# Primordial non-Gaussianities from string theory: multifield inflation with non-standard actions

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IAP, Paris, 21.09.09

# Outline

## I : Motivation

## II : Inflation and non-Gaussianities

## III : Generalized multifield inflation

## IV : Multifield DBI inflation

# I Motivation

# Inflation

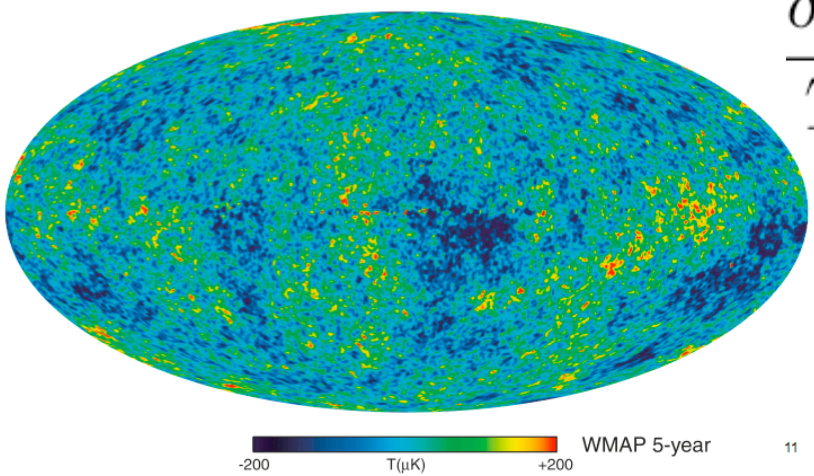
- A period of accelerated expansion before the radiation era that solves the problems of the ‘standard’ Hot Big-Bang model.

- Quasi exponential expansion $H \simeq cte \qquad \epsilon = -\frac{\dot{H}}{H^2} \ll 1$

- Simplest implementation: single field with very flat potential. Its predictions perfectly match the observations:

$$\frac{\delta T}{T} = -\frac{\zeta}{5}$$

← Primordial curvature perturbation:




- nearly scale invariant
- nearly adiabatic
- nearly Gaussian



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# More?

- Simplest models surprisingly difficult to embed in high-energy physics models (eta-problem).

- Many high energy physics models involve several scalar fields. If several scalar fields are light enough during inflation
  ⇨ **multifield inflation**, changes a lot the predictions !

- D-brane action: **non-standard kinetic terms**.

- Alternatives: curvaton, ekpyrotic…

- They are all degenerate at the linear level.

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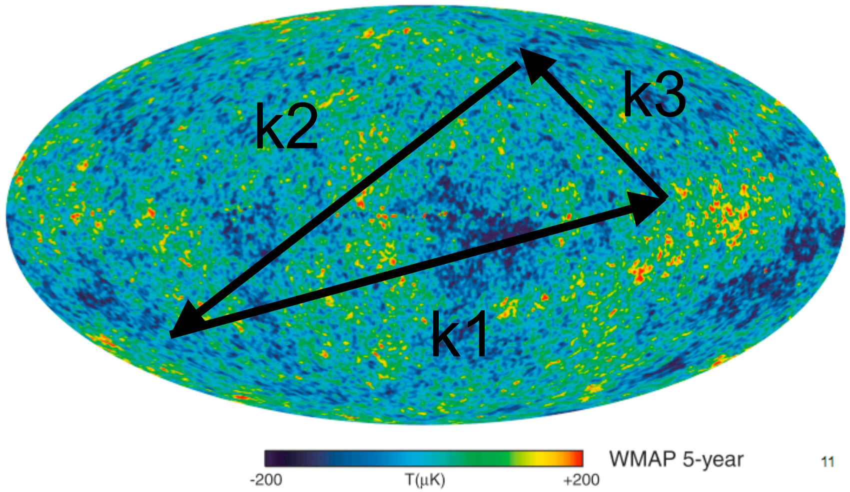
NON GAUSSIANITIES

probe field interactions

# Non-Gaussianities

Beyond the power spectrum:
higher-order, connected,
n-point functions.

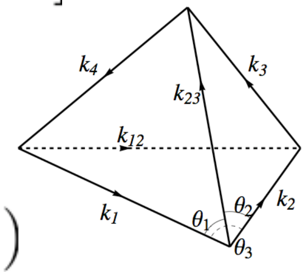



3-point function, the bispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv B_\zeta(k_1, k_2, k_3)(2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$$

$$\equiv \frac{6}{5} f_{NL} \left[ P_\zeta(k_1) P_\zeta(k_2) + \text{perm.} \right]$$

Connected 4-point function of zeta, the trispectrum




$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_c \equiv T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)(2\pi)^3 \delta^3 \left(\sum_{\mathbf{i}} \mathbf{k_i}\right)$$

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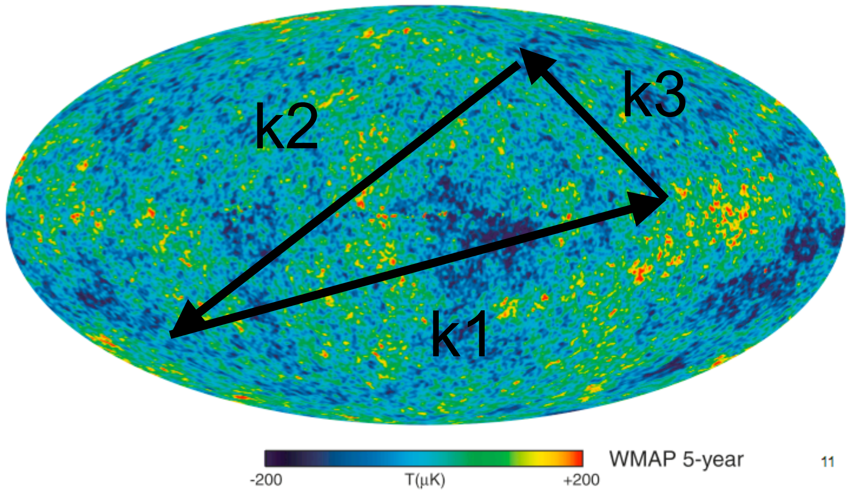
# The Bispectrum

$$B_\zeta(k_1, k_2, k_3)$$




- Amplitude
  Slow-roll single field $f_{NL} \sim 10^{-2}$
  Planck accuracy $\Delta f_{NL} \sim 5$
  Current constraints $f_{NL} = O(100)$

- Shape (largest for which triangles?)

Babich et al (04)
Fergusson & Shellard (08)

Each feature can rule out large classes of models

- Sign (more or less cold spots?)

- Scale-dependence (growing or shrinking on small scales?)

# General idea

Beyond single field slow-roll inflation:
multifield inflation, non standard kinetic terms,
non-inflationary scenarios.

⇨ Predictions for cosmological observables, especially non-Gaussianities.

Outcome:

• General formalisms that can be used in a wide variety of situations.

• Applications to interesting early universe models: multifield DBI inflation, ekpyrotic scenarios.

# II Inflation and non-Gaussianities

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# Standard single-field inflation

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right)$$

- Sasaki-Mukhanov variable

$$v = z\mathcal{R}$$

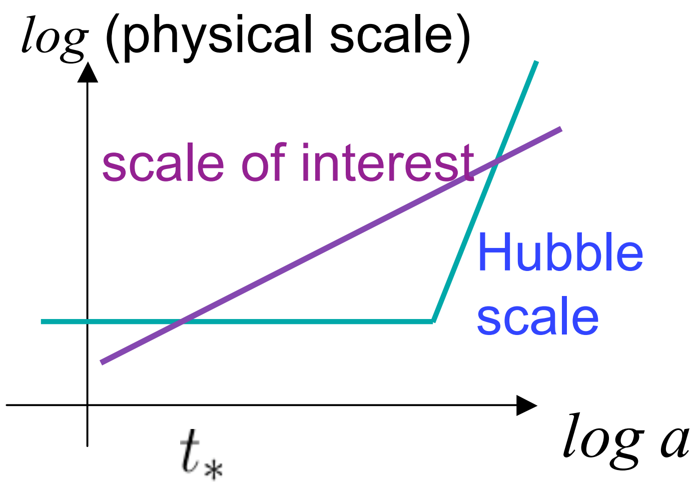
$$v'' + \left( k^2 - \frac{z''}{z} \right) v = 0$$

- Harmonic oscillator with a time-dependent frequency

$$v'' + \left( k^2 - 2a^2H^2 \right) v \simeq 0$$

- The quantum fluctuations of the scalar field are amplified at **Hubble crossing**

$$\mathcal{P}_{\mathcal{R}_*} = \frac{1}{8\pi^2 \epsilon} \left( \frac{H}{M_P} \right)^2 \Bigg|_{k=aH}$$




