# Probing backreaction effects with supernova data

Marina Seikel

Universität Bielefeld

in collaboration with Nan Li and Dominik J. Schwarz

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# Outline

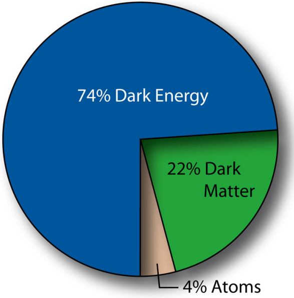
1. Motivation
2. Averaging
3. Fluctuation of the Hubble rate
4. Supernova type Ia data
5. Results

# Outline

# Standard model of cosmology

Assuming a homogeneity and isotropy, observations indicate:

- universe is approximately flat ($\Omega_k \simeq 0$)
- only $\sim 4\%$ baryonic matter
- universe dominated by dark energy with $w \simeq -1$

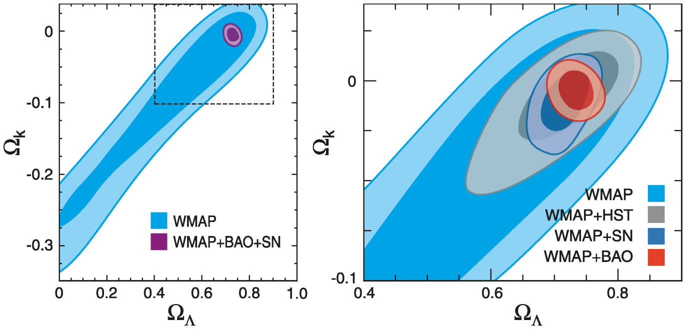



from WMAP

These results are obtained by a combination of observations at high (CMB) and low redshifts (SNe, BAO and $H_0$).

When giving up the assumption of homogeneity (e.g. LTB models), no dark energy is needed to explain the observations.

# Importance of low redshift observations




from Komatsu et al. (2008)

- low redshift observations are crucial for proving the existence of dark energy
- homogeneity scale is $\sim$ 100 Mpc
- largest structure $\sim$ 400 Mpc (Sloan Great Wall)
- assumption of homogeneity and isotropy might not be justified for low redshift observations

# Evidence for acceleration using supernovae

Again, we assume homogeneity and isotropy.

- Null hypothesis: the universe has never expanded accelerated

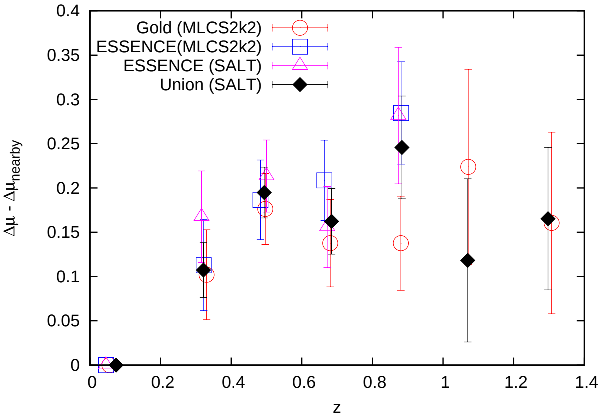
$$\Rightarrow \quad d_L(z) \leq d_{L,q=0}(z) = \frac{c}{H_0}(1+z)\ln(1+z)$$

or with $\mu(z) = m(z) - M = 5\log(d_L(z)) + 25$:

$$\Delta\mu(z) = \mu_{\text{obs}}(z) - \mu_{q=0}(z) \leq 0$$

- If $\Delta\mu$ is significantly positive, the null hypothesis can be rejected $\Rightarrow$ evidence for accelerated expansion
- In order to make the test calibration-independent, we consider $\Delta\mu - \Delta\mu_{\text{nearby}}$ instead of $\Delta\mu$.

# Evidence for acceleration using supernovae




- Using the Union data set, we find a $7\sigma$ evidence for acceleration in the case of a flat or closed universe and $4\sigma$ for an open universe.
- Evidence depends strongly on nearby supernovae with $z < 0.1$.

