Separating expansion from collapse and generalizing TOV condition in spherically symmetric models with pressure, with Λ-CDM examples

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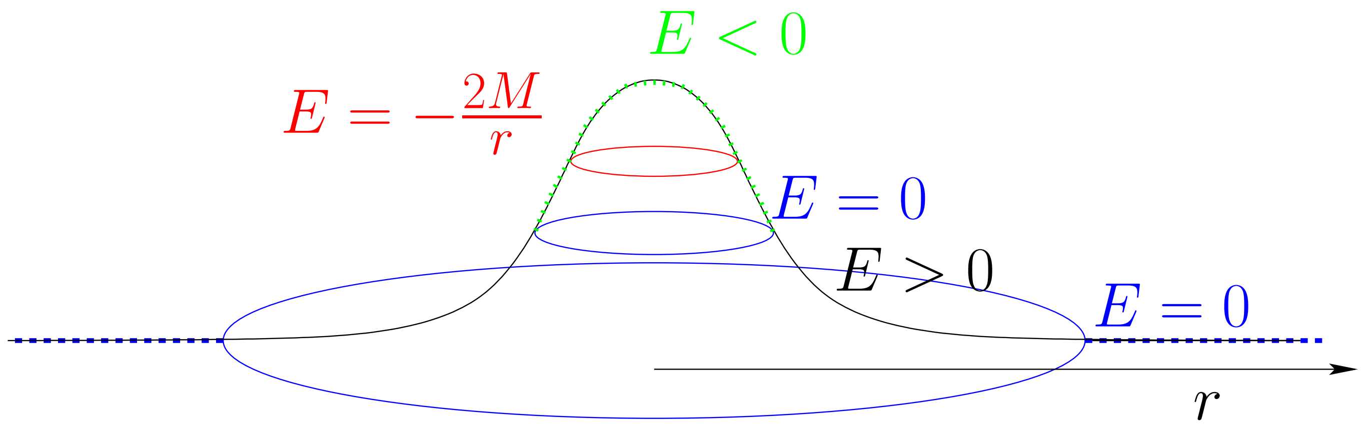
Mass/curvature perturbation in a flat background



[Le Delliou & Mimoso 2009]: AIP Conf.Proc.**1122** (2009) *316* arXiv:0903.4651,
to appear in [Le Delliou, Mena & Mimoso 2010] Proceedings of 'Invisible Universe', arXiv:0911.0241,
[Mimoso, Le Delliou & Mena 2010] submitted to Phys. Rev. D, arXiv:0910.5755.

Introduction-a

Context: •Structure formation in a Cosmology of Dark Energy (DE) and Dark Matter (DM) $\rightarrow$ spherical collapse with pressure and outer expansion •Press-Schechter Structure formation

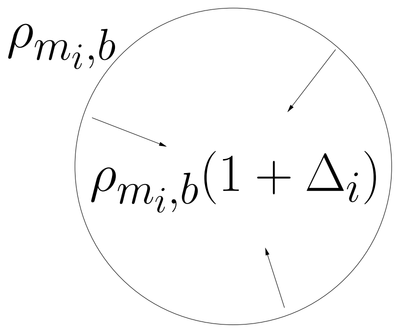
In the Press-Schechter scheme,

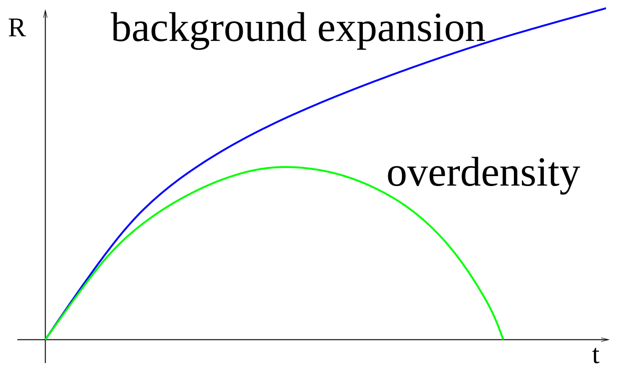
relativistic spherical collapse $\longrightarrow$ get the non-linear collapse time

for the simple minimally coupled homogeneous DE

top hat model **Einstein's Equations** - **with "Birkhoff's Theorem"**

FLRW model with "Top hat" model





Surface tension not treated!

Introduction-b

Context:

- Possibility of an analog/extension to Birkhoff's theorem in non-vacuum background
- trapped surfaces [Penrose, Hawking 1969; Cattoen 2005; Grundlach, Joshi 2007]

Birkhoff's theorem invoked in the Press-Schechter scheme
in the spherical collapse
to justify ignoring the backreactions of background vs overdensity

However,
the theorem is proved in the case of asymptotic flatness (vacuum at infinity),
not in cosmological expanding background.

An analog/extension would introduce some weaker sort of causal separation than the so-called trapped surfaces that capture all causal links.

Introduction-c

Goal: Does the expansion of the universe affect collapsing fluctuations? In spherical symmetry, can an analog/extension to Birkhoff's theorem be developed for cosmological boundary conditions? Can we define locally/globally matter-trapped surfaces separating cosmological expansion from collapsing regions?

Plan

I. ADM approach to LTB models in GPG system and GLTB: **General Description**

II. Definition of a separating shell: **Non-linear collapse model with P and expansion**

III. Initial fluctuation(s) with homogeneous pressure: the perturbed ΛCDM

I-a ADM in GPG:

- metric: with ADM 3+1 splitted along flow n^a, defined with

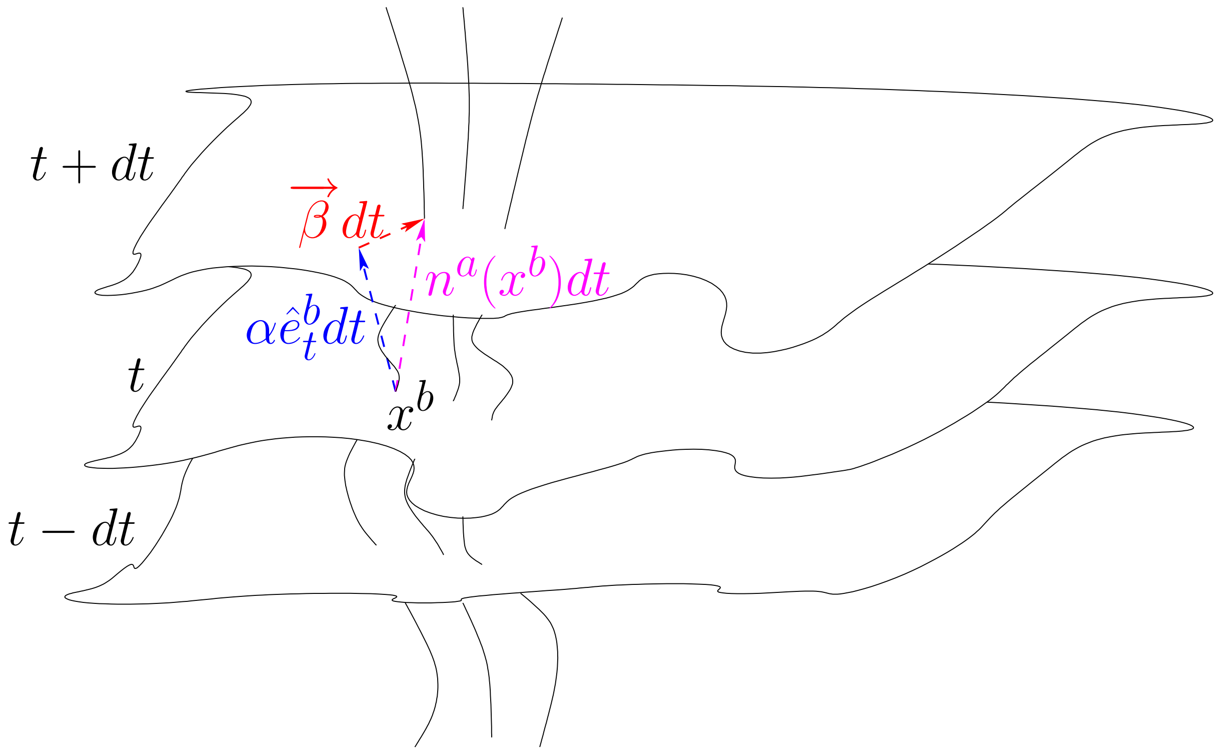
a lapse function α

a spherically symmetric shift vector $\overrightarrow{\beta} = \begin{pmatrix} \beta & 0 & 0 \end{pmatrix}$

a curvature/energy function E

in Generalised Painlevé-Gullstrand coordinates [Laski & Lun 2006]

$$ds^2 = -\alpha(t,r)^2 dt^2 + \frac{1}{1+E(t,r)}\left(\beta(t,r)dt + dr\right)^2 + r^2 d\Omega^2.$$



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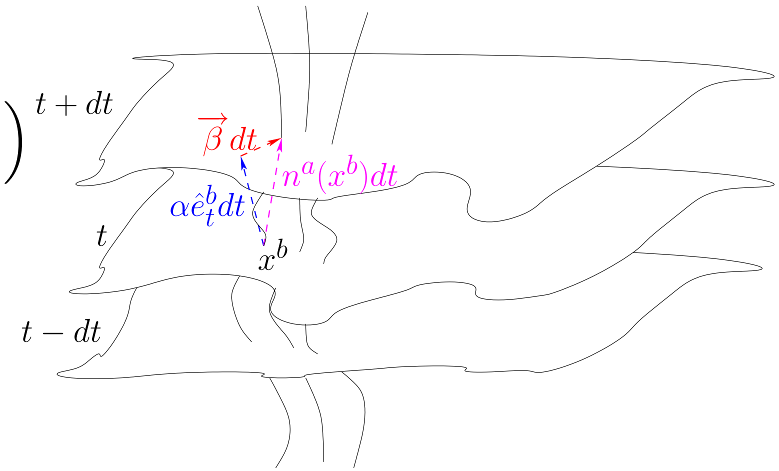

in Generalised Painlevé-Gullstrand coordinates

[Laski & Lun 2006]

$$ds^2 = -\alpha(t,r)^2 dt^2 + \frac{1}{1+E(t,r)}\left(\beta(t,r)dt + dr\right)^2 + r^2 d\Omega^2.$$