$\mathcal{R} \simeq -\zeta$ constant on large scales

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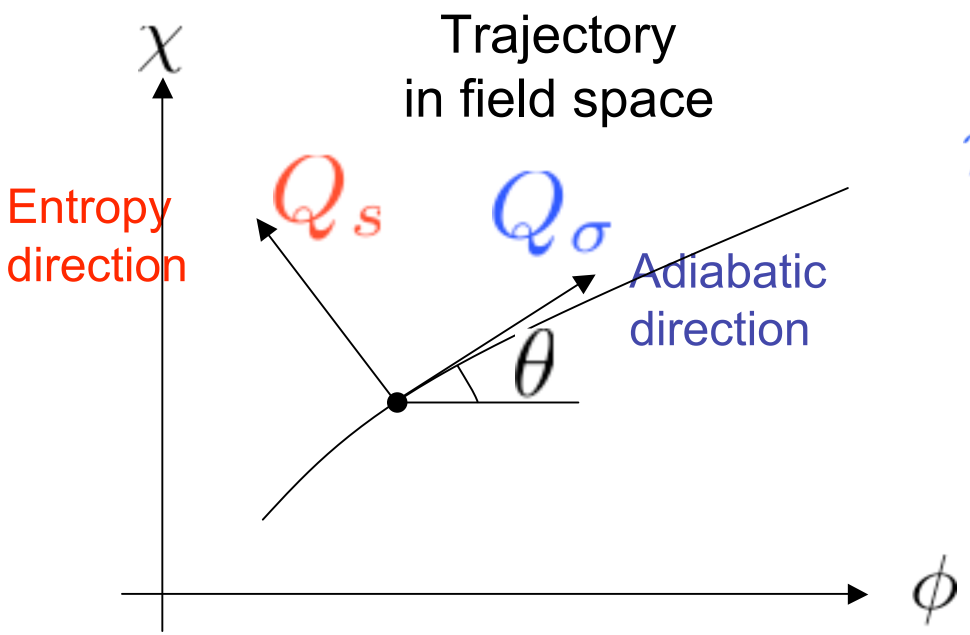
# Multifield inflation

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}G_{IJ}(\phi)\,\partial^\mu\phi^I\partial_\mu\phi^J - V(\phi)\right)$$




- Adiabatic / Entropy decomposition

$$\mathcal{R} = \frac{H}{\dot\sigma}Q_\sigma \quad \dot\sigma^2 = G_{IJ}\dot\phi^I\dot\phi^J$$

- Isocurvature perturbations decoupled from curvature perturbations

- Curvature perturbation is sourced by the isocurvature perturbation

$$\dot{\mathcal{R}} = \frac{2H}{\dot\sigma}\dot\theta\,Q_s + \mathcal{O}\left(\frac{k^2}{a^2H^2}\right)$$

⇨ Need to follow the super-Hubble evolution.

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# K-inflation

Garriga & Mukhanov (99)

- Single-field inflation with non-standard Lagrangian

$P(X, \phi)$ with $X = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi$

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- Perturbations: $v'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v = 0$

  with the speed of sound $\frac{1}{c_s^2} \equiv 1 + \frac{2XP_{,XX}}{P_{,X}}$

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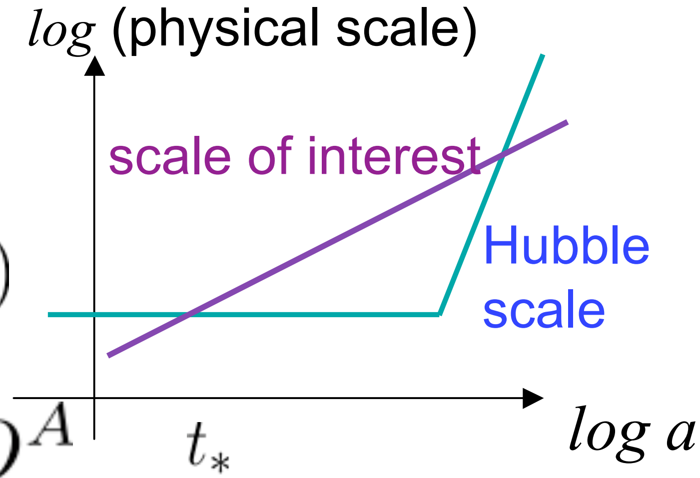
- Amplifications at **sound horizon crossing**

$$c_s k = aH$$

# Non-Gaussianities from the $\delta N$ formalism

M fields $\phi^A$ during inflation

Determine the number of e-folds $N(\phi^A_*)$




Quantum fluctuations $\bar{\phi}^A_* \longrightarrow \bar{\phi}^A_* + Q^A$

⇨ Fluctuation in local e-folding number = curvature perturbation

$$\zeta = \delta N \equiv N(\bar{\phi}^A_* + Q^A(\mathbf{x})) - N(\bar{\phi}^A_*)$$

Sasaki, Stewart (1996)
Sasaki, Tanaka (1998)
Lyth et al. (2005)

Taylor expansion

$$\zeta = N_A Q^A + \frac{1}{2} N_{AB} Q^A Q^B + \frac{1}{6} N_{ABC} Q^A Q^B Q^C + \dots$$

# Origin of the bispectrum

$$\langle \zeta_{\mathbf{k}_1} \, \zeta_{\mathbf{k}_2} \, \zeta_{\mathbf{k}_3} \rangle = N_A N_B N_C \langle Q^A_{\mathbf{k}_1} \, Q^B_{\mathbf{k}_2} \, Q^C_{\mathbf{k}_3} \rangle$$

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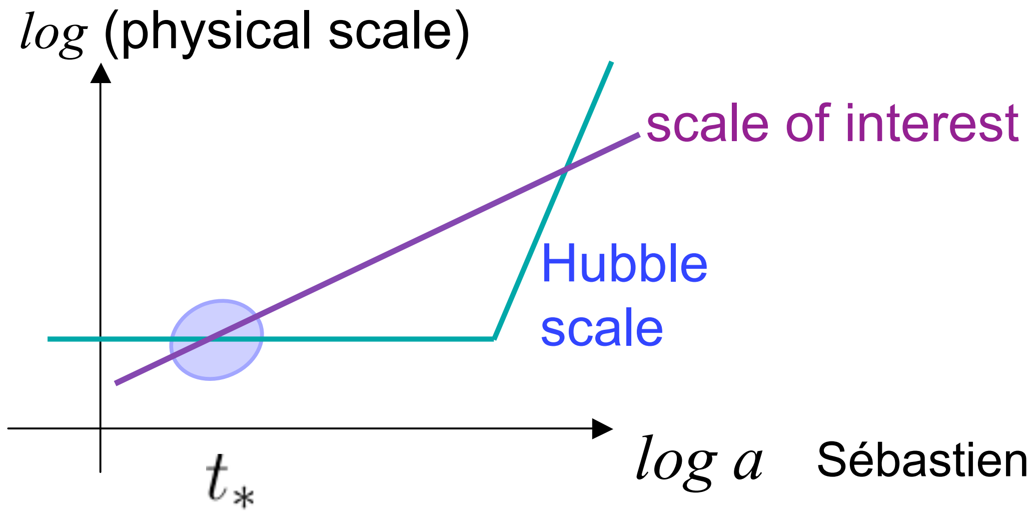
NG of the fields around horizon crossing $k_1 \sim k_2 \sim k_3$ $\quad \sim B^{ABC}(k_1, k_2, k_3)$

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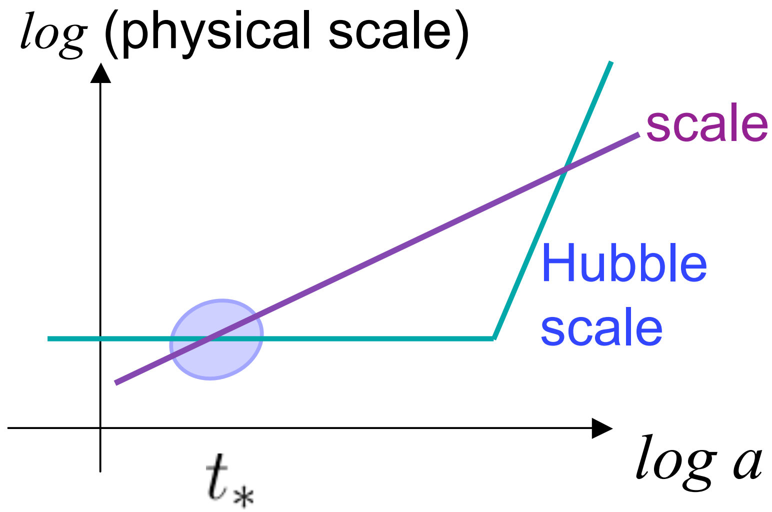
Suppressed by the flatness of the potential in slow-roll single and multifield models — Maldacena (03) Lidsey,Seery(05)

Important for models with non standard kinetic terms — Chen et al (06)

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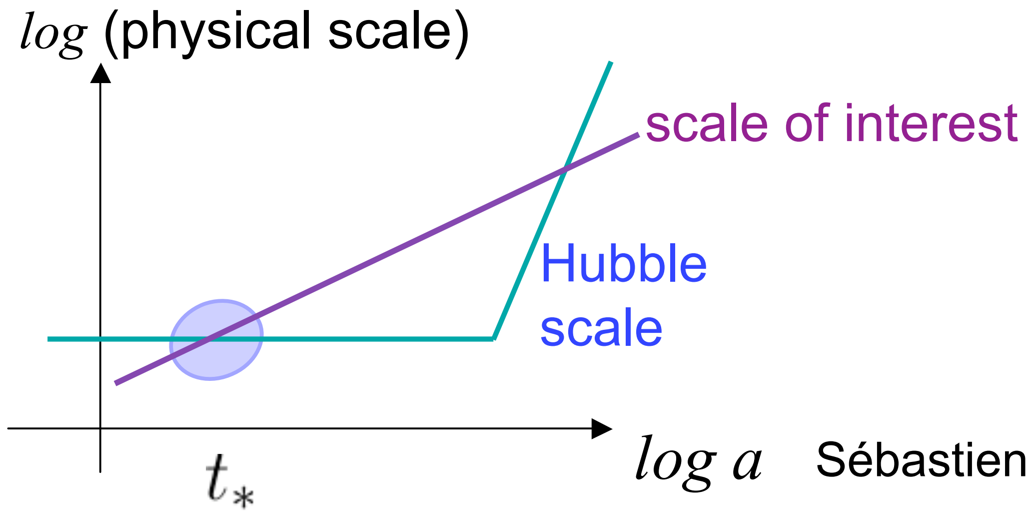
$$+\frac{1}{2} N_A N_B N_{CD} \langle Q^A_{\mathbf{k}_1} Q^B_{\mathbf{k}_2} (Q^C \star Q^D)_{\mathbf{k}_3} \rangle + 2\,\text{perms}$$

