The Dispersion of Growth of Matter Perturbations in f(R) Models (arXiv:0908.2669)

S. Tsujikawa[(1)], **R. Gannouji**[(2)], **B. Moraes**[(3)], **D. Polarski**[(3)]

(1) Department of Physics, Faculty of Science, Tokyo University of Science, 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan
(2) IUCCA, Post Bag 4, Ganeshkhind, Pune 411 007, India
(3) Laboratoire de Physique Théorique et Astroparticules, CNRS UMR 5207, Université Montpellier 2, Montpellier, France

26.10.2009

Séminaires GReCo - IAP/Paris

I. $f(R)$ Background : Action and background equations

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G_*}f(R) + \mathcal{L}_m\right],$$

$$3FH^2 = 8\pi G_*\,(\rho_m + \rho_{rad}) + \frac{1}{2}(FR - f) - 3H\dot{F}\ ,$$

$$-2F\dot{H} = 8\pi G_*\left(\rho_m + \frac{4}{3}\rho_{rad}\right) + \ddot{F} - H\dot{F},$$

$$F \equiv \frac{df}{dR}.$$

I. $f(R)$ Background : Definition of ρ_{DE}

- If we want to rewrite gravity modifications as an effective dark energy fluid, we want it to obey $\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0$. Therefore, we define :

$$8\pi G_* \ \rho_{DE} = \frac{1}{2}(FR - f) - 3H\dot{F} + 3(1 - F) \ H^2$$

- This way, we have the Friedmann Equations as usual.

$$3H^2 = 8\pi G_* \ [\rho_m + \rho_{rad} + \rho_{DE}]$$

$$-2\dot{H} = 8\pi G_* \left[\rho_m + \frac{4}{3}\rho_{rad} + \rho_{DE} + p_{DE}\right]$$

- Comment : There's an ambiguity in the definition of the energy densities Ω_i

I. $f(R)$ Background : Viability Conditions

- $F > 0$, $F_{,R} > 0$.
- Viable cosmological background history.(Amendola et al., 2007)
- $f(R) \to R - 2\Lambda$ for $R \gg R_0$, where R_0 is the Ricci scalar today.
- Stability of the late time de Sitter point : $F > RF_{,R}$ for $R = R_1$. (Starobinsky, 2007)
- Local Gravity Constraints (Chameleon Mechanism).

II. Linear Perturbations : Full Equations

The full linear perturbation equations in Fourier space for an f(R) model are given by (Hwang et al., 2005) :

$$\ddot{\delta}_m + \left(2H + \frac{\dot{F}}{2F}\right)\dot{\delta}_m - \frac{\rho_m}{2F}\delta_m$$
$$= \frac{1}{2F}\left[\left(-6H^2 + \frac{k^2}{a^2}\right)\delta F + 3H\dot{\delta F} + 3\ddot{\delta F}\right],$$

$$\ddot{\delta F} + 3H\dot{\delta F} + \left(\frac{k^2}{a^2} + \frac{F}{3F_{,R}} - \frac{R}{3}\right)\delta F$$
$$= \frac{1}{3}\rho_m\delta_m + \dot{F}\dot{\delta}_m\,,$$

II. Linear Perturbations : Approximations

For a conceptual analysis, we can use the following approximations :

- Sub-Hubble Approximation : $k^2 \gg a^2 H^2$
- Quasi-Static Approximation

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\ \rho_m\ \delta_m = 0\ ,$$

$$G_{\rm eff} = \frac{G_*}{F}\ \frac{1 + 4\frac{k^2 F_R}{a^2 F}}{1 + 3\frac{k^2 F_R}{a^2 F}}, \qquad F_R \equiv \frac{dF}{dR}.$$

(Tsujikawa et al., 2008)

II. Linear Perturbations : Asymptotical Regimes

Let's look more closely at the form of $G_{\rm eff}$. If we define $M^2 = \frac{F}{3F_R}$, then :

$$G_{\rm eff} = \frac{G_*}{F} \left(1 + \frac{1}{3}\frac{\frac{k^2}{a^2M^2}}{1+\frac{k^2}{a^2M^2}}\right).$$

