually the perturbations of the redshift are neglected.

$\hat{J}^i{}_j$ as a function of z_S

Usually the correlations of the shear and the convergence are evaluated on spheres of constant conformal time η.



We do not know how to select a sphere of constant η, but we can select a sphere of constant redshift z.

We measure correlations on distorted spheres $\Rightarrow$ additional contributions.

It has an effect on the convergence, but not on the shear.

κ and γ

$$\hat{J}^i_j(z_S,\mathbf{n}) = -\frac{\eta_O-\eta_S}{1+z_S}\begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 & 0 \\ -\gamma_2 & 1-\kappa+\gamma_1 & 0 \\ \mathbf{w}\cdot\mathbf{E}_1 & \mathbf{w}\cdot\mathbf{E}_2 & 0 \end{pmatrix}$$

$$\gamma_1 = \left(E^i_1E^j_1 - E^i_2E^j_2\right)\partial_i\partial_j\hat{\Psi}_S \qquad \gamma_2 = 2E^i_1E^j_2\partial_i\partial_j\hat{\Psi}_S$$

$$\begin{aligned}\kappa &= \Delta_\perp\hat{\Psi}_S - \Psi_S - 3\Psi_O + \frac{4}{\eta_O-\eta_S}\int_{\eta_S}^{\eta_O} d\eta\Psi + 2\int_{\eta_S}^{\eta_O} d\eta\frac{\eta-\eta_S}{\eta_O-\eta_S}\dot{\Psi} \\ &+ \left(1-\frac{1}{\mathcal{H}_S(\eta_O-\eta_S)}\right)\left[(\mathbf{v}_S-\mathbf{v}_O)\cdot\mathbf{n} + \Psi_S - \Psi_O + 2\int_{\eta_S}^{\eta_O} d\eta\dot{\Psi}\right] \\ \mathbf{w} &= \mathbf{v}_S - \mathbf{v}_O - \int_{\eta_S}^{\eta_O} d\eta\nabla\Psi\end{aligned}$$

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Effect of the third line

Assume $\Psi = 0$

$$\hat{J}^i_j(\eta_O, \eta_S) \;=\; -\frac{\eta_O - \eta_S}{1 + z_S} \begin{pmatrix} 1-\kappa & 0 & 0 \\ 0 & 1-\kappa & 0 \\ (\mathbf{v}_S - \mathbf{v}_O)\mathbf{E}_1 & (\mathbf{v}_S - \mathbf{v}_O)\mathbf{E}_2 & 0 \end{pmatrix}$$

$$\text{where} \quad \kappa = \left(1 - \frac{1}{\mathcal{H}_S(\eta_O - \eta_S)}\right)(\mathbf{v}_S - \mathbf{v}_O)\cdot\mathbf{n}$$

We consider a circular source of radius r and apply $\hat{J}^i_j$

$$(\theta_1\,,\,\theta_2) \quad \text{describes an ellipse:} \qquad \frac{\theta_1^2}{a^2} + \frac{\theta_2^2}{b^2} = 1$$

$$b = \frac{r}{1-\kappa} \qquad\qquad a = \frac{r}{(1-\kappa)\sqrt{1 + \left[(\mathbf{v}_S - \mathbf{v}_O)\mathbf{E}_1\right]^2}}$$

Effect of the third line NEGLIGIBLE

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$$\mathbf{w} = \mathbf{v}_S - \mathbf{v}_O - \int_{\eta_S}^{\eta_O} d\eta \nabla \Psi$$

Effect of peculiar motion on κ

$$\kappa_\Psi = \int_{\eta_S}^{\eta_O} d\eta \frac{(\eta - \eta_S)(\eta_O - \eta)}{\eta_O - \eta_S} \Delta_\perp \Psi$$

$$\kappa_{\mathbf{v}} = \left(1 - \frac{1}{\mathcal{H}_S(\eta_O - \eta_S)}\right)(\mathbf{v}_S - \mathbf{v}_O) \cdot \mathbf{n}$$

The convergence is not directly observable. The measurable quantity is the overdensity of galaxies

Broadhurst, Taylor and Peacock (1995)

$$\delta_g = \frac{n(f) - n_0(f)}{n_0(f)} \simeq 2\big(\alpha - 1\big)(\kappa_\Psi + \kappa_{\mathbf{v}})$$

α comes from the modelization of $n_0(f)$: it is known.

$\Rightarrow$ Measurements of δ_g corresponds to measurements of κ.

The angular power spectra

$\delta_g(z_S, \mathbf{n})$ depends on the direction of observation.