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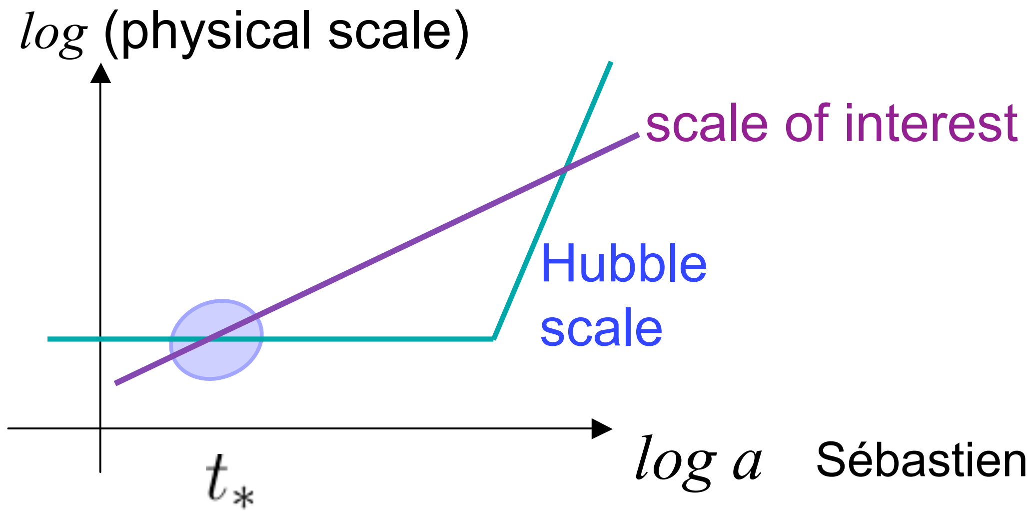


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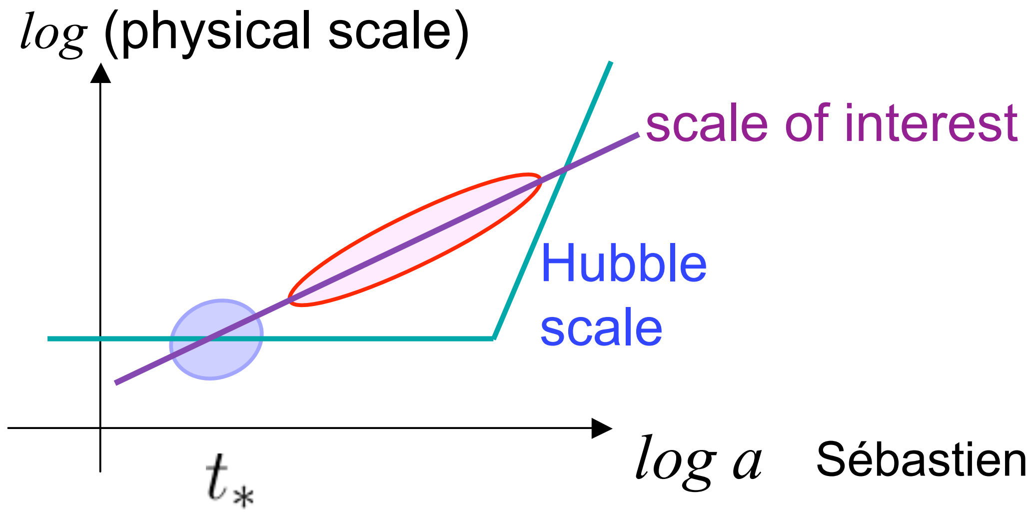
Superhorizon nonlinear relation between zeta and the fields $\qquad k_3 \ll k_1 \sim k_2$

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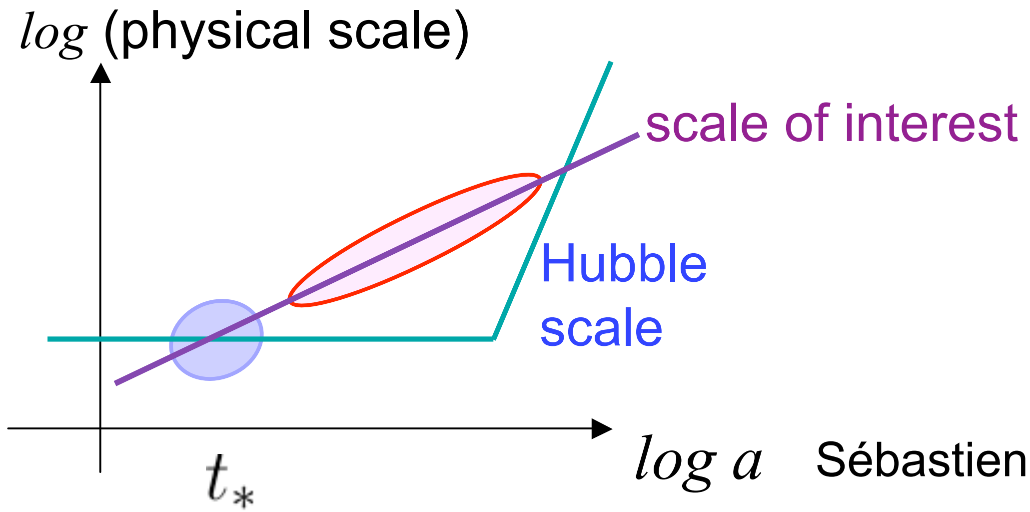
Because $\zeta = \mathrm{cte}$ on large scales in single field inflation, important only for multiple field models

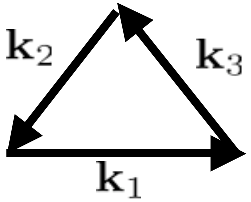


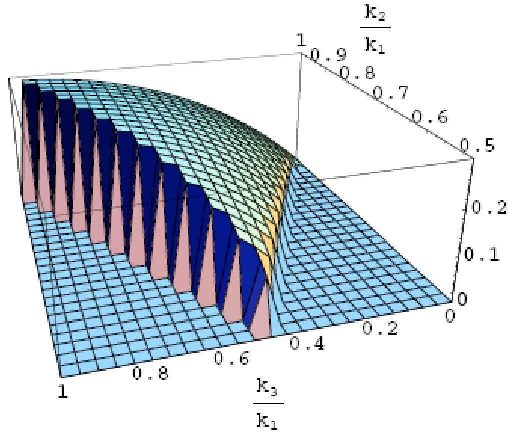
# The shape of the bispectrum, summary

## Equilateral type (quantum)

$$-151 < f_{NL}^{eq} < 253$$

WMAP5 (2008)



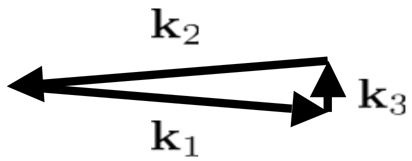

Non standard kinetic terms:

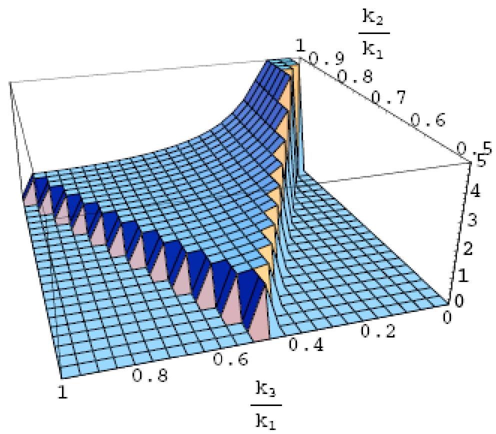
DBI inflation, Ghost inflation, low sound speed models.

## Local type (classical)

$$-4 < f_{NL}^{loc} < 80$$

Smith et al. (2009)




Multiple fields:

- Multifield inflation
- Curvaton
- Ekpyrotic

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# III Generalized multifield inflation

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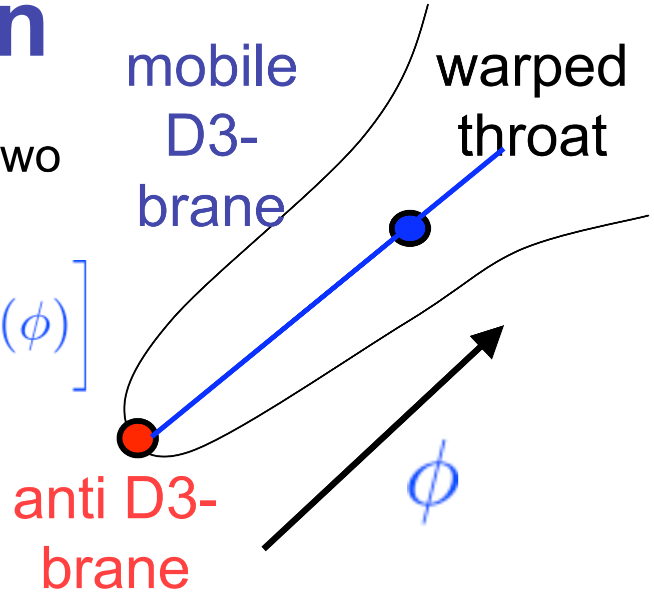
# DBI inflation