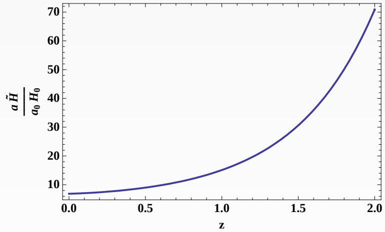
$$M^2 \gg k^2/a^2 \quad \Longrightarrow \quad G_{\rm eff} \approx \frac{G_*}{F}. \quad \text{(GR regime)}$$

$$M^2 \ll k^2/a^2 \quad \Longrightarrow \quad G_{\rm eff} \approx \frac{4}{3}\frac{G_*}{F}. \quad \text{("Scalar-Tensor" regime)}$$

The mass M introduces a critical length scale.

II. Linear Perturbations : Critical Length Scale

- In viable $f(R)$ models, M is a decreasing function of time.
- Therefore, the effect of the scalar degree of freedom is felt from small to large scales progressively.
- How can we study the effect of this behaviour in the growth of matter perturbations ?

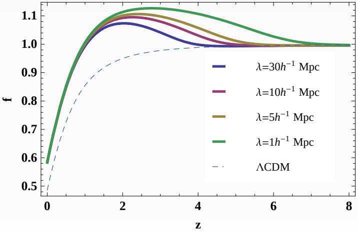


II. Linear Perturbations : Growth Rate

To study the growth of linear perturbations, we'd like to start by focusing on the growth rate :

$$s \equiv \frac{d \ln \delta}{d \ln a}$$

$s(z)$:

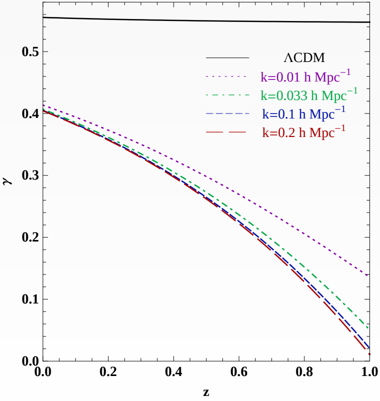


II. Linear Perturbations : γ Parameterization

We further make use of the standard parameterization.

$\gamma(z)$:

$$s(z) = \frac{d \ln \delta}{d \ln a} \equiv \Omega_m(z)^{\gamma(z)}$$



In GR, γ is nearly constant with $\gamma_0 \approx 0.55$. But this is not necessarily the case in $f(R)$ models (Gannouji, BM, Polarski, 2008).

III. Application : Some Viable Models

$$f(R) = R - \lambda R_c\, f_1(x)\,, \qquad x \equiv R/R_c\,, \tag{1}$$

We will study the following models :

- (A) $f_1(x) = x^{2n}/(x^{2n}+1) \quad (n > 0)$ (Hu & Sawicky, 2007) ,
- (B) $f_1(x) = 1 - (1+x^2)^{-n} \quad (n > 0)$ (Starobinsky, 2007) ,
- (C) $f_1(x) = 1 - e^{-x}$ (Linder, 2009) ,
- (D) $f_1(x) = \tanh(x)$ (Tsujikawa, 2008) .

III. Application : A Comment About The Solution

Let's take a closer look at the equation for δF. In the matter era, it can be written :

$$\delta\ddot{F} + 3H\delta\dot{F} + \left(\frac{k^2}{a^2} + M^2\right)\delta F \simeq \frac{1}{3}\delta\rho_m.$$

The homogeneous equation can be rewritten as :