# Outline

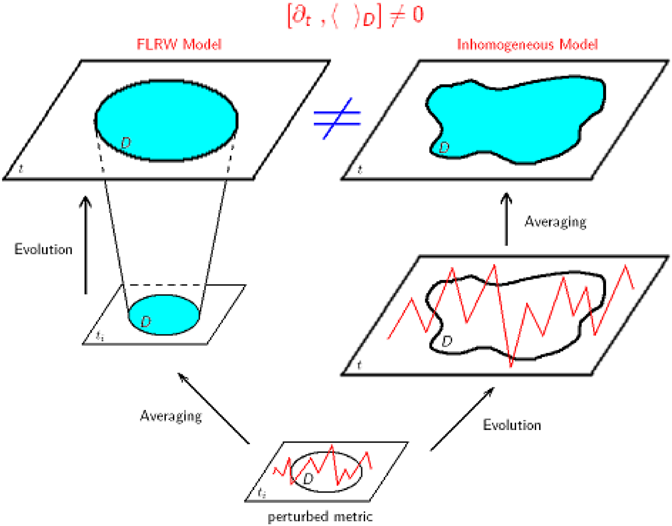
# Backreaction

- Universe can only be *statistically* homogeneous and isotropic.
- Einstein's equations are not linear in the metric $g_{\mu\nu}$.

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) \neq \langle G_{\mu\nu}(g_{\mu\nu})\rangle$$

- Local inhomogeneities and anisotropies affect the background universe via the backreaction mechanism.

# Averaging and time evolution do not commute




taken from the talk given by J. Larena on the 2nd Kosmologietag

# Averaging

- Most observables are averages.
- It is highly controversial how big the backreaction effects are.
- Unsolved problems:
  - tensor averaging
  - averages over the past lightcone
- We use:
  - scalar averages
  - spatial averages $\longrightarrow$ we are limited to small redshifts

# Spatial averaging

- Volume of a domain $D$:

$$V_D(t) \equiv \int W_D(\mathbf{x})\sqrt{\det g_{ij}}\mathrm{d}\mathbf{x} \ ,$$

  where $W_D(\mathbf{x})$ is the window function specifying the domain.
- Spatial average of an observable $O(t, \mathbf{x})$ at time $t$:

$$\langle O \rangle_D \equiv \frac{1}{V_D(t)} \int W_D(\mathbf{x}) O(t, \mathbf{x})\sqrt{\det g_{ij}}\mathrm{d}\mathbf{x}$$

- Effective scale factor $a_D$:

$$\frac{a_D}{a_{D_0}} \equiv \left(\frac{V_D}{V_{D_0}}\right)^{1/3}$$

- Effective Hubble rate:

$$H_D = \frac{\dot{a}_D}{a_D}$$

# Effective Friedmann equations

Following Buchert's formalism the effective Friedmann equations for a dust universe are obtained by averaging Einstein's equations:

$$\begin{aligned}\left(\frac{\dot{a}_D}{a_D}\right)^2 &= \frac{8\pi G}{3}\rho_{\rm eff} \\ -\frac{\ddot{a}_D}{a_D} &= \frac{4\pi G}{3}(\rho_{\rm eff} + 3p_{\rm eff})\end{aligned}$$

$\rho_{\rm eff}$ and $p_{\rm eff}$ are the energy density and pressure of an effective fluid:

$$\begin{aligned}\rho_{\rm eff} &\equiv \langle\rho\rangle_D - \frac{1}{16\pi G}\left(\langle Q\rangle_D + \langle\mathcal{R}\rangle_D\right) \\ p_{\rm eff} &\equiv -\frac{1}{16\pi G}\left(\langle Q\rangle_D - \frac{1}{3}\langle\mathcal{R}\rangle_D\right)\end{aligned}$$

$\langle Q\rangle_D$: kinematical backreaction, $\langle\mathcal{R}\rangle_D$: averaged spatial curvature

# Effective Friedmann equations

- $\langle Q\rangle_D$ and $\langle\mathcal{R}\rangle_D$ are related by an integrability condition:

$$\partial_t\left(a_D^6\langle Q\rangle_D\right)+a_D^4\,\partial_t\left(a_D^2\langle\mathcal{R}\rangle_D\right)=0$$

- The Friedmann equations and the integrability equation are not closed: There are four unknown variables $\langle Q\rangle_D$, $\langle\mathcal{R}\rangle_D$, $\langle\rho\rangle_D$ and $a_D$ constrained by only three equations.
- The equations can be closed by using cosmological perturbation theory (up to second order).
- The calculations are done in the comoving synchronous gauge as this matches the situation of a real observer.