- Brane inflation: inflaton as the distance between two branes

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{f} \left( \sqrt{1 + f \partial_\mu \phi \partial^\mu \phi} - 1 \right) - V(\phi) \right]$$




- Speed limit: $\dot{\phi}^2 < 1/f(\phi)$

- Slow-roll regime:

$f\dot{\phi}^2 \ll 1$ $\qquad$ $S = \int \mathrm{d}t a^3 \left[ \frac{1}{2} \dot{\phi}^2 - V(\phi) \right]$

KKLT models

Hubble friction

- "Relativistic" DBI regime:

$c_s^2 = 1 - f\dot{\phi}^2 \ll 1$

Silverstein, Tong (04); Alishahiha et al (04)

Large warping, inflation despite steep potential

$\Rightarrow$ $f_{NL}^{eq} = -\frac{35}{108} \frac{1}{c_{s*}^2}$

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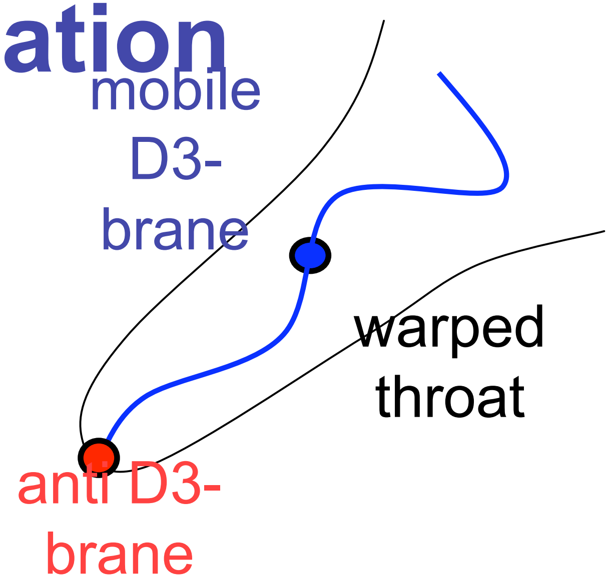
# Multifield DBI inflation




Aim : take into account all the internal coordinates

➡ Multi-field effective description

$$ds^2 = h^{-1/2}(y^K) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^K) G_{IJ}(y^K) dy^I dy^J$$

First try   Easson at al (07); Huang et al (07)

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{f} \left( \sqrt{1 + f G_{IJ} \partial_\mu \phi^I \partial^\mu \phi^J} - 1 \right) - V(\phi) \right]$$

➡ Not correct… but motivates generalized multifield inflation.

# Generalized multifield inflation

$$S = \int d^4x \sqrt{-g}\, P(X, \phi^I) \qquad \text{with} \qquad X = -\frac{1}{2} G_{IJ} \partial_\mu \phi^I \partial^\mu \phi^J$$

Langlois, S RP (08)

- Calculation of the full second-order action (ADM formalism)

$$S_{(2)} = \frac{1}{2} \int dt\, d^3x\, a^3 \left[ \left( P_X G_{IJ} + P_{,XX} \dot{\phi}_I \dot{\phi}_J \right) \mathcal{D}_t Q^I \mathcal{D}_t Q^J - \frac{P_X}{a^2} G_{IJ} \partial_i Q^I \partial^i Q^J \right.$$
$$\left. - M_{IJ} Q^I Q^J + 2 P_{,XJ} \dot{\phi}_I Q^J \mathcal{D}_t Q^I \right],$$

- Kinetic term

$$G_{IJ} + \frac{P_{,XX}}{P_{,X}} \dot{\phi}_I \dot{\phi}_J = \left( G_{IJ} - e_I^\sigma e_J^\sigma \right) + \frac{1}{c_s^2} e_I^\sigma e_J^\sigma$$

$$e_I^\sigma \propto \dot{\phi}_I, \qquad \frac{1}{c_s^2} \equiv 1 + \frac{2 X P_{,XX}}{P_{,X}}$$

# Adiabatic / entropy modes

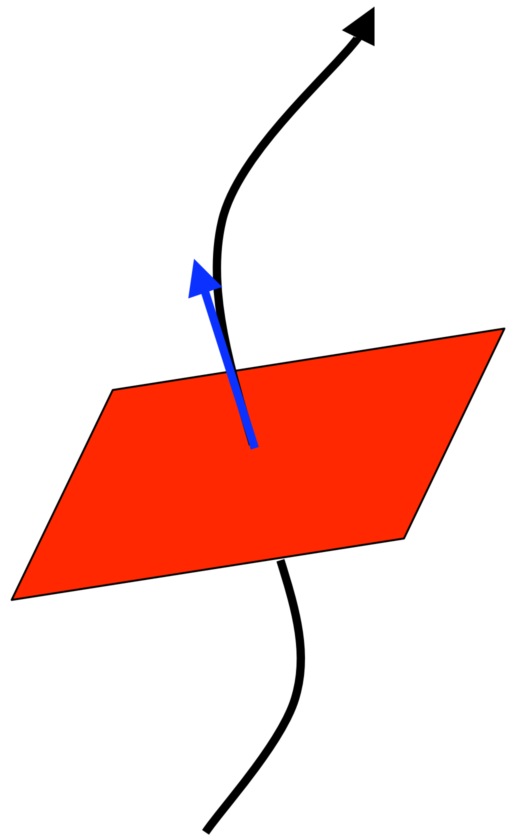
- Adiabatic d.o.f., parallel to the field trajectory

  **Propagation speed:** $c_s$

- Entropy d.o.f's, orthogonal to the field trajectory

  **Propagation speed:** $1$

⇨ Two horizons



# 2-field case

- Very simple equations of motion for the canonically normalized fields:

$$
\begin{aligned}
v_\sigma'' - \xi v_s' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_\sigma - \frac{(z\xi)'}{z} v_s &= 0\,. \\
v_s'' + \xi v_\sigma' + \left(k^2 - \frac{\alpha''}{\alpha} + a^2 \mu_s^2\right) v_s - \frac{z'}{z} \xi v_\sigma &= 0\,.
\end{aligned}
$$

# 2-field case

- Very simple equations of motion for the canonically normalized fields:

$$\xi = 0$$

$$\begin{aligned} v_\sigma'' + \left( c_s^2 k^2 - \frac{z''}{z} \right) v_\sigma &= 0\,. \\ v_s'' + \left( k^2 - \frac{\alpha''}{\alpha} + a^2 \mu_s^2 \right) v_s &= 0\,. \end{aligned}$$

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$$

- Coupling: one parameter. Lots of intuition in a wide variety of situations.

# 2-field case

- Huge work: very simple equations of motion for the canonically normalized fields:

$$\begin{aligned} v_\sigma'' - \xi v_s' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_\sigma - \frac{(z\xi)'}{z} v_s &= 0\,. \\ v_s'' + \xi v_\sigma' + \left(k^2 - \frac{\alpha''}{\alpha} + a^2\mu_s^2\right) v_s - \frac{z'}{z}\xi v_\sigma &= 0\,. \end{aligned}$$

- Coupling: one parameter. Lots of intuition in a wide variety of situations.

⇨ $\mathcal{R}$ **is generically non constant on large scales, even if the trajectory is straight!**

# Generalized multifield inflation

$$S = \int d^4x \sqrt{-g}\, P(X, \phi^I) \qquad \text{with} \qquad X = -\frac{1}{2} G_{IJ} \partial_\mu \phi^I \partial^\mu \phi^J$$

- Non-Gaussianities studied (although for $c_s \simeq 1$ only )

  Gao (08)

- Used in the context of curvaton, quintessence...

  Li et al (08), Sur et al (08)

- A precise model? At some point, it was believed that multifield DBI inflation does the job.

  Easson at al (07); Huang et al (07)

- Not quite…, with important observational consequences.

# IV Multifield DBI inflation

# DBI action

Dirac-Born-Infeld action: Nambu-Goto action
(neglecting bulk and gauge fields)

$$L_{DBI} = -\frac{1}{f}\sqrt{-\det\left(g_{\mu\nu} + f\, G_{IJ}\, \partial_\mu \phi^I \partial_\nu \phi^J\right)}$$

$$\Rightarrow \quad S = \int d^4x \sqrt{-g}\left(\frac{M_p^2}{2} R^{(4)} - \frac{1}{f}\left(\sqrt{\mathcal{D}} - 1\right) - V(\phi^I)\right)$$

with $\mathcal{D} = \det\left(\delta^\mu_\nu + f G_{IJ} \partial^\mu \phi^I \partial_\nu \phi^J\right)$

Background : homogeneous fields

$$\mathcal{D} = 1 - f G_{IJ} \dot{\phi}^I \dot{\phi}^J$$

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# DBI action

Multiple inhomogeneous fields

Lorentz covariance allows to consider

$$X^{IJ} = -\frac{1}{2}\partial^\mu \phi^I \partial_\mu \phi^J \qquad X_I^J = G_{IK} X^{KJ}$$

$$\mathcal{D} = 1 - 2fG_{IJ}X^{IJ}$$
$$+4f^2 X_I^{[I} X_J^{J]} - 8f^3 X_I^{[I} X_J^J X_K^{K]} + 16 f^4 X_I^{[I} X_J^J X_K^K X_L^{L]}$$

Terms which vanish for:

- one field

- multiple homogeneous fields.