We expand it in spherical harmonics, and determine the angular power spectrum

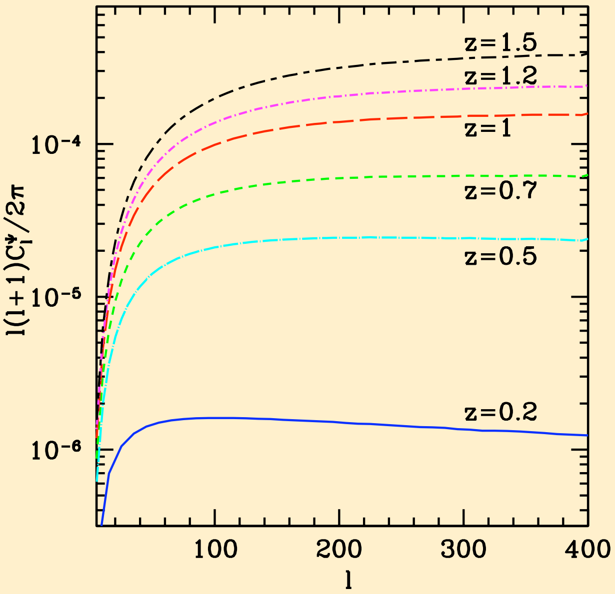
$$\langle \delta_g(z_S, \mathbf{n}) \delta_g(z_{S'}, \mathbf{n}') \rangle = \sum_\ell \frac{2\ell + 1}{4\pi} C_\ell(z_S, z_{S'}) P_\ell(\mathbf{n} \cdot \mathbf{n}')$$

The angular power spectrum contains two contributions

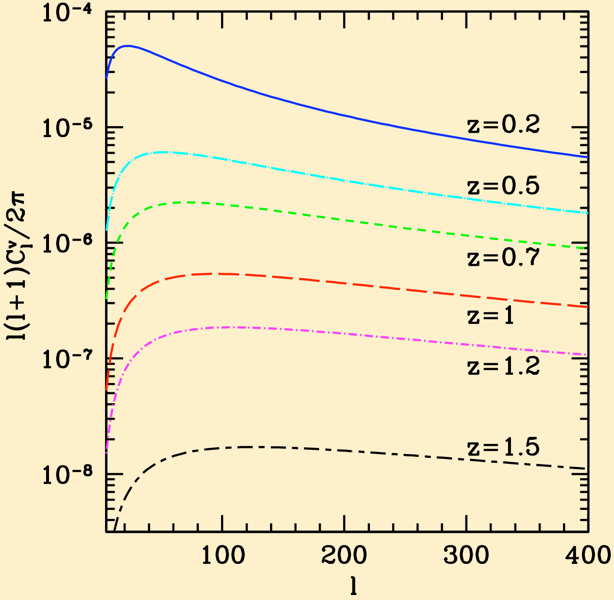
$$C_\ell = C_\ell^\Psi + C_\ell^{\mathbf{v}}$$

We choose a gaussian primordial power spectrum for Ψ and we use Einstein's equations to relate $\mathbf{v}$ to Ψ.