# Outline

# Fluctuation of the Hubble rate

- Fluctuation of the Hubble rate:

$$\delta_H \equiv \frac{H_D - H_0}{H_0}$$

- Without backreaction effects:
  - $\overline{\delta_H} = 0$
  - variance only due to empirical variance
- With backreaction effects:
  - $\overline{\delta_H}$ becomes negative
  - backreaction increases the variance of $\delta_H$
  - size of the effect depends on the window function specifying the domain

# Backreaction effects

Ensemble mean:

$$\overline{\delta_H} = -\frac{41}{162}\frac{1}{(1+z)^2}\frac{R_H^4}{V_D^2}\int \mathrm{d}\mathbf{x}_1 \mathrm{d}\mathbf{x}_2 \frac{\mathrm{d}\mathbf{k}}{32\pi^4} k\mathcal{P}_\varphi(k) W_D(\mathbf{x}_1) W_D(\mathbf{x}_2) e^{i\mathbf{k}\cdot(\mathbf{x}_1+\mathbf{x}_2)}$$

Variance:

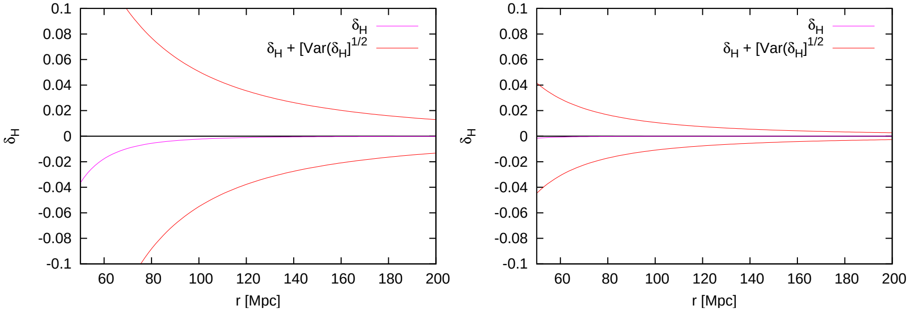
$$\mathrm{Var}(\delta_H) = \frac{25}{81}\frac{1}{(1+z)^2}\frac{R_H^4}{V_D^2}\int \mathrm{d}\mathbf{x}_1 \mathrm{d}\mathbf{x}_2 \frac{\mathrm{d}\mathbf{k}}{32\pi^4} k\mathcal{P}_\varphi(k) W_D(\mathbf{x}_1) W_D(\mathbf{x}_2) e^{i\mathbf{k}\cdot(\mathbf{x}_1+\mathbf{x}_2)}$$

where $\mathcal{P}_\varphi(k) = \mathcal{P}_\varphi \left(\frac{k}{k_0}\right)^{n_s-1}$ is the dimensionless power spectrum. The values for $\mathcal{P}_\varphi$, $k_0$ and $n_s$ are obtained from WMAP measurements.

We consider the following spherically symmetric window functions:

- Tophat: $W_D(r) = \Theta(r - R_D)\,\Theta\left(\frac{5}{3}R_D - r\right)$
- Gaussian: $\frac{1}{\sqrt{2\pi}R_D}\exp\left(-\frac{r^2}{2R_D^2}\right)$

# Tophat vs. Gaussian




- $r$ is the average distance corresponding to the window function:

$$r = \frac{1}{V_D(t)} \int_0^\infty r' \, W_D(r') \, 4\pi r'^2 dr'$$

- Effects are much larger for the tophat window function

# Outline

# Supernova data

Supernovae type Ia:

- considered to be good standard candles $\rightarrow$ they have approximately the same absolute magnitude $M$
- distance modulus: $\mu_i = m_i - M$

**Calibration**

There is a controversy about the correct calibration of $M$:

- Riess calibration
- Sandage calibration

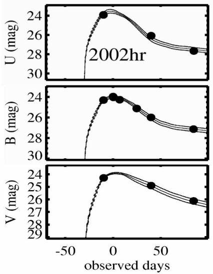
- Problem: $H_i$ depends on the calibration of $M$.
- The obtained $H_i$ cannot be compared to a global $H_0$ determined by other measurements (e.g. WMAP).