Essential for perturbations

$$P(X^{IJ}, \phi^K)$$

Langlois, S R-P, Steer, Tanaka (08)
Arroja et al (08)
Gao et al (09)

# DBI action

$$P = -\frac{1}{f(\phi^I)}\left(\sqrt{1-2f(\phi^I)\tilde{X}}-1\right) - V(\phi^I)$$

where $X$ and $\tilde{X}$ differ only by spatial gradients

Generalized multifield inflation very useful

$$\Rightarrow \quad \begin{aligned} v_\sigma'' - \xi v_s' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_\sigma - \frac{(z\xi)'}{z} v_s &= 0\,, \\ v_s'' + \xi v_\sigma' + \left(c_s^2 k^2 - \frac{\alpha''}{\alpha} + a^2 \mu_s^2\right) v_s - \frac{z'}{z}\xi v_\sigma &= 0\,. \end{aligned}$$

All modes propagate at the common speed of sound.

Intuitive geometrical understanding Mizuno et al (09)

# Isocurvature perturbations

$$\mathcal{L}_2 = \frac{a^3}{2c_s^3}\left[\dot{Q}_\sigma^2 - c_s^2\left(\frac{1}{a}\nabla Q_\sigma\right)^2\right] + \frac{a^3}{2c_s}\left[\dot{Q}_s^2 - c_s^2\left(\frac{1}{a}\nabla Q_s\right)^2\right]$$

$$v_\sigma = \frac{a}{c_s^{3/2}} Q_\sigma \qquad v_s = \frac{a}{c_s} Q_s$$

$$v_\sigma \simeq v_s$$

$$\mathcal{P}_{Q_\sigma} = \left(\frac{H}{2\pi}\right)^2 \qquad \mathcal{P}_{Q_s} = \left(\frac{H}{2\pi c_s}\right)^2$$

$$Q_s \simeq \frac{1}{c_s} Q_\sigma$$

Enhancement of isocurvature perturbations

# Primordial spectra

- Curvature perturbation $$\mathcal{P}_{\mathcal{R}_*} = \frac{1}{8\pi^2 \epsilon c_s}\left(\frac{H}{M_P}\right)^2\Bigg|_{kc_s=aH}$$

  [ same as single-field k-inflation:
  Garriga & Mukhanov (99) ]

- In the multi-field case, $\mathcal{R}$ can evolve on large scales

$$\mathcal{R} = \mathcal{R}_* + T_{\mathcal{RS}}\mathcal{S}_* \quad \left[\mathcal{S} = c_s \frac{H}{\dot{\sigma}} Q_s\right] \quad \mathcal{P}_{\mathcal{R}} = (1 + T_{\mathcal{RS}}^2)\mathcal{P}_{\mathcal{R}_*} = \frac{\mathcal{P}_{\mathcal{R}_*}}{\cos^2\Theta}$$

Feeding of curvature perturbation by entropy perturbations

- Tensor modes

$$\mathcal{P}_{\mathcal{T}} = \left(\frac{2H^2}{\pi^2}\right)_{k=aH} \quad \Rightarrow \quad r = \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon\, c_s \cos^2\Theta$$

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# Non-Gaussianities : influence of isocurvature perturbations

- Third order action

$$S_3^{(\mathrm{main})} = \int dt d^3x \left\{ \frac{a^3}{2c_s^5\dot{\sigma}} \left[ \dot{Q}_\sigma^3 + c_s^2 \dot{Q}_\sigma \dot{Q}_s^2 \right] - \frac{a}{2c_s^3\dot{\sigma}} \left[ \dot{Q}_\sigma (\nabla Q_\sigma)^2 + c_s^2 \dot{Q}_\sigma (\nabla Q_s)^2 - 2c_s^2 \dot{Q}_s \nabla Q_\sigma \nabla Q_s \right) \right] \right\}$$

- Shape of $f_{\mathrm{NL}}$ unaltered

- Amplitude $f_{NL}^{(\mathrm{equil})} = -\frac{35}{108}\frac{1}{c_s^2}\frac{1}{1+T_{\mathcal{RS}}^2} = -\frac{35}{108}\frac{1}{c_s^2}\cos^2\Theta$

Equilateral non-Gaussianities reduced by entropy perturbations

Very important for model-building

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# Multifield DBI inflation ctd

- Calculation of the spectral index, running of non-Gaussianities.

  Langlois, S.RP,Steer, Tanaka (08)

- New consistency relation.

- Bulk fields (NS-NS and R-R) and gauge field on the brane included.

  Langlois, S.RP,Steer (09)

- Revisited gravitational waves constraints on DBI inflation.

- Loop corrections. Gao & Xu (09)

# Multifield DBI inflation

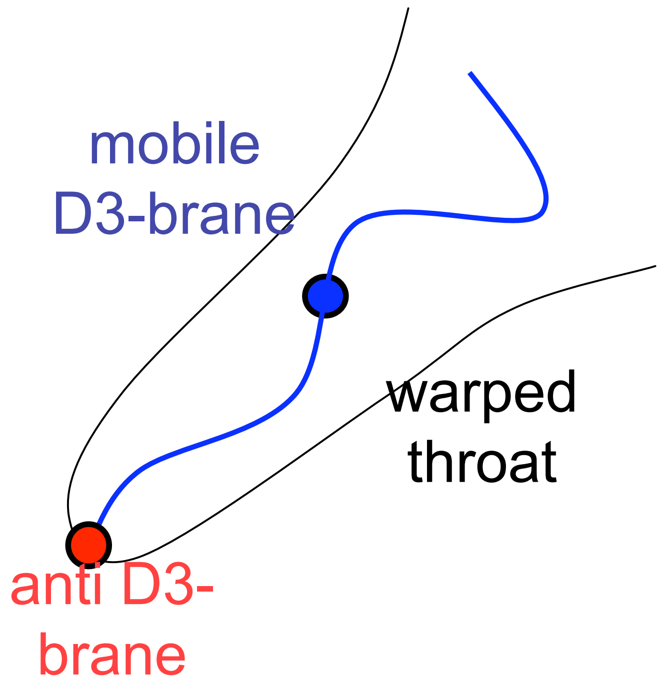
• Brane inflation: moving D3-brane in higher dimensions, non-standard kinetic terms.

$\Rightarrow \quad f_{NL}^{eq}$

• Inflaton: position of the D-brane, multifield.

• Multiple field effects reduce the amplitude of equilateral non-Gaussianities

Langlois, S R-P, Steer, Tanaka (08,09)




NEW: superhorizon nonlinear evolution in a simple model. $\Rightarrow \quad f_{NL}^{loc}$

S R-P (09)

Combined local and equilateral non-Gaussianities

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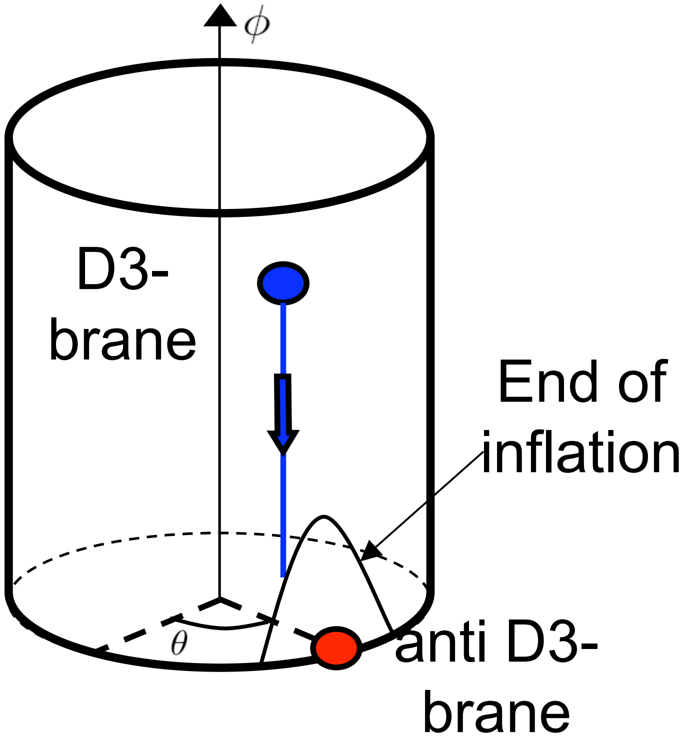
# A simple stringy model

• Radial D-brane motion
and D-brane angular fluctuations.