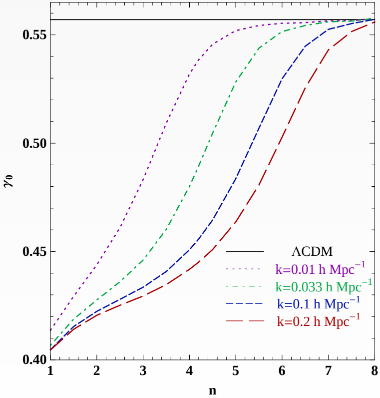
$$(a^{3/2}\delta F_{\rm osc})^{\cdot\cdot} + M^2(a^{3/2}\delta F_{\rm osc}) \simeq 0.$$

and for a specific model :

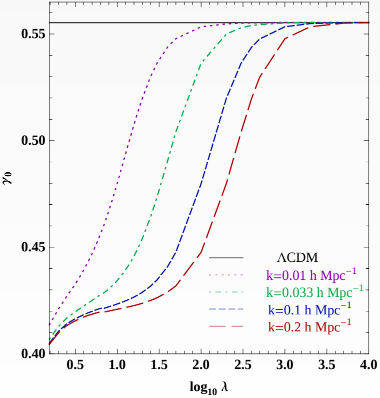
$$\delta R_{\rm osc} \simeq c\, t^{-3n-4}\cos(c_0\, t^{-2n-1})\,. \quad \text{(Tsujikawa, 2007)}$$

III. Application : Models (A) and (B)

$\gamma_0(n)$ (for $\lambda = 1.55$) :

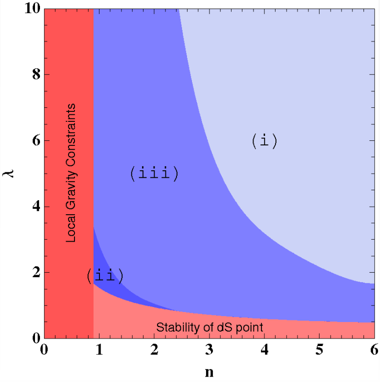


$\gamma_0(\lambda)$ (for $n = 1$) :

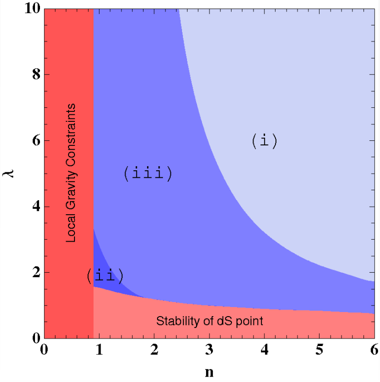


III. Application : Models (A) and (B)

Starobinsky :

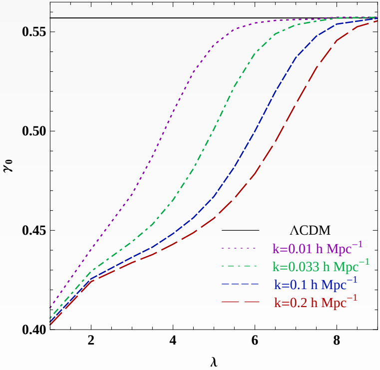


Hu & Sawicky :

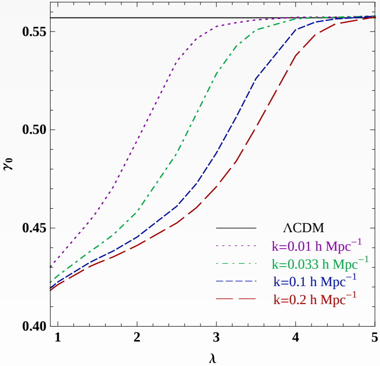


III. Application : Models (C) and (D)

Linder :



Tsujikawa :



IV. Conclusions

- Viable $f(R)$ models present an enhanced growth of matter perturbations.
- This growth can be characterized according to the typical length scale introduced by the scalar degree of freedom, which defines two different asymptotic regimes. The transition between the two regimes (if it occurs today) is responsible for dispersion in the growth.
- For a viable range of parameters, all $f(R)$ models studied present a strong observational signature, and this could be detected in the near future.