The angular power spectra

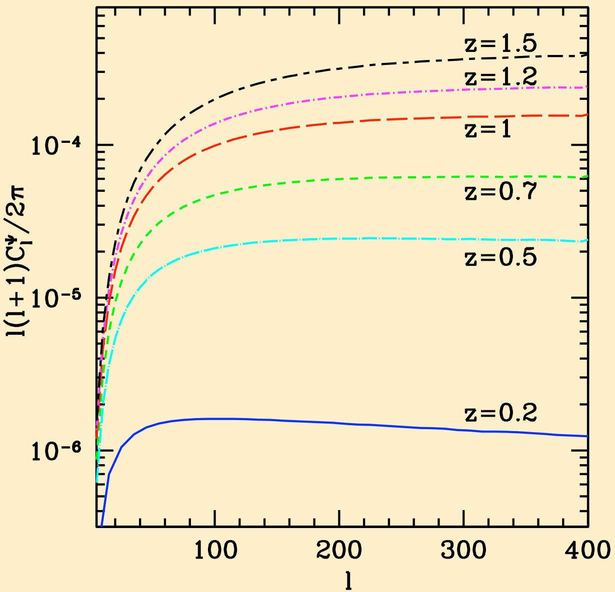


Contribution from the potential κ_Ψ

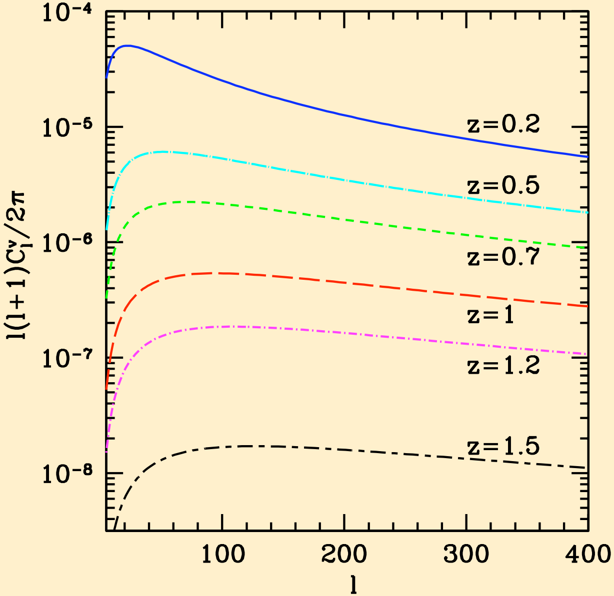


Contribution from the velocity $\kappa_{\mathbf{v}}$

The angular power spectra OBSERVABLE

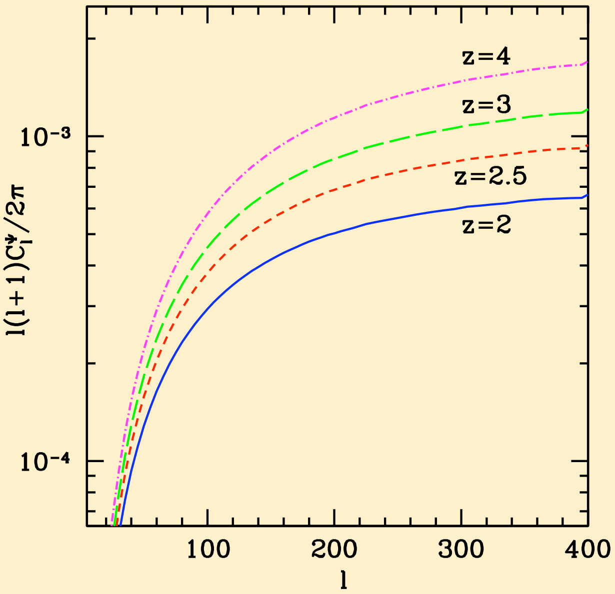


Contribution from the potential κ_Ψ

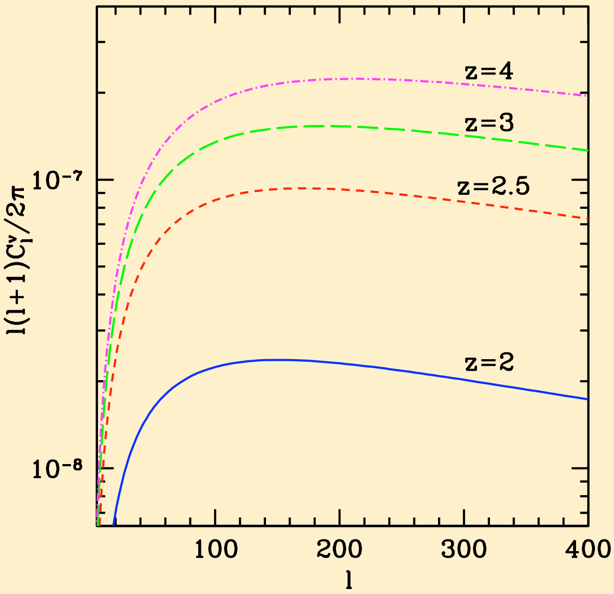


Contribution from the velocity $\kappa_{\mathbf{v}}$

The angular power spectra



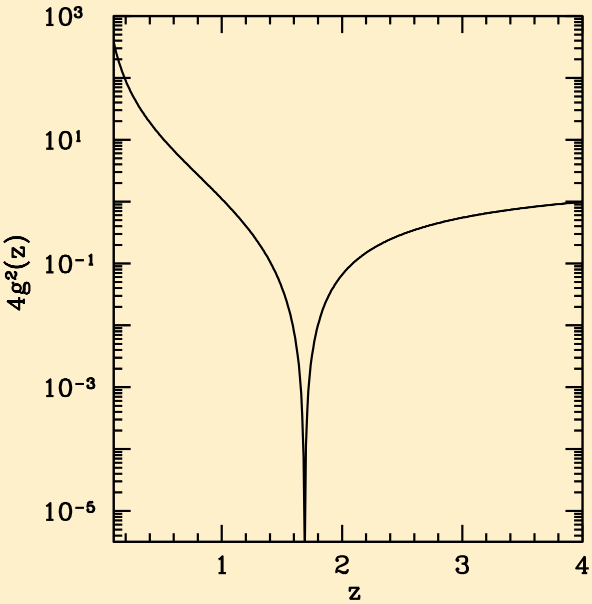
Contribution from the potential κ_Ψ

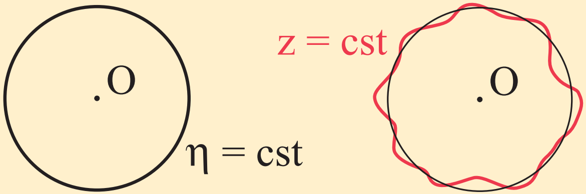


Contribution from the velocity $\kappa_\mathbf{v}$

Evolution of the velocity term

$$\kappa_{\mathbf{v}} = \left(1 - \frac{1}{\mathcal{H}_S(\eta_O - \eta_S)}\right)(\mathbf{v}_S - \mathbf{v}_O)\cdot\mathbf{n}$$





Two opposite effects:

Further away → smaller angle → demagnification

Smaller scale factor → larger stretch → magnification

Summary

The peculiar velocities have two effects on weak lensing:

- The plane of the source differs from the plane of the image
  $\longrightarrow$ New contribution to the shear, that is second order in the velocity.
- The sphere of constant redshift is distorted
  $\longrightarrow$ New contribution to the convergence, that is first order in the velocity. It is measurable up to redshift 1.

Two questions

What is the effect on the reduced shear ?

What is the effect on the consistency relation ?

Reduced shear

Experiments measure the reduced shear

$$g = \frac{\gamma}{1-\kappa} \simeq \gamma + \gamma\kappa$$

$$\langle g(z_S, \mathbf{n}) g(z_S, \mathbf{n}') \rangle = \langle \gamma(z_S, \mathbf{n}) \gamma(z_S, \mathbf{n}') \rangle + 2 \langle \kappa(z_S, \mathbf{n}) \gamma(z_S, \mathbf{n}) \gamma(z_S, \mathbf{n}') \rangle$$

$\langle \kappa_\Psi \gamma\gamma \rangle \longrightarrow$ impact on parameter estimation

Dodelson, Shapiro and White (2006)

$\langle \kappa_{\mathbf{v}} \gamma\gamma \rangle = 0$

γ is perpendicular to $\mathbf{n}$ and $\kappa_{\mathbf{v}}$ is along $\mathbf{n} \Rightarrow$ no correlations.