• End of inflation:

$$d(D3, \overline{D3}) = \text{ string length}$$




Lyth, Riotto (06)
Leblond, Shandera (06)

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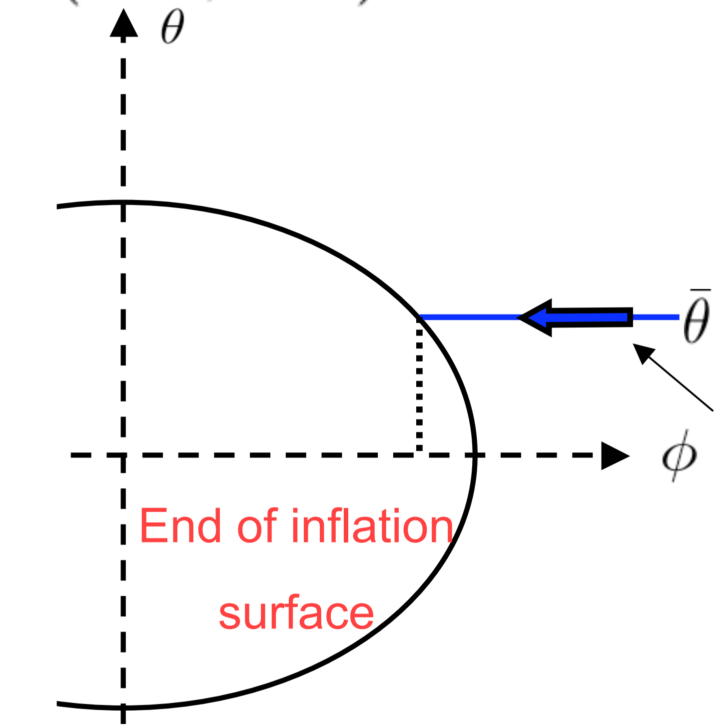


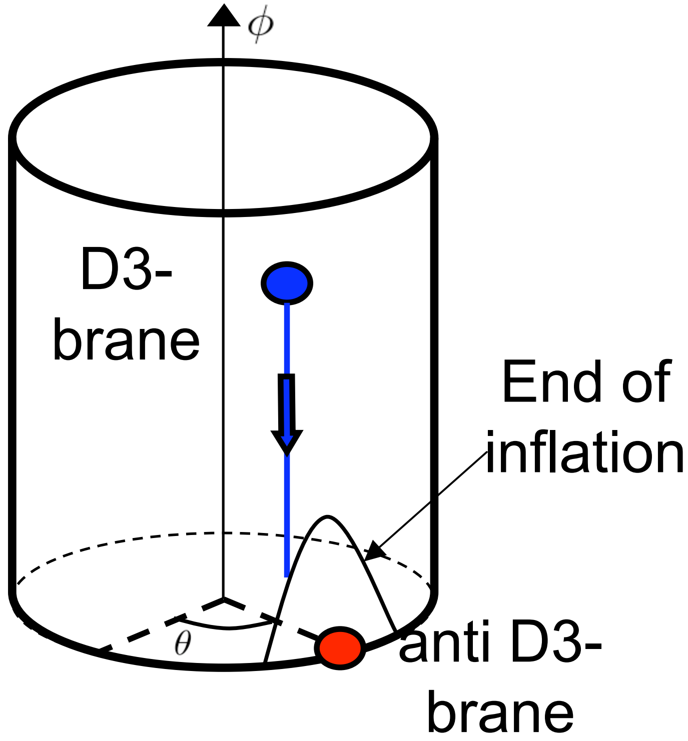


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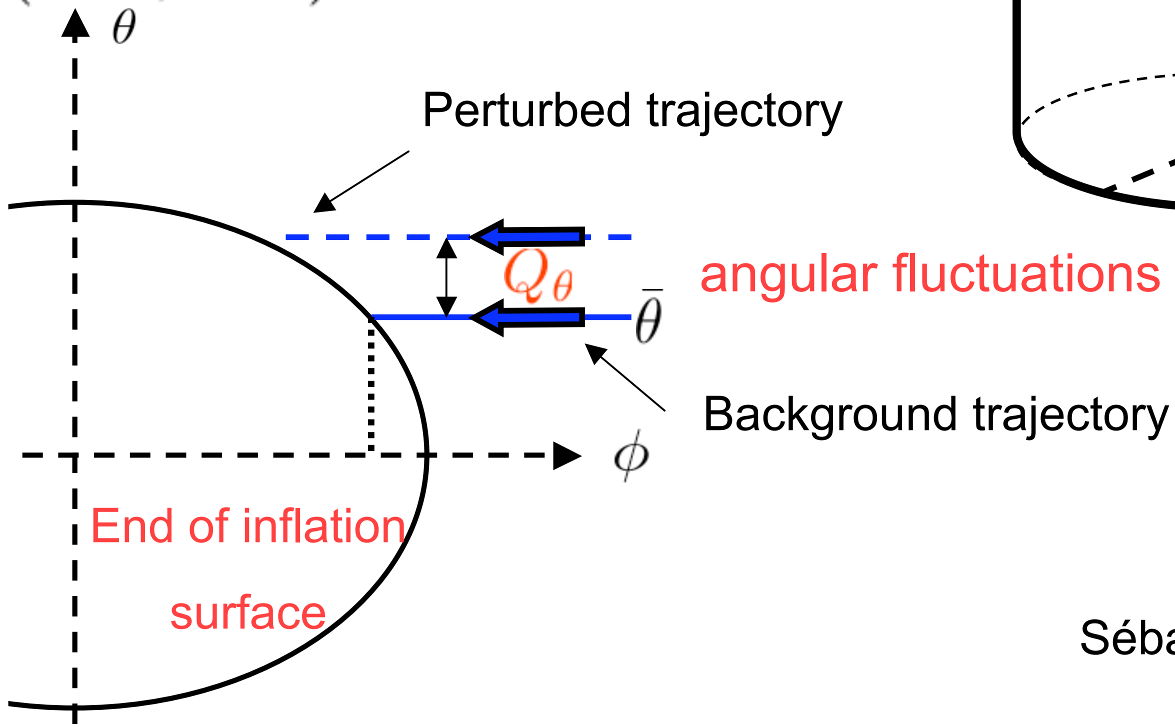


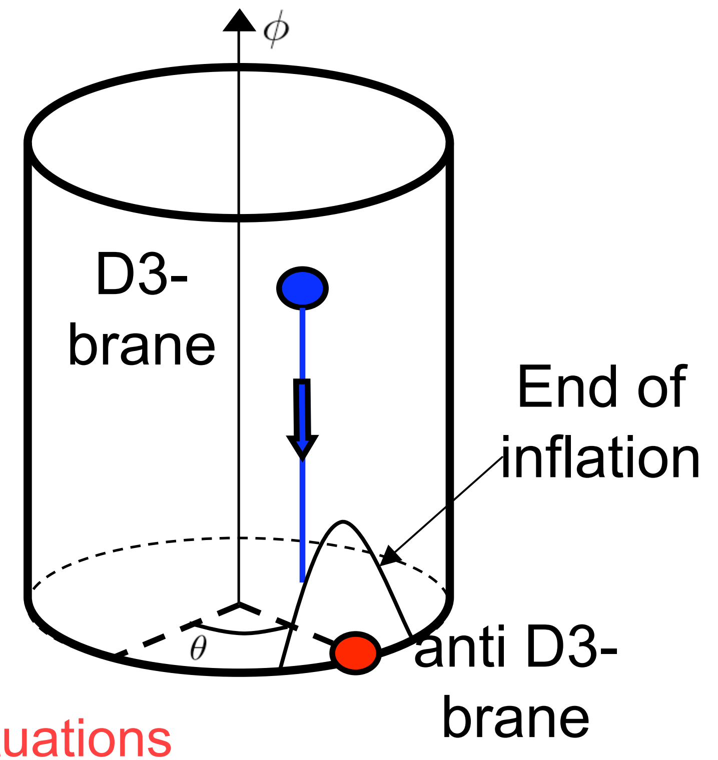


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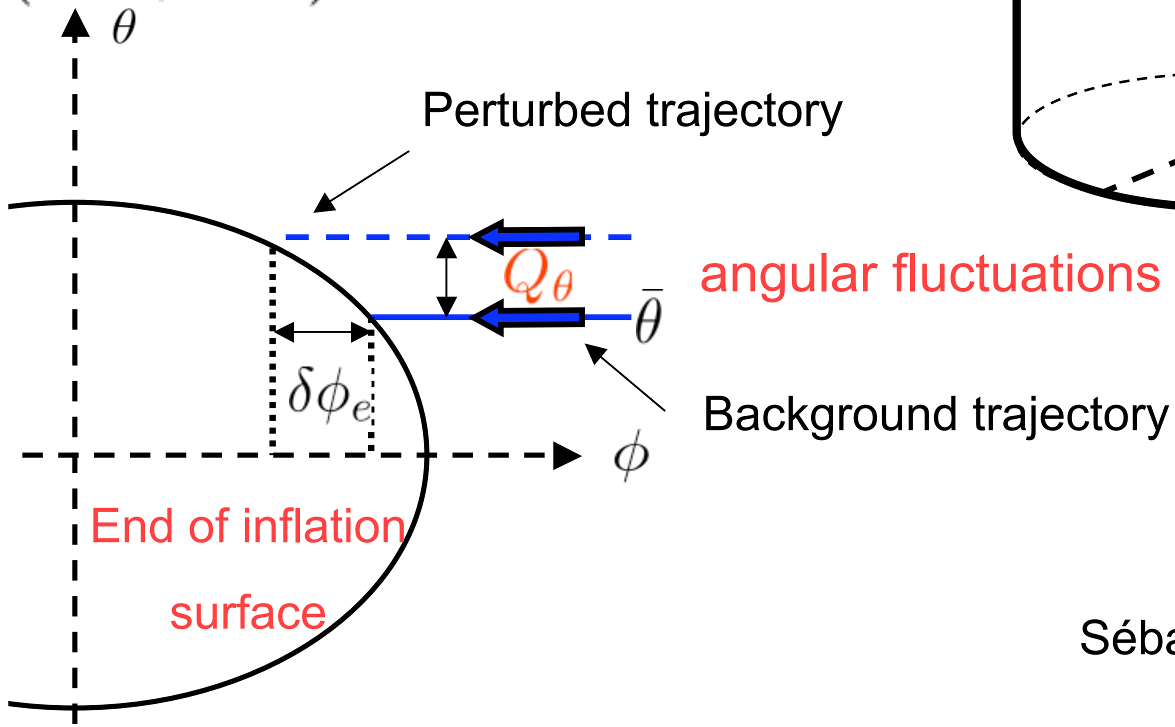


