Backreaction as an alternative to dark energy and modified gravity

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A factor of 2 in distance

- The early universe is well described by a model which is homogeneous and isotropic, contains only ordinary matter and evolves according to general relativity.
- However, such a model underpredicts the distances measured in the late universe by a factor of 2.
- This is interpreted as faster expansion.
- There are three possibilities:

1) There is matter with negative pressure.
2) General relativity does not hold.
3) The universe is not homogeneous and isotropic.

Backreaction

- **The average evolution of an inhomogeneous and/or anisotropic spacetime is not the same as the evolution of the corresponding smooth spacetime.**
- At late times, non-linear structures form, and the universe is only statistically homogeneous and isotropic, on scales above 100 Mpc.
- Finding the model that describes the average evolution of the clumpy universe was termed **the fitting problem** by George Ellis in 1983.

Backreaction, exactly

- Consider a dust universe. The Einstein equation is

$$G_{\alpha\beta} = 8\pi G \rho u_\alpha u_\beta .$$

- The dynamics can be written in terms of the gradient

$$\nabla_\beta u_\alpha = \frac{1}{3} h_{\alpha\beta}\theta + \sigma_{\alpha\beta} + \omega_{\alpha\beta} .$$

- The scalar part of the Einstein equation is:

$$\begin{cases} \dot{\theta} + \frac{1}{3}\theta^2 = -4\pi G\rho - 2\sigma^2 + 2\omega^2 \\ \frac{1}{3}\theta^2 = 8\pi G\rho - \frac{1}{2}{}^{(3)}R + \sigma^2 - \omega^2 \end{cases}$$

$$\dot{\rho} + \theta\rho = 0$$

- Here θ is the expansion rate, ρ is the energy density, $\sigma^2 \geq 0$ is the shear, $\omega^2 \geq 0$ is the vorticity and ${}^{(3)}R$ is the spatial curvature. We take $\omega^2 = 0$.

- The Buchert equations (1999):

$$\begin{cases} 3\frac{\ddot a}{a} = -4\pi G\langle\rho\rangle + Q \\ 3\frac{\dot a^2}{a^2} = 8\pi G\langle\rho\rangle - \frac{1}{2}\langle {}^{(3)}R\rangle - \frac{1}{2}Q \\ \partial_t\langle\rho\rangle + 3\frac{\dot a}{a}\langle\rho\rangle = 0 \end{cases}$$

$$\dot\theta + \frac{1}{3}\theta^2 = -4\pi G\rho - 2\sigma^2$$
$$\frac{1}{3}\theta^2 = 8\pi G\rho - \frac{1}{2}{}^{(3)}R + \sigma^2$$
$$\dot\rho + \theta\rho = 0$$

- Here $a(t) \propto \left(\int d^3x\sqrt{{}^{(3)}g}\right)^{1/3} \Leftrightarrow \langle\theta\rangle = 3\frac{\dot a}{a}.$

$$\langle f\rangle \equiv \frac{\int d^3x\sqrt{{}^{(3)}g}\,f}{\int d^3x\sqrt{{}^{(3)}g}}$$

- The backreaction variable is

$$Q \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle.$$

- The average expansion can accelerate, even though the local expansion decelerates.

Understanding acceleration

- The average expansion rate can increase, because the fraction of volume occupied by faster regions grows.
- Structure formation involves overdense regions slowing down and underdense regions decelerating less.
- Acceleration can be explicitly demonstrated using a toy model with one overdense and one underdense region.

$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 = v_1 H_1 + v_2 H_2$$

$$\frac{\ddot{a}}{a} = v_1 \frac{\ddot{a}_1}{a_1} + v_2 \frac{\ddot{a}_2}{a_2} + 2 v_1 v_2 (H_1 - H_2)^2$$

- Expansion slows down as the overdense region becomes important, then accelerates as the void takes over.

Towards reality

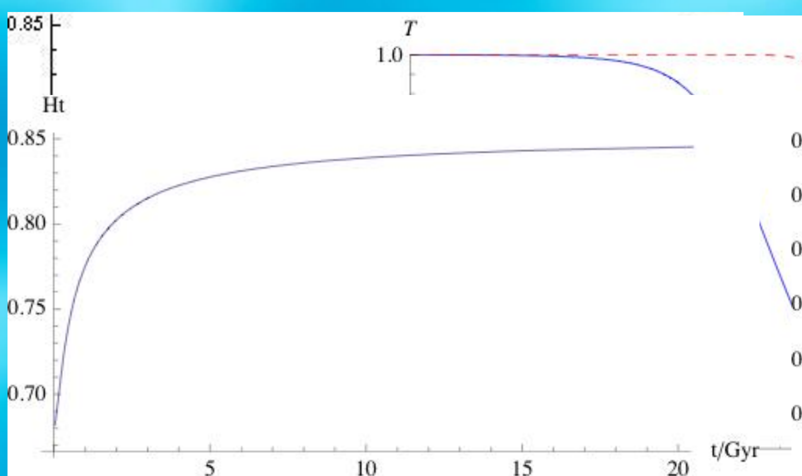
- Acceleration due to structures is possible: is it realised in the universe?
- The non-linear evolution is too complex to follow exactly.
- Because the universe is statistically homogeneous and isotropic, a statistical treatment is sufficient.
- We can evaluate the expansion rate with an evolving ensemble of regions.