- Perfect fluid⟶projected Bianchi identities:

along the flow, energy density conservation,

orthogonal to it, the Euler equation:

$$n^b T^a_{b;a} = -\mathcal{L}_n \rho - (\rho + P)\,{}^3\Theta = 0, \quad h_a^{\ b} T^c_{b;c} = 0 \Rightarrow P' = -(\rho + P)\,\frac{\alpha'}{\alpha}.$$

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- metric: with ADM 3+1 split, spherically symmetric, in GPG coordinates [Laski & Lun 2006]

$$ds^2 = -\alpha(t,r)^2 dt^2 + \frac{1}{1+E(t,r)}\left(\beta(t,r)dt + dr\right)^2 + r^2 d\Omega^2.$$

- Einstein Field Equations read as Lie derivatives along the flow, of
  curvature E (in explicit presence of a Λ)
  a Misner-Sharp Mass $M \equiv r^2 (1+E)(\ln \alpha)' - 4\pi P r^3 + \frac{1}{3}\Lambda r^3 + r^2 \mathcal{L}_n \left(\frac{\beta}{\alpha}\right)$

$$\mathcal{L}_n E = \pm 2\sqrt{2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E}\,\frac{1+E}{\rho+P}P' = 2\frac{\beta}{\alpha}\frac{1+E}{\rho+P}P',$$

$$\mathcal{L}_n M = \pm 4\pi P r^2 \sqrt{2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E} = 4\pi P r^2 \frac{\beta}{\alpha}.$$

with the radial evolution

$$E + 2\frac{M}{r} + \frac{1}{3}\Lambda r^2 = \left(\frac{\beta}{\alpha}\right)^2.$$

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$$E + 2\frac{M}{r} + \frac{1}{3}\Lambda r^2 = \left(\frac{\beta}{\alpha}\right)^2.$$

We also isolated a gTOV parameter (generalised Tolman-Oppenheimer-Volkoff)

$$\text{gTOV} = \left[\frac{1+E}{\rho+P}P' + 4\pi P r + \frac{M}{r^2} - \frac{1}{3}\Lambda r\right] = \mathcal{L}_n\left(\frac{\beta}{\alpha}\right).$$

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- EFEs read as Lie along the flow

$$\mathcal{L}_n E = \pm\, 2\sqrt{2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E}\,\frac{1+E}{\rho+P}P' = 2\frac{\beta}{\alpha}\frac{1+E}{\rho+P}P',$$

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along the flow, energy density conservation,

$$n^b T^a_{b;a} = -\mathcal{L}_n\rho - (\rho+P)\,{}^3\Theta = 0,$$

orthogonal to it, the Euler equation:

$$h_a^b T^c_{b;c} = 0 \Rightarrow P' = -(\rho+P)\frac{\alpha'}{\alpha}.$$

System is closed with equation of state $f(\rho, P) = 0$.

I-a ADM in GPG:

- metric: with ADM 3+1 split, spherically symmetric, in GPG coordinates [Laski & Lun 2006]

$$ds^2 = -\alpha(t,r)^2 dt^2 + \frac{1}{1+E(t,r)}\left(\beta(t,r)dt + dr\right)^2 + r^2 d\Omega^2.$$

- EFEs: In terms of GPG time derivatives it reads

$$\dot{M} = \pm\,\alpha\sqrt{2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E}\left(M' + 4\pi P r^2\right) = \beta\left(M' + 4\pi P r^2\right),$$

$$\dot{E} = \pm\,\alpha\sqrt{2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E}\left(E' + 2\frac{1+E}{\rho+P}P'\right) = \beta\left(E' + 2\frac{1+E}{\rho+P}P'\right).$$

with radial evolution

$$E + 2\frac{M}{r} + \frac{1}{3}\Lambda r^2 = \left(\frac{\beta}{\alpha}\right)^2.$$

I-b ADM in GLTB:

- metric: **Generalised Lemaître-Tolman-Bondi**: choosing $\beta = -\dot{r}$ in GPG, we can get almost Lemaître-Tolman-Bondi coordinates [Laski & Lun 2006]

$$ds^2 = -\alpha(T,R)^2 \, (\partial_T t)^2 \, dT^2 + \frac{(\partial_R r)^2}{1 + E(T,R)} dR^2 + r^2 d\Omega^2,$$

I-b ADM in GLTB:

- metric: **Generalised LTB**: choosing $\beta = -\dot{r}$ we can get almost LTB coordinates [Laski & Lun 2006]

$$ds^2 = -\alpha(T,R)^2 \, (\partial_T t)^2 \, dT^2 + \frac{(\partial_R r)^2}{1+E(T,R)} dR^2 + r^2 d\Omega^2,$$

- EFEs: the Lie derivatives in GPG become time derivatives in GLTB

$$\dot{M} = \beta 4\pi P r^2 = \pm\alpha\sqrt{2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E} 4\pi P r^2,$$

$$\dot{E} r' = 2\beta \frac{1+E}{\rho+P} P' = \pm 2 \frac{1+E}{\rho+P} P' \alpha \sqrt{2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E}$$

with the radial evolution

$$E + 2\frac{M}{r} + \frac{1}{3}\Lambda r^2 = \left(-\frac{\dot{r}}{\alpha}\right)^2.$$

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- EFEs: the Lie becomes time derivatives

$$\dot{M} = \beta 4\pi P r^2 = \pm\alpha\sqrt{2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E} 4\pi P r^2,$$

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with radial evolution

$$E + 2\frac{M}{r} + \frac{1}{3}\Lambda r^2 = \left(-\frac{\dot{r}}{\alpha}\right)^2.$$

- **Many fluids** mass equation (absorbs Λ term): each component i uses the $\frac{\beta}{\alpha}$ term from the overal sum of the masses.

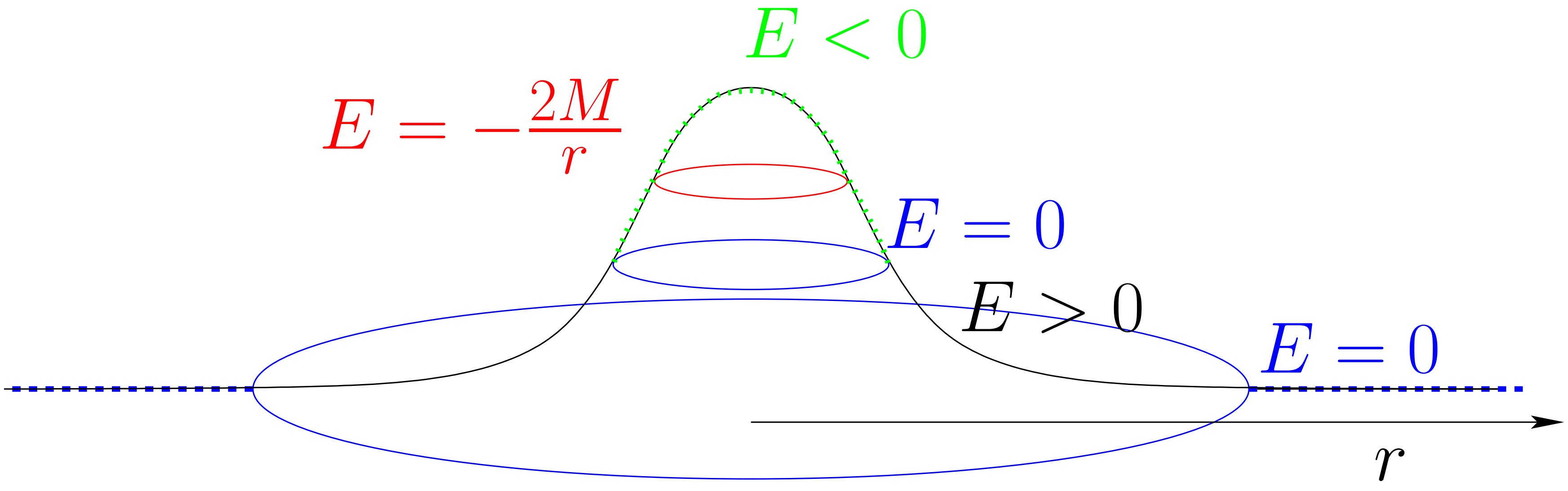
$$\dot{M}_i = \beta 4\pi P_i r^2 = \pm\alpha\sqrt{2\frac{M}{r} + E} 4\pi P_i r^2, \qquad \text{where } M = \sum M_i.$$

II Definitions of a separating shell

Equations of motion and behaviour of dust suggest to focus on shells with dust-like null mass/energy flow, that is:

$$\forall t,\ \mathcal{L}_n M(t, r_\star(t)) = 0 \qquad \Leftrightarrow \forall t,\ E = -2\frac{M}{r_\star} < 0!!$$

Mass/curvature perturbation in a flat background



II Definitions of a separating shell

Equations of motion and behaviour of dust suggest shells with dust-like null mass/energy flow:

$$\forall t,\ \mathcal{L}_n M(t, r_\star(t)) = 0 \qquad \Leftrightarrow \forall t,\ E = -2\frac{M}{r_\star} < 0!!$$

Radial behaviour of that shell is then similar to a turnaround shell

$$r_\star = -\frac{2M_\star}{E_\star}, \qquad \dot{r}_\star = 0, \qquad \ddot{r}_\star = -\alpha^2 \left[\mathrm{gTOV}_\star - r_\star^2 \frac{\mathrm{gTOV}_\star^2}{M_\star} \right].$$

The only difference in LTB coordinates is acceleration:

$$\ddot{r}_{LTB,\star} = -\alpha^2 \mathrm{gTOV}_\star.$$

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Locally, gTOV=0 gives the TOV equation on the limit shell

$\Leftrightarrow$ static condition

$$\mathrm{gTOV}_\star = 0 \qquad \Leftrightarrow -\frac{1}{\rho + P}P' = \left[\frac{4\pi P r + \frac{M}{r^2}}{1 - \frac{2M}{r}}\right]_\star .$$