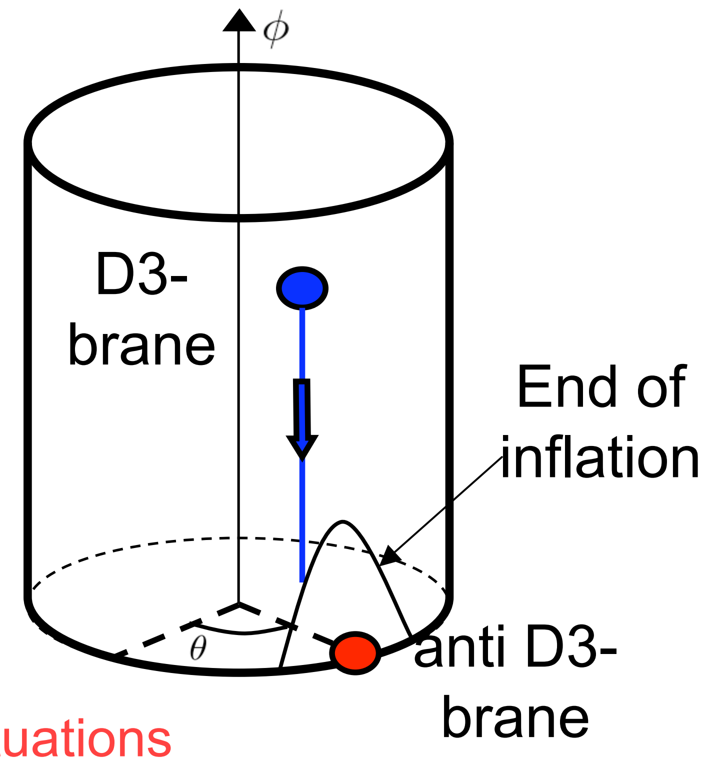


Lyth, Riotto (06)
Leblond, Shandera (06)

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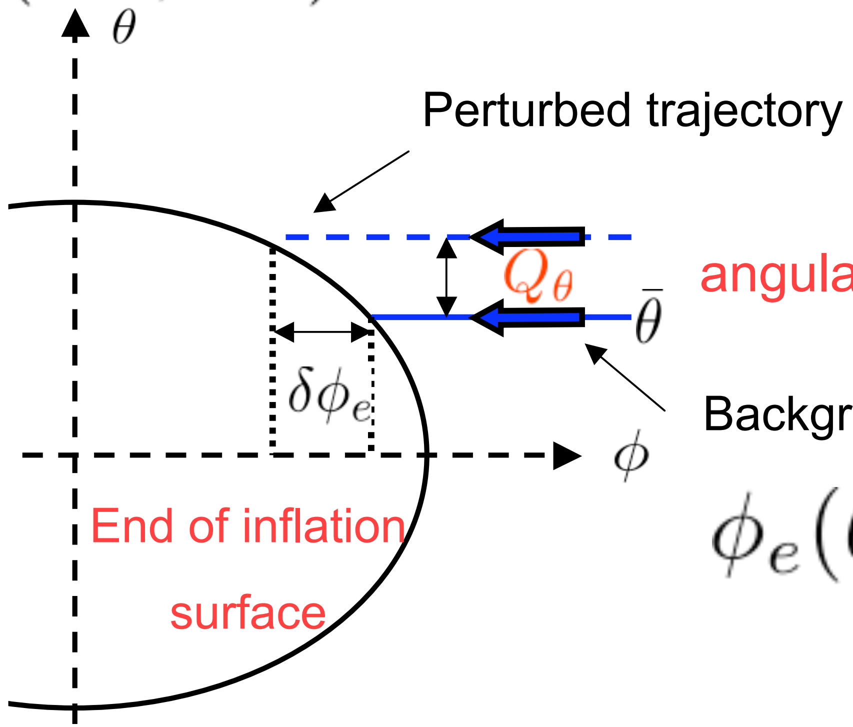
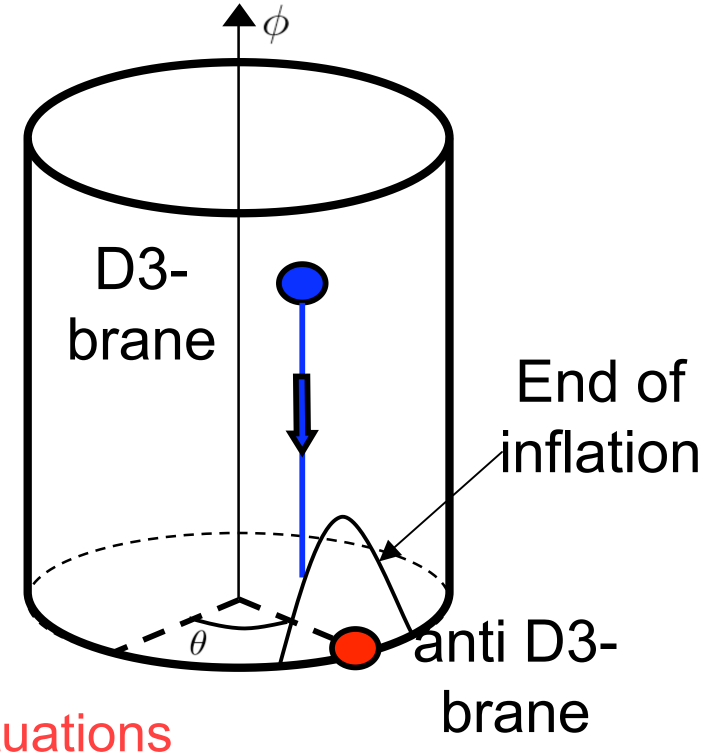



angular fluctuations




Lyth, Riotto (06)
Leblond, Shandera (06)

$\phi_e(\theta)$ nonlinear $\Rightarrow$ $f_{NL}^{loc} \sim \frac{h_{tip} m_s}{H_e}$

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# The trispectrum

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# The quantum trispectrum

$$T_\zeta \quad = N_A N_B N_C N_D \langle Q^A(\mathbf{k}_1) Q^B(\mathbf{k}_2) Q^C(\mathbf{k}_3) Q^D(\mathbf{k}_4) \rangle_c + \ldots$$

Connected 4-point function of the fields around horizon crossing (quantum)

Similar to $f_{NL}^{eq}$ for the bispectrum

Mizuno et al (09)

# The local trispectrum

1

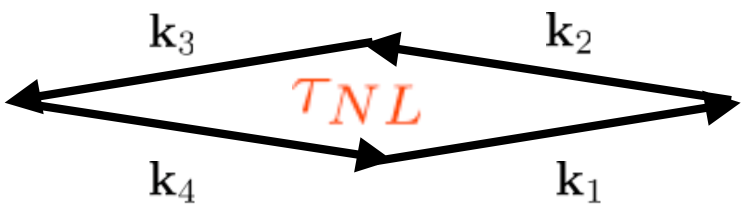
$$
\begin{aligned}
T_\zeta \quad = \quad & \ldots + N_{AB} N_{CD} N_E N_F \left[ C^{BD}(k_{13}) C^{AE}(k_3) C^{CF}(k_4) + 11 \text{ perms} \right] \\
+ \quad & N_{ABC} N_D N_E N_F \left[ C^{AD}(k_2) C^{BE}(k_3) C^{CF}(k_4) + 3 \text{ perms} \right] + \ldots
\end{aligned}
$$

2

where $\langle Q^A_{\mathbf{k}} Q^B_{\mathbf{k}'} \rangle = C^{AB}(k)(2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}')$

Superhorizon nonlinear evolution, (classical)

1

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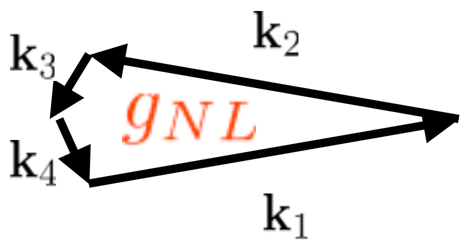


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# Combined local and equilateral non-Gaussianities in the trispectrum

$$T_\zeta = \ldots + N_{AB}N_C N_D N_E \left[C^{AC}(k_1)B^{BDE}(k_{12},k_3,k_4) + 11 \text{ perms}\right]$$

has always been neglected

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# Combined local and equilateral non-Gaussianities in the trispectrum

$$T_\zeta = \ldots + N_{AB} N_C N_D N_E \left[ C^{AC}(k_1) B^{BDE}(k_{12}, k_3, k_4) + 11 \text{ perms} \right]$$

Superhorizon nonlinear evolution, (classical)

Field three-point function (quantum)

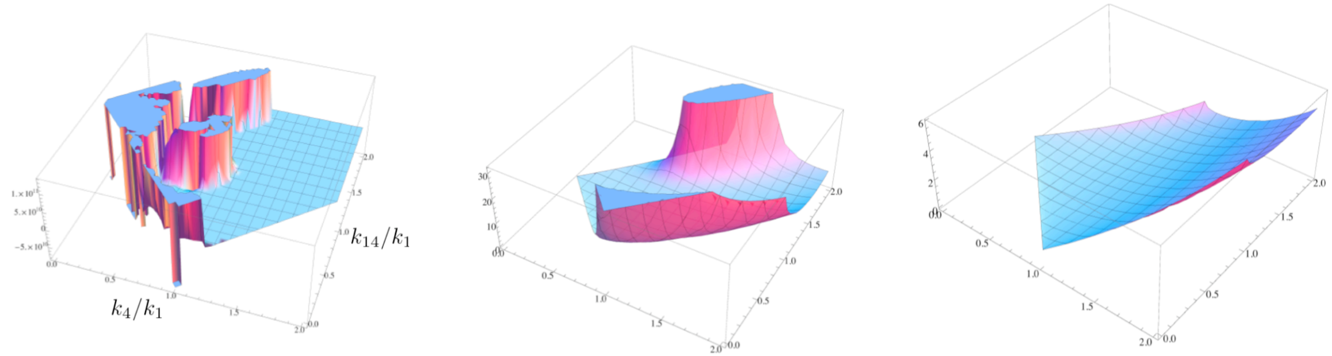
Requires light fields other than the inflaton and with non standard kinetic terms