# Supernova data

## Fitting method

There are different methods to fit the light-curves of the supernovae in order to determine the distance modulus:

- SALT II
- MLCS2k2




- MLCS2k2: assumption that dust in the universe has the same properties as dust in the Milky Way
- SALT II: no assumptions about dust properties

Data set: Constitution set (SALT II) with 178 SNe up to $z = 0.1$

# Determining the Hubble rate

- Determine luminosity distance for each supernova:

$$d_{L,i} = 10^{(\mu_i - 25)/5}\,\mathrm{Mpc}$$

- Hubble rate for a theory without backreaction effects assuming ΛCDM:

$$H_i = (1+z_i)\frac{c}{d_{L,i}}\int_0^{z_i}\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}\,dz'$$

- Hubble rate for a theory with backreaction effects assuming CDM:

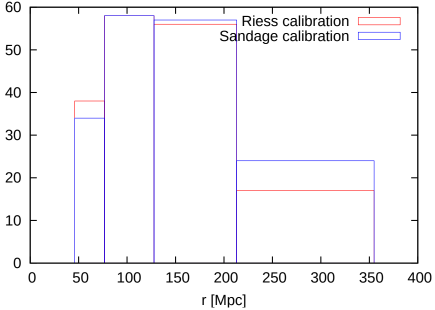
$$H_i = (1+z_i)\frac{c}{d_{L,i}}\int_0^{z_i}\sqrt{(1+z')^3}\,dz'$$

- Comoving distance:

$$r_i = \frac{d_{L,i}}{1+z_i}$$

# Averaging the Hubble rate

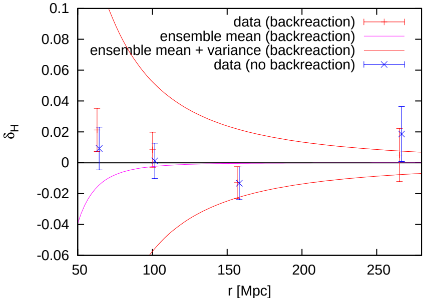
- Tophat window function: $W_D(r) = \Theta(r - R_D)\,\Theta\left(\frac{5}{3}R_D - r\right)$
- Distribution of supernovae:



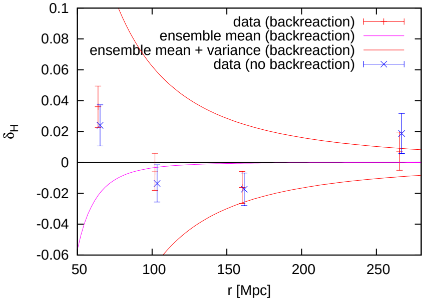

- Use all SNe within a bin to calculate the weighted averages of $H_D(\Lambda\mathrm{CDM})$, $H_D(\mathrm{CDM})$ and $r_D$ for the corresponding domain.
- $\delta_H = (H_D - H_0)/H_0$ depends on the choice of $H_0$.

# Outline

# Results for tophat window function




Riess calibration




Sandage calibration

Global Hubble rate $H_0$ in km/(s Mpc):

| | Riess | Sandage |
|---|---|---|
| backreaction (CDM) | 68.7 | 62.7 |
| no backreaction ($\Lambda$CDM) | 70.1 | 64.0 |

# Likelihoods

Likelihoods for the theories with and without backreaction effects for the following tophat window functions:

- window 1: $W_D(r) = \Theta(r - R_D)\,\Theta\left(\frac{5}{3}R_D - r\right)$
- window 2: $W_D(r) = \Theta(r - R_D)\,\Theta\left(\frac{3}{2}R_D - r\right)$
- window 3: $W_D(r) = \Theta(r - R_D)\,\Theta\left(2R_D - r\right)$

| | | Riess | Sandage |
|---|---|---|---|
| window 1 | backreaction (CDM) | 13.2% | 61.1% |
| | no backreaction ($\Lambda$CDM) | 86.8% | 38.9% |
| window 2 | backreaction (CDM) | 25.7% | 43.2% |
| | no backreaction ($\Lambda$CDM) | 74.3% | 56.8% |
| window 3 | backreaction (CDM) | 28.1% | 17.7% |
| | no backreaction ($\Lambda$CDM) | 71.9% | 82.2% |

# Different light-curve fitters

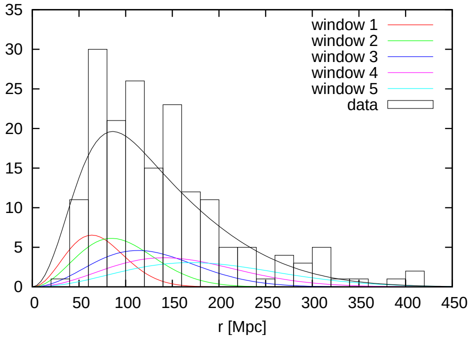
- SALT II:

| | Riess | Sandage |
|---|---|---|
| backreaction (CDM) | 13.2% | 61.1% |
| no backreaction ($\Lambda$CDM) | 86.8% | 38.9% |

- MLCS2k2:

| | Riess | Sandage |
|---|---|---|
| backreaction (CDM) | 3.6% | 4.7% |
| no backreaction ($\Lambda$CDM) | 96.4% | 95.3% |

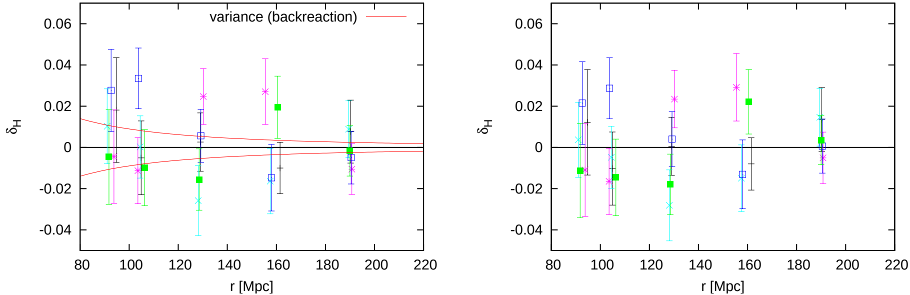
# Gaussian window function




- Number density of supernovae needs to be constant within the considered domain
- Number of SNe in the distance interval $[r, r+\mathrm{d}r]$ must be proportional to $r^2 W_D(r)\mathrm{d}r$

- As the five domains overlap, the assignment of supernovae to the five subsets representing the domains is not unique
- Therefore, we consider different realisations, i.e. different assignments of supernovae to the subsets
- Within *one* realisation all five subsets are statistically independent.

# Results for the Gaussian window function




with backreaction | without backreaction

(Riess calibration)

- Riess calibration: in 24 out of 100 realisations the model with backreaction effects is slightly favoured
- Sandage calibration: in 17 out of 100 realisations the model with backreaction effects is slightly favoured

# To do ...

Until now we have only

- used spherically symmetric domains
- considered the radial but not the angular distribution of the supernovae

We need to adjust the theory to the actual distribution of supernovae.

- non-spherically symmetric domains
- more data points at a certain distance
- large spread in $\delta_H$ at small distances would indicate backreaction effects

# Conclusions

- Backreaction influences the measurement of the Hubble rate by decreasing its mean value for small domain sizes and by increasing its variance
- But our local measurements could by chance be consistent with a model that does not include backreaction effects
- The test can potentially prove the existence of backreaction effects, but it cannot prove that there are no such effects
- We have not found any evidence for backreaction (yet).
- There is a chance to find evidence by using
  - non-spherically symmetric domains
  - larger data sets that will be available in the future