Note that

$$\mathcal{L}_n r = -\,\frac{\beta}{\alpha} \qquad \Leftrightarrow \mathcal{L}_n^2 r = -\mathrm{gTOV}.$$

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Alternate approach: turnaround shell? (null expansion Θ),
gauge invariant definition (Θ linked with shear a)

$$\Theta = -3\left(a + \frac{\beta}{\alpha}\frac{1}{r}\right), \qquad \Theta_\star + 3a_\star = 0.$$

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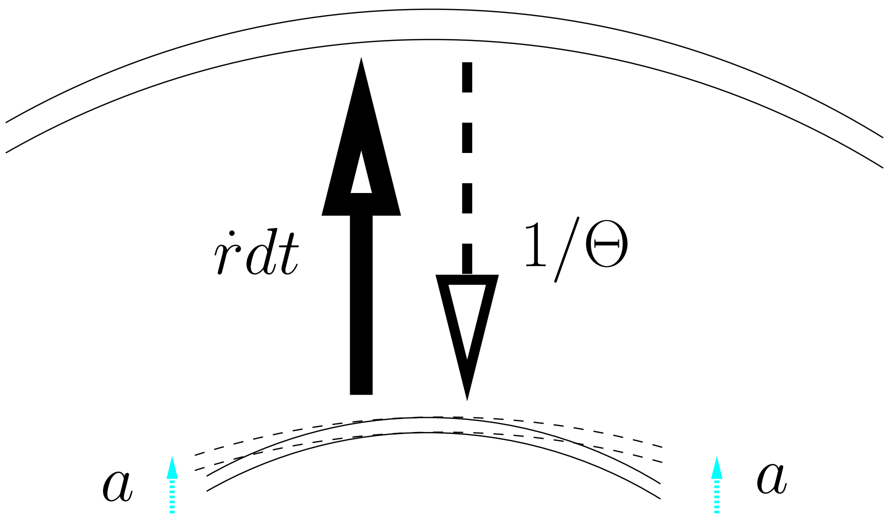
Non Zero Shear in spherical symmetry due to non-flatness:

$$\boxed{a \neq 0 \text{ in spherical symmetry}}$$



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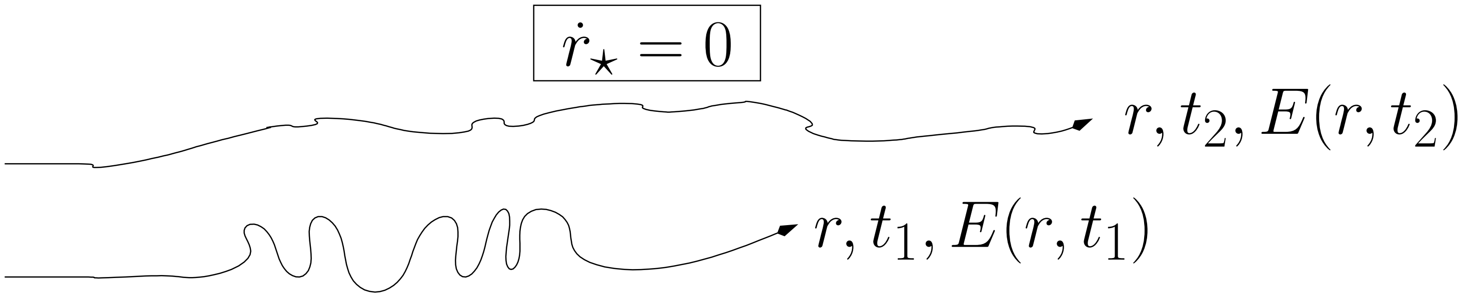
Constant Areal radius doesn't imply global staticity:



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Dynamics of shells governed by the Raychaudhuri equation. Reformulated:

$$-\mathcal{L}_n\Theta - \Theta^2 - \frac{6}{r}\frac{\beta}{\alpha}\left[\frac{2\Theta}{3} + \frac{1}{r}\frac{\beta}{\alpha}\right] = \underbrace{4\pi\left(\rho + 3P\right)}_{\text{FLRW Friedmann source}} - \frac{P'}{2\left(\rho + P\right)}E'$$

$$+\left(\underbrace{\text{gTOV} - \tfrac{4\pi}{3}r\left(\langle\rho\rangle + 3P\right)}_{\text{FLRW-like source}}\right)' + \left(\frac{2}{r} - \frac{P'}{\rho + P}\right)\left(\underbrace{\text{gTOV} - \tfrac{4\pi}{3}r\left(\langle\rho\rangle + 3P\right)}_{\text{FLRW-like source}}\right),$$

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Alternate approach: gauge invariant definition (expansion Θ linked with shear a)

$$\Theta = -3\left(a + \frac{\beta}{\alpha}\frac{1}{r}\right), \qquad \Theta_\star + 3a_\star = 0.$$

Examples= Dynamics: Raychaudhuri equation, shear free

$$-3\mathcal{L}_n\frac{\Theta}{3} - 3\left(\frac{\Theta}{3}\right)^2 = \underbrace{4\pi\left(\rho + 3P\right)}_{\text{FLRW Friedmann source}} - \frac{P'}{2\left(\rho + P\right)}E'$$

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Ex.=Dynamics: Raychaudhuri equation, homogeneous pressure ($P' = 0$)

$$-\mathcal{L}_n\Theta - \Theta^2 - \frac{6}{r}\frac{\beta}{\alpha}\left[\frac{2\Theta}{3} + \frac{1}{r}\frac{\beta}{\alpha}\right] = \underbrace{4\pi\left(\rho + 3P\right)}_{\text{FLRW Friedmann source}},$$

$$\text{gTOV} = \underbrace{\tfrac{4\pi}{3} r\left(\langle\rho\rangle + 3P\right)}_{\text{FLRW-like source}}$$

$$\text{gTOV}_r = 0 \Rightarrow P = -\frac{\langle\rho\rangle_r}{3}.$$

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Ex.=Dynamics: Raychaudhuri equation, FLRW model (also defined with $\alpha = 1, \beta = -\dot{r}, E = -kx^2, r = ax = \partial_x r.x$)

$$-3\mathcal{L}_n\frac{\Theta}{3} - 3\left(\frac{\Theta}{3}\right)^2 = -3\dot{H} - 3H^2 = -3\frac{\ddot{a}}{a} = \underbrace{4\pi\left(\rho + 3P\right)}_{\text{Friedmann source}}$$

$$\langle\rho\rangle = \rho \Rightarrow \quad -\ddot{r} = \underbrace{\tfrac{4\pi}{3}r\left(\rho + 3P\right)}_{\text{FLRW source}} = \text{gTOV},$$

$$\text{gTOV} = 0 \Rightarrow P = -\frac{\rho}{3}, \text{ only dark radiation}$$

III-a Non-linearly perturbed ΛCDM

Simplest example of non-pressureless perfect fluid away from dust:

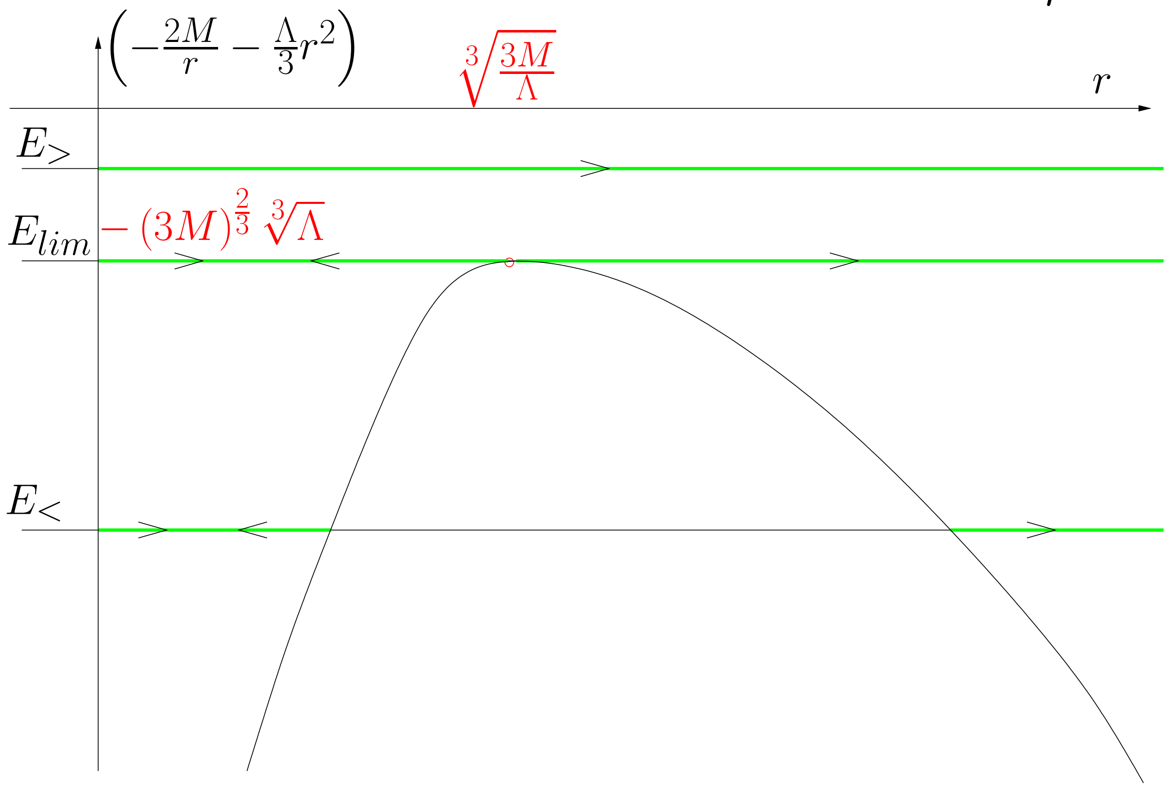
spherically symmetric non-linear perturbation in a ΛCDM background.