➡ Multifield DBI inflation

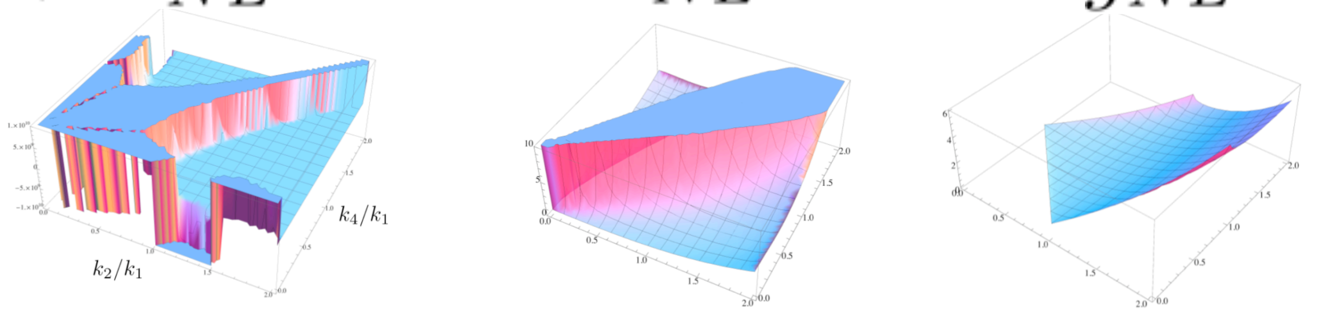
All quantities appeared in lower order correlation functions

# Combined local and equilateral NG in the trispectrum from multifield DBI

- Momentum-dependence, 6-dim parameter space




$$\frac{\langle\zeta^4\rangle_c}{\langle\zeta^2\rangle^3} \rightarrow \quad s_{NL} \qquad \tau_{NL} \qquad g_{NL}$$




- Amplitude $\boxed{s_{NL} = f_{NL}^{loc}\, f_{NL}^{eq}}$ for every multifield DBI model

# Conclusions

- Non-Gaussianities: key-discriminant amongst early universe scenarios.
- My work so far:

Early universe physics models from high energy physics ⟷ Primordial non-Gaussianities

- Cosmological perturbation theory
- Very general formalisms
- Observable predictions

# Future work

Realistic models embedded
in high-energy physics

- Moduli stabilization
- Tachyon condensation
- Reheating

…

Early universe physics models
from high energy physics

Primordial non-
Gaussianities

- Secondary non-Gaussianities
- Statistical estimators and data
- LSS

…

Cosmological
observations

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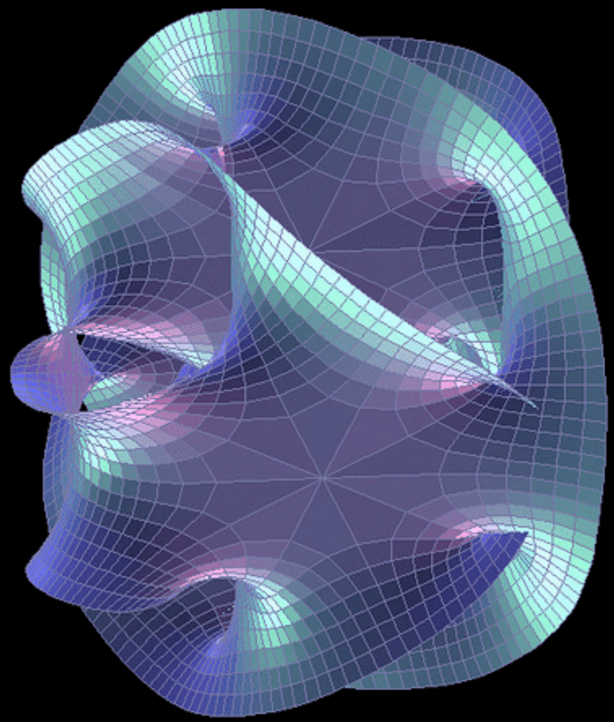

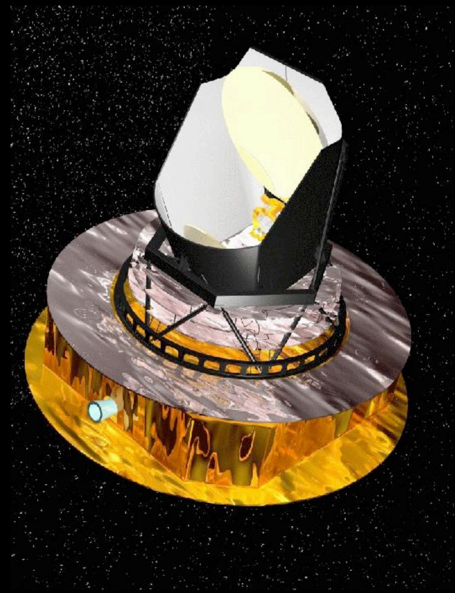

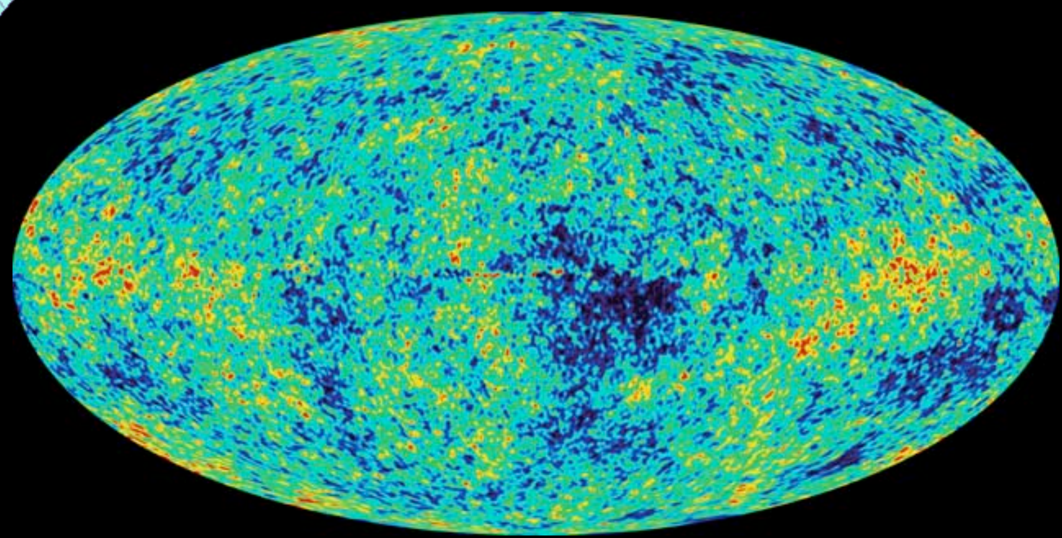

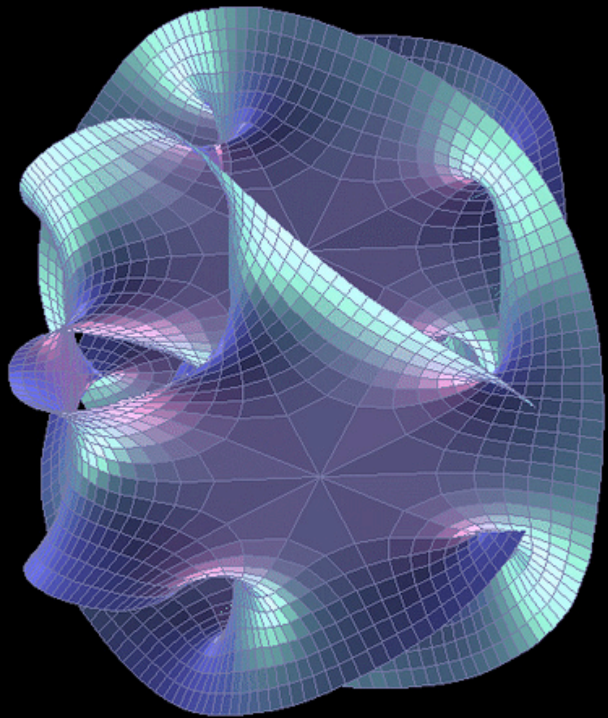

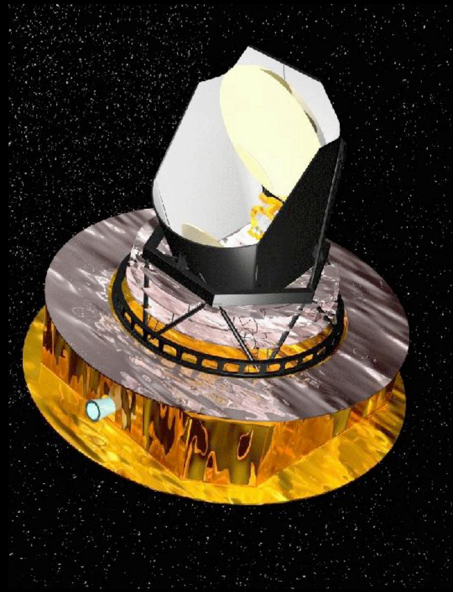

Thank you