Peculiar velocities have no effect on the reduced shear correlations.

Consistency relation

In the flat sky approximation

$$\gamma(\boldsymbol{\beta}) = \gamma_1(\boldsymbol{\beta}) + i\gamma_2(\boldsymbol{\beta}) = \frac{1}{\pi}\int d^2\beta' D(\boldsymbol{\beta} - \boldsymbol{\beta}')\kappa(\boldsymbol{\beta}') \ ,$$

$$\text{where} \quad D(\boldsymbol{\beta}) = \frac{\beta_2^2 - \beta_1^2 - 2i\beta_1\beta_2}{|\boldsymbol{\beta}|^4} \ .$$

The power spectra satisfy $P_\gamma(k) = P_{\kappa_{tot}}(k) - P_{\kappa_{\mathbf{v}}}(k)$

In the all sky calculation

$$\frac{\ell(\ell+1)}{(\ell+2)(\ell-1)} C_\ell^\gamma = C_\ell^{\kappa_{tot}} - C_\ell^{\kappa_{\mathbf{v}}}$$

We can use this relation to measure the peculiar velocities of galaxies.

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$$\text{where} \qquad D(\boldsymbol{\beta}) = \frac{\beta_2^2 - \beta_1^2 - 2i\beta_1\beta_2}{|\boldsymbol{\beta}|^4} \; .$$

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Characteristic of experiment

In order to detect the effect of peculiar motion, an experiment must satisfy 3 criteria

- cover a large part of the sky: $C_\ell^{\mathbf{v}}$ pics at $\ell \sim 30 - 150$, i.e. $\theta \sim 70$ arcmin -6 degrees.

  CFHTLS already up to 4 degrees, near future 8 degrees. EUCLID, SKA 20'000 square degrees.

- remove intrinsic clustering $\rightarrow$ precise measurements of z

  SKA: 21cm emission line

  other: photometric measurements.

- cover redshifts smaller than 1.

Conclusion

- The peculiar velocity of galaxies affect weak lensing observations.
- The effect on the shear is second order in the velocity $\rightarrow$ negligible for measurements of the power spectrum.

  It could play a role in higher order correlations (bispectrum).
- The effect on the convergence is first order in the velocity $\rightarrow$ important for measurements, especially for $z \leq 1$.
- The consistency relation between the shear and the convergence is modified by peculiar velocities.
- This could be used to measure the velocity of galaxies.