$\longrightarrow$Almost dust: no P gradients $\Rightarrow$ no shell crossing, $\alpha' = 0 \Rightarrow \alpha = 1$,
E and M conserved:

Use radial equation in GLTB and its time derivative per shell

$$\dot{r}^2 = 2\frac{M}{r} + \frac{1}{3}\Lambda r^2 + E, \qquad \text{with } \ddot{r} = -\frac{M}{r^2} + \frac{\Lambda}{3} r,$$

we perform Kinematic analysis, using $E = V(r) \equiv -\frac{2M}{r} - \frac{\Lambda}{3} r^2$.



III-a Non-linearly perturbed ΛCDM

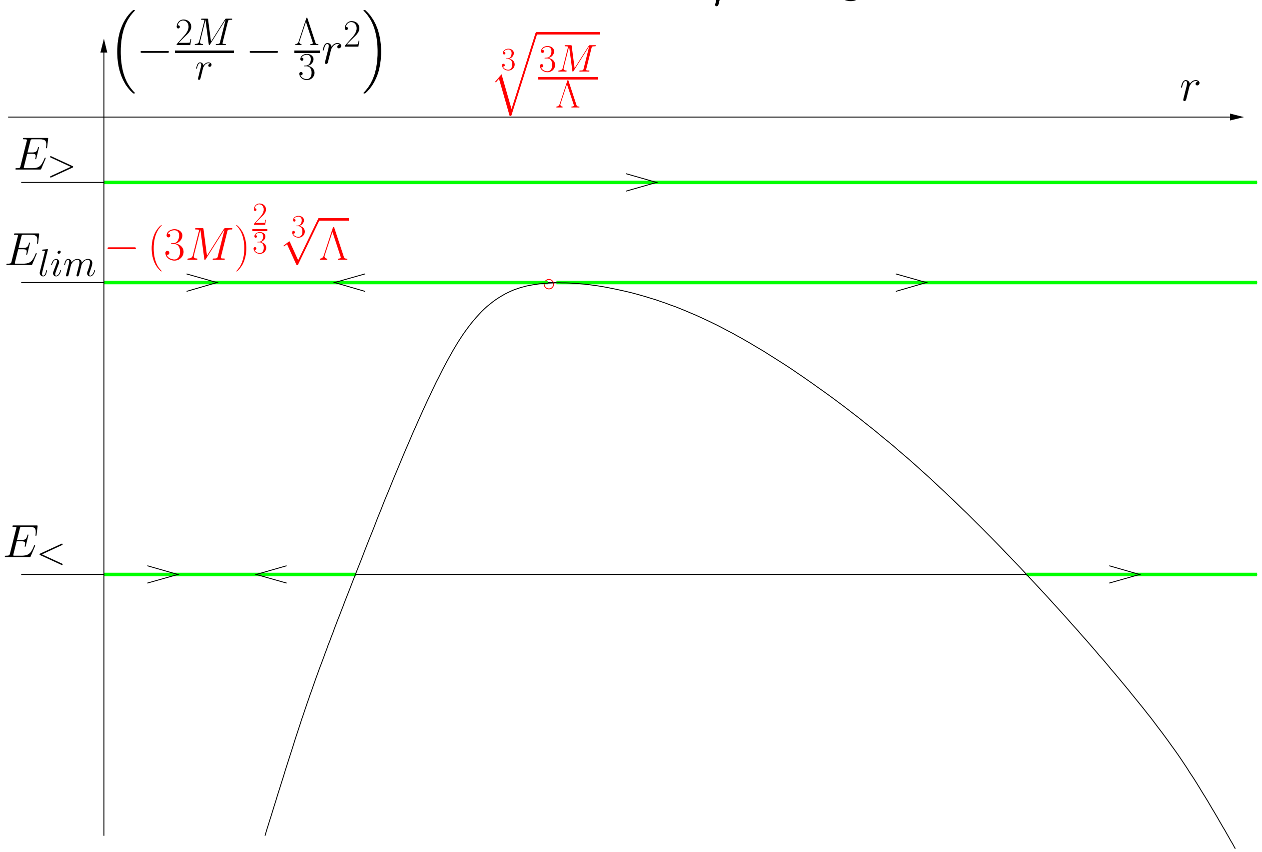
Kinematic analysis per shell: for each shell of a given E and $V(r)$, turnaround is reached when $\dot{r} = 0 \Leftrightarrow E = V(r)$.

Motion in effective potential V.

Possibility of a stable shell: for $\ddot{r} = 0 \Rightarrow \quad r = r_{lim} = \sqrt[3]{\frac{3M}{\Lambda}}$ and

$$E = E_{lim} = -(3M)^{\frac{2}{3}}\Lambda^{\frac{1}{3}}$$

Remark that for the ΛCDM, gTOV $= \frac{M}{r^2} - \frac{\Lambda}{3}r = -\ddot{r}$!



III-a Non-linearly perturbed ΛCDM

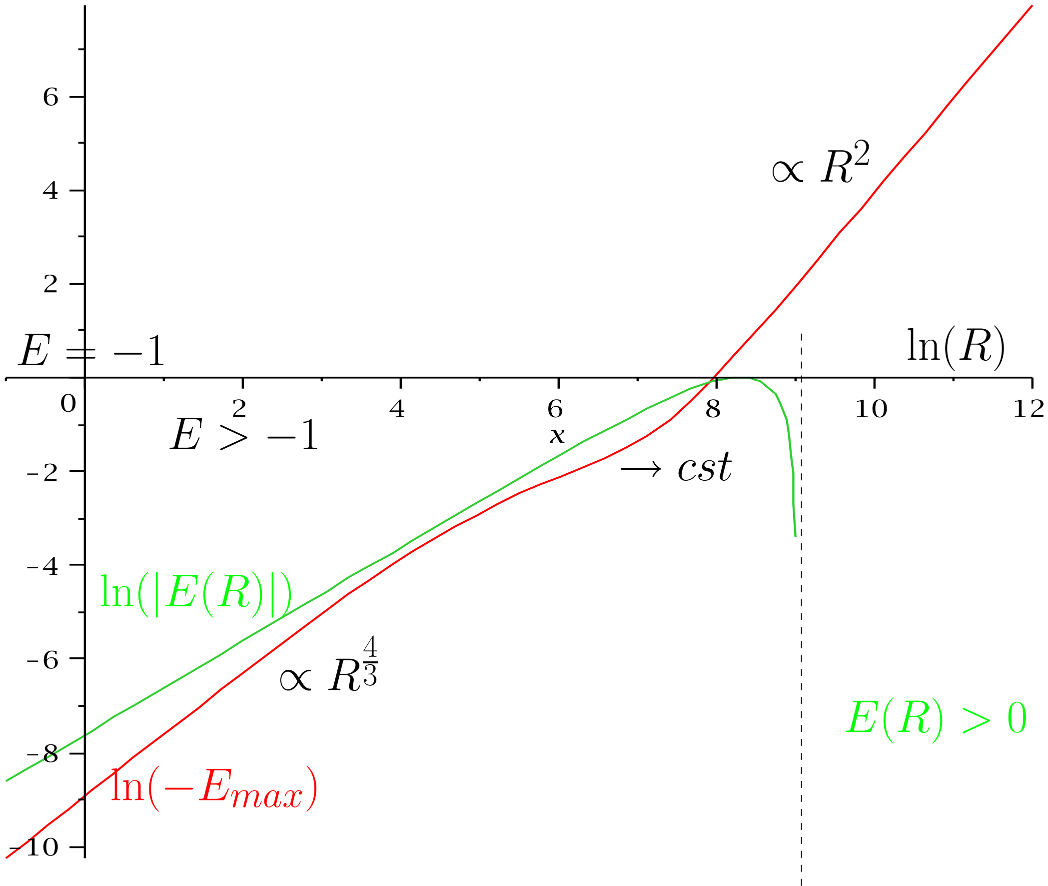
Apply this to whole system:

choose initial ρ_i profile sets the profile of E_{lim}s, where shells are stable,
v_i sets the actual E_i.

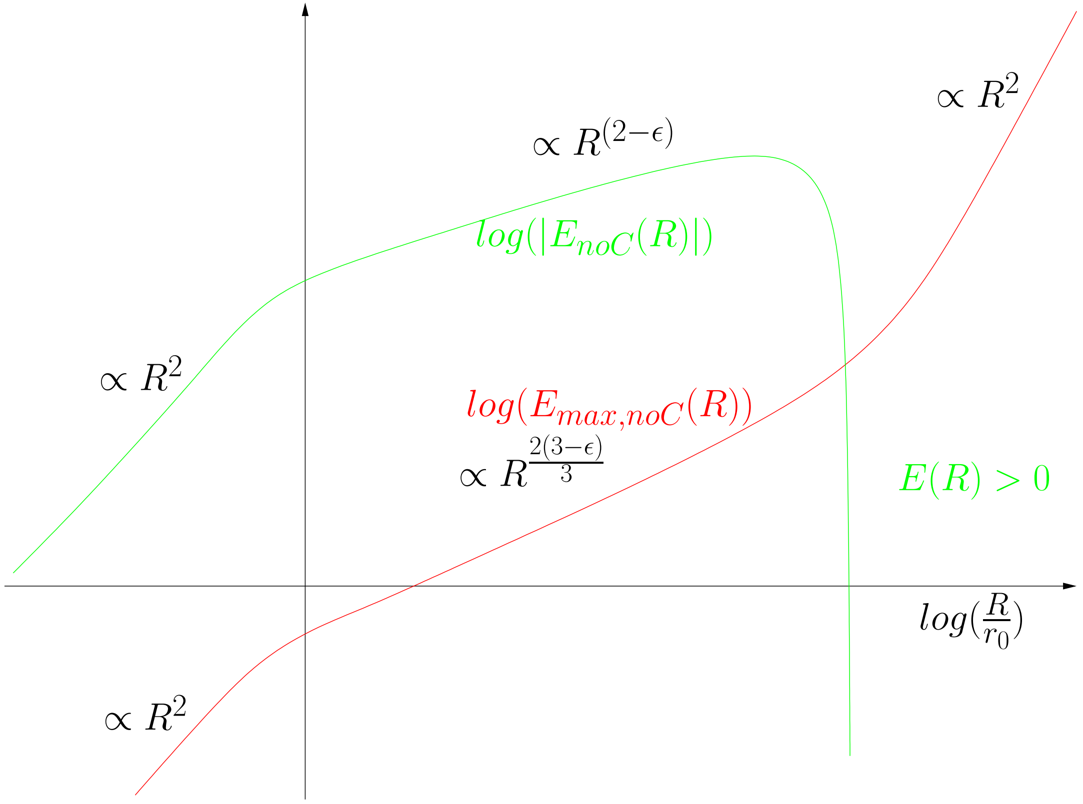
Intersection of E_{lim} and E_i gives the actual stable separating shell