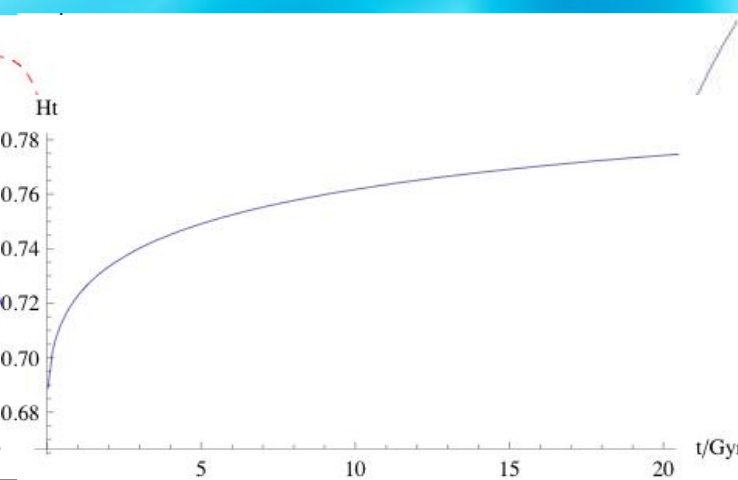
The peak model

- We start from a FRW background of dust with an initial Gaussian linear density field.
- We identify structures with spherical isolated peaks of the smoothed density field. (BBKS 1986)
- We keep the smoothing threshold fixed at $\sigma(t,R)=1$, which gives the time evolution $R(t)$.
- Each peak expands like a separate FRW universe.
- The peak number density as a function of time is determined by the primordial power spectrum and the transfer function.
- We take a scale-invariant spectrum with CDM transfer function.

- The expansion rate is $H(t) = \int_{-\infty}^{\infty} d\delta \; v_\delta(t) H_\delta(t).$
- There are no parameters to adjust.
- Consider two approximate transfer functions.

  Bonvin and Durrer — BBKS (with f_b=0.2)





Ht as a function of t (in Gyr, with t_{eq}=50 000 yr)

Ht as a function of R/R_{eq}

Light propagation

- The average expansion rate is evaluated on the spatial hypersurface of proper time.
- Most cosmological observations are made along the past lightcone, and measure the redshift and the luminosity distance.
- In a general spacetime, these quantities are not determined only by expansion.
- However, in a statistically homogeneous and isotropic dust space, the average expansion rate does give the redshift and the distance.

The redshift

- The redshift+1 is proportional to the photon energy:

$$1+z \propto E = -u_\alpha k^\alpha ,$$ where $$k^\alpha = E\left(u^\alpha + e^\alpha\right).$$

- The change of the energy along the null geodesic is

$$\partial_\lambda E = k^\alpha \nabla_\alpha E = -k^\alpha k^\beta \nabla_\alpha u_\beta = -E^2\left(\frac{1}{3}\theta + \sigma_{\alpha\beta}e^\alpha e^\beta\right)$$

$$\Rightarrow 1+z = \exp\left[\int_\lambda^{\lambda_o} d\lambda E\left(\frac{1}{3}\theta + \sigma_{\alpha\beta}e^\alpha e^\beta\right)\right] = \exp\left[\int_t^{t_o} dt\left(\frac{1}{3}\theta + \sigma_{\alpha\beta}e^\alpha e^\beta\right)\right].$$

- Assuming statistical homogeneity and isotropy, the redshift is given by the scale factor:

$$1+z = \exp\left[\int_t^{t_o} dt\left(\frac{1}{3}\langle\theta\rangle + \frac{1}{3}\Delta\theta + \sigma_{\alpha\beta}e^\alpha e^\beta\right)\right] = \exp\left(\int_t^{t_o} dt\,\frac{1}{3}\langle\theta\rangle\right) = a(t)^{-1}.$$

The distance

- The exact angular diameter distance is given by

  $$\partial_\lambda^2 D_A = -\left(4\pi G_N \rho E^2 + \tilde{\sigma}^2\right) D_A ,$$

  from which we get for the average

  $$H\partial_{\langle z\rangle}\left[\left(1+\langle z\rangle\right)^2 H\partial_{\langle z\rangle}\langle D_A\rangle\right] = -\left(4\pi G_N\langle\rho\rangle + \langle E\rangle^{-2}\langle\tilde{\sigma}^2\rangle\right)\langle D_A\rangle .$$

- Due to conservation of mass, $\langle\rho\rangle \propto a^{-3} \propto \left(1+\langle z\rangle\right)^{-3}$.
- Apart from the null geodesic shear, the distance equation in terms of *H* is the same as in FRW ΛCDM.
- The spatial curvature enters only via *H*, so the CMB is consistent with large spatial curvature.
- Unless *H* is the same as in the FRW ΛCDM model, the relation between the expansion rate and the distance is different than in the FRW case.

Conclusion

- Observations of the late universe are inconsistent with a homogeneous and isotropic model with ordinary matter and gravity.
  - FRW models do not include non-linear structures.
- The Buchert equations show that the average expansion of a clumpy dust space can accelerate.
  - The acceleration has been understood physically.
  - The expansion rate Ht has been found to rise by the right order of magnitude around $10^5\, t_{eq}$.
  - The relationship of the average expansion rate to distance observations has been determined.
- Much work remains to be done to get detailed predictions with quantified errors.