We use cosmological or plausible cosmological initial conditions

ρ_i is NFW and E_i a parabola



ρ_i: cusspless power law; v_i: Hubble flow



III-a Non-linearly perturbed ΛCDM

Intersection of E_{lim} and E_i gives the actual stable separating shell

We use cosmological or plausible cosmological initial conditions

ρ_i is NFW and E_i a parabola

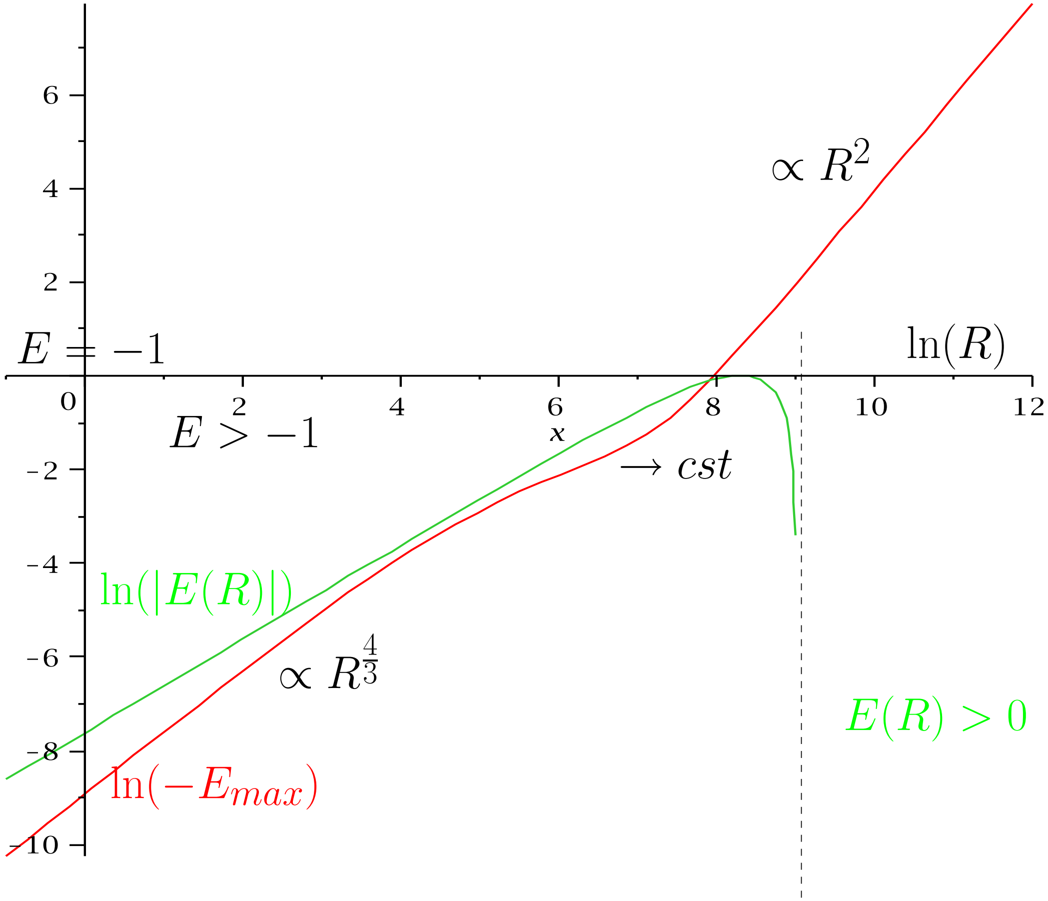
$$\rho_i(R) = \frac{\rho_0}{\frac{R}{R_0}\left(1+\frac{R}{R_0}\right)^2} + \rho_b$$

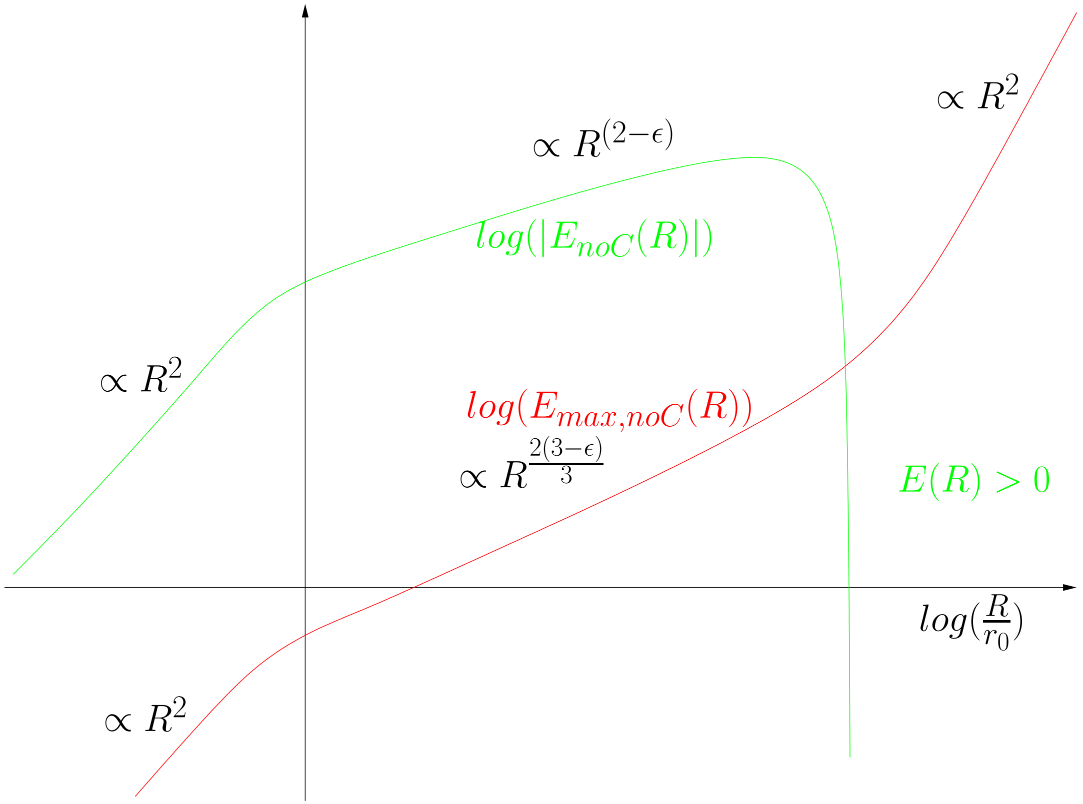
$$E(R) = E_i = -4E_{min}\left(\frac{R}{r_1}\right)\left(\frac{R}{r_1} - 1\right)$$

ρ_i: cuspless power law; v_i: Hubble flow

$$\rho_i(R) = \rho_0\left(1+\frac{R}{R_0}\right)^{-\epsilon} + \rho_b$$

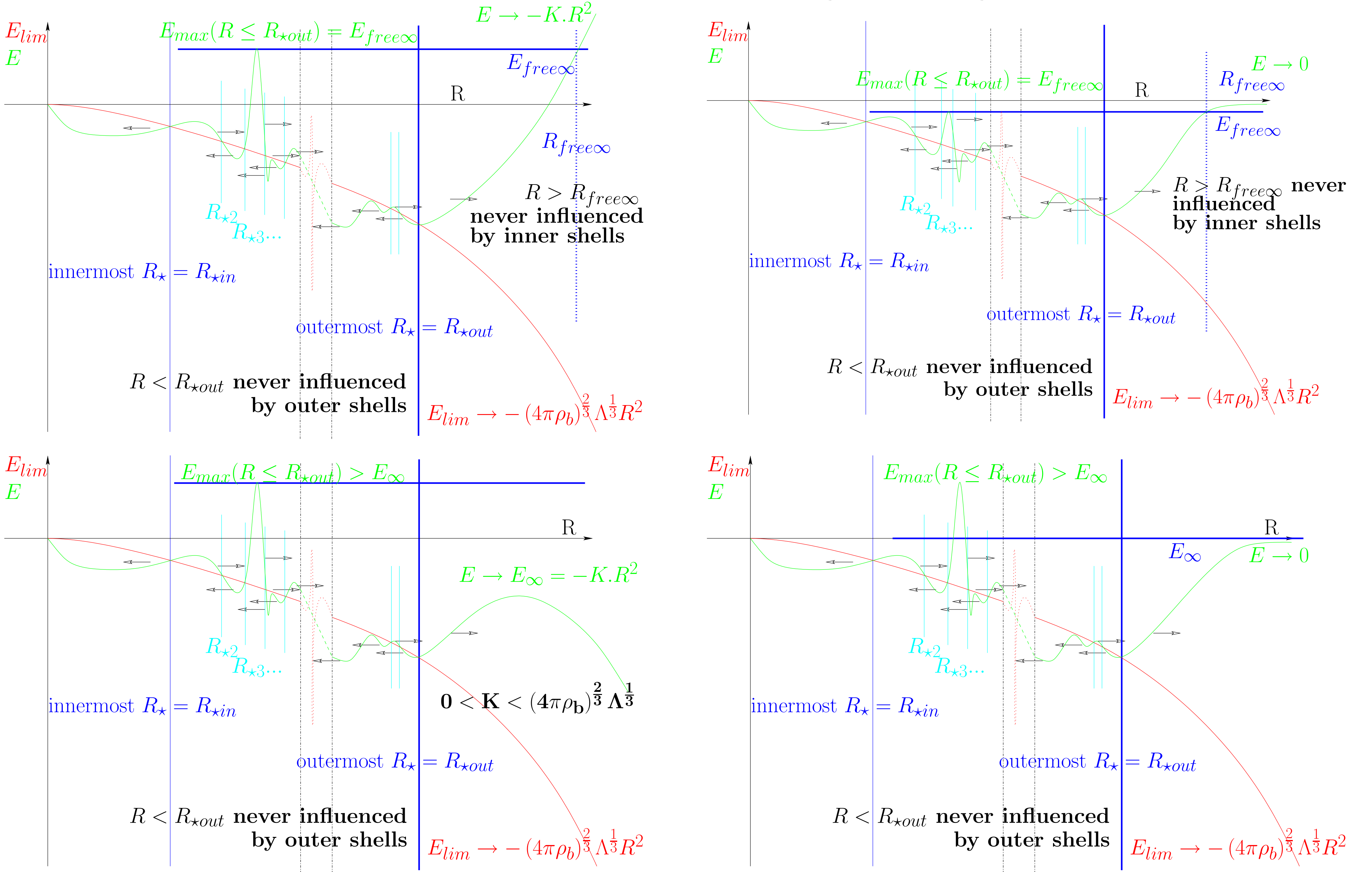
$$E(R) = \left(H_i^2 - \frac{\Lambda}{3}\right)R^2 - \frac{2M}{R}, \qquad \text{from } v_i = H_i.R$$





III-b Non-linearly perturbed ΛCDM

More general initial conditions: Local E_{lim} and E_i → stable shells, $\overline{\text{w}}$ shell crossing, + intersection part local futures; Global in. cond. w cosmological settings at ∞ set global future

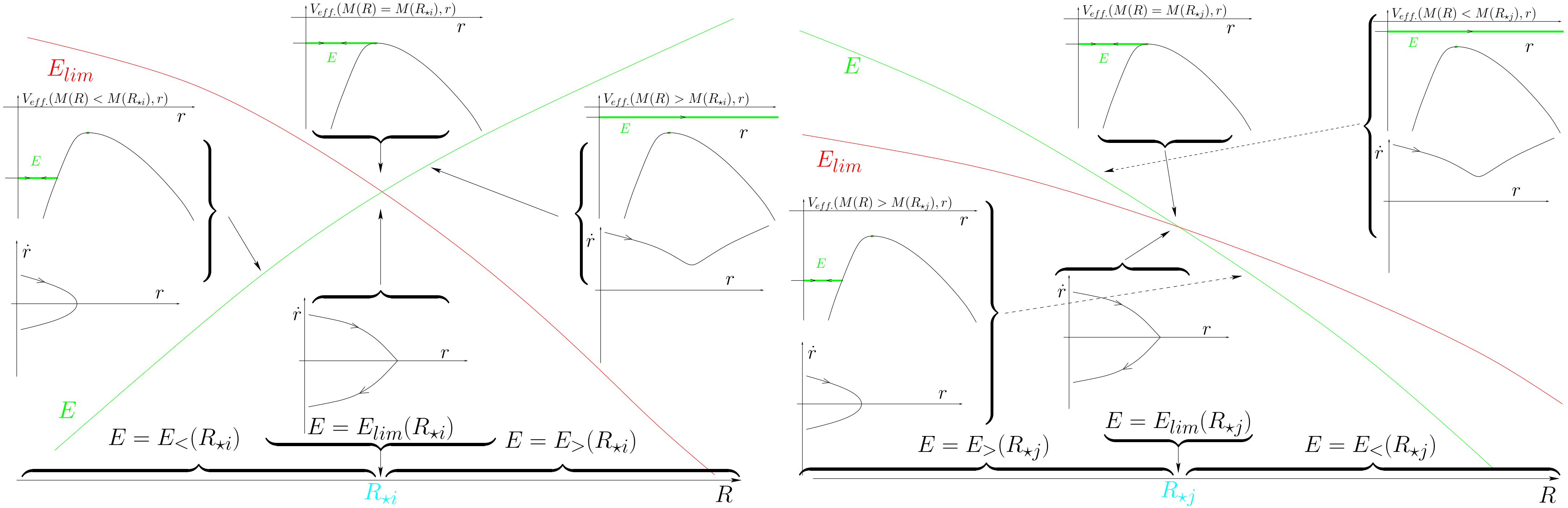


III-b Non-linearly perturbed ΛCDM

1. Global limit shells $\overline{\text{w}}$ shell crossing

Then $\dot{E} = 0 = \dot{M}$ and thus E_{lim} and E_i are conserved: each shell is integrable from initial conditions

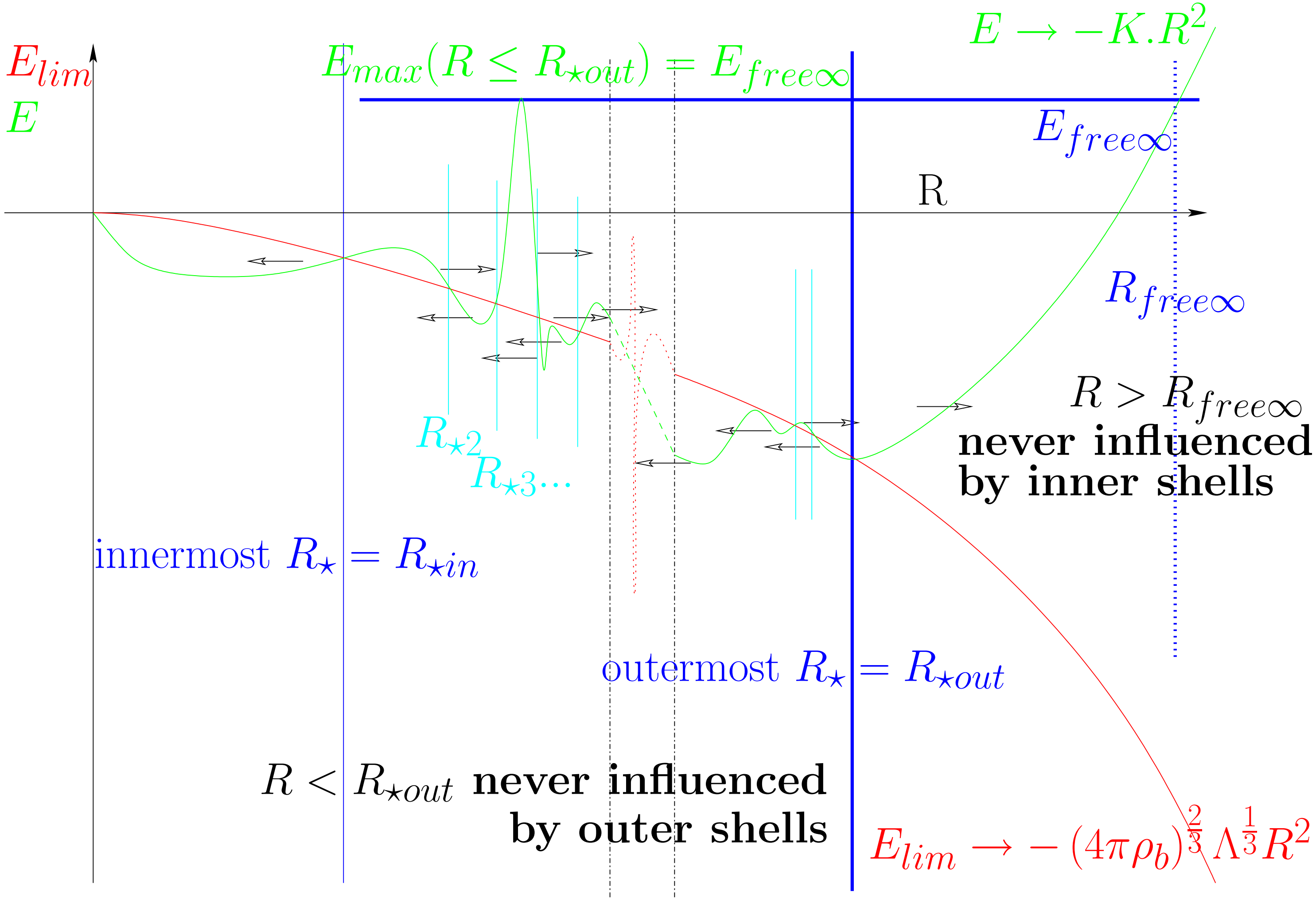
Local evolution: projections in time from the 2 possible local intersection configurations of E_{lim} and E_i



III-b Non-linearly perturbed ΛCDM

More general initial conditions: **1. Global limit shells w̄ shell crossing**

cosmological open outer limit

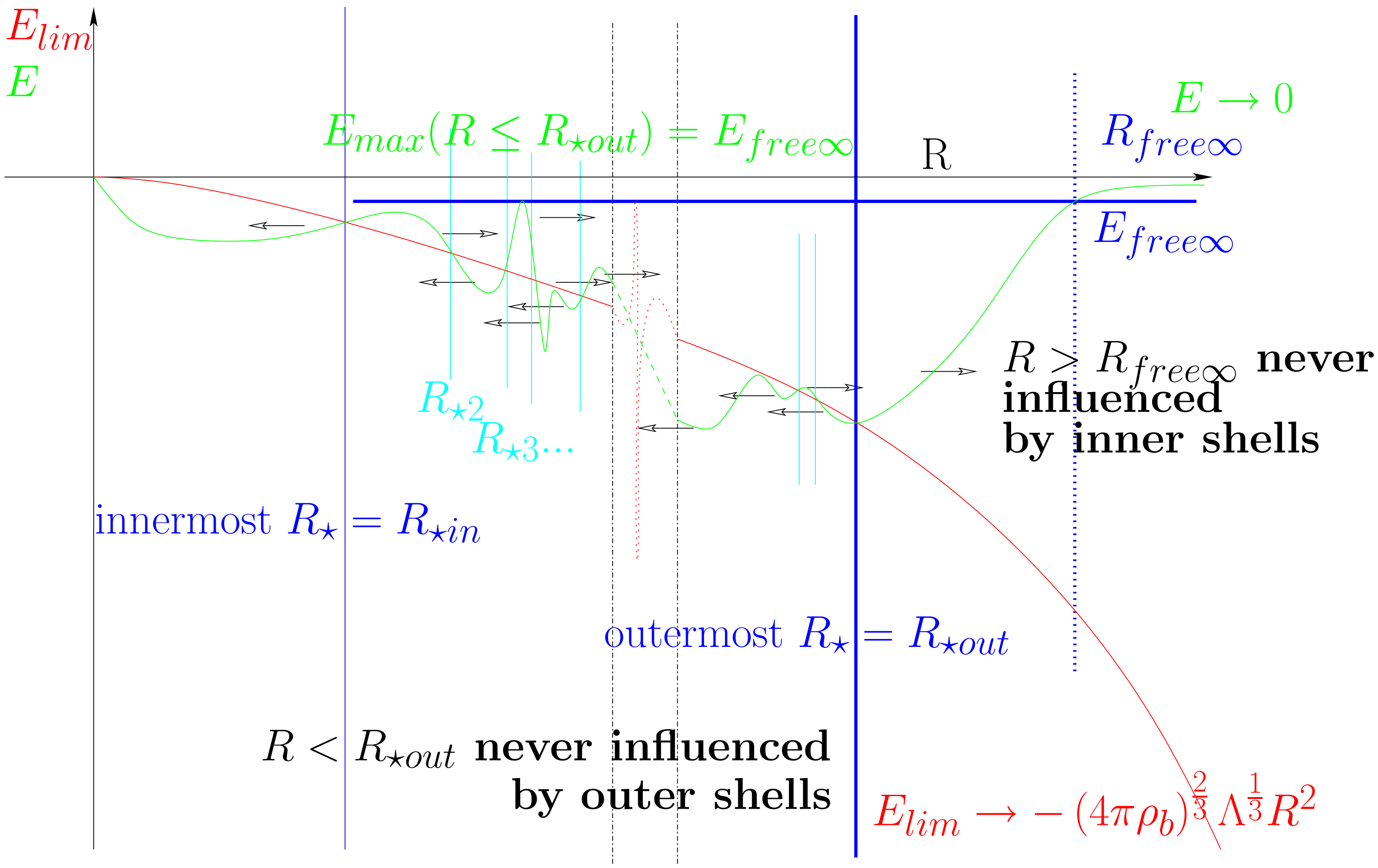


split into inner and outer boundaries

III-b Non-linearly perturbed ΛCDM

More general initial conditions: **1. Global limit shells $\overline{\mathbf{w}}$ shell crossing**

cosmological flat outer limit, case 1

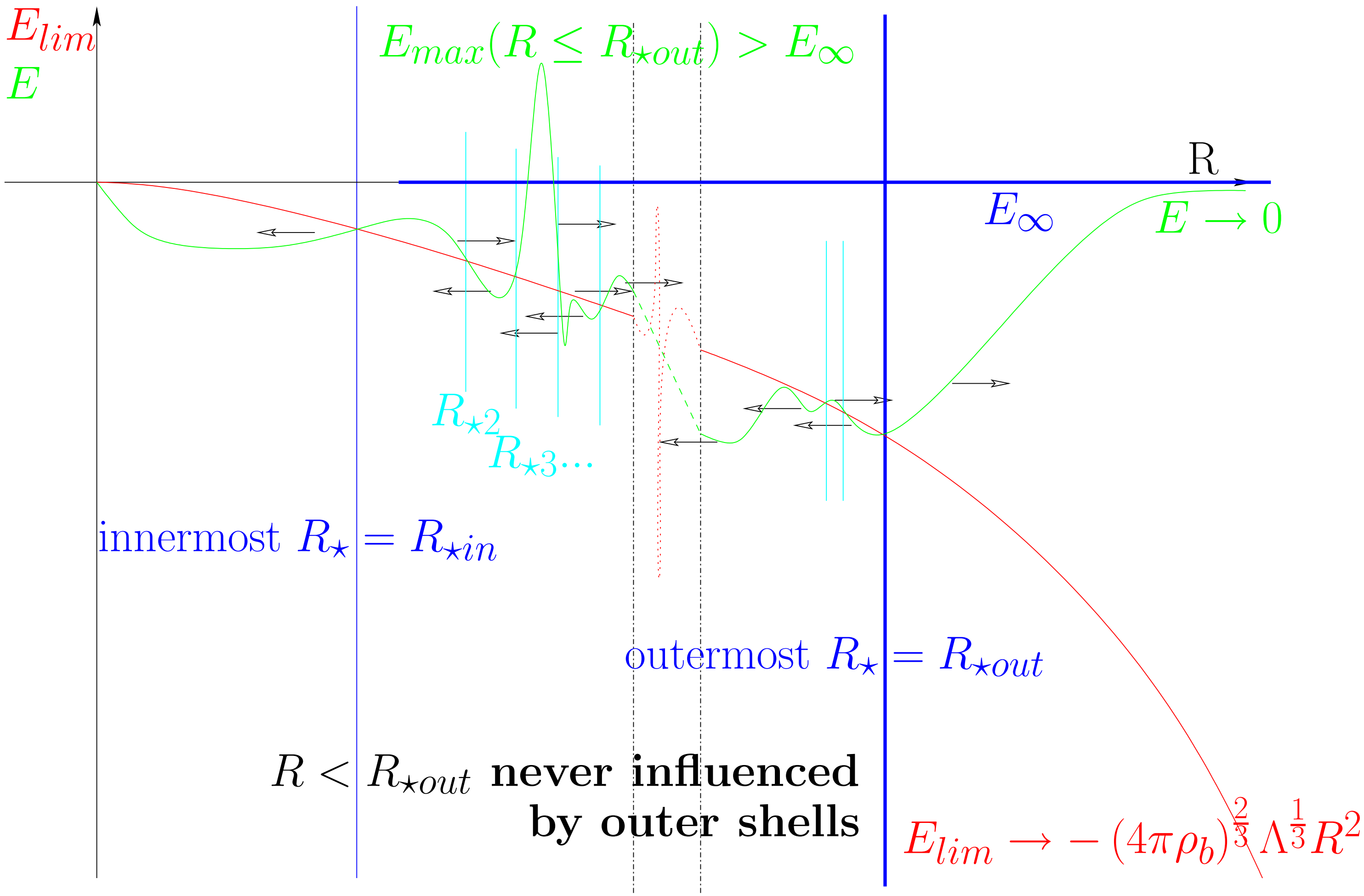


still split into inner and outer boundaries

III-b Non-linearly perturbed ΛCDM

More general initial conditions: **1. Global limit shells $\overline{\mathbf{w}}$ shell crossing**

cosmological flat outer limit, case 2

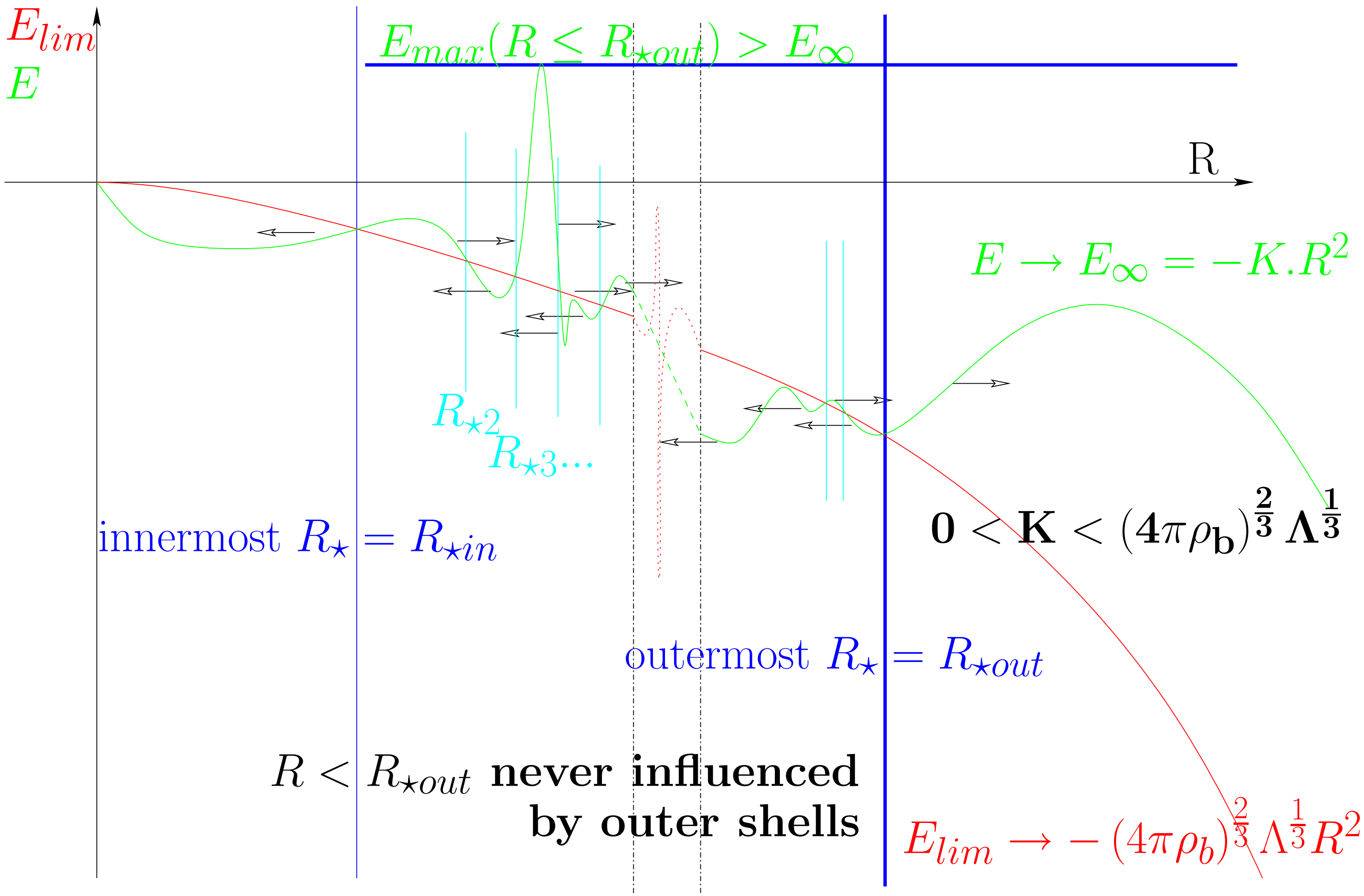


only exists inner boundary

III-b Non-linearly perturbed ΛCDM

More general initial conditions: **1. Global limit shells $\overline{\text{w}}$ shell crossing**

cosmological closed outer limit

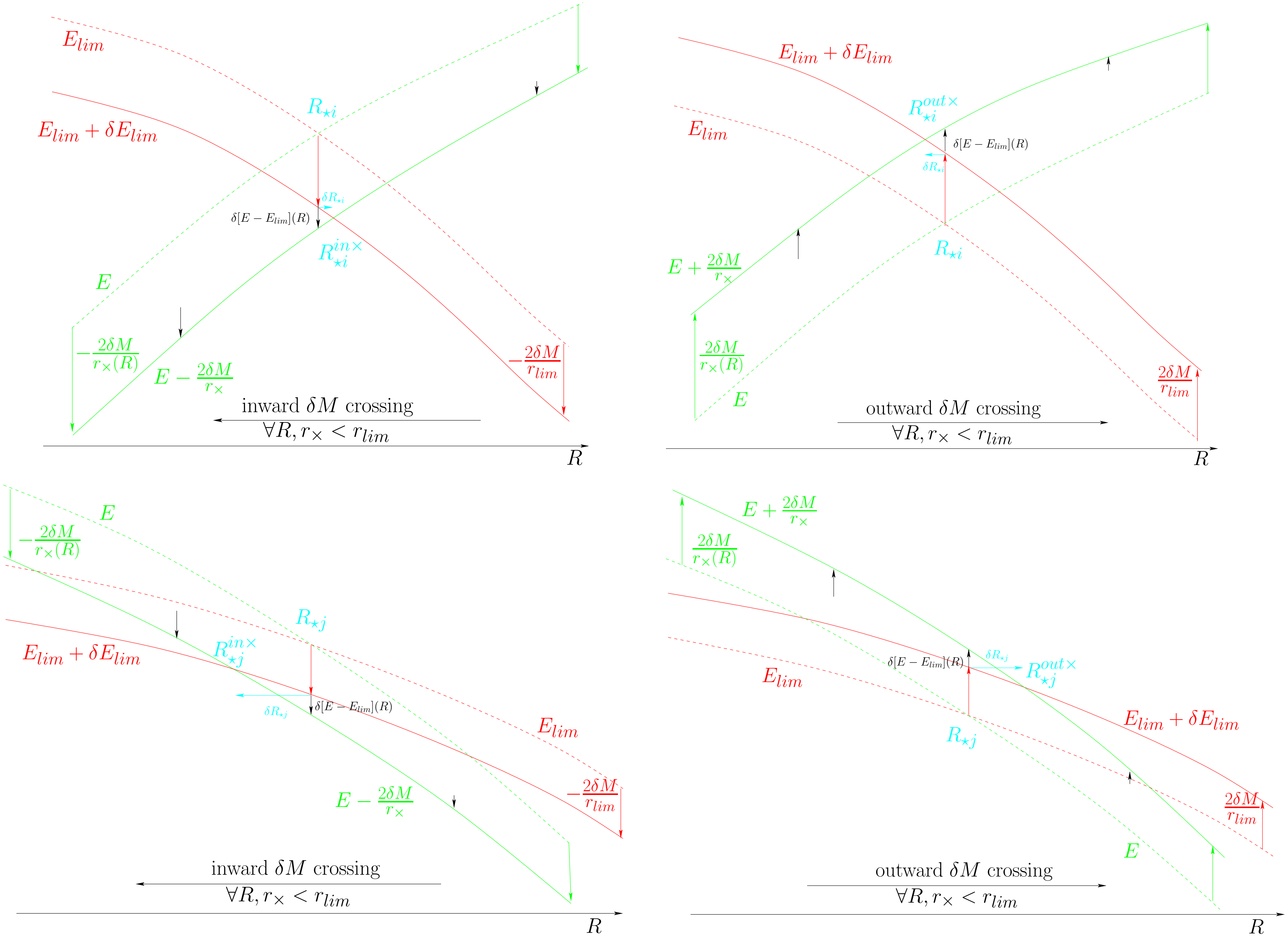


only exists inner boundary

III-b Non-linearly perturbed ΛCDM

2. Global limit shells w shell crossing

Shell crossing is generic. Locally, infinitesimal shell crossing causes differential shift: $\delta\left[E - E_{lim}\right] \simeq 2\delta M \left(\frac{1}{r_{lim}} - \frac{1}{r_\times}\right) < 0$ for $\delta M > 0$ as $r_{lim} > r_\times$.



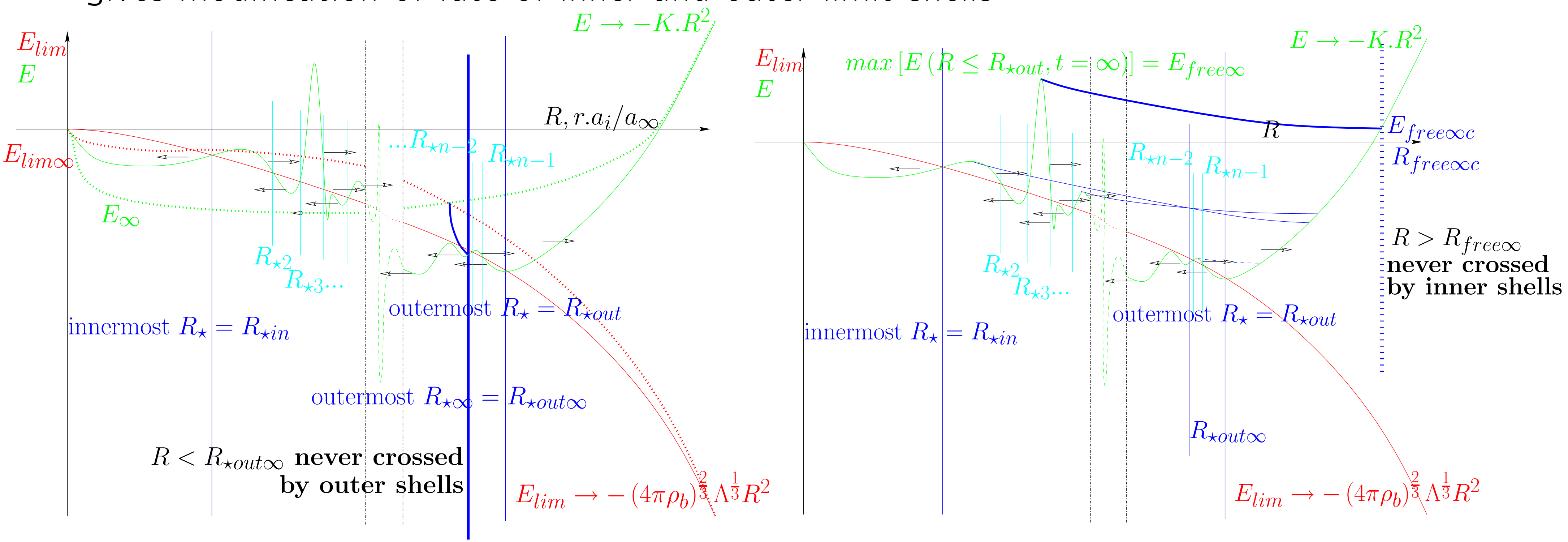
III-b Non-linearly perturbed ΛCDM

2. **Global** in. cond. **w shell crossing**: example of open background setting

Global in. cond. with cosmological settings at ∞ set global future: qualitative picture unchanged. Qualitative integration w time of local shell crossing effect

$$E_{+\delta} = E - \frac{2\delta M}{r_\times}, \qquad E_{lim+\delta} \simeq E_{lim} + \frac{2}{3}\frac{\delta M}{M}E_{lim},$$

gives modification of fate of inner and outer limit shells



Conclusions

Using non-singular, **Generalized Painlevé-Gullstrand** coordinate formulation of the ADM *spherically symmetric*, perfect fluid system [Laski & Lun 2006] allows full description without junction conditions (required in Einstein-Straus models)

we found evidence [Mimoso, Le Delliou & Mena 2010] of
possible local separating shells between inner and outer regions,
only located in elliptic $(E < 0)$ regions,
and where expansion and shear are dependent

formulated as either

- Misner-Sharp mass flows or
- (gauge invariant) expansion/shear flows.

Moreover we have linked the conditions for staticity on these shells to the Tolman-Oppenheimer-Volkoff equation via a

- function of Pressure and Mass we coined gTOV

and pointed out in the Raychaudhuri equation this link, together with the *FLRW source* of acceleration.

Conclusions

We **argue** that -this local condition is global in a cosmological context (FLRW match at radial asymptote).
-Given appropriate initial conditions, this translates into global separations between an expanding outer region and an eventually collapsing inner region.

We present simple but physically interesting illustrations of the results,

a model of Lemaître-Tolman dust with Λ = spherical perturbations in a ΛCDM with two different initial sets of cosmologically interesting conditions

$\rightarrow$ consistent with known phenomenological constraints [refs in Mimoso, Le Delliou & Mena 2010]

- an NFW density profile with a *simple curvature profile* going from bound to unbound conditions
- a non cuspy power law fluctuation with *initial Hubble flow*

We show, for these models, the existence of a global separation.

We also show, for generalised but asymptotically cosmological initial conditions that in these models, the existence of a global separation is split into collapsed and expansion regions separation and that in closed, and some flat, cases, only the former may survive. Shell crossing only modifies quantitatively this picture.

Conclusions

We **argue** that these shells are

- trapped *matter* surfaces [Mimoso, Le Delliou & Mena 2010]

and that they constitute the validity locus to

- an analog to *Birkhoff's theorem*.

Remark: Since, in the classic LTB and ΛLTB models, $\dot{M} = 0$ over all spacetime,

⇒the extended *Birkhoff's theorem* valid globally on them.